Reliable Tracking Control of Dynamic Positioning Ships Based on Aperiodic Measurement Information

Guoquan Chen*,, Yongfeng Suo*, Minjie Zheng*, Shenhua Yang*, Lina Li***

**Navigation College, Jimei University, Jimei, Xiamen 361021, PR China*
***China Southampton Business School, University of Southampton, Southampton SO17 1BJ, UK.*
Corresponding author: Yongfeng Suo (e-mail: yfsuo@jmu.edu.cn)

Abstract: This study investigates the reliable tracking control problem for a dynamic positioning ship (DPS) based on aperiodic measurement information. Firstly, the actuator-failure model for the DPS is established, and the values of the actuator failures are random and the upper and lower bounds of the delay are considered. By introducing a new Lyapunov-Krasovskii functional (LKF), the mean square exponential stability criteria are obtained by means of linear matrix inequalities. And the real sampling patterns can be fully captured by constructing a suitable Lyapunov functions. Finally, an illustrative example is exhibited that the sampled-data controller can ensure the states of the DPS and have excellent tracking performances.

Keywords: dynamic positioning system; tracking control; aperiodic sampling; stochastic actuator failures

1. INTRODUCTION

Dynamic positioning ship (DPS) is a kind of ship that relies on the thrust generated by its own propulsion device to resist the influence of external interference and make the ship keep a certain position along a fixed track. It has widely used in offshore oil drilling platforms, salvage ships, engineering ship and so on (see T. I. Fossen, 2002). Therefore, DP technology improves the efficiency and safety of marine development, which is very important for marine operation. At present, the issue about H_∞ control, optimal control, backstep integral control, fuzzy control, hybrid switching control etc. have been reported for DPS (see Sorensen, 2011; Ngongi et al., 2015; Du et al., 2016; Bidikli et al., 2017; Fu et al., 2018; Zhang et al., 2018; Peng et al., 2019). For example, in (Dung et al., 2019), according to the characteristics that the DPS is easily affected by the environment, a multi-level fuzzy model is proposed to improve the quality of DPS. In (Xu et al., 2019), a PID controller based on fuzzy rules for DPS is proposed, which takes the estimated positioning error and low-frequency speed as input.

With the development of digital computer, sampled-data control system has appeared in all fields of national production and life, national defense construction and scientific experiment (see Fridman, 2006). One of the most remarkable characteristics of this system is that both continuous and discrete signals are existing in one system, which have difficulties in analysing the stability and designing the controller. Now, the analysis of control problem for sampled-data system has received much attentions, and considerable research results have been developed in multiagent systems (Zhang et al., 2018), multiagent systems (Yu et al., 2022), fuzzy systems (Wang et al., 2018; Zhang et al., 2021; Liu et al., 2016; Zheng et al., 2018), chaotic systems (Wang et al., 2016; Chen et al., 2012), neural networks systems (Wu et al., 2013; Wu et al., 2014; Yucel et al., 2017), and so on. For example, In (Liu et al., 2016), the quantization problem of fuzzy sampled-data systems is studied by the improved input delay method and wirtinger integral inequality, and the design method of corresponding quantization controller is given. In (Yucel et al., 2017), the filtering problem of uncertain fuzzy neural networks is studied by using the input time-delay approach, and the fuzzy sampled-data filters are designed.

Sampled-data system exists in the DPS (see Zheng et al., 2019), firstly, it uses various digital sensors, such as GPS, electric compass, wind speed sensor etc. to obtain ship's positioning data, heading, the change of wind speed, etc. respectively, then the sensors signals are sampled by the digital computer to determine the motion of the ship. In recent year, sampled-data control theory has been adopted for solving the DP problem. In (Katayama, 2010), by using integrator backstepping technique, the nonlinear control problems of sampled-data DPS are discussed, and the output and state feedback controllers are designed. In (Katayama et al., 2014), the nonlinear sampled-data control theory is used for DPS to deal with the problem about straight-line trajectory tracking. In (Zheng et al., 2021), the sampled-data controllers for DPS are designed to make the system stable and achieve good H_∞ performance. (Yang et al., 2018) discusses the robust fault-tolerant control issue for sampled-data DPS with actuator-failure. In (Zheng et al., 2018), the fuzzy control issues for the nonlinear sampled-data DPS are discussed, and the free-weighting matrices are used for guaranteeing the system's stability.

However, few literatures have been reported for aperiodic sampled-data DPS with actuator failures. Besides, the LKF has room for improving to fully capture the real sampling patterns and get less conservative result, which are the motivations of this paper.

In this paper, the stabilization of DPS with aperiodic sampled-data is discussed. The motion models of DPS with random actuator failures are established. Then the upper and lower bounds of delay are considered. Secondly, Lyapunov theorem are introduced to fully capture the sampling patterns in LMI. And the conditions of mean square exponential stability are given. Then the design methods of sampled-data controller are introduced, which guarantee that the system has excellent performance even if the stochastic actuator failure, extern disturbances and aperiodic measurements exists. Finally, a specific example is given to illustrate that the method is effective.

Notations: $Sym\{M\} = M + M^T$."*" is the symmetric term of a matrix. $I_i, i = 1,2,3$ represents defined as block entry matrices. $\lambda_{\max}(P), \lambda_{\min}(P)$ represents the maximum and minimum eigenvalues of the matrix P, E represents the mathematical expectation.

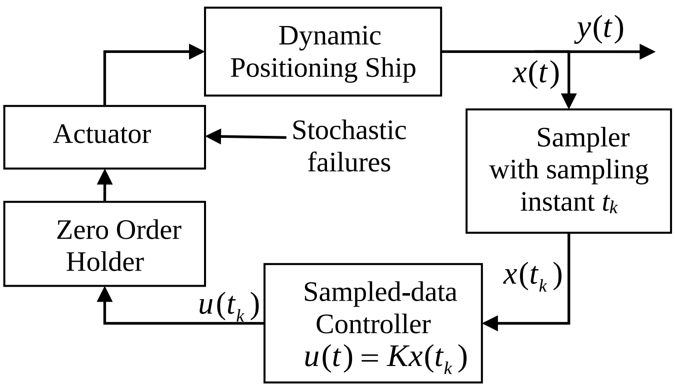

Fig. 1. The schematic of sampled-data DPS control system.

2. PROBLEM FORMULATION

The mathematical model of DPS is considered as follow.

$$\begin{aligned} M\dot{v}(t) + Dv(t) &= u^f(t) + w(t), \\ \dot{\eta}(t) &= J(\psi)v(t), \end{aligned} \tag{1}$$

where

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix},$$

$$D = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix},$$

$$J(\psi(t)) = \begin{bmatrix} \cos(\psi(t)) & -\sin(\psi(t)) & 0 \\ \sin(\psi(t)) & \cos(\psi(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

where $\eta(t) = [x_a(t) \quad y_a(t) \quad \psi(t)]^T$ represents the positions $x_a(t)$, $y_a(t)$ and yaw angle $\psi(t)$ in earth-fixed frame. $\upsilon(t) = [p(t) \quad v(t) \quad r(t)]^T$ represents the ship's velocity of surge, sway and yaw in body-fixed frame. $J(\psi)$ denotes the transformation matrix. $u^f(t)$ represents the control input with stochastic failures; $w(t)$ denotes the extern disturbance; M and D represent the inertia matrix and damping matrix respectively.

where m represents the mass of the ship; I_z is the moment of inertia; x_G is the distance from the origin of the ship's coordinate system to the center of gravity; $X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}}$ are the added mass of hydrodynamic forces in the three directions of surge, sway and yaw; $Y_{\dot{r}}$ is the added mass due to the coupling of sway and yaw;

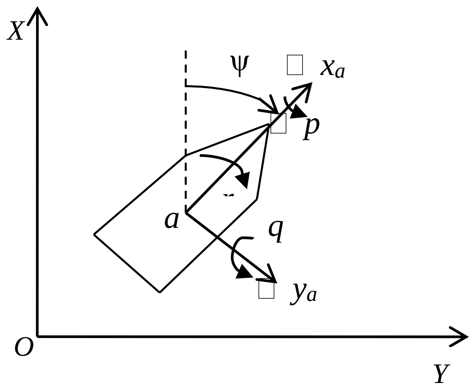

Fig. 2. Body-fixed coordinate systems.

Assumed that yaw angle $\psi(t)$ is small enough, which means that

$\cos(\psi(t)) \approx 1, \sin(\psi(t)) \approx 0$, then

$$J(\psi(t)) \approx I. \tag{2}$$

Under the condition (2), we define

$$\begin{aligned} x(t) &= [\eta(t) \quad \upsilon(t)]^T \\ &= [x_a(t) \quad y_a(t) \quad \psi(t) \quad p(t) \quad v(t) \quad r(t)]^T, \end{aligned}$$

Then the exquation (1) is converted to

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t), \tag{3}$$

where

$$A=\begin{bmatrix}0_{3\times3} & I_{3\times3}\\ 0_{3\times3} & -M^{-1}D\end{bmatrix},\quad B=\begin{bmatrix}0_{3\times3}\\ M^{-1}\end{bmatrix},\quad B_w=\begin{bmatrix}0_{3\times3}\\ M^{-1}\end{bmatrix}.$$

Consider the output signal that

$$y(t)=Cx(t)=\psi(t), \tag{4}$$

where

$$C=\begin{bmatrix}0 & 0 & 1 & 0 & 0 & 0\end{bmatrix},$$

From (3) and (4), we have

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu^f(t)+B_w w(t),\\ y(t)&=Cx(t).\end{aligned} \tag{5}$$

The control object is to consider the sampled-data tracking control problem when actuator faults exist in the DP ships (see Fig.1). The following actuator fault model is adopted as follows.

$$u^f(t)=\theta u(t)=\sum_{i=1}^{m}\theta_i\Delta_i u(t), \tag{6}$$

where $\theta=diag\{\theta_1,\theta_2,\dots\theta_i\},\quad i=1,,\dots m$ are m unrelated random variables and

$$\Delta_i=diag\left\{\underbrace{0,\dots,1}_{i-1},\underbrace{0,\dots0}_{m-i}\right\}.$$

It is assumed that θ takes values in a finite set, that is $\theta_i\in diag\{\pi_{i1},\pi_{i2},\dots\pi_{in}\}$. In addition, the process $\{\theta_i\}$. is assumed to be independent and identically distributed, with the probabilities given by

$$Prob\left\{\theta_i=\pi_{ij}\right\}=\alpha_{ij},i=1,\dots,m,\ j=1,\dots,n. \tag{7}$$

where α_{ij} is as positive scalar and $\sum_{j=1}^{n}\alpha_{ij}=1$.

Remark 1 It is noted that random values in a finite set are used to describe the actuator fault for the sampled-data DPS in this paper, which is more practical than (Yang et al., 2018), as the variable θ in the reference is assumed to be an unknown constant with known upper and lower bounds. Therefore, the assumption in (4) is better in describing the failure characteristics. The failure characteristics are listed in Table 1.

Table 1. Failure characteristics.

failure mode	θ
complete failure	$\theta=0$
partial degradation	$0<\theta<1$
no failure	$\theta=1$
forward drift failure	$\theta>1$

Define the tracking error as follow

$$e(t)=r(t)-y(t). \tag{8}$$

Where $r(t)$ represents the reference signal. Noted that a controller's tracking error integral plays a key role to eliminate the tracking error. Then, similar to [28], the integral term $\delta(t)=\int_0^t e(t)dt$ is defined, then the DPS with actuator faults can be written as

$$\dot{\varsigma}(t)=\bar{A}\varsigma(t)+\bar{B}\theta u^f(t)+\bar{B}_w\bar{w}(t), \tag{9}$$

where

$$\varsigma(t)=\begin{bmatrix}x^T(t) & \delta^T(t)\end{bmatrix}^T,\bar{w}(t)=\begin{bmatrix}w^T(t) & r^T(t)\end{bmatrix}^T,$$

$$\bar{A}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix},\quad \bar{B}=\begin{bmatrix}B\\ 0\end{bmatrix},\quad \bar{B}_w=\begin{bmatrix}B_w & 0\\ 0 & I\end{bmatrix},$$

Assume that the signal of DPS can be obtained at sampling instant t_k with $0=t_0<t_1<t_2<\dots<t_k<\dots$.The sampling period is assumed that

$$d_1\le t_{k+1}-t_k=d_k\le d_2,\quad \forall k\ge0,d_1\ge0,d_2>0, \tag{10}$$

Considered the state-feedback control law as follow,

$$u(t)=K\varsigma(t_k)=\begin{bmatrix}K_1 & K_2\end{bmatrix}\begin{bmatrix}x(t_k)\\ \delta(t_k)\end{bmatrix},\quad t_k\le t<t_{k+1}, \tag{11}$$

where K is a controller matrix. Combining (11) and (5), then

$$\dot{\varsigma}(t)=\bar{A}\varsigma(t)+\bar{B}\theta K\varsigma(t_k)+\bar{B}_w\bar{w}(t). \tag{12}$$

The object of the paper is designing a sampled-data controller to guarantee that

1) Under the conditions of stochastic actuator failure and aperiodic measurement, the system (10) with $w(t)=0$ is mean square exponential stable, and the output $y(t)$ can track the reference signal $r(t)$.

2) Despite the extern disturbances, it is required to satisfy that $E\left\{\int_0^t\varsigma^T(s)\varsigma(s)ds\right\}\le\gamma^2\int_0^t w^T(s)w(s)ds$ for all

nonzero $w(t) \in L_2[0,\infty)$ under zero condition, where $\gamma > 0$.

3. MAIN RESULTS

In this section, the linear matrix inequality method is introduced. Then the stability additions and the design method of controller for sampled-data DPS is given by establishing LKF in terms of LMI.

3.1 Linear matrix inequality

Compared with the traditional Riccati equation method, linear matrix inequality method is more convenient and effective. It can solve many problems of control system by solving the feasibility problem of LMI or the convex optimization problem of LMI with constraints. Besides, LMI can be used to obtain multiple optimal solutions of multiple parts of the system respectively, so that the control system can meet the performance index requirements of multiple objectives.

The form of LMI is that:

$$F(x) = F_0 + x_1 F_1 + \cdots + x_n F_n < 0$$

where $x_1, \cdots, x_m$ are decision variables, $F_0, F_1, \cdots, F_n$ are given real symmetric matrix.

Then we can adopt the LMI toolbox of MATLAB to deal with the convex optimization problem of LMI.

3.2 Stability analysis

Theorem 1: For scales $\varepsilon_1, \varepsilon_2, \lambda > 0, \gamma > 0$ and $d_2 > d_1 \geq 0$, the system (12) is mean square exponential stable with γ-disturbance attenuation when $w(t) = 0$, if there exist matrices $Y_{11}, Y_{22}, M, P > 0, \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0,$ such that

$$\tilde{\Xi}_1(d) = \begin{bmatrix} \Xi_1 + de^{-2\lambda d_2} I_3 Q_{22} I_3^T & \Xi_2 \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{13}$$

$$\tilde{\Xi}_2(d) = \begin{bmatrix} \Xi_1 & -de^{-2\lambda d_2}\Xi_3 & \Xi_2 \\ * & -de^{-2\lambda d_2} Q_{11} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, d \in \{d_1, d_2\}, \tag{14}$$

$$\Theta = \begin{bmatrix} P + d_2\left(Y_{11} + Y_{11}^T\right) & -d_2\left(Y_{11} + Y_{22}\right) \\ * & d_2\left(Y_{22} + Y_{22}^T\right) \end{bmatrix} > 0, \tag{15}$$

where

$$\begin{aligned}
\Xi_1 = Sym\{ & I_1 P I_2^T + \lambda I_1 P I_1^T - I_1 Y_{11} I_1^T \\
& + I_1 Y_{11} I_3^T + I_1 Y_{22} I_3^T - \varepsilon_1 I_1 MA I_1^T - I_1 A^T M^T I_2^T \\
& - \varepsilon_1 I_1 MB\theta K I_3^T + I_2 M I_2^T - I_3 Y_{22} I_3^T + \varepsilon_1 I_1 M I_2^T \\
& - \varepsilon_1 I_1 A^T M^T I_3^T - I_2 MB\theta K I_3^T + \varepsilon_2 I_2 M^T I_3^T \\
& - \varepsilon_1 I_3 MB\theta K I_3^T \} + I - de^{-2\lambda d_2} I_3 Q_{22} I_3^T,
\end{aligned}$$

$$\begin{aligned}
\Gamma_1 = Sym\{ & I_1\left(Y_{11} + Y_{11}^T\right) I_2^T - I_2\left(Y_{11} + Y_{22}\right) I_3^T + I_2 Q_{11} I_2^T \\
& + I_2 Q_{12} I_3^T + \lambda I_1\left(Y_{11} + Y_{11}^T\right) I_1^T - \lambda I_1\left(Y_{11} + Y_{22}\right) I_3^T \\
& + I_3 Q_{22} I_3^T + \lambda I_3\left(Y_{22} + Y_{22}^T\right) I_3^T \},
\end{aligned}$$

$$\Xi_2 = \begin{bmatrix} -\varepsilon_1 B_w^T M^T & -B_w^T M^T & -\varepsilon_2 B_w^T M^T \end{bmatrix}^T,$$

$$\Xi_3 = \begin{bmatrix} 0 & 0 & Q_{12} \end{bmatrix}^T.$$

Proof. The LKF is constructed as

$$V(\varsigma_t, t) = \sum_{i=1}^{3} V_i(\varsigma_t, t), \quad t \in [t_k, t_{k+1}) \tag{16}$$

$$V_1(\varsigma_t, t) = \varsigma(t)^T P \varsigma(t),$$

$$V_2(\varsigma_t, t) = h_2(t)\int_{t_k}^{t} e^{2\lambda(s-t)} \begin{bmatrix} \dot{\varsigma}(s) \\ \varsigma(t_k) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} \dot{\varsigma}(s) \\ \varsigma(t_k) \end{bmatrix} ds,$$

$$V_3(\varsigma_t, t) = h_2(t) \begin{bmatrix} \varsigma(t) \\ \varsigma(t_k) \end{bmatrix}^T \begin{bmatrix} Y_{11} + Y_{11}^T & -Y_{11} - Y_{22} \\ * & Y_{22} + Y_{22}^T \end{bmatrix} \begin{bmatrix} \dot{\varsigma}(t) \\ \varsigma(t_k) \end{bmatrix}.$$

It can be seen that except for the sampling instants $t_k\ (k = 0,1,2,\ldots)$, $V(t)$ is continuous on $[0,\infty)$. When $t = t_k^+$, except the first term of $V_1(t)$, the other terms of $V(t)$ are equal to 0 when $t = t_k^+$ and not less than 0 when $t \to t_k^-$. Hence, $\lim_{t \to t_k^-} V(t) \geq V(t_k^+)$, which means that $V(t)$ does not increase along t_k.

Then, according to the condition (12), it can be got that

$$\begin{aligned}
& V_1(\varsigma_t, t) + V_3(\varsigma_t, t) \\
& = \frac{d_2 - h_2(t)}{d_2} \varsigma^T(t) P \varsigma(t) + \frac{h_2(t)}{d_2} \varsigma^T(t) P \varsigma(t) \\
& + \frac{h_2(t)}{d_2} d_2 \begin{bmatrix} \varsigma(t) \\ \varsigma(t_k) \end{bmatrix}^T \begin{bmatrix} Y_{11} + Y_{11}^T & -Y_{11} - Y_{22} \\ * & Y_{22} + Y_{22}^T \end{bmatrix} \begin{bmatrix} \dot{\varsigma}(t) \\ x(t_k) \end{bmatrix} \\
& = \frac{d_2 - h_2(t)}{d_2} \varsigma^T(t) P \varsigma(t) + \frac{h_2(t)}{d_2} \begin{bmatrix} \varsigma(t) \\ \varsigma(t_k) \end{bmatrix}^T \Theta \begin{bmatrix} \dot{\varsigma}(t) \\ \varsigma(t_k) \end{bmatrix} \\
& \geq 0.
\end{aligned} \tag{17}$$

The infinitesimal operator L is defined for the random process $\{\varsigma(t), t \ge 0\}$. Using the operator for $V(\varsigma_t, t)$, it can be obtained that:

$$\begin{aligned}
&LV_1(\varsigma_t,t)+2\lambda V_1(\varsigma_t,t)\\
&=2\varsigma(t)^T P\dot{x}(t)+2\lambda\varsigma(t)^T P\varsigma(t),\\
&LV_2(\varsigma_t,t)+2\lambda V_2(\varsigma_t,t)\\
&=h_2(t)\begin{bmatrix}\dot{\varsigma}(t)\\ \varsigma(t_k)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}\begin{bmatrix}\dot{\varsigma}(t)\\ \varsigma(t_k)\end{bmatrix}\\
&-e^{-2\lambda d_2}\int_{t_k}^{t}\begin{bmatrix}\dot{\varsigma}(s)\\ \varsigma(t_k)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}\begin{bmatrix}\dot{\varsigma}(s)\\ \varsigma(t_k)\end{bmatrix}ds,\\
&LV_3(\varsigma_t,t)+2\lambda V_3(\varsigma_t,t)\\
&=2h_2(t)\begin{bmatrix}\varsigma(t)\\ \varsigma(t_k)\end{bmatrix}^T\begin{bmatrix}Y_{11}+Y_{11}^T & -Y_{11}-Y_{22}\\ * & Y_{22}+Y_{22}^T\end{bmatrix}\begin{bmatrix}\dot{\varsigma}(t)\\ 0\end{bmatrix}\\
&-\begin{bmatrix}\varsigma(t)\\ \varsigma(t_k)\end{bmatrix}^T\begin{bmatrix}Y_{11}+Y_{11}^T & -Y_{11}-Y_{22}\\ * & Y_{22}+Y_{22}^T\end{bmatrix}\begin{bmatrix}\varsigma(t)\\ \varsigma(t_k)\end{bmatrix}.\\
&+2\lambda h_2(t)\begin{bmatrix}\varsigma(t)\\ \varsigma(t_k)\end{bmatrix}^T\begin{bmatrix}Y_{11}+Y_{11}^T & -Y_{11}-Y_{22}\\ * & Y_{22}+Y_{22}^T\end{bmatrix}\begin{bmatrix}\varsigma(t)\\ \varsigma(t_k)\end{bmatrix}
\end{aligned} \quad (18)$$

Using the Leibniz-Newton formula, for any free matrix M and scalars $\varepsilon_1, \varepsilon_2$, it can be got that

$$\begin{aligned}
&2[\varepsilon_1\varsigma^T(t)M+\dot{\varsigma}^T(t)M+\varepsilon_2\varsigma^T(t_k)M]\\
&\times\left[\dot{\varsigma}(t)-\left(Ax(t)+B\theta Kx(t_k)+B_w w(t)\right)\right]=0.
\end{aligned} \quad (19)$$

Remark 2 Note that the free weighting matrix M is introduced in equation (19), and the matrix is obtained by solving the corresponding LMI, which avoids the conservatism caused by giving the weighting matrix in advance, and greatly reduces the conservatism of the results.

Combine (18) with (19), the H_∞ performance index can be considered that

$$\begin{aligned}
&LV(\varsigma_t,t)+2\lambda V(\varsigma_t,t)+\varsigma^T(t)\varsigma(t)-\gamma^2 w^T(t)w(t)\\
&=\frac{h_2(t)}{d_k}\psi^T(t)\tilde{\Xi}_1(d_k)\psi(t)\\
&+\frac{1}{d_k}\int_{t_k}^{t}\psi^T(t,s)\tilde{\Xi}_2(d_k)\psi(t,s)ds.
\end{aligned} \quad (20)$$

where

$$\psi(t,s)=\begin{bmatrix}\varsigma^T(t) & \dot{\varsigma}^T(t) & \varsigma^T(t_k) & \dot{\varsigma}^T(s) & w^T(t)\end{bmatrix}^T,$$

From (13) and (14), it can be obtained

$$\begin{aligned}
\tilde{\Xi}_1(d_k)&=\frac{d_k-d_1}{d_2-d_1}\Xi_1(d_2)+\frac{d_2-d_k}{d_2-d_1}\Xi_1(d_1)<0,\\
\tilde{\Xi}_2(d_k)&=\frac{d_k-d_1}{d_2-d_1}\Xi_2(d_2)+\frac{d_2-d_k}{d_2-d_1}\Xi_2(d_1)<0,
\end{aligned} \quad (21)$$

which implies

$$\begin{aligned}
&LV(\varsigma_t,t)+2\lambda V(\varsigma_t,t)+\varsigma^T(t)\varsigma(t)-\gamma^2 w^T(t)w(t)^T<0,\\
&t\in[t_k,t_{k+1}).
\end{aligned} \quad (22)$$

When $w(t)=0$, it can be got

$$\begin{aligned}
\lambda_{\min}(P)E\left\{\|\varsigma(t)\|^2\right\}&\le E\left\{V(\varsigma_t,t)\right\}\\
&<e^{-2\lambda(t-t_k)}E\left\{V(\varsigma_{t_k},t_k)\right\}\\
&<e^{-2\lambda(t-t_{k-1})}E\left\{V(\varsigma_{t_{k-1}},t_{k-1})\right\}\\
&<\cdots<e^{-2\lambda t}E\left\{V(\varsigma_{t_0},0)\right\}\\
&\le\lambda_{\max}(P)e^{-2\lambda t}\|\varsigma(t_0)\|^2.
\end{aligned} \quad (23)$$

Then

$$E\left\{\|\varsigma(t)\|\right\}\le\left(\frac{\sqrt{\lambda_{\max}(P)}}{\sqrt{\lambda_{\min}(P)}}\right)e^{-\lambda t}\|\varsigma(t_0)\|. \quad (24)$$

It means that the system (12) is mean square exponentially stable when $w(t)=0$. Similar with the proofs above, we can establish the γ-disturbance attenuation performance easily. The proof is completed.

Remark 3 In the paper, the stability criterions for system (12) are given to achieve the performance of disturbance attenuation. Besides, a quadratic function $V_3(\varsigma_t,t)$ is added in LKF (16) to obtain the sampling patterns fully, and more free matrices are introduced in $V_3(\varsigma_t,t)$ which can be non-positive, and they can reduce the conservativeness to some extent.

Remark 4 In the paper, both upper and lower bounds of the sampling period have been discussed. if $d_1=d_2=d$, the proposed mechanism can be used to deal with the periodic sampled-data control issue for DPS like (Yang et al., 2018), which omits the lower bound of the sampling period and it will lead to conservatism to some extern. Thus, the proposed method has wider application scopes than (Yang et al., 2018).

Remark 5 Note that successions of equations are introduced in the paper to get the LMIs (13)(14)(15) of theorem1 and LMIs (25)(26)(27) of theorem 2, which can be calculated by

MATLAB toolbox easily, and this method has been widely applied in the analysis of many control systems.

3.3 Sampled-data controller design for DPS

According to theorem 2, the design method of the sampled-data controller is given.

Theorem 2: For scales $\varepsilon_1, \varepsilon_2, \lambda > 0, \gamma > 0$, the system (12) is mean square exponentially stable with γ-disturbance attenuation ,if there exist matrices $\bar{Y}_{11}, \bar{Y}_{22}, \bar{M}, \bar{P} > 0, \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} \\ * & \bar{Q}_{22} \end{bmatrix} > 0$, such that:

$$\tilde{\bar{\Xi}}_1(d) = \begin{bmatrix} \bar{\Xi}_1 + de^{-2\lambda d_2} I_3 \bar{Q}_{22} I_3^T & \bar{\Xi}_2 & \bar{\Xi}_4 \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (25)$$

$$\tilde{\bar{\Xi}}_2(d) = \begin{bmatrix} \bar{\Xi}_1 & -de^{-2\lambda d_2} \bar{\Xi}_3 & \bar{\Xi}_2 & \bar{\Xi}_4 \\ * & -de^{-2\lambda d_2} \bar{Q}_{11} & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (26)$$

$d \in \{d_1, d_2\}$,

$$\bar{\Theta} = \begin{bmatrix} \bar{P} + d_2\left(\bar{Y}_{11} + \bar{Y}_{11}^T\right) & -d_2\left(\bar{Y}_{11} + \bar{Y}_{22}\right) \\ * & d_2\left(\bar{Y}_{22} + \bar{Y}_{22}^T\right) \end{bmatrix} > 0. \quad (27)$$

where

$$\bar{\Xi}_1 = Sym\{I_1\bar{P}I_2^T + \lambda I_1\bar{P}I_1^T - I_1\bar{Y}_{11}I_1^T$$
$$+I_1\bar{Y}_{11}I_3^T + I_1\bar{Y}_{22}I_3^T - \varepsilon_1 I_1 A\bar{M}I_1^T - I_1\bar{M}A^T I_2^T$$
$$-\varepsilon_1 I_1 B\theta\bar{K}I_3^T + I_2\bar{M}I_2^T - I_3\bar{Y}_{22}I_3^T + \varepsilon_1 I_1\bar{M}I_2^T$$
$$-\varepsilon_1 I_1\bar{M}A^T I_3^T - I_2 B\theta\bar{K}I_3^T + \varepsilon_2 I_2\bar{M}^T I_3^T$$
$$-\varepsilon_1 I_3 B\theta\bar{K}I_3^T\} - de^{-2\lambda d_2} I_3\bar{Q}_{22}I_3^T,$$

$$\bar{\Gamma}_1 = Sym\{I_1\left(\bar{Y}_{11} + \bar{Y}_{11}^T\right)I_2^T - I_2\left(\bar{Y}_{11} + \bar{Y}_{22}\right)I_3^T + I_2\bar{Q}_{11}I_2^T$$
$$+I_2\bar{Q}_{12}I_3^T + \lambda I_1\left(\bar{Y}_{11} + \bar{Y}_{11}^T\right)I_1^T - \lambda I_1\left(\bar{Y}_{11} + \bar{Y}_{22}\right)I_3^T$$
$$+I_3\bar{Q}_{22}I_3^T + \lambda I_3\left(\bar{Y}_{22} + \bar{Y}_{22}^T\right)I_3^T\},$$

$$\bar{\Xi}_2 = \begin{bmatrix} -\varepsilon_1 B_w^T & -B_w^T & -\varepsilon_2 B_w^T \end{bmatrix}^T,$$

$$\bar{\Xi}_3 = \begin{bmatrix} 0 & 0 & \bar{Q}_{12} \end{bmatrix}^T,$$

$$\bar{\Xi}_4 = \begin{bmatrix} \bar{M}^T & 0 & 0 \end{bmatrix}^T.$$

The controller gain matrix K is computed that

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \bar{K}\bar{M}^{-T} \quad (28)$$

Proof: Let

$$\chi_1 = diag\{\bar{M}, \bar{M}\}, \chi_2 = diag\{\chi_1, \bar{M}\},$$
$$\chi_3 = diag\{\chi_2, \bar{M}, I\}.$$

Defining

$$\bar{M} = M^{-1}, \bar{P} = \bar{M}P\bar{M}^T, Q_{11} = \bar{M}Q_{11}\bar{M}^T,$$
$$Q_{12} = \bar{M}Q_{12}\bar{M}^T, Q_{22} = \bar{M}Q_{11}\bar{M}^T, Y_{11} = \bar{M}Y_{11}\bar{M}^T,$$
$$Y_{22} = \bar{M}Y_{11}\bar{M}^T, \bar{K} = K\bar{M}^T.$$

Pre- and post-multiplying (13)-(15) by χ_1, χ_2 and χ_3 respectively, (25)-(27) can be obtained by Schur complement. This completed the proof.

4. NUMERICAL EXAMPLES

To validate the effectiveness of the developed method, a simulation example for a multipurpose supply vessel is given (see Fig.3). Similar with (Tannuri et al.,2010), the ship's length is 76.2 m and mass is $4.591*10^6$ kg, the other model parameters are considered in Table 2.

Table 2. Ship model parameters.

Items	Value	Items	Value
X_u	$5.0242*10^4$	$Y_{\dot v}$	$-3.6921*10^6$
I_z	$3.7454*10^9$	$-N_{\dot r}$	$3.7454*10^9$
Y_v	$2.7229*10^5$	N_r	$4.1894*10^8$
$X_{\dot u}$	$-0.7212*10^6$	Y_r	$-4.3933*10^6$
$Y_{\dot r}$	$-1.0234*10^6$	N_v	$-4.3821*10^6$

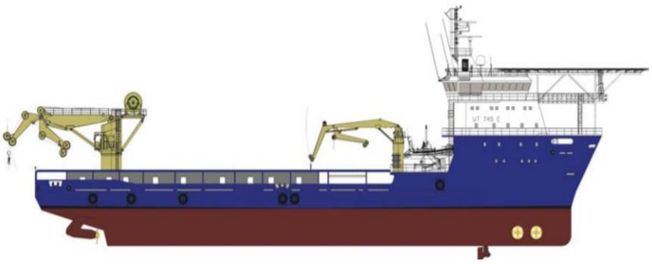

Fig. 3. The multipurpose supply vessel.

And the *M* and *D* in model (1) are

$$M = \begin{bmatrix} 5.3122\times10^6 & 0 & 0 \\ 0 & 8.2831\times10^6 & 0 \\ 0 & 0 & 3.7454\times10^9 \end{bmatrix},$$

$$D=\begin{bmatrix}5.0242\times10^4 & 0 & 0\\ 0 & 2.7229\times10^5 & -4.3933\times10^6\\ 0 & -4.3933\times10^6 & 4.1894\times10^8\end{bmatrix}.$$

Noted that

$$A=\begin{bmatrix}0 & I\\ 0 & -M^{-1}D\end{bmatrix},\quad B=\begin{bmatrix}0\\ M^{-1}\end{bmatrix},\quad B_w=\begin{bmatrix}0\\ M^{-1}\end{bmatrix},$$

then

$$A=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&-0.0095&0&0\\0&0&0&0&-0.0329&0.5304\\0&0&0&0&0.0012&-0.1119\end{bmatrix},$$

$$B=B_w=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\0.1882&0&0\\0&0.1207&0\\0&0&0.0003\end{bmatrix}\times10^{-6}.$$

The probabilistic actuator failures are considered that

$\theta_1\in\{0.3,1,1.2\}$ and $\theta_2\in\{0.4,1,1.5\}$ with

$$Prob\{\theta_1=0.3\}=Prob\{\theta_2=0.4\}=0.2,$$
$$Prob\{\theta_1=1\}=Prob\{\theta_2=1\}=0.6,$$
$$Prob\{\theta_1=1.2\}=Prob\{\theta_2=1.5\}=0.2.$$

Consider the ship's initial state $x_s(t)=\begin{bmatrix}1m & 1m & 0° & 0m/s & 0m/s & 0°/s\end{bmatrix}$

and $d_1=0.8s, d_2=1.6s, \lambda=0.01, \varepsilon_1=1, \varepsilon_2=0.01, \gamma=10$. Then, the controller gain can be computed that

$$K_1=\begin{bmatrix}-163.6565 & 0.0569 & -1.6445 & -188.8556 & 0.0598 & -2.1451\\ 0.0170 & -245.7164 & 10.9512 & 0.1187 & -324.8716 & 24.8897\\ -0.0820 & 5.8243 & -12.2825 & -0.2125 & 7.3309 & -27.2819\end{bmatrix},$$

$$K_2=\begin{bmatrix}1.6423\\ -10.9513\\ 12.2825\end{bmatrix}.$$

Consider the extern disturbance

$$w(t)=0.2\cos(0.4t). \tag{29}$$

The simulation results are shown in Figs. 4-5. Fig. 4 is the responses curve of the position (x, y) and yaw angle, Fig.6 is the responses curve of the ship's velocity, which show that the proposed control strategy can make the DPS stabile and reach the desired target even if the stochastic actuator failure, extern disturbances and aperiodic measurements exists.

Then the tracking command $r(t)$ is set that $r(t)=1$, then the tracking responses of yaw angle ψ and the error between yaw angle ψ and $r(t)$ are illustrated in Figs. 6 and Fig. 7 respectively. It is shown that when there exist aperiodic measurements, extern disturbances and actuator failure, yaw angle ψ has good tracking ability for $r(t)$ .Then we set $r(t)=\sin 4t$, the simulation result of Figs. 8-9 can further verify that the yaw angle ψ can also track $r(t)$ well. Hence, it is shown that the developed tracking strategy is effective so that the DPS has an acceptable tracking performance.

Then, we compared with (Yang et al., 2018) which uses different methods for DPS to deal with the fault-tolerant sampled-data control issue. Choose the same sampling interval $d_1=0$, then according to theorem 2, the maximum sampling period can be obtained that $d_2=1.1024s$, which improves (Yang et al., 2018) about 131.2%. It is illustrated that the sampling period in the paper is longer than (Yang et al., 2018). Besides, the lower bound of the sampling interval is not considered in (Yang et al., 2018), which will lead to conservatism.

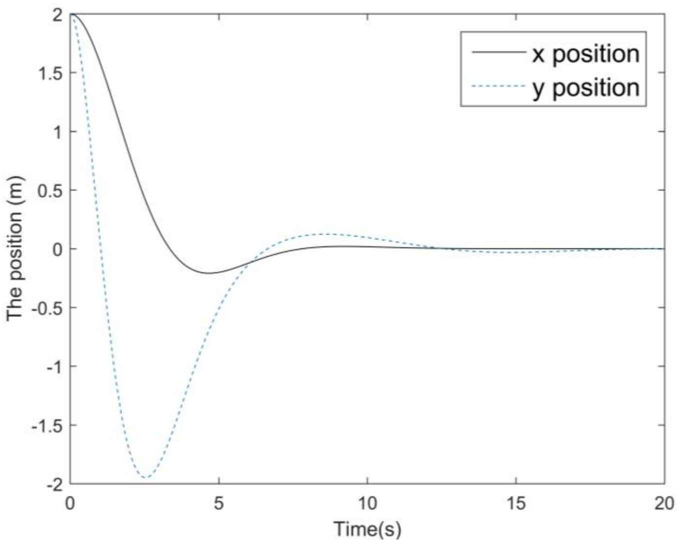

Fig. 4. Responses of the x, y positions for the DPS.

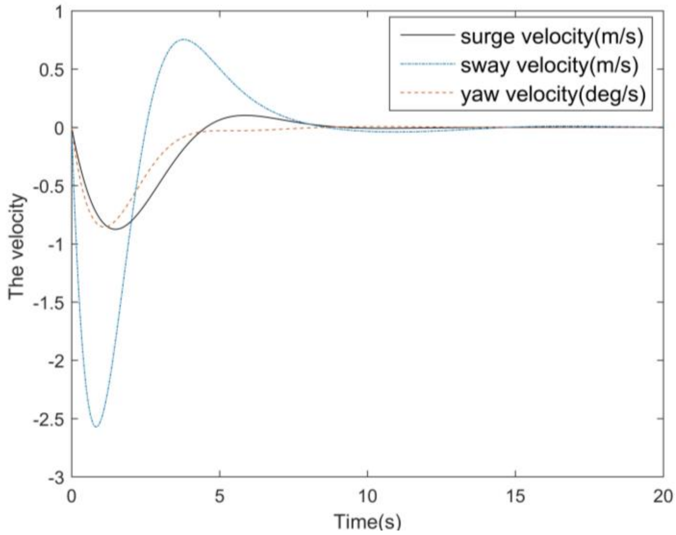

Fig. 5. Responses of the velocity for the DPS.

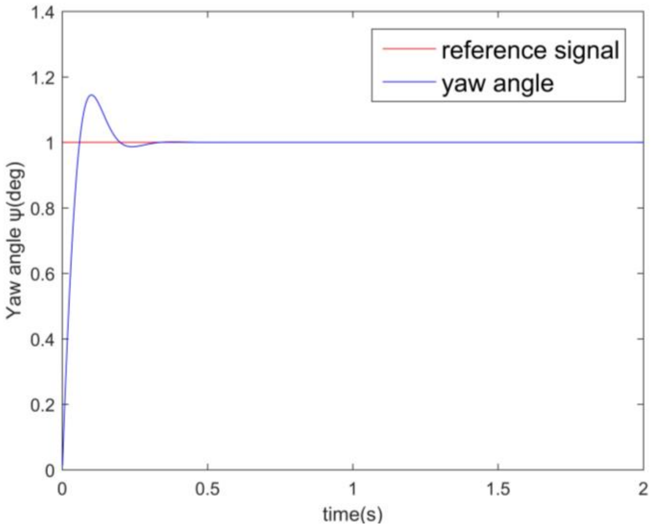

Fig. 6. Tracking responses of yaw angle ψ when $r(t)=1$.

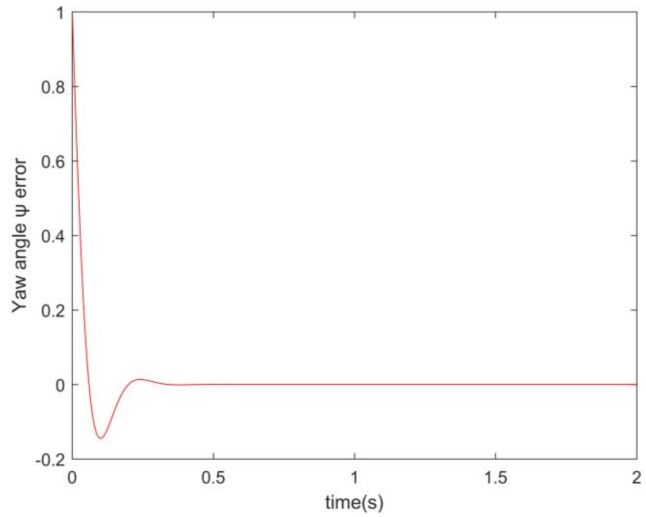

Fig. 7. The error between yaw angle ψ and $r(t)$ when $r(t)=1$.

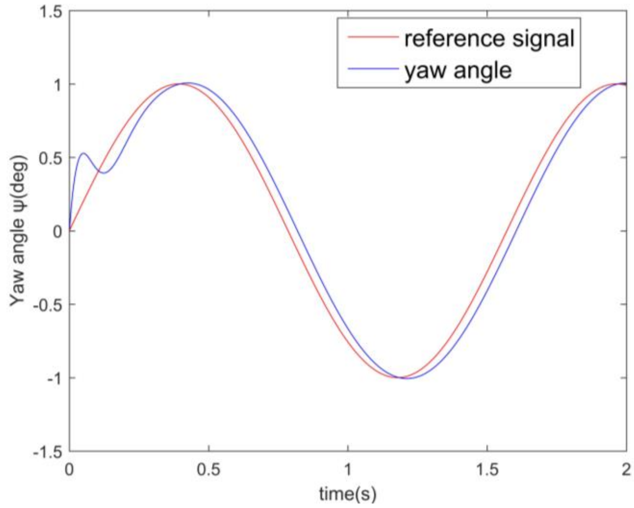

Fig. 8. Tracking responses of yaw angle ψ when $r(t)=\sin 4t$.

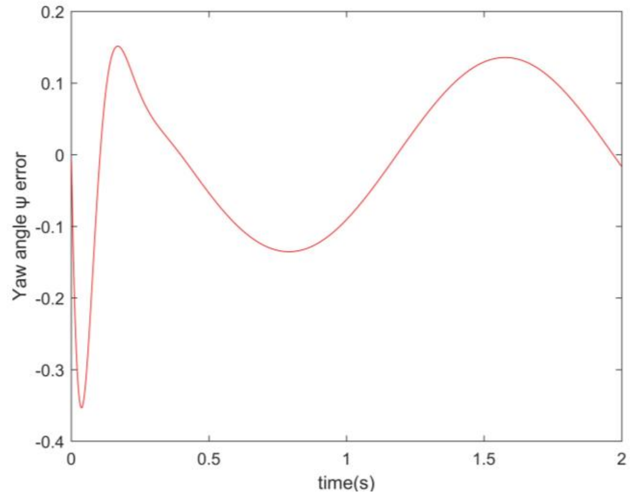

Fig. 9. the error between yaw angle ψ and $r(t)$ when $r(t)=\sin 4t$.

5. CONCLUSIONS

The aperiodic sampled-data tracking issue for DPS is discussed in the paper. Stochastic actuator-failure models of the DPS have been established, and both upper and lower bounds of the sampling period are considered. Mean square exponential stability criteria are given. By constructing a suitable LKF, the sampling pattern can be obtained fully. The simulation results illustrate that the designed sampled-data controllers are effective to provide excellent tracking performances for the DPS despite extern disturbances, aperiodic measurements and stochastic actuator failure.

ACKNOWLEDGEMENT

This work was supported by National Natural Science Foundation of China (51579114, 51879119); The Natural Science Foundation of Fujian Province (2018J01484, 2021J01822, 2022J01323, 2020J01660); The Project of Young and Middle-aged Teacher Education of Fujian Province (JAT170311); Scholarship of studying abroad in

Education Department of Fujian Province; Youth Innovation Foundation of Xiamen (3502Z20206019); The Fuzhou-Xiamen-Quanzhou In-dependent Innovation Region Cooperated Special Foundation (No: 3502ZCQXT2021007).

REFERENCES

B. Bidikli, E. Tatlicioglu, and E. Zergeroglu, (2017). Compensating of added mass terms in dynamically positioned surface vehicles: A continuous robust control approach, *Ocean Eng.*,139,198-204.

B. Shen, Z. Wang, and X. Liu (2012). Sampled-data synchronization control of dynamical networks with stochastic sampling. *IEEE Trans. Autom. Control*, 57(10), 2644-2650.

E.A. Tannuri, A. C. Agostinho, H. M. Morishita, and L. Moratelli Jr (2010). Dynamic positioning systems: An experimental analysis of sliding mode control, *Control Eng. Pract*., 18(10), 1121-1132.

Fossen, T. I. (2002). *Marine Control Systems: Guidance, navigation and control of ships*, Rigsand Underwater Vecicles, Marine Cybernetics, Trondheim, Norway, lst edition.

Fridman, E.(2006). Robust sampled-data H_∞ control of linear singularly perturbed systems. *IEEE Transactions on Automatic Control*, 51(3), 470-475.

Fridman, E.(2010). A refined input delay approach to sampled-data control. *Automatica*, 46(2), 421-427.

Fu M, Sun J, Wang D. Research on Thrust Allocation of Dynamic Positioning Ship with Cycloidal Propeller (2018). *2018 37th Chinese Control Conference (CCC)*. IEEE, 2018, 620-624.

H. Katayama (2010). Nonlinear Sampled-Data Stabilization of Dynamically Positioned Ships. *IEEE Trans. on Control Systems Technology*, 18,463-468

H. Katayama and H. Aoki (2014). Straight-line trajectory tracking control for sampled-data underactuated ships, *IEEE Trans. Control Syst. Technol*., 22(4), 1638-1645.

Hao L Y, Zhang H, Guo G, et al.(2019). Quantized Sliding Mode Control of Unmanned Marine Vehicles: Various Thruster Faults Tolerated With a Unified Model. *IEEE Transactions on Systems, Man, and Cybernetics*.

J. Du, X. Hu, M. Krsti ´ c, and Y. Sun (2016). Robust dynamic positioning of ships with disturbances under input saturation, *Automatica*, 73, 207-214.

K. Gu, V.L. Kharitonov, J. Chen, (2003). Stability of Time-Delay Systems, Birkhäuser, Boston,.

K. Liu, E. Fridman (2012). Wirtinger's inequality and Lyapunov-based sampled-data stabilization. *Automatica*, 48(1): 102-108.

L. Hu, P. Shi, and P (2006). Frank, Robust sampled-data control for Markovian jump linear systems. *Automatica*, 42(11): 2025-2030.

Liu Y, Lee S M. Stability and stabilization of Takagi–Sugeno fuzzy systems via sampled-data and state quantized controller. *IEEE Transactions on Fuzzy Systems*, *24*, 635-644 (2016)

Liu Z, Zhang Y, Yuan C, et al. (2018). Adaptive Path Following Control of Unmanned Surface Vehicles Considering Environmental Disturbances and System Constraints. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*.

Martin S C, Whitcomb L L. (2017). Nonlinear model-based tracking control of underwater vehicles with three degree-of-freedom fully coupled dynamical plant models: Theory and experimental evaluation. *IEEE Transactions on Control Systems Technology*,26(2),404-414.

M. Chen, S. S. Ge, B. V. E. How, and Y. S. Choo (2013). Robust adaptive position mooring control for marine vessels, *IEEE Trans. Control Syst. Technol.*, 21(2), 395-409.

Minjie Z, Yujie Z, Shenhua Y, et al. Sampled-data control for nonlinear singular systems based on a Takagi–Sugeno fuzzy model (2018). *Transactions of the Institute of Measurement and Control*, 40(14): 4027-4036.

N. E. Kahveci and P. A. Ioannou (2013). Adaptive steering control for uncertain ship dynamics and stability analysis, *Automatica*, 49(3), 685-697.

Ngongi, W., Du, J. (2015). Linear fuzzy controller design for dynamic positioning system of surface ships. *International Journal of Intelligent Systems Technologies and Applications*, 14(1), 1-26.

Peng X, Zhang B, Rong L.(2019). A robust unscented Kalman filter and its application in estimating dynamic positioning ship motion states. *Journal of Marine Science and Technology*, 1-15.

P.G. Park, J.W. Ko, C. Jeong, (2011). Reciprocally convex approach to stability of systems with time-varying delays, *Automatica*, 47, 235-238.

Qin H, Li C, Sun Y, et al.(2018). Trajectory tracking control of unmanned surface vessels with input saturation and full-state constraints. *International Journal of Advanced Robotic Systems*,15(5): 1729881418808113.

R.-Y. Ren, Z.-J. Zou, and X.-G. Wang (2014). A two-time scale control law based on singular perturbations used in rudder roll stabilization of ships, *Ocean Eng.*, 88, 488-498.

Sorensen A J (2011). A survey of dynamic positioning control systems. *Annual Reviews in Control*, 35(1), 123-136.

Tijani I, Budiyono A. (2016). Robust control of an unmanned underwater vehicle with parametric uncertainty. *J Instrum Autom Syst*, ,2, 72-80.

Wang, Y., Karimi, H. R., Lam, H. K., & Shen, H (2018). An improved result on exponential stabilization of sampled-data fuzzy systems. *IEEE Transactions on Fuzzy Systems*, 26, 3875-3883

W.-H. Chen, Z. Wang, and X. Lu (2012). On sampled-data control for masterslave synchronization of chaotic Lur' e systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, 59, 515－519

Wu Zheng-Guang; Shi Peng; Su Hongye (2013). Stochastic Synchronization of Markovian Jump Neural Networks With Time-Varying Delay UsingSampled Data, *IEEE transactions on cybernetics*, *43*, 1796-1806

Yang S, Zheng M. H-infinity Fault-Tolerant Control for Dynamic Positioning Ships based on Sampled-data (2018). *Journal of Control Engineering and Applied Informatics*, 20(4): 32-39.

Yueying Wang, Weixiang Zhou, Jun Luo, Huaicheng Yan, Huayan Pu, Yan Peng (2019). Reliable intelligent path following control for a robotic airship against sensor faults, *IEEE/ASME Transactions on Mechatronics*, 24(6), 2572-2582.

Yueying Wang, Yuanqing Xia, Pingfang Zhou (2016). Fuzzy-Model-Based Sampled-Data Control of Chaotic Systems: A Fuzzy Time-Dependent Lyapunov-Krasovskii Functional Approach, *IEEE Transactions on Fuzzy Systems*, *25*, 1672-1684

Yucel E, Ali M S, Gunasekaran N, et al (2017). Sampled-data filtering of Takagi–Sugeno fuzzy neural networks with interval time-varying delays. *Fuzzy Sets and Systems*, *316*, 69-81

Yoneyama, J. (2012). Robust sampled-data stabilization of uncertain fuzzy systems via input delay approach. *Inf. Sci.*, 198, 169-176.

Y. Wang, S. Wang, and M. Tan (2015). Path generation of autonomous approach to a moving ship for unmanned vehicles, *IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS* , 62(9), 5619-5629.

Y.-L. Wang and Q.-L. Han (2017). Network-based heading control and rudder oscillation reduction for unmanned surface vehicles, *IEEE Trans. Control Syst. Technol.*, 25(5), 1609-1620

Y.-L. Wang and Q.-L. Han (2016). Network-based fault detection filter and controller coordinated design for unmanned surface vehicles in network environments, *IEEE Trans. Ind. Inf.*, 12(5), 1753-1765.

Yucel E, Ali M S, Gunasekaran N, et al(2017). Sampled-data filtering of Takagi-Sugeno fuzzy neural networks with interval time-varying delays. *Fuzzy Sets and Systems*, 316: 69-81.

Yu L, Cui Y, Lu Z, Liu Y (2022). Sampled-based bipartite tracking consensus of nonlinear multiagents subject to input saturation. *Complex Eng Syst* , http://dx.doi.org/10.20517/ces.2022.08

Zhang, H., Yang, D., Chai, T.Y. (2007). Guaranteed costnetworked control for T-S fuzzy systems with time delays. IEEE Trans. Syst. Man Cybern., Part C, Appl. Rev.,37(2),160-172

Zhang, Dan, Lu Liu, and Gang Feng (2018). Consensus of heterogeneous linear multiagent systems subject to aperiodic sampled-data and DoS attack. *IEEE transactions on cybernetics*, *49*,1501-1511

Zhang G, Huang C, Zhang X, et al.(2018). Practical constrained dynamic positioning control for uncertain ship through the minimal learning parameter technique. *IET Control Theory & Applications*, 12(18), 2526-2533.

Zhang Z, Song X, Li C, Sun X (2021). Fuzzy reduced-order filtering for nonlinear parabolic PDE systems with limited communication. *Complex Eng Syst.* http://dx.doi.org/10.20517/ces.2021.10, 1-9

Zheng M, Zhou Y, Yang S (2018). Robust Fuzzy Sampled-Data Control for Dynamic Positioning Ships. *Journal of Shanghai Jiaotong University (Science)*, 23(2), 209-217.

Zheng M, Zhou Y, Yang S, et al (2019). Robust H_∞ control of neutral system for sampled-data dynamic positioning ships. *IMA Journal of Mathematical Control and Information*, 36(4): 1325-1345.

Zheng M, Yang S, Li L (2021). Stabilisation of Dynamic Positioning Ships Based on Sampled-data Control. *Journal of Control Engineering and Applied Informatics*, 23(1): 22-31.

Z. Wu, P. Shi, H. Su, and J. Chu (2013). Stochastic synchronization of markovian jump neural networks with time-varying delay using sampled data. *IEEE Trans. Cybern*., 99, 1-11.

Z.-G. Wu, P. Shi, H. Su, and J. Chu (2014). Local synchronization of chaotic neural networks with sampled-data and saturating actuators, *IEEE Trans. Cybern*., *44*, 2635– 2645