Evaluation and Optimization of Nonlinear Central Pattern Generators for Robotic Locomotion

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Abstract: With regard to the optimization of Central Pattern Generators (CPGs) for bipedal locomotion in robots, this paper investigates how the different cases of CPGs such as uncoupled, unidirectional, bidirectional two CPGs are used to produce rhythmic patterns for one leg with two degrees of freedom (DOF). This paper also discusses the stability analysis of CPGs and attempts to utilize genetic algorithms with the hybrid function and adapts the CPGs to robotic systems that perform one-leg movement, by utilizing the bidirectional two CPGs. The results show far greater improvement than in the other cases. CPGs not only enhance movement but also control locomotion without any sensory feedback.

Keywords: Central Patterns Generators (CPGs), one leg model, a strategy to couple CPGs, stability analysis, optimizing CPGs.

1. INTRODUCTION

Nowadays, locomotion of robot is one of the most widely discussed topics with basic locomotors at the heart of investigations. In biological systems, there are some different patterns are produced by a central nervous system such as, running, walking, swimming and crawling, the central nervous system is called Central Pattern Generators (CPGs). The central nervous system of vertebrate and invertebrate animals locals in the spinal cord is shown by Some researches (Ijspeert, 2008; Sillar, 1996). Some studies have shown that there are functions in the human body, for instance breathing and digestion, that cannot be controlled consciously, but they are controlled by CPGs (Ijspeert, 2008; Billard & Ijspeert, 2000).

Biologically, CPGs can be defined as inspired networks of nonlinear oscillating neurons capable of generating rhythmic patterns without higher control centres or sensory feedback. A neural oscillator comprises a pair of neurons with inhibitive connections between them. Two such neurons suppress one another, which results in two different neurons, they are a flexor and an extensor neuron (Bucher, Haspel, Golowasch, & Nadim, 2000; Casasnovas & Meyrand, 1995; Van Vreeswijk, Abbott, & Ermentrout, 1994). It should be known that a single CPG is not a small neural network. Rather, it receives inputs from the higher parts of control centres (Ijspeert, 2008; Sillar, 1996).

Various mathematical and physical structures of the legs and limbs robots have been modeled (Williamson, 1999; Arikan & Irfanoglu, 2011; Abdalftah Elbori et al 2017) and the control systems have been copied to reproduce patterns of movement in robots similar to those in certain biological organisms. When the body initiates movement, CPGs commence synchronization and send signals to neurons simultaneously in a movement cycle (Kimura, Fukuoka, & Cohen, 2007; Wu et al., 2013).

Interestingly, various mathematical structures in the literature have modelled and mimicked biological control parameters (Williamson, 1999; Amrollah & Henaff, 2010; Ijspeert & Cabelguen, 2006; Jiaqi, Masayoshi, Qijun, & Chengju, 2011). Various CPG models, including the connectionist models, have been implemented in the robotic systems (Arena, 2000; Lu, Ma, Li, & Wang, 2005). Also, there are some studies discussing how systems of oscillators can be coupled (Crespi & Ijspeert, 2006; Ijspeert & Crespi, 2007; Ijspeert, Crespi, Ryczko, & Cabelguen, 2007; Kimura, Akiyama, & Sakurama, 1999).

Recently, various CPGs models have been implemented and designed in order to control bipedal locomotion in the humanoid robot see (Jiaqi et al., 2011; Taga, 1998; Taga, Yamaguchi, & Shimizu, 1991). Also, different kinds of robots and modes of locomotion have been controlled using CPGs. The locomotion of hexapod and octopod robots are employed by Some different CPG models (Arena, Fortuna, Frasca, & Sicurella, 2004; Inagaki, Yuasa, Suzuki, & Arai, 2006; Inagaki, Yuasa, & Arai, 2003). Also, to control swimming robots, for instance swimming lamprey or eel robots (Ijspeert & Crespi, 2007; Inagaki et al., 2006; Arena, 2001; Crespi & Ijspeert, 2008), as well as to understand how CPGs can control quadruped robots, see (Billard & Ijspeert, 2000; Brambilla, Buchli, & Ijspeert, 2006; Fukuoka, Kimura, & Cohen, 2003). Other studies explored how one can use CPGs to control bipedal locomotion (Ishiguro, Ono, Imai, & Kanda, 2003; Pinto & Golubitsky, 2006; Wang, Chevallereau, & Tlalolini, 2014; Arikan & Irfanoglu, 2011; Inada & Ishii, 2003; Abdalftah Elbori et al

2017; Aoi & Tsuchiya, 2005; Endo, Nakanishi, Morimoto, & Cheng, 2005; Torres-Huitzil & Girau, 2008; Nachstedt, Tetzlaff, & Manoonpong, 2017). Some of these studies have been considered the Van der Pol and the Hopf oscillator. Others used different mathematical structures of CPGs to control bipedal locomotion.

This paper draws on and derives support from the studies mentioned above and investigates how CPGs can best be optimized for bipedal locomotion via an adaptation to the robotic systems that perform one leg movement. In particular, this paper investigates a nonlinear limit-cycle oscillator similar to the Van der Pol oscillator. It combines both oscillators to control the swing phase of the kinematic model of a single leg system. Based on the cost function, this paper uses a developmental algorithm to find the optimum values of the parameters for two CPGs in three different cases of two CPGs, it is predicted that the bidirectional two CPGs will give the best results.

Also, the kinematic model has been discussed in this paper for simulations and gaits design, and analyzes the stability of CPGs with the ultimate goal of adapting optimized CPG structures to the Test Bench in order to generate one-leg movement. Finally, the paper draws conclusions and offers suggestions for future research.

2. KINEMATIC MODEL

Economically speaking, the kinematic model is designed to conduct a base analysis. Fig 1 presents the leg structure in two cases of motion a swing and stance modes. L_1 and L_2 are the lengths from the hip joint to the knee joint, and from the knee joint to the end effector, respectively; and θ_1 is the angular position of the hip and θ_2 is the angular position of the knee, where y_g is the angular the distance between the lower body and the ground. The coordinates of the lowest part of the hip and knee are denoted by (x_A, y_A) and (x_f, y_f), respectively.

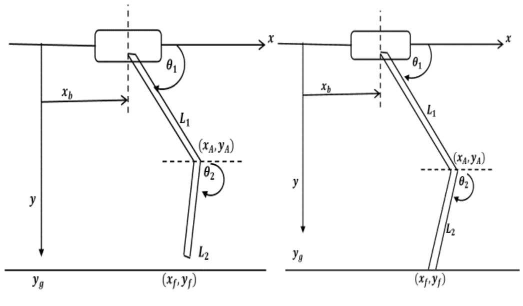

Fig 1. Swing and stance modes of the leg

In fact, there are two cases to consider during motion. The first case is when $y_f = y_g$, that is, the leg touches the ground. This case is known as the stance mode in which case the leg behaves as a revolute joint. In stance mode, the hip joint angle θ_1 is computed in terms of the knee angle θ_2, which is established by the CPGs. Thus, in this mode, the kinematic model has one degree of freedom (DOF). Moreover, only in stance mode will the body move in the x-direction. The second case is when $y_f < y_g$, which is the time when the leg does not touch the ground. This mode is known as the swing mode. In this mode, the DOF is two, and the angles of both the hip and knee are calculated by two different CPGs.

The simple kinematic equations for the first coordinates are

$$x_A = x_b + L_1 \cos\theta_1, \quad y_A = L_1 \sin\theta_1,$$

and for the second coordinates

$$x_f = x_A + L_2 \cos\theta_2, \quad y_f = y_A + L_2 \sin\theta_2.$$

Simple kinematic equations are considered in the sagittal plane. Additionally, the lower body is parallel to the ground during the locomotion, as in the Test Bench, where the hip joint height is fixed and the stability during the locomotion is guaranteed, consequently, the dynamic equations are not needed (See Fig 2 below.)

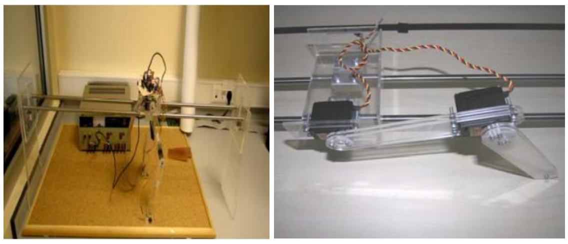

Fig 2. The physical system and a closer view of the actuators of one leg of the Test Bench.

3. CENTRAL PATTERN GENERATORS (CPGs)

As stated previously, the CPG unit is responsible for generating the required angular signals for both joints. Such CPGs are furthered in mathematical equations and presented in general formulas for details, see (Marbach, 2004; Van den Kieboom, 2009). In their paper Marbach and Van den Kieboom claim that a number of experiments reveal that sinusoidal signals are the appropriate function for locomotion control:

$$x = A \sin(2\pi f t + \varphi)$$

where A, f and φ are the amplitude, frequency and phase respectively. Differentiating x twice yields the system

$$\tau\dot{x} = v, \quad \tau\dot{v} = -x, \text{ where } \tau = \frac{1}{2\pi f}.$$

Here, the amplitude is implicitly defined by the system. In fact, it relies on the initial conditions. Therefore, to force the system to lead to a limit cycle with the desired amplitude, a new term should be included in the system. For example, the new system of differential equations may look like

$$\left.\begin{aligned} \tau\dot{x} &= v \\ \tau\dot{v} &= -\frac{\alpha}{E}(x^2 + v^2 - E)v - x \end{aligned}\right\} \tag{1}$$

in which the parameters τ, α and E are positive constants. In (1), the term $x^2 + v^2$ stands for the actual energy and the difference $x^2 + v^2 - E$ denotes the error in the energy of the oscillator. Therefore, the newly added term may be considered the normalized energy. Below, two oscillators are coupled.

3.1 A Strategy to couple CPGs

A simple coupling strategy given in (Ijspeert & Cabelguen, 2006) is considered where each CPG sends a signal proportional to the states variables to every other CPG. More specifically, the system

$$\left.\begin{aligned}\tau\dot{x}_i &= v_i\\ \tau\dot{v}_i &= -\frac{\alpha}{E_i}(x_i^2+v_i^2-E_i)v_i - x_i + \sum_j \frac{a_{ij}x_j + b_{ij}v_j}{\sqrt{{x_j}^2+{v_j}^2}}\end{aligned}\right\} \quad (2)$$

will be considered. In (2), a_{ij} and b_{ij} are positive constants that determine how oscillator j influences oscillator i. Specific forms of outputs can still occur by means of changing the numerical values of the parameters (Amrollah & Henaff, 2010). Depending on the values of a_{ij} and b_{ij}, the coupling of CPGs may be called uncoupled, unidirectional and bidirectional. The corresponding systems of differential equations are provided below:

$$\left.\begin{aligned}\tau\dot{x}_1 &= v_1\\ \tau\dot{v}_1 &= -\frac{\alpha_1}{E_1}(x_1^2+v_1^2-E_1)v_1 - x_1\\ \tau\dot{x}_2 &= v_2\\ \tau\dot{v}_2 &= -\frac{\alpha_2}{E_2}(x_2^2+v_2^2-E_2)v_2 - x_2\end{aligned}\right\}, \quad (3)$$

$$\left.\begin{aligned}\tau\dot{x}_1 &= v_1\\ \tau\dot{v}_1 &= -\frac{\alpha_1}{E_1}(x_1^2+v_1^2-E_1)v_1 - x_1 + \frac{a_{12}x_2 + b_{12}v_2}{\sqrt{{x_2}^2+{v_2}^2}}\\ \tau\dot{x}_2 &= v_2\\ \tau\dot{v}_2 &= -\frac{\alpha_2}{E_2}(x_2^2+v_2^2-E_2)v_2 - x_2\end{aligned}\right\} \quad (4)$$

and

$$\left.\begin{aligned}\tau\dot{x}_1 &= v_1\\ \tau\dot{v}_1 &= -\frac{\alpha_1}{E_1}(x_1^2+v_1^2-E_1)v_1 - x_1 + \frac{a_{12}x_2 + b_{12}v_2}{\sqrt{{x_2}^2+{v_2}^2}}\\ \tau\dot{x}_2 &= v_2\\ \tau\dot{v}_2 &= -\frac{\alpha_2}{E_2}(x_2^2+v_2^2-E_2)v_2 - x_2 + \frac{a_{21}x_1 + b_{21}v_1}{\sqrt{{x_1}^2+{v_1}^2}}\end{aligned}\right\} \quad (5)$$

System (3) is called uncoupled because the two CPGs occurring in the system work independently. On the other hand, in system (4), there is a signal transfer from the second CPG to the first one, and that transfer is one way. This is why it is called unidirectional. Finally, in system (5), each CPG gives reference to the other CPG. Therefore, it is called bidirectional coupling. In any case, the outputs are $x_1 = \theta_1$ and $x_2 = \theta_2$, where θ_1 and θ_2, as defined in the kinematic model, represent the angular positions of the hip and knee, respectively.

4. STABILITY ANALYSIS

In this part, the stability analysis for each type of coupling given in the previous section will be discussed. For each case, a figure will be provided to illustrate the existence of a limit cycle.

4.1 Uncouple Two CPGs

As stated previously, uncoupled two CPGs consist of four differential equations given in (3). These four differential equations represent two independent CPGs. Therefore, it is sufficient to analyze one of them to understand the stability of the system. For (1), which is a nonlinear system, clearly the origin is the only fixed point and the linearized system at the fixed point has the Jacobian matrix

$$J(0,0) = \begin{bmatrix} 0 & 1/\tau \\ -1/\tau & \alpha/\tau \end{bmatrix}$$

whose eigenvalues are $\lambda_{1,2} = \left(\alpha \pm \sqrt{\alpha^2-4}\right)/(2\tau)$. Since $\alpha/\tau > 0$, the fixed point is locally unstable. Let's consider (1) in polar coordinates: $x = r\cos\theta$, $v = r\sin\theta$. Then, (1) becomes

$$\begin{cases}\dot{r} = \dfrac{\alpha}{\tau E} r(E - r^2)\sin^2\theta\\ \dot{\theta} = -\dfrac{1}{\tau} - \dfrac{\alpha}{\tau E}(r^2 - E)\cos\theta\sin\theta\end{cases}$$

It is now obvious that all solutions with $r(0) > 0$ tend to $\sqrt{E}$ as $t \longrightarrow \infty$. Translating this into the x, y plane, it means that $x^2 + v^2$ increases as long as $x^2 + v^2 < E$, and it decreases when $x^2 + v^2 > E$. Thus, there is a stable limit cycle with radius $\sqrt{E}$ and an unstable equilibrium point at the origin. In Fig 3, two solutions corresponding to different values of the parameters for the first CPG are shown. The limit cycle for one CPG can be observed in that figure.

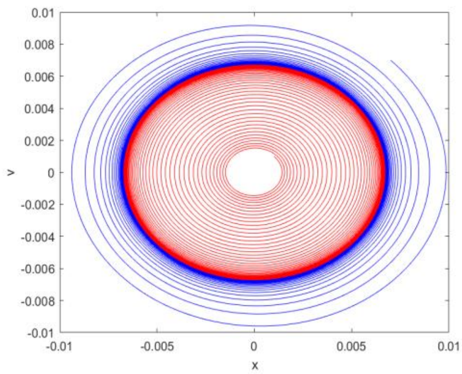

Fig 3. Stable limit cycle of uncoupled single CPGs

4.2 Unidirectional Two CPGs

Unidirectional two CPGs consist of four differential equations given in (4). These four differential equations represent two dependent CPGs, one of which represents by the first and second equations, and the other being illustrated by the third and fourth equations. It is clear that there is no equilibrium point for unidirectional two CPGs. To check whether there is a periodic solution, the change of variables

$$x_1 = r_1\cos\theta_1, \quad v_1 = r_1\sin\theta_1,$$
$$x_2 = r_2\cos\theta_2, \quad v_2 = r_2\sin\theta_2.$$

are used. Then, in the new coordinate system, (4) becomes

$$\begin{cases}\dot{r}_1 = -\dfrac{\alpha}{\tau E_1}(r_1^2 - E_1)r_1\sin^2\theta_1 + \dfrac{1}{\tau}(a_{12}\cos\theta_2 + b_{12}\sin\theta_2)\sin\theta_1\\ \dot{\theta}_1 = -\dfrac{1}{\tau} - \dfrac{\alpha}{\tau E_1}(r_1^2 - E_1)\cos\theta_1\sin\theta_1 + \dfrac{1}{\tau r_1^2}(a_{12}\cos\theta_2 + b_{12}\sin\theta_2)\\ \dot{r}_2 = -\dfrac{\alpha}{\tau E_2}(r_2^2 - E_2)r_2\sin^2\theta_2\\ \dot{\theta}_2 = -\dfrac{1}{\tau} - \dfrac{\alpha}{\tau E_2}(r_2^2 - E_2)\cos\theta_2\sin\theta_2\end{cases}$$

Clearly, $r_2 = \sqrt{E_2}$ is the periodic solution for the second CPG. The limit cycle for the first CPG can be observed in Fig 4. Therefore, as in the case of uncoupled two CPGs, this system has a limit cycle.

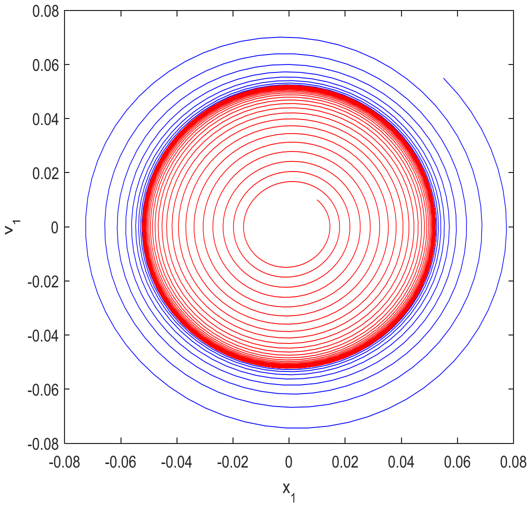

Fig 4. Two trajectories of the first CPG in the case of unidirectional two CPGs.

4.3 Bidirectional Two CPGs

The four differential equations in (5) above represent two dependent CPGs; the first and second equations stand for the first CPG, and the second is delineated by the third and fourth equations. Note that for an equilibrium point the first and the third equations give $v_1 = v_2 = 0$ which leads to $-x_1 \pm a_{12} = 0$ and $-x_2 \pm a_{21} = 0$. Note that x_1 and x_2 should have the same signs. Therefore, the equilibrium points are $(a_{12}, 0, a_{21}, 0)$ and $(-a_{12}, 0, -a_{21}, 0)$.

Different signs of x_1 and x_2 cannot be the equilibrium points for Equation (5). In other words, the points $(a_{12}, 0, -a_{21}, 0)$ and $(-a_{12}, 0, a_{21}, 0)$ are not the equilibrium points. The Jacobian matrix at the fixed point is

$$J(\pm a_{12}, 0, \pm a_{21}, 0)$$
$$= \begin{bmatrix} 0 & 1/\tau & 0 & 0 \\ -1/\tau & \alpha(E_1 - a_{12}^2)/(\tau E_1) & 0 & \pm b_{12}/\tau a_{21} \\ 0 & 0 & 0 & 1/\tau \\ 0 & \pm b_{21}/\tau a_{12} & -1/\tau & -\alpha(a_{21}^2 - E_2)/(\tau E_2) \end{bmatrix}.$$

Therefore, the eigenvalues for both equilibrium points are the roots of the characteristic equation

$$\left(\lambda^2 + \frac{\alpha\lambda}{\tau E_1}(a_{12}^2 - E_1) + \frac{1}{\tau^2}\right)\left(\lambda^2 + \frac{\alpha\lambda}{\tau E_2}(a_{21}^2 - E_2) + \frac{1}{\tau^2}\right) - \frac{\lambda^2 b_{12} b_{21}}{\tau^2 a_{12} a_{21}} = 0.$$

It is difficult to solve this equation manually or to find the eigenvalues analytically. Instead, more conditions can be imposed in order to extract some information from this equation in some particular cases.

For example, one can let $E_1 = a_{12}^2$ and $E_2 = a_{21}^2$. These conditions are used to simplify and reduce the number of parameters in the equation above. In this case, the characteristic equation is

$$\lambda^4 + \frac{1}{\tau^2}\left(2 - \frac{b_{12} b_{21}}{a_{12} a_{21}}\right)\lambda^2 + \frac{1}{\tau^4} = 0$$

Setting $\mu = b_{12} b_{21}/(a_{12} a_{21})$, the above equation becomes

$$\lambda^4 + \frac{2-\mu}{\tau^2}\lambda^2 + \frac{1}{\tau^4} = 0.$$

Therefore, the eigenvalues are

$$\lambda_{1,2} = \frac{\sqrt{\mu} \pm \sqrt{\mu - 4}}{2\tau} \quad \text{and} \quad \lambda_{3,4} = -\frac{\sqrt{\mu} \pm \sqrt{\mu - 4}}{2\tau}.$$

If $\mu = 0$, then the eigenvalues demonstrate multiplicity, suggesting that the fixed points are unstable.

If $\mu < 0$, then the eigenvalues are

$$\lambda_{1,2} = \pm\frac{\sqrt{-\mu} + \sqrt{4-\mu}}{2\tau} i \ \text{ and } \ \lambda_{3,4} = \pm\frac{-\sqrt{-\mu} + \sqrt{4-\mu}}{2\tau} i.$$

Clearly, the fixed points are stable in that case. This can also be observed in Fig 5.

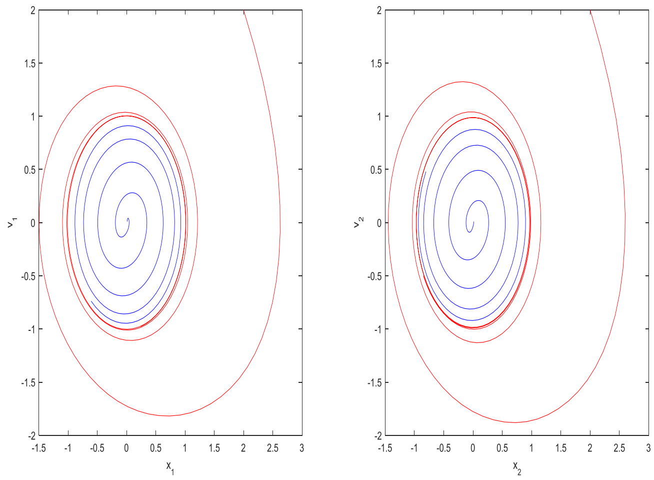

Fig5. Two different solutions for each CPG: Red curve correspond to the initial condition $(x_1(0), v_1(0), x_2(0), v_2(0)) = (2, 2, 2, 2)$, and the blue solution corresponds to the initial condition $(x_1(0), v_1(0), x_2(0), v_2(0)) = (0.1, 0.1, 0.1, 0.1)$. These different solutions correspond to the values $\tau = 0.1$, $\alpha = 0.01$, $a_{12} = 0.1$, $a_{21} = -0.1$, $b_{12} = 0.02$, $b_{21} = 0.01$, $E_1 = a_{12}^2$ and $E_2 = a_{21}^2$.

If $\mu > 0$, then there exist a two-dimensional stable manifold and two-dimensional unstable manifold, both of them are corresponding to the eigenvalues with a negative real part, and a positive real part respectively.

5. OTIMIZING CPGs

It is important to mention here that each CPG produces angular patterns as outputs for each joint. To estimate gait generation, it is essential for computing the optimal parameter sets for each CPG, in other words, it is necessary to get the idea how the angular position of both joint would change in time in order to produce rhythmic patterns in the x-direction.

The parameter sets in the cases of uncoupled, unidirectional and bidirectional two CPGs are $\{\alpha_1, \tau_1, E_1, \alpha_2, \tau_2, E_2\}$, $\{\alpha_1, \tau_1, E_1, a_{12}, b_{12}, \alpha_2, \tau_2, E_2\}$ and $\{\alpha_1, \tau_1, E_1, a_{12}, b_{12}, \alpha_2, \tau_2, E_2, a_{21}, b_{21}\}$, respectively. The Genetic Algorithm (GA) will be used to find the values of the parameters that optimize the objective function given below. The different walking patterns rely on this cost function:

$$J = -C_1 \sum_{k=1}^{n} x_b(k) + C_2 \left(\sum_{k=1}^{n} (\theta_1^2(k) + \theta_2^2(k)) \right) / N, \quad C_1, C_2 \in [0,1] \quad (6)$$

where $C_1 + C_2 = 1$ and n is the number of elements of the position vector that is simulated with the objective of establishing patterns in order to maximize the displacement or velocity, and N is the length of time. When $C_2 = 0$, the energy consumption is ignored; hence, the displacement is maximized. Otherwise, when $C_1 = 0$, the energy consumption is minimized ignoring the displacement. It is obvious that for $C_1 = 0$, the minimum displacement is obtained when there is no locomotion. Therefore, throughout this study, it is assumed that $C_1 \neq 0$. Moreover, the requirement $C_1 + C_2 = 1$ is used just to balance the energy consumption and the total displacement.

The values of C_1 and C_2 are varied to clarify the relationship between the velocity and the energy constraints during CPG optimization. The second term in (6) is used to provide patterns, including smaller angular displacements for the legs. Consequently, this reduces the energy consumption. The ultimate goal here is to minimize the energy while altering the position for more details on biological locomotion (Alexander, 1996; Nolfi & Floreano, 2000).

The two constraints revealed here are $0 \leq \theta_1, \theta_2 \leq \pi$ due to physical reasons. The present study uses a hybrid function during the optimization process defined as an optimization function which runs after the GA terminates for improving the value of the fitness function**.** Another advantage of using the hybrid function is that, this function uses the final point from the GA as its initial point and it can be specified in the Hybrid function options.

With regard to the stability aspect, it is assumed that the leg is unable to move normally when both cases of the uncoupled and the unidirectional two CPGs are applied to simple kinematic equations, as shown in Figs 6 and 7, where locomotion can be achieved with the Bidirectional two CPGs in 80 seconds, as per Figs 8 and 9.

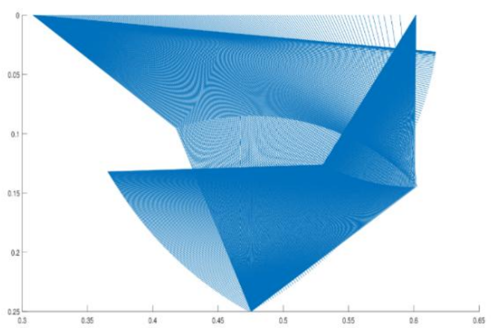

Fig 6. One leg animation in the case of uncoupled two CPGs.

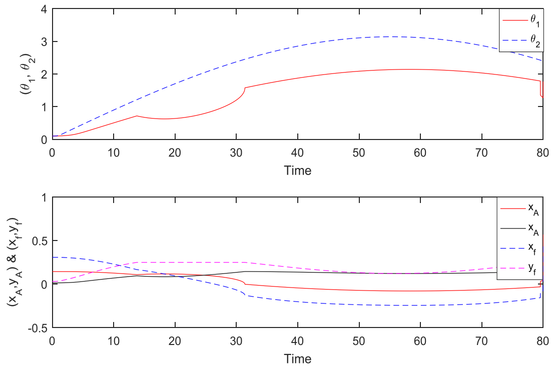

Fig7. The output of CPGs corresponding to the values $\tau_1 = 28.9834$, $\alpha_1 = 36.2518$, $E_1 = 4.5906$, $\tau_2 = 111.2763$, $\alpha_2 = 35.3121$ and $E_2 = 9.8696$, in the uncoupled case.

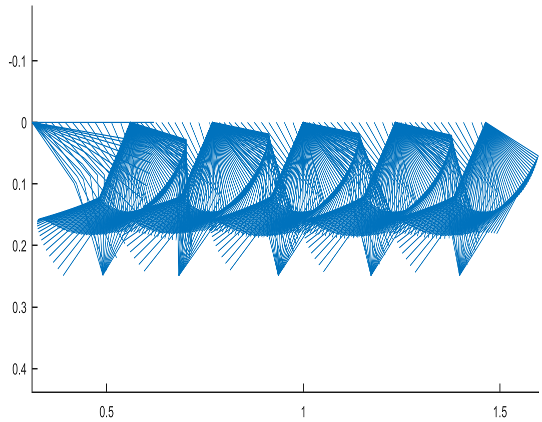

Fig 8. One leg animation for bidirectional two CPGs.

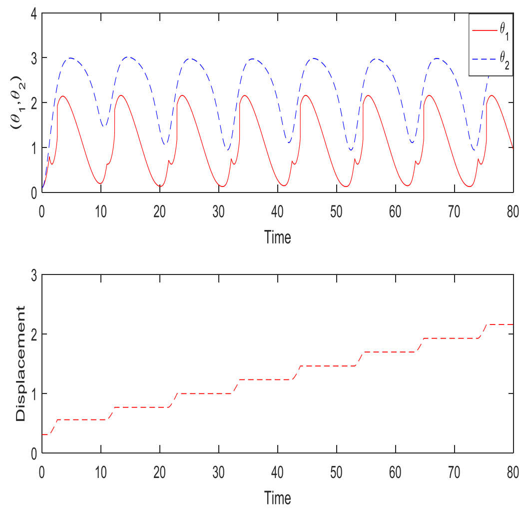

Fig 9. Angles and displacement against time for bidirectional two CPGs.

In both cases of the uncoupled and unidirectional two CPGs, either the leg does not move or it takes only a few steps in an unusual manner. More specifically, since the equilibrium points are stable in the case of bidirectional two CPGs when $\mu < 0$, the best gait pattern is obtained. In fact, these optimization results supply the conclusion given in the stability analysis. Tables 1-6 summarize the results of this optimization for the three cases under investigation: uncoupled, unidirectional and bidirectional two CPGs, respectively. In all cases, J, x_b and E denote the values of the cost function, total displacement and energy, respectively.

Table 1. Optimization of uncoupled two CPGs when $C_1, C_2 \neq 0$.

Solver	Optimization of uncoupled two CPGs in 80 seconds			
	Parameter values	J	x_b	E
GA	28.9834 36.2518 4.5906 111.2763 35.3121 9.8696	−2048.5	0.5271	8.5951
GA	31.5979 36.3647 4.5536 110.8210 35.3333 9.8660	−2050.4	0.5299	8.5763
GA with Hybridfcn at pattern search	32.9827 36.6254 4.5648 110.8486 35.2849 9.8694	−2052.7	0.5257	8.5868
GA with Hybridfcn at fmincon	52.6429 61.4651 8.2096 18.6831 28.2615 6.9612	−1998	0.6037	6.8465

Table 2. Optimization of uncoupled two CPGs when $C_2 = 0$.

Solver	Optimization of uncoupled two CPGs in 80 seconds			
	Parameter values	J	x_b	E
GA	43.4261 79.7966 14.7043 96.3819 41.6429 9.8691	−3916.3	0.5942	9.2125
GA	44.4261 78.7966 14.2043 98.0069 41.7679 9.8691	−3915.6	0.5940	9.1781
GA with Hybridfcn at patternsearch	46.6724 79.0434 14.4197 100.0917 41.0695 9.8547	−3934	0.5951	9.2887
GA with Hybridfcn at fmincon	54.3031 83.7118 11.7086 16.1890 33.7975 7.5507	−3870	0.6038	6.9387

According to Tables 1 and 2, increasing optimization time or extending the region renders the leg unable to walk. Similarly, Tables 3 and 4 support the results given previously such that the leg is unable to walk normally.

Table 3. Optimization of unidirectional two CPGs when $C_1, C_2 \neq 0$.

Solver	Optimization unidirectional two CPGs in 80 seconds			
	Parameter values	J	x_b	E
GA	23.8309 37.8704 2.2214 39.5901 20.4244 26.1536 33.2256 9.8616	−2367.3	0.6130	9.3244
GA	57.1658 38.1815 2.1811 4.8573 24.7362 58.3004 47.0796 9.8694	−2257.4	0.6150	7.6594
GA with Hybridfcn at patternsearch	57.7655 38.1815 2.1811 4.8534 24.9237 58.3017 47.0796 9.8694	−2257.6	0.6151	7.6566
GA with Hybridfcn at fmincon	59.5006 34.2580 2.2877 7.7744 30.2325 60.7280 39.4452 9.8668	−2.336.9	0.6141	8.3105

Table 4. Optimization of unidirectional two CPGs when $C_2 = 0$.

Solver	Optimization unidirectional two CPGs in 80 seconds			
	Parameter values	J	x_b	E
GA	19.1593 37.3610 2.4442 29.2822 23.7775 39.8811 31.5082 9.8595	−4798.6	0.6103	9.3739
GA	16.6046 12.9354 0.2630 29.7203 25.9645 41.3522 38.5263 9.8690	−4722	0.6202	8.3526
GA with Hybridfcn at patternsearch	17.4443 12.7218 0.2635 32.9492 26.5549 59.7024 38.5626 9.8659	−4741	0.6194	8.4787
GA with Hybridfcn at fmincon	16.8698 14.1344 0.2599 30.8606 26.4618 43.3883 42.3464 9.8660	−5259.9	0.6201	8.3873

Table 5. Optimization of bidirectional Two CPGs when $C_1, C_2 \neq 0$.

Solver	Optimization of bidirectional Two CPGs			
	Parameter values	J	x_b	E
GA	0.9999 1.9982 0.5233 2.4831 1.6538 0.8678 2.2644 2.6215 2.4274 1.6276	−877.5649	0.6204	10.0515
GA	1.0158 2.0092 0.5724 2.5768 1.6678 0.8976 2.2644 3.1261 2.4401 1.6481	−1810.7	0.6209	11.3672
GA with Hybridfcn at pattern search	1.0517 1.2945 2.0737 1.3461 0.7428 2.1834 1.9535 3.3817 3.2864 3.2522	−5078.9	2.1939	7.1134
GA with Hybridfcn at fmincon	1.0520 1.2952 2.0771 1.3466 0.7448 2.1885 1.9547 3.3832 3.2922 3.2542	−5068.4	2.1966	7.1075

Table 6. Optimization of bidirectional Two CPGs when $C_2 = 0$.

Solver	Optimization of bidirectional Two CPGs			
	Parameter values	J	x_b	E
GA	0.9961 1.0081 0.5403 2.3114 0.4843 0.9942 1.9538 2.3390 2.9803 2.6291	−2411.2	0.6211	13.0062
GA	1.0033 1.0088 0.5403 2.3375 0.4844 0.9942 1.9539 2.3381 2.9809 2.6721	−4904.6	0.6222	13.7206
GA with Hybridfcn at pattern search	0.9855 1.2981 2.0748 1.3475 0.7401 2.1861 1.9233 3.3868 3.3011 3.2758	−10286	2.1852	7.4392
GA with Hybridfcn at fmincon	0.9940 1.3046 2.1067 1.3529 0.7216 2.2720 1.9025 3.4102 3.3188 3.3236	−10332	2.2126	7.3614

In Tables 5 and 6, in contrast, the leg walks normally and moves when bidirectional two CPGs are utilized. Of course, this movement depends entirely on the values of τ and α. As shown in Tables 5 and 6, a change in the values of τ and α result in either a decrease or an increase in the displacement and velocity, based on these two parameters, respectively. Experiments relying on the value of $\mu = b_{12}b_{21}/(a_{12}a_{21})$ show that when $|\mu|$ and τ are close to zero, the velocity and displacement increase sharply. Moreover, when the angles are kept in the region $[0, 6\pi/7]$, to be more consistent with real life, the locomotion becomes better than the angles kept between $[0, \pi]$, as shown in Figs 10 and 11 and illustrated Tables 7 and 8, when $\alpha = \alpha_1 = \alpha_2$ and $\tau = \tau_1 = \tau_2$.

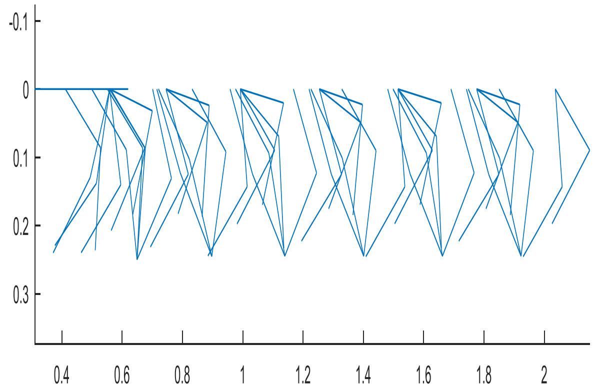

Fig10. One leg animation for Bidirectional two CPGs in 5sec.

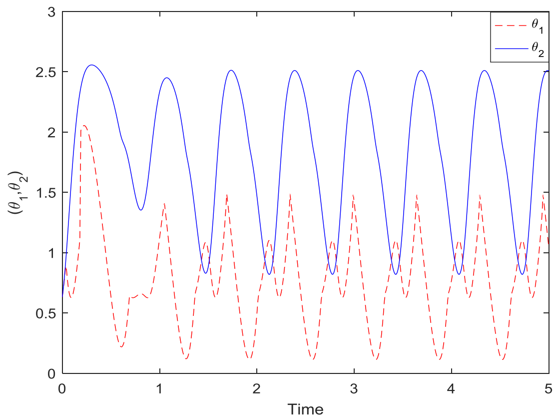

Fig11. Angles against time for Bidirectional two CPGs in 5sec when $\tau = 0.1$

Table 7. Optimization of bidirectional Two CPGs when $\mu \leq 0$ and the initial conditions are fixed

Optimization of bidirectional Two CPGs in 80s							
M	C	Parameter values	Constraints	μ	J	x_b	E
0.05	0.2	2.9799 1.9701 1.5022 0.0000 2.4996 -0.3959	$0 \leq \theta_1, \theta_2 \leq \pi$	0	−5012.7	0.6801	8.0224
0.05	0.8	0.4725 0.1013 0.7215 2.7683 2.0108 -2.9652	$0 \leq \theta_1, \theta_2 \leq \pi$	−5.6581	−23541	4.2837	4.5490
0.05	0.2	0.5266 0.1002 0.7631 3.6360 1.9427 -2.5072	$0 \leq \theta_1, \theta_2 \leq \pi$	−6.1489	−21600	4.2153	4.2755
0.05	0.8	0.5262 0.1002 0.7596 3.6372 1.9458 -2.5150	$0 \leq \theta_1, \theta_2 \leq \frac{7\pi}{8}$	−6.1887	−10897	4.2788	4.2831

Table 8. Optimization of bidirectional Two CPGs when $\mu < 0$ and the initial conditions are not fixed

Optimization of bidirectional Two CPGs in 80s							
M	C	Parameter values		μ	J	x_b	E
0.5	0.2	0.1471 0.1007 1.0387 1.2284 1.8850 -1.7978 0.4902 0.5756 1.0000 0.1999	$0 \leq \theta_1, \theta_2 \leq \frac{5\pi}{6}$	−1.1279	−91875	22.9083	4.2892
0.5	0.8	0.1608 0.1013 0.7540 1.6696 1.9418 -2.3228 0.9878 0.5171 0.9840 0.6064	$0 \leq \theta_1, \theta_2 \leq \pi$	−2.6490	−68824	16.7631	4.3538
0.05	0.2	0.2740 0.1008 0.9226 1.5559 2.0109 -1.5199 0.9839 0.5405 0.9512 0.6559	$0 \leq \theta_1, \theta_2 \leq \frac{6\pi}{7}$	−1.2747	−90186	22.1309	4.4187

In these two tables *M* and *C* stand for the mutation rate and crossover rate, respectively. These results are also consistent with the conclusions given in the stability section.

6. CONCLUSIONS

This paper was focused on three cases of CPGs, these cases uncoupled, unidirectional and bidirectional two CPGs are used to produce rhythmic patterns for one leg with two degrees of freedom. When the optimization is performed in the first two cases, CPGs are unable to provide the basic locomotor rhythm patterns for the leg. In contrast, by utilizing the bidirectional two CPGs, the results show far greater improvement than in the other cases. The bidirectional coupling yields the best performance level. Results also reveal that when $|\mu|$ and τ are close to zero, the velocity and displacement increase. That is, changes in the CPG parameters may produce different results. These results are important not only because of what they may contribute to the ongoing discussion of locomotion but also for the support that they give to bidirectional two CPGs as a critical aspect of locomotion. These results also pave the way for further research into CPGs as potential controllers of upper limbs for different movements.

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