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Coordinate transformations and stabilization of some switched control systems with application to hydrostatic electrohydraulic servoactuators

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Abstract: The main result of the paper is a sufficient condition for existence of controllers that stabilize the zero solution for some switched nonlinear control systems in the critical case of a zero eigenvalue in the spectrum of the Jacobian matrix calculated in zero. The control synthesis is based on a condition on the relative degree in the equilibrium point and subsequent coordinates transformations. An application to a pump controlled electrohydraulic servoactuator is given.

Keywords: switched systems, Lyapunov stability, relative degree, local coordinates transformations, common Lyapunov function

1. INTRODUCTION

The paper continues the study that started in [7], of switched systems in a critical case for stability theory, when a zero eigenvalue is present in the spectrum of the Jacobian matrix of each component of the switched system calculated in a common equilibrium point.

Let a set of switched systems of differential equations, indexed by a parameter $\mu \in \Omega$, have the form (Malkin canonical form, see [13])

$$\begin{aligned} \dot{y} &= Y_\mu^{(l)}(y,\xi) \\ \dot{\xi} &= D^{(l)}(\mu)\xi + F_\mu^{(l)}(y,\xi) \end{aligned}, \; l = 1,2 \tag{1.1}$$

$\xi = (\xi_1, \ldots, \xi_n)^\tau \in \mathbf{R}^n$, $y \in \mathbf{R}$, $D^{(l)} : \Omega \to \mathcal{M}_n(\mathbf{R})$ continuous, $l = 1,2$, $\Omega \in \mathbf{R}^p$, $Y_\mu^{(l)}$ and $F_\mu^{(l)}$, $l = 1,2$, contain only powers of y and ξ_i, $i = 1,\ldots,n$, of order greater or equal to two for every $\mu \in \Omega$ and $F_\mu^{(l)}(y,0) = 0, Y_\mu^{(l)}(y,0) = 0$, $\forall y$, $l = 1,2$ (τ means transpose). Ω is a set of parameters and a specific switched system is obtained when such a parameter is fixed . In the above it is not supposed that the functions are defined on the whole $\mathbf{R}^{n+1}$. The following result is proved in [7].

Theorem 1.1. *Suppose there exists $P = P^\tau > 0$ such that*

$$D^{(l)}(\mu)^\tau P + PD^{(l)}(\mu) \le -cI < 0, \; \forall \mu \in \Omega, \; l = 1,2. \tag{1.2}$$

Then the zero solution for any switched system

$$\begin{aligned} \dot{y} &= Y_k^{(l)}(y,\xi) \\ \dot{\xi} &= D^{(l)}(\mu_k)\xi + F_k^{(l)}(y,\xi) \end{aligned}, \; k = 1,\ldots,N, \; l = 1,2 \tag{1.3}$$

($\mu_1, \ldots, \mu_N \in \Omega$) is uniformly stable by Lyapunov. Moreover, there exists $\delta > 0$ such that, if $||(y(0),\xi(0))|| < \delta$ then $\lim_{t\to\infty} \xi_i(t) = 0$, $\forall i = 1,\ldots,n$ whenever (y,ξ) is a solution of (1.3).

For stability in the case of switched systems, see [11], [12], [14], [22].

The proof relies on the existence, due to (1.2), of a Common Lyapunov Function (CLF) for the switched system. Actually, most results on stability for switched systems are based on existence of various types of CLF (see [3], [5], [15], [16], [20], [21]).

The problem to be approached in this paper is feedback control synthesis for stabilization of switched control systems of type (1.1). Namely

$$\dot{\zeta} = f^{(l)}(\zeta) + g(\zeta)u_l, \quad l = 1,2 \tag{1.4}$$

with $\zeta = (y,\xi)$, $g(\zeta) = (0,0,\ldots,0,1)^\tau \in \mathbf{R}^{n+1}$ and the controllers u_1, u_2 are scalar. There is also a hidden parameter μ that was not written in order to keep the notations less complicated, so

$$f_1^{(l)}(y,\xi) = Y_\mu^{(l)}(y,\xi),$$

$$(f_2,\ldots,f_{n+1})(y,\xi) = D^{(l)}(\mu)\xi + F_\mu^{(l)}(y,\xi)$$

and $Y_\mu^{(l)}$, $F_\mu^{(l)}$ satisfy the previous assumptions. The main result is that if (1.4) has relative degree n in zero (see [10]) then there exist feedback controllers u_1 and u_2 such that the zero solution is simple stable for the switched system (1.4) asymptotically with respect to variables $\xi_1, \ldots, \xi_n$. In order to achieve this two coordinate transformations are used. After specifically defining the controllers u_1 and u_2 the system (1.4) is turned into

$$\begin{aligned} \dot{y} &= q^{(l)}(y,\tilde{z}) \\ \dot{\tilde{z}} &= D\tilde{z} \end{aligned}, \quad l=1,2 \tag{1.5}$$

where $\tilde{z}=(z_2,\ldots,z_{n+1})^\tau$, D is Hurwitz, $q^{(l)}$ contain only powers of $y, z_2,\ldots,z_{n+1}$ in its Taylor development around zero and $q^{(l)}(y,0)=0$, $\forall y$, $l=1,2$.

To system (1.5) one can apply Theorem 1.1 with P the unique solution of the Lyapunov equation $D^\tau P+PD=-I$ (see, e.g. [4]).

This situation of a relative degree one unit less then the order of the system is encountered in the case of valve actuated electrohydraulic servomechanisms (see [1]). Even when the relative degree is smaller it might still be possible to find coordinate transformations and controllers u_1 and u_2 that bring the switched system (1.4) to the form (1.5) making thus applicable the Malkin theorem for switched systems (see [2]).

The paper is organised as follows. In section 2 the main result on stabilizability is proved. In section 3 the mathematical model of a hydrostatic electrohydraulic servoactuator is investigated. We end with some concluding remarks.

2. RELATIVE DEGREE, COORDINATE TRANSFORMATIONS AND STABILIZATION

Consider a switched control system of type (1.4) with $f^{(l)}$, $l=1,2$, as in (1.1). Leaving apart the parameter μ the system is

$$\begin{aligned} \dot{y} &= Y^{(l)}(y,\xi) \\ \dot{\xi} &= D^{(l)}\xi + F_k^{(l)}(y,\xi)+(0,\ldots,0,1)^\tau u_l \end{aligned} \tag{2.1}$$

$l=1,2$, $Y^{(l)}, F^{(l)}$ contain only powers of y and $\xi_1,\ldots,\xi_n$ of order greater of equal to two in their Taylor development around zero and $Y^{(l)}(y,0)=F^{(l)}(y,0)=0$, $\forall y$, $D^{(l)}\in\mathcal{M}_n(\mathbf{R})$. Looking at the application in Section 3 we suppose that switching takes place when one specific component of ξ changes sign so condition $u_1(0)=u_2(0)$ is to be imposed.

Recall from [10] the definition of the relative degree

Definition. A simple-input single-output nonlinear system

$$\begin{aligned} \dot{x} &= f(x)+g(x)u \\ y &= h(x) \end{aligned} \tag{2.2}$$

has relative degree r at a point x^0 if

$$\begin{aligned} &(L_gL_f^k h)(x)=0,\ \forall x \text{ in a neighbourhood of } x^0, \\ &\text{for } k=0,1,\ldots,r-2 \end{aligned} \tag{2.3}$$

$$(L_gL_f^{r-1}h)(x^0)\neq 0 \tag{2.4}$$

L_fh is the Lie derivative of h along $f=(f_1,\ldots,f_n)$, $L_fh=\sum_{i=1}^n \frac{\partial h}{\partial x_i}f_i$. Recall also from [10] that $(ad_fg)(x)=g'(x)f(x)-f'(x)g(x)$.

The following theorem is an adaptation of the results in [10] ch. 4, in particular of Theorem 2.6 and Remark 2.9.

Theorem 2.1. *Suppose a system of order n, $\dot{x}=f(x)+g(x)u$, is given in $D\subset\mathbf{R}^n$. There exists an output function $y=h(x)$ for which the system has relative degree $(n-1)$ at $x^0\in D$ if and only if the following conditions are satisfied*:

(i) *The matrix $[g(x^0)(ad_fg)(x^0)\ldots(ad_f^{n-2}g)(x^0)]$ has rank $(n-1)$*

(ii) *For the distribution $D=Span\{g,ad_fg,\ldots,ad_f^{n-3}g\}$ there exists a closed form among the generators of the orthogonal codistribution $D^\perp$ near x^0.*

Proof. Suppose h satisfies the two conditions (2.3) and (2.4) for $(n-1)$ in x^0. By Lemma 1.3 in [10], ch. 4 the system (2.3) is equivalent to the following system of first order partial differential equations

$$(L_gh)(x)=0,(L_{ad_fg}h)(x)=0,\ldots,(L_{ad_f^{n-3}g}h)(x)=0 \tag{2.5}$$

for x in a neighbourhood of x^0 and the nontriviality condition (2.4) is equivalent to

$$(L_{ad_f^{n-2}g}h)(x^0)\neq 0. \tag{2.6}$$

Condition (i) is proved in [10], Lemma 1.2 in Ch. 4 . Thus the distribution D is nonsingular and $(n-2)$ dimensional in a neighbourhood of x^0. Equations (2.5) can be rewritten as

$$dh(x)[g(x)(ad_fg)(x)\ldots(ad_f^{n-3}g)(x)]=0$$

and this implies dh is among the generators of the two-dimensional codistribution $D^\perp$ around x^0 and since dh is a closed form, (ii) results.

Conversely, suppose (i) and (ii) hold. Then the distribution D is nonsingular and $(n-2)$-dimensional in a neighbourhood of x^0. Let $\omega(x)$, a closed form defined in U, a neighbourhood of x^0, be one of the generators of $D^\perp$. Then $\omega(x)=dh(x)$ since it is closed and

$$dh(x)[g(x)(ad_fg)(x)\ldots(ad_f^{n-3}g)(x)]=0 \text{ since } \omega\in D^\perp.$$

It follows that h satisfies (2.5) that is equivalent to (2.3). h can be choosed to satisfy also (2.6) since otherwise the distribution would not be $(n-2)$-dimensional in x^0.

□

Suppose that the systems in (2.1), of order $n+1$, have relative degree n in $(y,\xi)=(0,0)$ and let h_1,h_2 satisfy (2.3), (2.4) for $r=n$ and $g=(0,0,\ldots,0,1)^\tau\in\mathbf{R}^{n+1}$. One can always choose h_1 and h_2 such that

$$h_1(0)=h_2(0)=0 \tag{2.7}$$

(see also [10], pag. 169). Define

$$u_l=\frac{1}{L_gL_{f^{(l)}}^{n-1}h_l}\left(-L_{f^{(l)}}^n h_l+\sum_{i=1}^n c_iL_{f^{(l)}}^{i-1}h_l\right) \tag{2.8}$$

From (2.7) we infer that $u_1(0)=u_2(0)=0$. Remark that $L_gh_l=0$ implies $\frac{\partial h_l}{\partial\xi_n}=0$, $l=1,2$. Define the following coordinate transformations. For $\zeta=(y,\xi)$

$$z=\Phi^{(l)}(\zeta),\quad \zeta=\Psi^{(l)}(z) \tag{2.9}$$

is given by

$$\begin{aligned} &z_1=y,\ z_2=h_l(\zeta), \\ &z_3=(L_f^{(l)}h_l)(\zeta),\ldots,z_{n+1}=(L_{f^{(l)}}^{n-1}h_l)(\zeta). \end{aligned} \tag{2.10}$$

Condition (i) in Theorem 2.1 and Lemma 1.3 in [10], Ch. 4 show that $\Phi^{(l)}$ defined in (2.9) are locally invertible around $\zeta=0$. By (2.7), $\Phi^{(l)}(0)=0$.

Recall now that, from Lemma 1.3 in [10], ch. 4, h_l is a solution of (2.5) with n replaced by $n+1$. Denote $g_i = ad_f^i g$, $f = f^{(l)}$, $l = 1,2$. (2.5) becomes

$$\frac{\partial h}{\partial \xi_n} = 0, \frac{\partial h}{\partial y} g_{i1} + \frac{\partial h}{\partial \xi_1} g_{i2} + \ldots + \frac{\partial h}{\partial \xi_{n-1}} g_{in} = 0, \; i = 1, \ldots, n-2.$$

We choose h with $\frac{\partial h}{\partial y} = 0$ and show that there exists a nonzero solution of

$$\frac{\partial h}{\partial \xi_1} g_{i2} + \ldots + \frac{\partial h}{\partial \xi_{n-1}} g_{in} = 0, \quad i = 1, \ldots, n-2. \quad (2.11)$$

Since $f(0) = 0$ and there are no linear terms in f_1 ($l = 1,2$), it follows that $g_{i1} = 0$, $\forall i = 1, \ldots, n-2$. Then, by condition (i) in Theorem 2.1 and by Lemma 1.3 in [10], ch. 4 applied to $g = (0, \ldots, 0, 1)^\tau$ it follows that

$$\text{rank} \begin{bmatrix} 0 & g_{11} & \cdots & g_{(n-1)1} \\ 0 & g_{12} & \cdots & g_{(n-1)2} \\ \vdots & \vdots & \ldots & \vdots \\ 0 & g_{1n} & \cdots & g_{(n-1)n} \\ 1 & g_{1(n+1)} & \cdots & g_{(n-1)(n+1)} \end{bmatrix} = n$$

so

$$\text{rank} \begin{bmatrix} g_{12} & \cdots & g_{(n-2)2} \\ \vdots & \ldots & \vdots \\ g_{1n} & \cdots & g_{(n-2)n} \end{bmatrix} = n-2 =$$

$$= \text{rank} \begin{bmatrix} g_{12} & \cdots & g_{1n} \\ \vdots & \ldots & \vdots \\ g_{(n-2)2} & \cdots & g_{(n-2)n} \end{bmatrix}$$

in a neighbourhood of zero and this implies that indeed (2.11) has a nonzero solution.

In the new coordinates defined by (2.10) the system (2.1) becomes

$$\begin{aligned} \dot z_1 = \dot y = q_l(z),\ \dot z_2 = z_3, \ldots, \dot z_{n+1} = \\ c_1 z_1 + \ldots + c_n z_n,\ l = 1,2 \end{aligned} \quad (2.12)$$

with

$$q_l(z) = q_l(y, z_2, \ldots, z_{n+1}) = Y^{(l)}[\Psi^{(l)}(z)].$$

Theorem 2.2. *If $c_1, \ldots, c_n$ are choosed such that the matrix*

$$D = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ c_1 & c_2 & c_3 & \ldots & c_n \end{bmatrix}$$

is Hurwitz, then the zero solution of the switched system (2.12) *is simple stable by Lyapunov and is asymptotically stable with respect to state variables $z_2, \ldots, z_{n+1}$.*

Proof. With $\tilde z = (z_2, \ldots, z_{n+1})$, the switched system (2.12) becomes

$$\begin{aligned} \dot z_1 &= q_l(z_1, \tilde z) \\ \dot{\tilde z} &= D\tilde z, \end{aligned} \quad l = 1,2. \quad (2.13)$$

From the condition that D is Hurwitz it follows that the Lyapunov equation $D^\tau P + PD = -I$ has a unique solution $P > 0$. To apply Theorem 1.1 one has to verify that $q_l(z_1, 0) = 0$, $\forall z_1$, $l = 1,2$

$$q_l(z_1, 0) = Y^{(l)}[\Psi^{(l)}(z_1, 0)].$$

We show that $\Psi^{(l)}(z_1, 0) = (y, 0)$. This is equivalent to $\Phi^{(l)}(y, 0) = (z_1, 0)$. If we take $\xi = 0$ in (2.10) then $z_2 = h_l(y, 0) = h_l(0) = 0$ since h_l do not depend on y, $l = 1,2$.

$$z_3 = (L_f^{(l)} h_l)(y, 0) = \sum_{i=1}^{n-1} \frac{\partial h_l}{\partial \xi_i} f_{i+1}^{(l)}(y, 0) = 0$$

by (2.1) and the hypotheses on $F^{(l)}$. The same holds for $z_4, \ldots, z_{n+1}$ (recall h_l do not depend on ξ_n, $l = 1,2$). It follows that $q_l(z_1, 0) = Y^{(l)}(y, 0) = 0$ so, by Theorem 1.1, the zero solution is stable for the switched system (2.13) and $\lim_{t\to\infty} z_i(t) = 0$, $i = 2, \ldots, n+1$. Then the zero solution is stable for the switched system (2.1) and since $L_{f^{(l)}}^k h_l$, $0 \le k \le n-1$, $l = 1,2$, do not depend on y, it follows that $z_2, \ldots, z_{n+1}$ depend only on $\xi_1, \ldots, \xi_n$ so $\xi_1, \ldots, \xi_n$ depend only on $z_2, \ldots, z_{n+1}$ through $\Psi^{(l)}$ and since $\Psi^{(l)}(0) = 0$ and $\Psi^{(l)}$ are local diffeomorphisms we infer that $\lim_{t\to\infty} \xi_i(t) = 0$, $i = 1, \ldots, n$.

□

3. THE MODEL OF A HYDROSTATIC ELECTROHYDRAULIC SERVOACTUATOR

Hydrostatic electrohydraulic servoactuators (EHSA) have the specificity that are pump controlled (see [6], [18],[19], [23]). The physical and the mathematical models of such an EHSA are described in [18] and in [8]. In [8] the stability of equilibria is investigated. We refer to the papers [18] and [8] for all details.

Denote the load displacement by x_1, the load velocity by x_2, an internal friction state variable by x_3, the pressures in the cylinder chambers $p_1 = x_4$, $p_2 = x_5$ and introduce two more state variables $x_6 = \xi$, $x_7 = \dot\xi$ related to dynamics of an electric motor that drives the pump. Then the switched system of control differential equations that describes the dynamics of the hydrostatic EHSA is

$$\begin{aligned}
\dot x_1 &= x_2 \\
\dot x_2 &= \frac{1}{m}[-kx_1 - (f_r + f_\nu + \sigma_1)x_2 - \sigma_0 x_3 + S(x_4 - x_5) + \\
&+ \sigma_1 \frac{|x_2| x_3}{F_c + (F_s - F_c)e^{-(\frac{x_2}{\nu})^2}} \\
\dot x_3 &= x_2 - \frac{|x_2| x_3}{F_c + (F_s - F_c)e^{-(\frac{x_2}{\nu})^2}} \\
\dot x_4 &= \frac{B}{V_{01} + Sx_1}[D_p b_0 x_6 + D_p b_1 x_7 - (C_{ip} + C_{ep} + C_{ec})x_4 + \\
&+ C_{ip} x_5 + C_{ep} p_r - Sx_2] \\
\dot x_5 &= \frac{B}{V_{02} - Sx_1}[-D_p b_0 x_6 - D_p b_1 x_7 + (C_{ip} - C_{ep})x_4 - \\
&- (C_{ip} + C_{ec})x_5 + C_{ep} p_r + Sx_2] \\
\dot x_6 &= x_7, \quad \dot x_7 = -a_0 x_6 - a_1 x_7 + u(x_1, \ldots, x_7)
\end{aligned} \quad (3.1)$$

One has always $F_s > F_c$.

The system (3.1) corresponding to $x_2 \geq 0$ will be denoted by $\mathcal{S}_1$ and the one for $x_2 \leq 0$ by $\mathcal{S}_2$. Both $\mathcal{S}_1$ and $\mathcal{S}_2$ can be considered as defined in the whole domain described by $-\frac{V_{01}}{S} < x_1 < \frac{V_{02}}{S}$, $(x_2, x_3, \ldots, x_7) \in \mathbf{R}^6$. When u is set to zero, (3.1) has the family of equilibria parametrized by $w \in (-R, R) \subset \left(-\frac{V_{01}}{S}, \frac{V_{02}}{S}\right)$,

$$\begin{aligned} &\hat{x}_1 = w,\ \hat{x}_2 = 0,\ \hat{x}_3 = -\frac{kx}{\sigma_0}, \\ &\hat{x}_4 = \hat{x}_5 = \frac{C_{ep}p_r}{C_{ep} + C_{3c}},\ \hat{x}_6 = \hat{x}_7 = 0 \end{aligned} \tag{3.2}$$

Suppose $u(\hat{x}) = 0$ and translate (3.2) to zero through $y_i = x_i - \hat{x}_i$, $i = 1, \ldots, 7$, $\tilde{u}(y) = u(y + \hat{x})$. System (3.1) becomes

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \frac{1}{m}[-ky_1 - (fr + f_\nu + \sigma_1)y_2 - \sigma_0 y_3 + \\ &+ \sigma_1 \frac{|y_2|(y_3 + \hat{x}_3)}{F_c + (F_s - F_c)e^{-(\frac{y_2}{\nu})^2}} + S(y_4 - y_5)] \\ \dot{y}_3 &= y_2 - \frac{|y_2|(y_3 + \hat{x}_3)}{F_c + (F_s - F_c)e^{-(\frac{y_2}{\nu})^2}} \\ \dot{y}_4 &= \frac{B}{V_{01} + Sy_1 + Sx}[D_p b_0 y_6 + D_p b_1 y_7 - \\ &- (C_{ip} + C_{ep} + C_{ec})y_4 + C_{ip}y_5 - Sy_2] \\ \dot{y}_5 &= \frac{B}{V_{02} - Sy_1 - Sx}[-D_p b_0 y_6 - D_p b_1 y_7 + \\ &= (C_{ip} - C_{ep})y_4 - (C_{ip} + C_{ec})y_5 + Sy_2] \\ \dot{y}_6 &= y_7, \quad \dot{y}_7 = -a_0 y_6 - a_1 y_7 + \tilde{u}(y) \end{aligned} \tag{3.3}$$

The two components of (3.3) will be denoted by Σ_l, $l = 1, 2$ and will be considered for $y_2 \in \mathbf{R}$.

If $A^{(1)}$ and $A^{(2)}$ are the Jacobian matrices for (3.3) calculated in zero then their characteristic polynomials are $Q^{(l)}(\lambda) = \lambda Q_1^{(l)}(\lambda)$. The controllers u_1 and u_2 are to be designed such that for $x \in (-R, R)$

$$\tilde{u}_1(0) = \tilde{u}_2(0) = 0 \tag{3.4}$$

and

$$Q_1^{(l)}(0) \neq 0. \tag{3.5}$$

It follows that the systems Σ_l are in a critical case for stability theory covered by Malkin Theorem (see [13]). Introduce new state variables through

$$y^{(l)} = -a_{32}^{(l)} y_1 + y_3 \tag{3.6}$$

$$a_{32}^{(1)} = \frac{\partial f_3^{(1)}}{\partial y_2}(0) = 1 + \frac{kx}{\sigma_0 F_s}, \quad a_{32}^{(2)} = \frac{\partial f_3^{(2)}}{\partial y_2}(0) = 1 - \frac{kx}{\sigma_0 F_s}.$$

The new switched systems with components Σ_l', $l = 1, 2$ will have no linear terms in the equation for $\dot{y}$.

$$\begin{aligned} \dot{y} &= Y^{(l)}(y, y_1, y_2, y_4, y_5, y_6, y_7) \\ \dot{y}_1 &= y_2 \\ \dot{y}_2 &= Y_2^{(l)}(y, y_1, y_2, y_4, y_5, y_6, y_7) \\ \dot{y}_4 &= f_4(y_2, y_4, y_5, y_6, y_7) \\ \dot{y}_5 &= f_5(y_2, y_4, y_5, y_6, y_7) \\ \dot{y}_6 &= y_7 \\ \dot{y}_7 &= Y_7^{(l)}(y, y_1, y_2, y_4, y_5, y_6, y_7) \end{aligned} \tag{3.7}$$

To eliminate y from the linear part of the last six equations in (3.7) one applies the Implicit Function Theorem (IFT) to the algebraic systems

$$\begin{aligned} &y_2 = 0,\ Y^{(l)} = 0,\ f_4 = 0,\ f_5 = 0, \\ &y_7 = 0,\ Y_7^{(l)} = 0, \quad l = 1, 2. \end{aligned} \tag{3.8}$$

The condition imposed to $Q_1^{(l)}$ assures that the conditions in IFT are satisfied so, in a neighbourhood of $y = 0$ there exist C^1-functions $\varphi_1^{(l)}$, $\varphi_2^{(l)}$, $\varphi_3^{(l)}$ and $\varphi_4^{(l)}$, $l = 1, 2$, $\varphi_i^{(l)}(0) = 0$, $i = 1, 2, 3, 4$; $l = 1, 2$ and $y_1 = \varphi_1^{(l)}(y)$, $y_2 = 0$, $y_4 = \varphi_2^{(l)}(y)$, $y_5 = \varphi_3^{(l)}(y)$, $y_6 = \varphi_4^{(l)}(y)$, $y_7 = 0$, $l = 1, 2$, solve (3.8).

The new change of variables

$$\begin{aligned} &\xi_1 = y_1 - \varphi_1(y),\ \xi_2 = y_2,\ \xi_3 = y_4 - \varphi_2^{(l)}(y), \\ &\xi_4 = y_5 - \varphi_3^{(l)}(y),\ \xi_5 = y_6 - \varphi_4^{(l)}(y),\ \xi_6 = y_7 \end{aligned} \tag{3.9}$$

turn Σ_l', $l = 1, 2$, into

$$\begin{aligned} \dot{y} &= \tilde{Y}^{(l)}(y, \xi) \\ \dot{\xi} &= D^{(l)}(x)\xi + F^{(l)}(y, \xi),\ l = 1, 2. \end{aligned} \tag{3.10}$$

The system (3.10) has the same form as (2.1). If the relative degree of (3.10) in zero is 6 one can apply the constructions in section 2 and synthesize controllers u_1 and u_2 by (2.7) and it results directly from this definition and the special choose of h_1, h_2 that u_1 and u_2 satisfy (3.4) and (3.5). Remark that the conditions for h_1 and h_2 to exist are independent of u_1, u_2 so we eventually can find h_1 and h_2 using the terms that do not depend on control, construct then u_1 and u_2 by (2.8) and impose then to satisfy the condition that D is Hurwitz. The invariance of the spectrum of a matrix to similarities (see, e.g. [9]) implies (3.5) is satisfied.

The relative degree is preserved by coordinate transformations ([10], Ch. 4, Lemma 2.4) thus one can compute it for systems (3.3) in zero and apply the theory in Section 2 to show eventually that the zero solution of the switched system is stabilizable through coordinate transformations. As for the construction of the controllers, it depends on solving the systems (2.11).

A numerical calculation was performed with the following values for the constants in (3.1): $m = 60[Kg]$; $f_r = 10^4[Ns/m]$; $k = 10^6[N/m]$; $S = 2 \cdot 10^{-4}[m^2]$; $D_p = 1.7 \cdot 10^{-7}[m^3/rad]$; $V_{01} = V_{02} = 6 \cdot 10^{-6}[m^3]$; $B = 6 \cdot 10^8[Pa]$; $C_{ec} = 1.7 \cdot 10^{-13}[m^3/(Pa \cdot s)]$; $C_{ip} = 2 \cdot 10^{-13}[m^3/(Pa \cdot s)]$; $C_{ep} = 2 \cdot 10^{-13}[m^3/Pa \cdot s]$; $\sigma_0 = 2 \cdot 10^4[N/m]$; $\sigma_1 = 306[Ns/m]$; $f_\nu = 60[Ns/m]$; $\nu_s = 0.1[m/s]$; $F_s = 6 \cdot 10^{-3}[m]$; $F_c = 5 \cdot 10^{-3}[m]$; $a_0 = 17300.14[s^{-2}]$; $a_1 = 8600[s^{-1}]$; $b_0 = 230000[rad/(V \cdot s^3)]$; $b_1 = 1600[rad/(V \cdot$

$s^2)$].With $w = \frac{1}{100}[m]$,$w = \frac{1}{1000}[m]$ and $w = 0[m]$,it revealed that the rank of the matrix

$$[g(\hat{x})(ad_f g)(\hat{x}) \dots (ad_f^5 g)(\hat{x})] \quad \text{is} \quad 6.$$

As concerns the numerical calculations, one remarks that although there are some large differences between the order of the constants involved, that might induce the ideea of an ill conditioned system, the agregated coefficients prove to be in a tractable magnitude interval.

4. CONCLUDING REMARKS

Based on the results in [7] a specific control synthezis is proposed for stabilization of the zero solution for switched control systems in Malkin canonical form

$$\begin{aligned} \dot{y} &= Y_l(y, \xi) \\ \dot{\xi} &= D_l \xi + F_l(y, \xi) + (0, \dots, 0, 1)^\tau u, \quad l = 1, 2 \end{aligned} \tag{4.1}$$

$\xi = (\xi_1, \dots, \xi_n)$, $y \in \mathbf{R}$, Y_l and F_l contain only powers of y and $\xi_1, \dots, \xi_n$ of order greater or equal to two in their Taylor development around zero and

$$Y_l(y, 0) = F_l(y, 0) = 0 \quad \forall y, \quad l = 1, 2.$$

This control synthesis relies on the condition that the matrix

$$[g(0)(ad_{f_l} g)(0) \dots (ad_{f_l}^{n-2} g)(0)]$$

has the rank equal to $n - 1$ ($g(y, \xi) = (0, 0, \dots, 0, 1)^\tau \in \mathbf{R}^{n+1}$,

$$f_l(y, \xi) = (Y_l(y, \xi), D_l \xi + F_l(y, \xi))^\tau).$$

This condition ensures that the relative degree of (4.1) in zero is $(n - 1)$. An application is given to a model of a pump controlled EHSA.

It must be mentioned that stabilization can eventually be obtained using the same results from [7] even if the relative degree is $r < n-1$ if one can calculate h_1, h_2 and then find a completion to local diffeomorphisms of $\Phi_1 = z_1 = h_l$, $\Phi_2 = z_2 = L_{f^{(l)}} h_l, \dots, \Phi_r = z_r = L_{f^{(l)}}^{r-1} h_l$ such that, for the new systems

$$\begin{aligned} \dot{z}_1 &= \tilde{Y}^{(l)}(z_1, \xi) \\ \dot{\tilde{z}} &= \tilde{D}^{(l)} \tilde{z} + \tilde{F}^{(l)}(z_1, \tilde{z}), \end{aligned} \qquad \tilde{z} = (z_2, \dots, z_{n+1}),$$

still in the critical case covered by Malkin Theorem, a Common Lyapunov Function exists. For this it is enough to have $\tilde{D}^{(l)} = D$, $l = 1, 2$, with D a Hurwitz matrix. A situation when this happens is presented in [2].

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