Model Free Adaptive Double Sliding Mode Control Based on U-Control Framework

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Abstract: This paper investigates the high-precision control problem of nonlinear continuous systems with strong uncertainty, and proposes a model free adaptive double sliding mode control based on the U-model control framework. On the basis of using a double sliding mode controller as the dynamic inversion (DI) in the U-model control framework, this paper designs a novel adaptive controller. Compared with the traditional double sliding mode controller, the controller designed in this paper no longer needs to select gain parameters based on experience, but adopts an adaptive method of integrating the sliding mode function to adaptively adjust the equivalent control gain and switching control gain contained in the traditional double sliding mode controller. Meanwhile, this paper demonstrates the stability of this novel adaptive controller using Lyapunov stability theory. Therefore, this controller can solve the high-precision control problem of a class of SISO nonlinear continuous systems with completely mismatched models. The results show that the proposed adaptive controller can adaptively adjust the gain parameters and ensure system stability, while having better accuracy than traditional double sliding mode controllers. Finally, the superiority of this adaptive controller was demonstrated through simulation experiments.

Keywords: Adaptive control; Model free sliding mode control; Double sliding mode controller; Dynamic inversion; SMPID

1. INTRODUCTION

With the development of technology, more and more production control demands have strong nonlinear characteristics. In order to meet the needs of production control, researchers have conducted extremely in-depth research on nonlinear models in recent years. Due to the inherent complexity of nonlinear systems, establishing a universal mathematical model is crucial and also one of the research challenges.

There are various types of early nonlinear models, including Wiener model, Hammerstein model, NARMAX model, etc. The Wiener model proposed by (Wills et al., 2013) and Hammerstein model proposed by (Ding et al., 2005) belong to a class of nonlinear models with relatively simple structures, both of which are composed of a series combination of linear systems and nonlinear static gains. Compared to the previous two, the NARMAX model can more effectively describe any nonlinear system and has a certain degree of generality. The U-model is a nonlinear system description model proposed by Professor (Zhu, 2002) based on the NARMAX model. The core concept of U-model control is the dynamic inversion concept proposed by (Miller, 2011). According to the U-model transformation rules, a class of nonlinear models is transformed into linear systems that are easy to design, and the inverse system of the transformed linear system is obtained through methods such as Newton iterative formula. The U-model obtains the actual control quantity by solving a univariate equation about the actual control quantity. This method does not require complex feedback linearization design, reducing the complexity of controller design. However, traditional U-control dynamic inversion still has some drawbacks. Firstly, the complexity of the inversion system depends on the controlled plant itself. If the controlled plant itself is complex, the inversion system of the U-model is also difficult to calculate. Secondly, the calculation method of continuous time dynamic inversion can be applied to continuous time systems, but the perfect match between the inversion system and the model is still an idealized assumption in the academic research field. Therefore, in recent years, researchers have adopted various approaches—such as internal model control (Li et al., 2021), neural network control (Xu et al., 2020), active disturbance rejection control (Wei et al., 2021), and sliding mode control (Kant et al., 2025; Lone et al., 2024) - to enhance the robustness of nonlinear dynamic inversion. However, the above-mentioned U-control is based on controlled systems with model matching or model mismatch. For systems where the model is completely mismatched or completely unknown, the above methods become difficult to implement. So it is quite important to design a good control scheme for systems where the model is completely mismatched or completely unknown.

Sliding mode control, due to its strong anti-interference ability, has good robustness against system model mismatch and is widely used in various fields (Chen et al., 2016; Deng et al., 2019; Lu et al., 2019; Mustafa et al., 2025). Based on the advantages of sliding mode control, many researchers have adopted sliding mode control to study model free systems. (Precup et al., 2021) combine the advantages of fuzzy PID and sliding mode control to propose a new data-driven sliding mode controller design. (Liu et al., 2023) propose a new data-driven adaptive sliding mode control scheme using model free adaptive theory for unmanned surface vehicle heading control in the presence of disturbances. (Wang et al., 2016) propose a model free terminal sliding mode control (MFTSMC) strategy to control the attitude and position of quadcopters including parameter variations, uncertainties, and external disturbances. (Yonezawa et al., 2025) incorporated a model-free scheme for virtual controlled objects into the sliding mode control system and simultaneously addressed the chattering issue via an adaptive fuzzy inference approach. However, the above-mentioned model free sliding mode control still requires neural networks or observers to fit the uncertain terms in the controlled plant, which makes the control structure complex and specific, and is not conducive to universal application.

Based on the above discussion, the sliding mode controller is used as the dynamic inversion in the U-control framework, which enables the system to be resistant to external disturbances and achieve good performance without needing to know the estimated values of uncertain terms (Zhu, 2021). This controller greatly reduces the complexity of the control structure and has a very good suppression effect on the inherent chattering phenomenon of sliding mode control. However, this control scheme has the problem of selecting parameters based on experience, which is not conducive to universal application. Therefore, this paper conducts research on the model free adaptive double sliding mode controller based on the U-control framework, which is of great significance for the widespread application of the new model free sliding mode controller (Zhu, 2023).

The structure of this paper is as follows: Section 1 provides a basic discussion on nonlinear U-model and sliding mode control; Section 2 provides the mathematical expression of the U model; Section 3 presents the block diagram of a double sliding mode control system based on the U-control framework, which includes the design of a linear invariant controller and a linear extended state observer; Section 4 presents the design process of the adaptive controller and proves the system stability based on Lyapunov stability theory; Section 5 presents simulation results and demonstrates the superiority of the adaptive controller designed in this study; Section 6 is the conclusion of this study.

2. U-MODEL

The U-model can represent the following SISO nonlinear continuous system(Zhu et al., 2019):

$$y^{(N)}(t) = \sum_{i=0}^{M} \alpha_i(t) f_i\left(u^{(M)}(t)\right) \tag{1}$$

In the equation (1), y(t) is the system output, and $y^{(N)}(t)$ is the Nth derivative of the system output. $u(t)$ is the system input, and $u^{(M)}(t)$ is the Mth derivative of the system input. $\alpha_i(t)$ is a coefficient that encompasses all functions associated with the system's inputs $\left[y^{(N-1)}(t), \dots, y(t), y^0(t)\right] \in R^N$ and outputs $\left[u^{(M-1)}(t), \dots, u(t), u^0(t)\right] \in R^M$. f_i is the function of input $u^{(M)}(t)$. In this way, nonlinear non affine systems can be represented as time-varying polynomials with respect to the input parameters of the system. For example, there are the following nonlinear systems:

$$\ddot{y} = (y-1)\dot{y} + y^2 u + (1+\dot{y}^2)u^2 + (y+\dot{y}^2)u^3 \tag{2}$$

The U model can be represented as:

$$\begin{gathered}\ddot{y} = \alpha_0 f_0(u^0) + \alpha_1 f_1(u) + \alpha_2 f_2(u^2) + \alpha_3 f_3(u^3) \\ \alpha_0 = (y-1)\dot{y}, f_0(u^0) = 1 \\ \alpha_1 = y^2, f_1(u) = u \\ \alpha_2 = (1+\dot{y}^2), f_2(u^2) = u^2 \\ \alpha_3 = (y+\dot{y}^2), f_3(u^3) = u^3\end{gathered} \tag{3}$$

The core idea of U-model control is dynamic inversion. Through the above modeling process, let the U-model G be a polynomial form mapping function $u \to y$. So the dynamic inversion $y \to u$ of the U model is the process of solving G^{-1}. This process proposed by (Li, 2020) can be described as:

$$G^{-1} \Leftrightarrow u^{(M)} \in y_d^{(N)}(t) - \sum_{i=0}^{M} \alpha_i(t) f_i\left(u^{(M)}(t)\right) = 0 \tag{4}$$

where y_d is the expected output, and G^{-1} is the inverse mapping of the U-model $u \to y$. By solving the polynomial equation of the U-model with a specified expected output, the control input is solved to counteract the nonlinearity of the controlled system.

The U-control system is functionally expressed as

$$\begin{aligned}\textstyle\sum(\mathcal{F}, \mathbb{C}(C_1, G^{-1}), G) &\Leftrightarrow \textstyle\sum\left(\mathcal{F}, C_1, \mathbb{C}(G^{-1}, G)\right) \\ &\Leftrightarrow \textstyle\sum(\mathcal{F}, C_1, I_{ip})\end{aligned} \tag{5}$$

where $\mathcal{F}$ is the U-control system configuration, $\mathbb{C}$ is a set of controllers, C_1 is a target open-loop transfer function, and $I_{ip} = \mathbb{C}(G^{-1}, G)$ is a unit constant or identity matrix. Fig. 1 and Fig. 2 shows model matched and model mismatched U-control configurations.

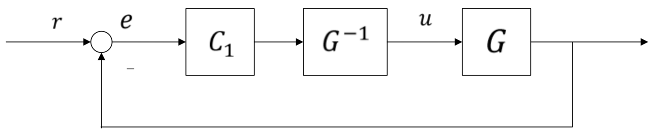

Fig. 1. Model matched U-control.

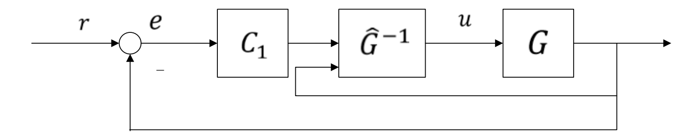

Fig. 2. Model mismatched U-control.

3. DOUBLE SLIDING MODE CONTROL FRAMEWORK

The double sliding mode control framework proposed based on the U-control framework (Zhu et al., 2022) as shown in Fig. 3, where C is a target open-loop transfer function, DSMC is the double sliding mode controller, G_p is the controlled plant, and LESO is a linear extended state observer (Qing Zheng et al., 2007). The target open-loop transfer function C specifies the outer loop performance and provides a reference signal x_d for DSMC; DSMC can be regarded as the dynamic inversion controlled by the U-model, ensuring that the controlled plant can follow the reference signal; LESO can obtain the various states of the controlled plant by obtaining its input and output, and then feedback the state information to DSMC.

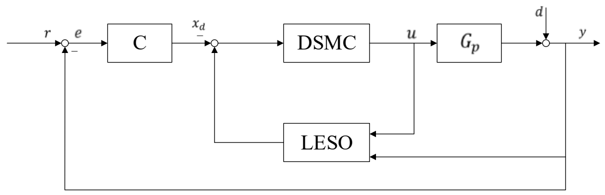

Fig. 3. Double sliding mode control framework.

It should be noted that DSMC is a dynamic inversion based on the U-model, satisfying the process represented by (4), and is not strictly the inverse system of the controlled plant.

3.1 Target closed-loop transfer function

The target closed-loop transfer function must specify the overall desired performance of the closed-loop control system, necessitating both favorable response speed and steady-state performance. For second-order controlled plants, this study designs the target open-loop transfer function as:

$$\mathrm{C} = \frac{\omega_n^2}{s^2+2\omega_n\xi s} \tag{6}$$

So the transfer function of the external closed-loop is:

$$G = \frac{\mathrm{C}}{1+\mathrm{C}} = \frac{\omega_n^2}{s^2+2\omega_n\xi s+\omega_n^2} \tag{7}$$

This study determines the undamped oscillation frequency and damping ratio of the entire outer loop by manually setting parameters ω_n and ξ , thereby determining the overall performance of the entire system.

3.2 Linear extended state observer

LESO can stably obtain various states of model free systems (Guo and Zhao, 2011). There is a state vector $x = [x_1, x_2, \dots, x_n]^T$, where $\{\dot{x}_1 = y^{(1)}, \dot{x}_2 = y^{(2)}, \dot{x}_3 = y^{(3)}, \dots, \dot{x}_n = y^{(n)}\}$. This study will write the model free system as the following state equation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \quad \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x_1, x_2, \dots, x_n, u, d) + u \\ y = x_1 \end{cases} \tag{8}$$

Assuming the state vector of the observer is $\hat{x} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n, \hat{x}_{n+1}]^T$, the LESO can be expressed as:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + {\omega_0}^1\beta_1(y-\hat{x}_1) \\ \quad \dots \\ \dot{\hat{x}}_n = \hat{x}_{n+1} + {\omega_0}^n\beta_n(y-\hat{x}_1) + u \\ \dot{x}_{n+1} = {\omega_0}^{n+1}\beta_{n+1}(y-\hat{x}_1) \end{cases} \tag{9}$$

where ω_0 is the observer bandwidth, and $\beta_i, i \in 1, \dots, n+1$ is an adjustable parameter. The literature suggests that ω_0 can be determined by trial and error or given in advance based on the system bandwidth, while β_i needs to satisfy the Hurwitz stability condition, which is determined by the following equation:

$$(s+1)^{n+1} = s^{n+1} + \beta_1 s^n + \dots + \beta_n s + \beta_{n+1} \tag{10}$$

where $\beta_i = \frac{(n+1)!}{i!(n+1-i)!}, i = 1,2, \dots, n+1$.

4. ADAPTIVE DOUBLE SLIDING MODE CONTROL SCHEME

Consider the general case of n-order SISO nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \quad \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x_1, x_2, \dots, x_n, u, d) + u \\ y = x_1 \end{cases} \tag{11}$$

In the nonlinear system (11), $x = [x_1, x_2, \dots, x_n]^T$ is the state vector of the system, where $\{\dot{x}_1 = y^{(1)}, \dot{x}_2 = y^{(2)}, \dot{x}_3 = y^{(3)}, \dots, \dot{x}_n = y^{(n)}\}$。 $u \in R$ is the input signal of the system, and $d \in R$ is defined as the disturbance signal and is assumed to be bounded. In order to design a sliding mode controller, this study defines the following error vector:

$$E = x - x_d = [e = x_1 - x_d, \dots, e^{(n-1)} = x_n - x_d^{(n-1)}]^T \tag{12}$$

Then design a sliding mode function:

$$\mathrm{S} = c_1 e + c_2\dot{e} + \dots + c_{n-1}e^{(n-2)} + e^{(n-1)} \tag{13}$$

where $c = [c_1, c_2, \dots, c_{n-1}]^T$ satisfies Hurwitz stability.

By taking the derivative of sliding mode function (13), we can obtain:

$$\dot{\mathrm{S}} = c_1\dot{e} + c_2\ddot{e} + \dots + c_{n-1}e^{(n-1)} + e^{(n)} \tag{14}$$

Afterwards, two sliding surfaces need to be designed, one is a banded sliding surface and the other is a linear sliding surface. The function represented by the banded sliding surface is called the global sliding banded function S_g, and the function represented by the linear sliding surface is called the local sliding line function S_l . The two sliding functions are expressed as follows:

$$\begin{cases} \mathrm{S}_g = S + \delta, |\delta| \geq 0 \\ \mathrm{S}_l = S \end{cases} \tag{15}$$

Therefore, the double sliding mode controller divides the system motion process into two parts:

$$\dot{S} = \begin{cases} k_1 + u & \forall |S| \geq |\delta| \\ k_2 S + \varepsilon + u & \forall |S| < |\delta| \end{cases} \tag{16}$$

When the sliding mode variable satisfies $|S| \geq |\delta|$, substituting Equation (14) yields:

$$\dot{S} = c_1\dot{e} + c_2\ddot{e} + \dots + c_{n-1}e^{(n-1)} + e^{(n)} = k_1 + u \tag{17}$$

Simplified:

$$\dot{S} = \sum_{i=1}^{n-1} c_i e^{(i)} + f(x, u, d) + u - x_d^{(n)} = k_1 + u \tag{18}$$

The equation (18) has $k_1 = \sum_{i=1}^{n-1} c_i e^{(i)} + f(x, u, d) - x_d^{(n)}$, where k_1 satisfies $m \leq k_1 \leq M$, and $|m| \leq |M|$.

Assign the equation:

$$\tilde{k} = \hat{k}_g - \bar{k}_1 \tag{19}$$

where $\hat{k}_g$ is the estimated value of the switching control coefficient k_g, $\bar{k}_1$ is the maximum value of k_1, and $\tilde{k}$ is the estimation error.

Theorem 1. Provided that k_1 and $\tilde{k}$ satisfies the bounded condition, the system maintains stability on the domain where $|S| \geq |\delta|$ when $\dot{\hat{k}}_g = \frac{1}{\alpha_1}|S+\delta|$.

Proof. The switching controller is:

$$u = u_{sw} = -\hat{k}_g sign(S+\delta) \tag{20}$$

Assign a Lyapunov function:

$$V_g = \frac{1}{2}(S+\delta)^2 + \frac{1}{2}\alpha_1\tilde{k}^2 \tag{21}$$

The derivative of the above equation includes:

$$\begin{aligned} \dot{V}_g &= (S+\delta)\dot{S} + \alpha_1\tilde{k}\dot{\tilde{k}} \\ &= (S+\delta)(k_1 - \hat{k}_g sign(S+\delta)) + \alpha_1(\hat{k}_g - \bar{k}_1)\dot{\hat{k}}_g \end{aligned} \tag{22}$$

If $\dot{\hat{k}}_g = \frac{1}{\alpha_1}|S+\delta|$ is given, then there is:

$$\begin{aligned} \dot{V}_g &= (S+\delta)(k_1 - \hat{k}_g sign(S+\delta)) + \\ &\quad \alpha_1(\hat{k}_g - \bar{k}_1)\frac{1}{\alpha_1}|S+\delta| \\ &= k_1(S+\delta) - \hat{k}_g|S+\delta| + (\hat{k}_g - \bar{k}_1)|S+\delta| \\ &= k_1(S+\delta) - \bar{k}_1|S+\delta| \leq 0 \end{aligned} \tag{23}$$

Q.E.D. *Remark 1.* The adaptive formula, when the system's sliding mode variable satisfies $|S| \geq |\delta|$, is given by:

$$\hat{k}_g = \int_0^t \frac{1}{\alpha_1}|S+\delta|\, dt \tag{24}$$

where α_1 is the adaptive rate.

Remark 2. It is noteworthy that the gain k_1, given by Equation (18), depends on both the system state x and the unknown disturbance d. Furthermore, based on the uniformly ultimately bounded stability property of the closed-loop system guaranteed by the proposed controller, it can be concluded that the control gain k_1 remains bounded throughout system operation.

When the sliding mode variable satisfies $|S| < |\delta|$, substituting Equation (14) yields:

$$\dot{S} = c_1\dot{e} + \cdots + c_{n-1}e^{(n-1)} + e^{(n)} = k_2 S + \varepsilon + u \tag{25}$$

Simplified:

$$\begin{aligned} \dot{S} &= \textstyle\sum_{i=1}^{n-1} c_i e^{(i)} + f(x,u,d) + u - x_d^{(n)} \\ &= k_2 S + \varepsilon + u \end{aligned} \tag{26}$$

Where $k_2 S + \varepsilon = \sum_{i=1}^{n-1} c_i e^{(i)} + f(x,u,d) - x_d^{(n)}$. There is a bounded term on the left side of (26) where $k_2 S$ is equivalent. ε is the deviation term, which includes the equivalent deviation quantity of $k_2 S$ and other neglected bounded terms. Both k_2 and ε are bounded.

There is an equation:

$$\tilde{k} = \hat{k}_l - \bar{k}_2 \tag{27}$$

Where $\hat{k}_l$ is the estimated value of the equivalent control coefficient k_l, $\bar{k}_2$ is the maximum value of k_2, and $\tilde{k}$ is the estimation error.

Theorem 2. Provided that k_2, ε and $\tilde{k}$ satisfies the bounded condition, the system maintains stability on the domain where $|S| < |\delta|$ when $\dot{\hat{k}}_l = \frac{1}{\alpha_2}S^2$.

Proof. The equivalent control is set as:

$$u = u_{eq} = -\hat{k}_l S - k_i \int_0^t S\, dt \tag{28}$$

We introduce an auxiliary state variable z, whose dynamics are defined by:

$$\dot{z} = S \tag{29}$$

with the initial condition $z(0) = 0$. This constitutes an extended dynamical system.

For the ideal state without the deviation term ε, let the Lyapunov function be:

$$V_l = \frac{1}{2}S^2 + \frac{1}{2}\alpha_2\tilde{k}^2 + \frac{1}{2}k_i z^2 \tag{30}$$

By taking the derivative of the (30):

$$\begin{aligned} \dot{V}_l &= S\dot{S} + \alpha_2\tilde{k}\dot{\tilde{k}} + k_i S\mathrm{z} \\ &= S\left(k_2 S - \hat{k}_l S - k_i \mathrm{z}\right) + \alpha_2\tilde{k}\dot{\hat{k}}_l + k_i S\mathrm{z} \\ &= S\left(k_2 S - \hat{k}_l S\right) + \alpha_2(\hat{k}_l - \bar{k}_2)\dot{\hat{k}}_l \end{aligned} \tag{31}$$

If $\dot{\hat{k}}_l = \frac{1}{\alpha_2}S^2$ is given, then there is:

$$\begin{aligned} \dot{V}_l &= S\left(k_2 S - \hat{k}_l S\right) + \alpha_2(\hat{k}_l - \bar{k}_2)\frac{1}{\alpha_2}S^2 \\ &= S\left(k_2 S - \hat{k}_l S\right) + (\hat{k}_l - \bar{k}_2)S^2 \\ &= (k_2 - \bar{k}_2)S^2 \leq 0 \end{aligned} \tag{32}$$

So for the ideal state where there is no deviation term ε, the system is asymptotically stable.

In practical situations where there is a deviation term ε:

$$\begin{aligned} \dot{S} &= k_2 S + \varepsilon + u = k_2 S + \varepsilon - \hat{k}_l S - k_i \int_0^t S\, dt \\ &= k_2 S + \varepsilon - (\tilde{k} + \bar{k}_2)S - k_i \int_0^t S\, dt \\ &= (k_2 - \bar{k}_2)S + \varepsilon_0 - k_i \int_0^t S\, dt \end{aligned} \tag{33}$$

where $k_2 - \bar{k}_2 < 0$, $\varepsilon_0 = \varepsilon - \tilde{k}S$, and ε_0 still satisfies the bounded condition. Solving (33):

$$\mathrm{S}(t) = \frac{a\varepsilon_0}{a-b}e^{-at} - \frac{b\varepsilon_0}{a-b}e^{-bt} \tag{34}$$

where $ab = k_i, a + b = \bar{k}_2 - k_2$, the system is asymptotically stable.

Q.E.D. *Remark 3.* The adaptive formula, when the system's sliding mode variable satisfies $|S| < |\delta|$, is given by:

$$\hat{k}_l = \int_0^t \frac{1}{\alpha_2}S^2\, dt \tag{35}$$

where α_2 is the adaptive rate.

5. SIMULATION ANALYSIS

5.1 Case 1

Regarding the rotary servo system model proposed by Ren et al. (2015):

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -25\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ 80\end{bmatrix}u+\begin{bmatrix}0\\ f_2(\theta,\dot{\theta},u)\end{bmatrix} \tag{36}$$

where $[\theta,\dot{\theta}]^T$ is the angular state vector, and $f_2(\theta,\dot{\theta},u)=32(\theta+\dot{\theta}+\arctan(u))$ is the non affine uncertainty term that exists at the control input. The target open-loop transfer function is set to:

$$\begin{bmatrix}\dot{\theta}_m\\ \ddot{\theta}_m\end{bmatrix}=\begin{bmatrix}0 & 1\\ -900 & -60\end{bmatrix}\begin{bmatrix}\theta_m\\ \dot{\theta}_m\end{bmatrix}+\begin{bmatrix}0\\ 900\end{bmatrix}c \tag{37}$$

where $[\theta_m,\dot{\theta}_m]^T$ is the angle reference state vector, and c is the outer loop input. That is to say, the target open-loop transfer function is set to $C=\frac{900}{s^2+60s}$, where $\omega_n=30$ and $\xi=1$.

The linear expansion observer is set to:

$$\begin{cases}\dot{\hat{x}}_1=\hat{x}_2+\omega_0\beta_1(y-\hat{x}_1)\\ \dot{x}_2=\hat{x}_3+\omega_0^2\beta_2(y-\hat{x}_1)+u\\ \dot{x}_3=\omega_0^3\beta_3(y-\hat{x}_1)\end{cases} \tag{38}$$

where $\beta_1=3$, $\beta_2=3$, and $\beta_3=1$. The observer bandwidth is set to $\omega_0=100$.

The sliding mode function is set to $S=20e+\dot{e}$. The bandwidth of the sliding mode function is $\omega_s=20$.

The sliding mode band thickness δ in the adaptive controller DSMCadaptive proposed in this study is empirically set to 2, with adaptive parameters $\alpha_1=150$, $\alpha_2=50$, and integral term coefficient $k_i=1$. The parameters of the traditional double sliding mode controller DSMC are set to $k_g=8$, $k_l=7$, $\delta=2$, and $k_i=1$.

The system input signal is:

$$r(t)=\begin{cases}2 & 0\le t<10\\ 6 & 10\le t\le 12\end{cases} \tag{39}$$

The simulation results are shown in Fig. 4 to Fig. 7, where DSMC is the traditional double sliding mode controller and DSMCadaptive is the adaptive controller proposed in this study. Fig. 4 shows the output results of plant under the action of input signal, Fig. 5 shows the corresponding error comparison chart, Fig. 6 shows the corresponding adaptive result chart, and Fig. 7 shows the corresponding control signal comparison chart.

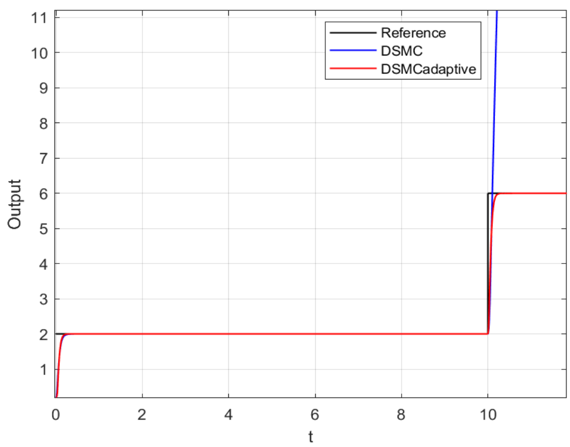

Fig. 4. Local comparison of system output results.

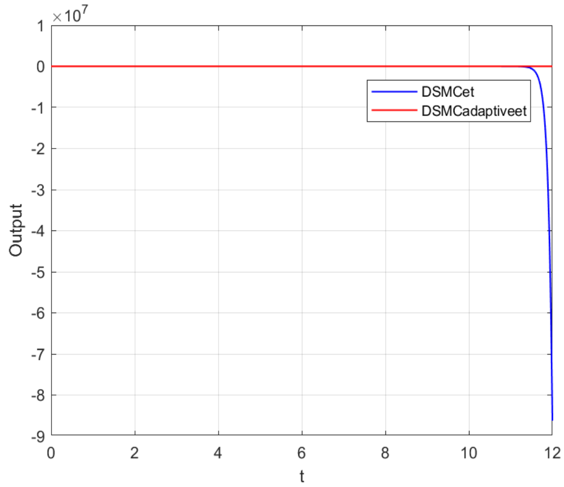

Fig. 5. Comparison of Input and Output Differences.

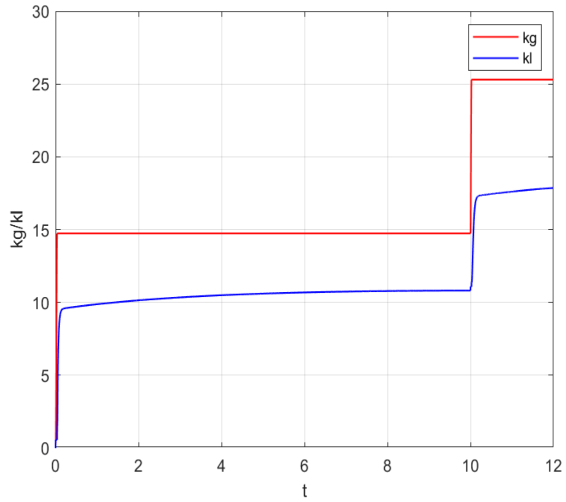

Fig. 6. Adaptive Results of k_g and k_l.

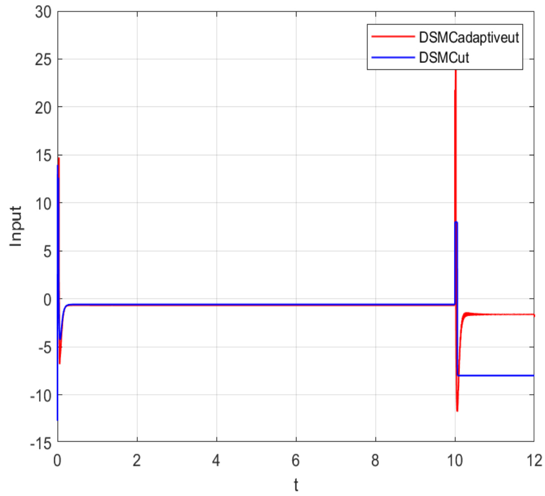

Fig. 7. Control signals of DSMCadaptive and DSMC controllers.

From Fig. 4 and Fig. 5, it can be seen that for the traditional double sliding mode controller, the coefficients are selected based on experience, and the entire system can remain stable under the input signal of $r(t)=2$. However, when the input signal suddenly changes to $r(t)=6$, the entire system becomes unstable. For the adaptive controller proposed in this study, the controller parameters can be adaptively adjusted. Despite sudden changes in the input signal, the adaptive controller can still ensure the stability of the entire system. From Fig. 6, it can be seen that when $r(t) = 6$, the parameters of the double sliding mode controller should satisfy the conditions $25 < k_g < 30$, $15 < k_l < 20$ in order to make the entire system stable. The parameters selected based on experience do not meet this requirement, resulting in divergent output results of the entire system. In summary, the adaptive controller proposed in this study has superior stability compared to traditional double sliding mode controllers.

5.2 Case 2

The required target closed-loop transfer function for the design is:

$$G_c = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{25}{s^2+7s+25} \tag{40}$$

That is to say, the target open-loop transfer function is set to $C = \frac{25}{s^2+7s}$, where $\omega_n = 5$ and $\xi = 0.7$. The reference signal required for the double sliding mode controller is set to $x_{d1} = \frac{1}{s}\frac{25}{s+7}$, and $\dot{x}_{d1} = x_{d2} = \frac{25}{s+7}$.

For the following second-order nonlinear non affine models:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.6x_2 - x_1x_2 - ux_2 + \sin(u) + 2u + u^3 \\ y = x_1 + d(t) \end{cases} \tag{41}$$

The external interference term $d(t) = 1$. The LESO related parameters and sliding mode function related parameters are of the same size as the Case 1. The sliding mode band thickness δ in the adaptive controller DSMCadaptive proposed in this study is empirically set to 2, with adaptive parameters $\alpha_1 = 120$, $\alpha_2 = 50$, and integral term coefficient $k_i = 1$. The parameters of the traditional double sliding mode controller DSMC are set to $k_g = 5$, $k_l = 4$, $\delta = 2$, and $k_i = 1$. This study conducted simulation experiments on two different input signals.

The system input signal is a square wave:

$$r(t) = \begin{cases} 2 & 0 \le t < 10 \\ 6 & 10 \le t < 20 \\ 0 & 20 \le t < 30 \\ -2 & 30 \le t \le 40 \end{cases} \tag{42}$$

The system input signal is a sine wave:

$$r(t) = 2\sin(0.3t) \tag{43}$$

The simulation results are shown in Fig. 8 to Fig. 14, where DSMC is the traditional double sliding mode controller and DSMCadative is the adaptive controller proposed in this study. Among them, Fig. 8 shows the output results of plant under the action of square wave, Fig. 9 shows the corresponding error comparison chart, Fig. 10 shows the corresponding adaptive result chart, and Fig. 11 shows the corresponding control signal comparison chart. Fig. 12 shows the output results of plant under the action of sine wave, Fig. 13 shows the corresponding adaptive result diagram, and Fig. 14 shows the corresponding control signal comparison diagram.

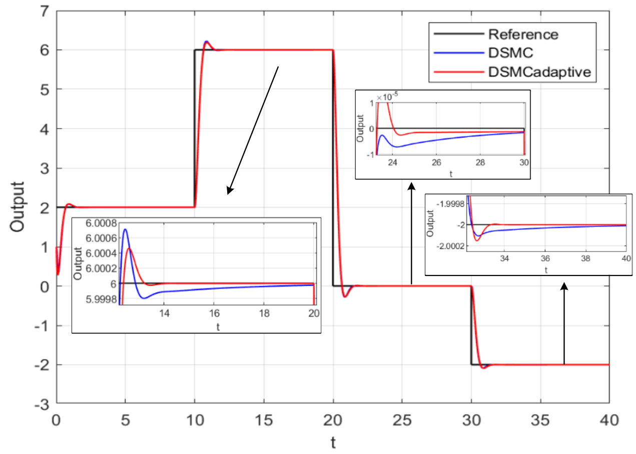

Fig. 8. Comparison of System Output Results.

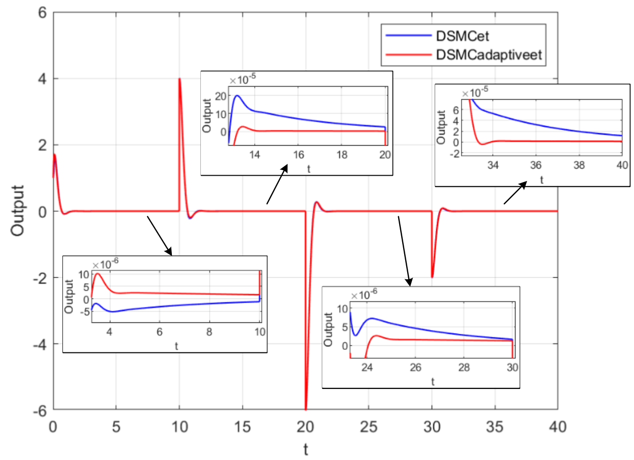

Fig. 9. Comparison of Input and Output Differences.

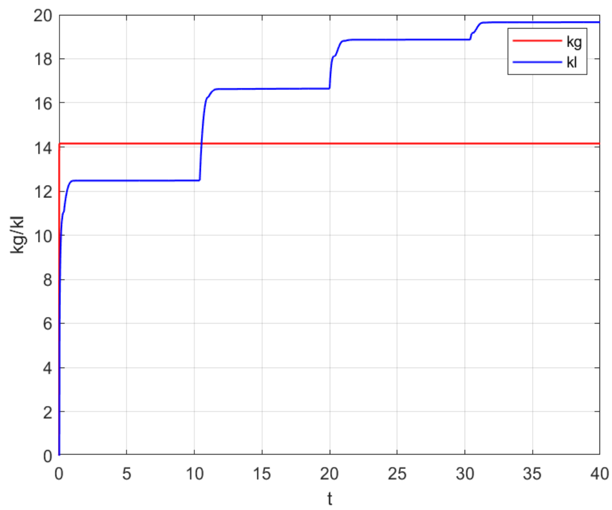

Fig. 10. Adaptive Results of k_g and k_l.

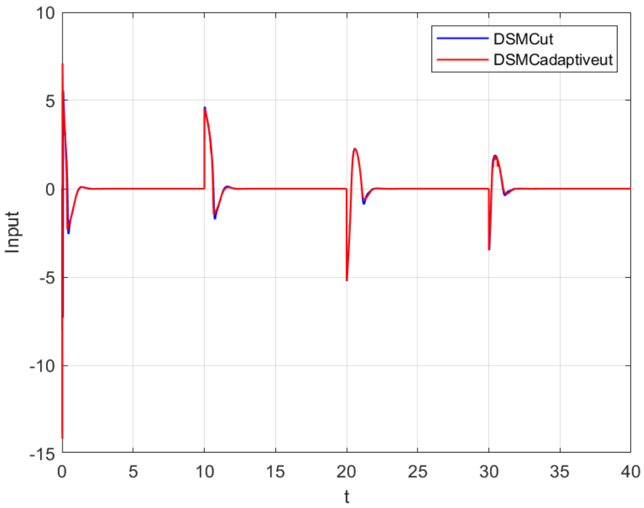

Fig. 11. Control signals of DSMCadaptive and DSMC controllers.

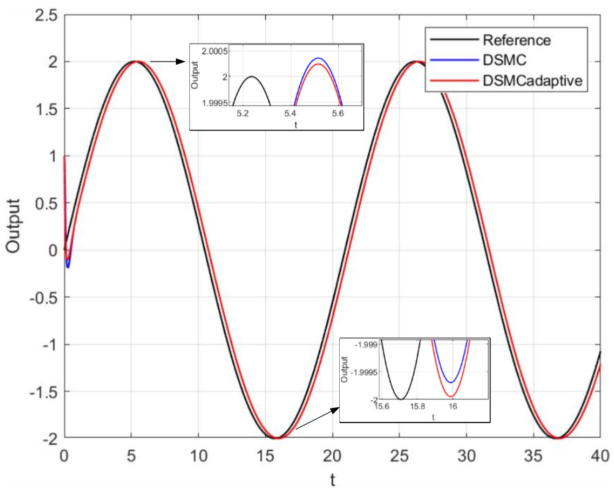

Fig. 12. Comparison of System Output Results.

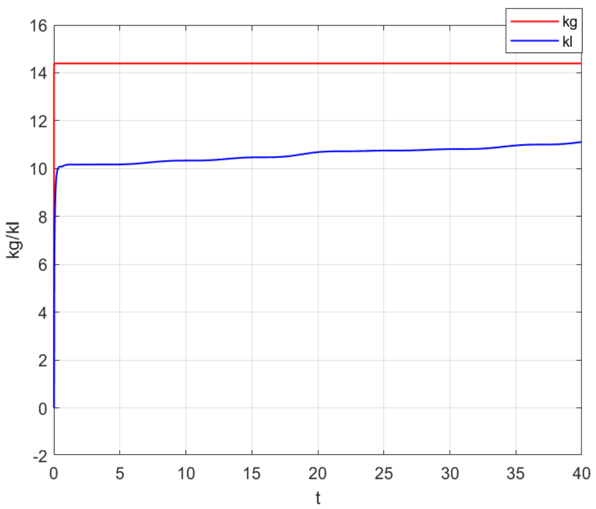

Fig. 13. Adaptive Results of k_g and k_l.

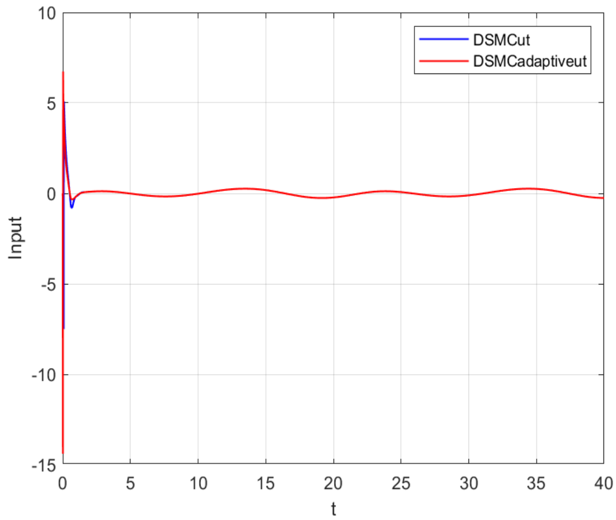

Fig. 14. Control signals of DSMCadaptive and DSMC controllers.

From Fig. 8, Fig. 9, and Fig. 12, it can be seen that the adaptive controller proposed in this study has higher accuracy than traditional DSMC. From Fig. 10 and Fig. 13, it can be seen that compared to traditional DSMC, the adaptive controller proposed in this study can select better switching control gain and equivalent control gain, without the need for empirical selection. From Fig. 11 and Fig. 14, it can be seen that the adaptive controller proposed in this study has a strong suppression effect on sliding mode chattering for complex nonlinear non affine models.

6. CONCLUSIONS

For SISO model free systems, this study combines the advantages of DSMC control and adopts DSMC adaptive controller for dynamic inversion in the inner loop of the U-control framework. Compared to traditional DSMC, the gain of the switching control and equivalent control in the adaptive controller designed in this study no longer requires experience to select, achieving self-adjustment of the gain and ensuring system stability. This is of great significance for industrial practical applications. Meanwhile, the adaptive controller proposed in this study further improves accuracy compared to traditional DSMC. Moreover, in the case where dynamic inversion of commonly used SISO nonlinear systems can be obtained, the adaptive controller does not require the structure and parameters of the model, further reducing the complexity of modeling the controlled system. Compared with traditional sliding mode controllers, the adaptive controller designed in this study does not contain switching terms during the convergence to zero, and does not require estimates of uncertain terms, thus having a stronger chattering suppression effect.

ACKNOWLEDGEMENTS

The authors thank the funds of Natural Science Foundation of Heilongjiang Province (JJ2025XQ0215) and Basic Scientific Research Business Expense Research Project of Heilongjiang Provincial Colleges and Universities (145409604).

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