CEAI, Vol. 4, No. 3, pp. 13-18, 2002 Printed in Romania

ADAPTIVE CONTROLLER FOR ROBOT ARMS

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***Abstract**: The case of an adaptive controller based on output feedback is developed for a two link robot manipulator. A simple non-linear observer using the desired velocity and bounded position tracking error is proposed to estimate the joint velocities. The closed loop system compound by the adaptive controller, observer and the robot system is shown to be semi-global asymptotically stable. Numerous simulations conducted on a two-link robot manipulator model confirm the effectiveness of the proposed controller-observer structure. The performance of the proposed scheme is evident by comparing the simulation results with a well-known passivity based control algorithm.*

***Keywords**: Adaptive control; Robot control; observers; Lyapunov stability.*

1. INTRODUCTION

The actual literature contains in great details adaptive control algorithms for robot arms based on complete state measurements. The feed-forward and passivity based algorithms for robot arms have been extensively used. Most of these algorithms need complete state measurements. The major drawback of such schemes is that both joint position are required for feedback control. Sensors for measuring robot joint velocities are too expensive. Further, measurements from the sensors are often mixed with noise. Velocity estimated feedback control of robot arms can be instead and the requirement of robots to be equipped with velocity sensors can be eliminated. Most of the robot adaptive schemes use velocity errors or modified velocity errors to drive the parameter adaptation algorithms. When the actual velocities are not available, estimated velocities and position errors have to be used to drive the parameter adaptation algorithms. This implies more difficulties in proving the stability of these algorithms.

Considerable research is done in the area of output feedback control of non-linear systems, especially on passivity based controller-observer design. A linear observer is designed to estimate the velocities that leads to a closed-loop system (formed by the controller-observer and the robot) that will be locally exponentially stable. The linear velocity observer is designed assuming complete knowledge of the structural parameters of the robot. Other research considered the controller with variable structure next to a non-linear observer, in the presence of the parameter uncertainties. A passivity based controller and a nonlinear sliding observer were also proposed to obtain local asymptotic convergence of the position tracking errors and velocity estimation error.

In this paper is proposed an adaptive feedback controller for robot arms using the partial feedback approach, meaning that only joint position measurements are needed to design the adaptive controller. Near this, a simple non-linear observer is designed to estimate the robot joint velocities. The closed-loop system formed by the adaptive controller, observer and the robot system is shown to be semi-global asymptotically stable so that the region of attraction can be increased arbitrarily by increasing the controller and the observer gains.

The estimated parameters convergence to the true parameters depends on weather the regression matrix satisfies the persistence of excitation condition. In the proposed adaptive controller the regression matrix entirely depends on the desired trajectory. The simulation made using the two-link planar arm model successfully show the validity of the proposed controller and observer. Further on, the proposed scheme is compared with a well known passivity based controller that assume all parameters exactly known and use a first-order numerical differentiation of joint position measurements to estimate velocities.

2. THE DYNAMICS OF THE ROBOT ARM MODEL

It is considered the dynamics of an *n* degree of freedom robot arm, as it follows:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &M(x_1)\dot{x}_2 + C(x_1,x_2)x_2 + g(x_1) = v_t \end{aligned} \tag{1}$$

where $x_1 \in \Re^n, x_2 \in \Re^n$ are the generalized position and velocity, respectively, $M(x_1) \in \Re^{n\times n}$ is the inertia matrix, $C(x_1,x_2) \in \Re^{n\times n}$ is the matrix composed of Coriolis and centrifugal terms, $g(x_1) \in \Re^n$ is the gravity vector, and $v_t \in \Re^n$ is the vector composed of joint torques. The structure of the robot dynamics satisfies the following properties:

The inertia matrix $M(x_1)$ is positive defined and is bounded from above and below by positive constants σ_M and σ_m, that is:

$$\sigma_m \le M(x_1) \le \sigma_M \tag{2}$$

The matrix $C(x_1,x_2)$ is bounded and satisfies the relation:

$$\begin{aligned} &\|C(x_1,x_2)\| \le c_m \|x_2\|, \forall x_1, x_2 \\ &C(x_1,x_2)z = C(x_1,x_2), \forall x_2, z. \end{aligned} \tag{3}$$

The matrix $\dot{M}(x_1) - 2C(x_1,x_2)$ is skew-symmetric.

The dynamics is linear in the unknown parameters and can be expressed as:

$$\begin{aligned} &M(x_1)\dot{x}_2 + C(x_1,x_2)x_2 + g(x_1) = \\ &\qquad = Y(x_1,x_2,\dot{x}_2)p_u \end{aligned} \tag{4}$$

where $p_u \in \Re^p$ is the unknown parameter vector and $Y(x_1,x_2,\dot{x}_2) \in \Re^{n\times p}$ is the known regression matrix.

For a desired trajectory of the robot, the main objective is to design a stable-tracking controller that only requires joint position measurements for feedback. To accomplish this purpose, it is proposed an adaptive controller together with a simple non-linear observer to estimate the joint velocities.

In these conditions, let $x_1^d(t)$ and $x_2^d(t)$ be the desired position and velocity, respectively. It must be assumed that the desired state trajectory is twice continuously differentiable, otherwise could appear some difficulties for the estimation process.. Let $\hat{x}_1(t)$ and $\hat{x}_2(t)$ denote the estimated position and estimated velocity, respectively. Let $\theta \in \Re^p$ denote the actual parameter vector given in accordance to model dynamics presented above; and let $\hat{\theta}(t)$ denote the estimate of θ. In these conditions, the tracking error and the estimation error can be defined by:

$$\begin{aligned} &e_1(t) := x_1(t) - x_1^d(t), \hat{e}_1(t) := x_1(t) - \hat{x}_1(t), \\ &e_2(t) := x_2(t) - x_2^d(t), \hat{e}_2(t) := x_2(t) - \hat{x}_2(t), \\ &e_v(t) := e_2(t) - e_c(t), \tilde{\theta}(t) = \hat{\theta}(t) - \theta, \\ &e_c(t) := \Lambda(e_1)e_1(t), \end{aligned}$$

where $e_1(t)$ and $e_2(t)$ are the position and the velocity tracking errors, $\hat{e}_1(t)$ and $\hat{e}_2(t)$ are the estimated position and estimated velocity errors, $e_v(t)$ is the reference velocity error, $\tilde{\theta}(t)$ is the parameter estimation error, $e_c(t)$ is an auxiliary bounded position tracking error, and $\Lambda(e_1(t))$ is a positive defined diagonal matrix given by:

$$\Lambda(e_1(t)) = diag\left(\frac{\lambda_c}{1+|e_{11}(t)|},...,\frac{\lambda_c}{1+|e_{1n}(t)|}\right), \quad (5)$$

where $e_{11}(t),...,e_{1n}(t)$ are the components of the position error vector $e_1(t)$ and λ_c is a positive gain. The choice of $\Lambda(e_1)$ renders $e_c(t)$ to be bounded by $\lambda _{c}$. In the next section, the adaptive controller, observer and the close-loop error dynamics are presented.

3. ADAPTIVE CONTROLLER AND OBSERVER

The control design proposed is as follows:

$$v_t = Y_d\left(x_1^d, x_2^d, \dot{x}_2^d\right)\hat{\theta} - K_d\left(e_v + \hat{e}_2\right) - K_p e_1, \quad (6)$$

where K_d, K_p are positive definite gain matrices and $\hat{\theta}$ is the estimated parameter vector of the robot. It must be observed that the second term in the control law is a function of estimated velocity, desired velocity, and actual position error, that is $e_v + \hat{e}_2 = \hat{x}_2 - x_2^d + e_c$ The desired regression matrix, $Y_d\left(x_1^d, x_2^d, \dot{x}_2^d\right)$ is given by:

$$\begin{aligned} &Y_d\left(x_1^d, x_2^d, \dot{x}_2^d\right)\hat{\theta} = \\ &= \hat{M}\left(x_1^d\right)\dot{x}_2^d + \hat{C}\left(x_1^d, x_2^d\right)x_2^d + \hat{g}\left(x_1^d\right) \end{aligned} \quad (7)$$

where: $\hat{M}\left(x_1^d\right), \hat{C}\left(x_1^d, x_2^d\right)$, and $\hat{g}\left(x_1^d\right)$ are the estimated of $M\left(x_1^d\right)$, $C\left(x_1^d, x_2^d\right)$, and $g\left(x_1^d\right)$ respectively. The desired regression matrix depends only on the desired trajectory and can be precomputed. The parameter adaptation law is chosen as below:

$$\hat{\theta}(t) = \hat{\theta}(0) - \Gamma\left\{Y_d^T e_r(t) - \int_0^t Y_d^T e_r(\omega)d\omega\right\}, \quad (8)$$

where $\hat{\theta}(0)$ denotes the initial estimate of unknown parameter vector, Γ is a positive definite gain matrix, and e_r is given by:

$$e_r(t) = e_1(t) - \hat{e}_1(t) + \int_0^t e_c(\varpi)d\varpi - \eta_1\int_0^t \hat{e}_1(\varpi)d\varpi \quad (9)$$

The next structure of observer is proposed to estimate the states:

$$\hat{x}_1 = -\eta_1\hat{e}_1 + \hat{x}_2, \quad (11)$$

$$\hat{x}_2 = x_2^d - \eta_1\hat{e}_1 + e_c. \quad (12)$$

where η_1 and η_2 are positive gains. Thus, by rearranging terms, the observer error dynamics is given by:

$$\begin{aligned} &\dot{\hat{e}} = -\eta_1\hat{e}_1 + \hat{e}_2, \\ &\hat{e}_2 = -\eta_2\hat{e}_2 + \eta_1\eta_2\hat{e}_1 + 2\dot{e}_2 - \dot{e}_v. \end{aligned} \quad (13)$$

4. THE EXPRESSION OF ERROR DYNAMICS

After the notation $\dot{e}_v = \dot{e}_2 + \dot{e}_c$ and $-e_v + e_2 + e_c = 0$ are made, $M(x_1)\dot{e}_v$ can be expressed as:

$$\begin{aligned} M(x_1)\dot{e}_v &= -C(x_1,x_2)e_v + M(x_1)\dot{e}_2 + \\ &+ C(x_1,x_2)e_2 - g(x_1) + M(x_1)\dot{e}_c + \\ &+ C(x_1,x_2)e_c \end{aligned} \quad (14)$$

From relation (1) and (14) it results that:

$$\begin{aligned} M(x_1)\dot{e}_v &= -C(x_1,x_2)e_v - M(x_1)\dot{x}_2^d + \\ &+ v_t + C(x_1,x_2)x_2^d + M(x_1)\dot{e}_c + \\ &+ C(x_1,x_2)e_c \end{aligned} \quad (15)$$

Using the control law and assigning that $\dot{e}_c = E_c^{'}e_2$, where $E_c^{'} = \lambda_c^{-1}\Lambda^2$ the error equation becomes:

$$\begin{aligned} M(x_1)\dot{e}_v &= -C(x_1,x_2)e_v + Y_d\tilde{\theta} - \Delta W - \\ &- K_d\left(e_v + \hat{e}_2\right) - K_p e_1 + M(x_1)E_c^{'}e_v - \\ &- M(x_1)E_c^{'}e_c \end{aligned} \quad (16)$$

where ΔW is given by the:

$$\begin{aligned} \Delta W &= \left[M(x_1) - M\left(x_1^d\right)\right] + \dot{x}_2^d + g(x_1) - \\ &- g\left(x_1^d\right) + [C(x_1,x_2)\left(x_2^d - e_c\right) - \\ &- C\left(x_1^d, x_2^d\right)x_2^d] \end{aligned} \quad (17).$$

Using the equation (13), the observer error equation can be derived as follows:

$$
\begin{aligned}
&M(x_1)\dot{\hat{e}}_2 = -\eta_2 M(x_1)\hat{e}_2 + \eta_1\eta_2 M(x_1)\hat{e}_1 + \\
&2M(x_1)\dot{e}_c - M(x_1)\dot{e}_v = \\
&= -C(x_1,x_2)\hat{e}_2 - \{M(x_1)\dot{e}_v + C(x_1,x_2)e_v\} - \\
&-\eta_2 M(x_1)\hat{e}_2 + C(x_1,x_2)(\dot{e}_2 + e_v) + \ldots \\
&\ldots + 2M(x_1)\dot{e}_c + \eta_1\eta_2 M(x_1)\hat{e}_1.
\end{aligned} \tag{18}
$$

On substitution of the robot error dynamics (16) and using the fact that $x_2 = e_v - e_c + x_2^d$, the error dynamics, finally, remains:

$$
\begin{aligned}
&M(x_1)\dot{\hat{e}}_2 = -C(x_1,x_2)\hat{e}_2 - Y_d\tilde{\theta} + \Delta W + \\
&+K_d(e_v + \hat{e}_2) + K_p e_1 + M(x_1)E_c^{'}e_v - \\
&-M(x_1)E_c^{'}e_c + \eta_2 M(x_1)\hat{e}_2 + . \\
&\eta_1\eta_2 M(x_1)\hat{e}_1 + \{C(x_1,e_v) - C(x_1,e_c)\} \\
&(\hat{e}_2 + e_v) + C(x_1,x_2^d)(\hat{e}_2 + e_v).
\end{aligned} \tag{19}
$$

5. SIMULATION RESULTS

5.1. *The adopted model*

The simulations are made using a two-link direct planar manipulator model shown in the Fig. 1. Each axes is driven by a direct servo-motor which is capable of up to 3 revolutions per second maximum velocity and position feedback resolution of up to 156400 counts per revolution. The simulation model output torque is suppose to be rated up to 245 Nm and the elbow motor rated up to 40Nm. This considerations turn the system into a stand-alone one that contains all the element needed for assure closed-loop servo motor control.

Theoretically, the motor can contain a high torque direct drive brush-less actuator, a high-resolution brush-less resolver, and a high precision bearing. The supposed direct drive actuator could eliminate the need for gear reduction, so repeatability is limited only by the resolution of the position feedback.

The inertia matrix $M(x_1)$ and the matrix composed of Coriolis and centrifugal terms, $C(x_1,x_2)$ for the two given manipulator are given by:

$$
M(x_1) = \begin{pmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{pmatrix},
$$

$$
C(x_1,x_2) = \begin{pmatrix} -p_3x_{21}s_2 & -p_3(x_{21}+x_{22})s_2 \\ p_3x_{21}s_2 & 0 \end{pmatrix}
$$

where $c_i = \cos(x_{1i})$, $s_i = \sin(x_{1i})$ and x_{1i} and x_{2i} denote the components of the vectors x_1 and x_2, respectively and p_1, p_2 and p_3 are coupled inertia parameters, which are treated as unknowns and estimated by the adaptive controller. The gravity term for this model robot simulation is considered zero, $.\, g(x_1) = 0$ The desired regression for the two-link manipulator is:

$$
Y_d(x_1^d, x_2^d, \dot{x}_2^d) = \begin{pmatrix} \dot{x}_{21}^d & \dot{x}_{22}^d & (2\dot{x}_{21}^d + \dot{x}_{22}^d)\cos(x_{12}^d) - \left((x_{22}^d)^2 + 2x_{21}^d x_{22}^d\right)\sin(x_{12}^d) \\ 0 & \dot{x}_{21}^d + \dot{x}_{22}^d & \dot{x}_{21}^d\cos(x_{12}^d) + (x_{21}^d)^2\sin(x_{12}^d) \end{pmatrix}.
$$

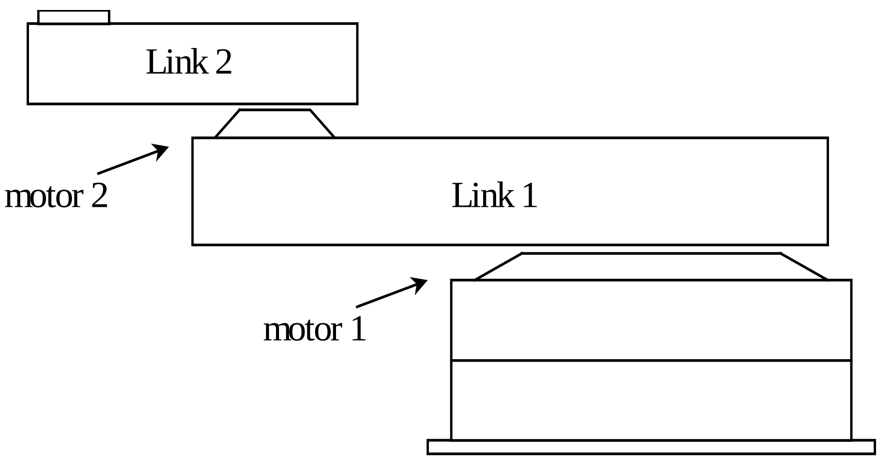

Fig. 1. Two-link robot manipulator.

The desired position, velocity, acceleration, and jerk (derivative of acceleration) trajectories for the two joint angles used in simulations are given in Fig. 2. These represent desired joint angles for 14 cycles of circle trajectory in the Cartesian space. The middle cycles are considered of 1 s duration, the first and the last of duration 2 s.

The proposed controller-observer is compared with the passivity based control algorithm. Exact knowledge of the parameters is assumed for the passivity based control scheme and a first order (one-step) numerical differentiation of the joint position measurements has been used to obtain joint velocities. The following passivity based control algorithm is chosen:

$$v_t = M(x_1)\dot{x}_{2r} + C(x_1, x_2)x_{2r} + F_v(x_{2r} - x_2)$$

where $\dot{x}_{2r} = \dot{x}_2^d - K_d\left(x_2 - x_2^d\right) - K_p\left(x_1 - x_1^d\right)$, and F_v, K_d, K_p are positive definite gain matrices.

5.2. *Graphic results*

Using the adaptive controller-observer with a sampling period of 4ms, the obtained position tracking errors are presented in Fig. 2 and Fig. 3.

6. CONCLUSIONS

Comparison with passivity based control algorithm, implemented with exact knowledge of the true parameters, shows that the proposed algorithm gives similar results even under large uncertainties in the robot parameters.

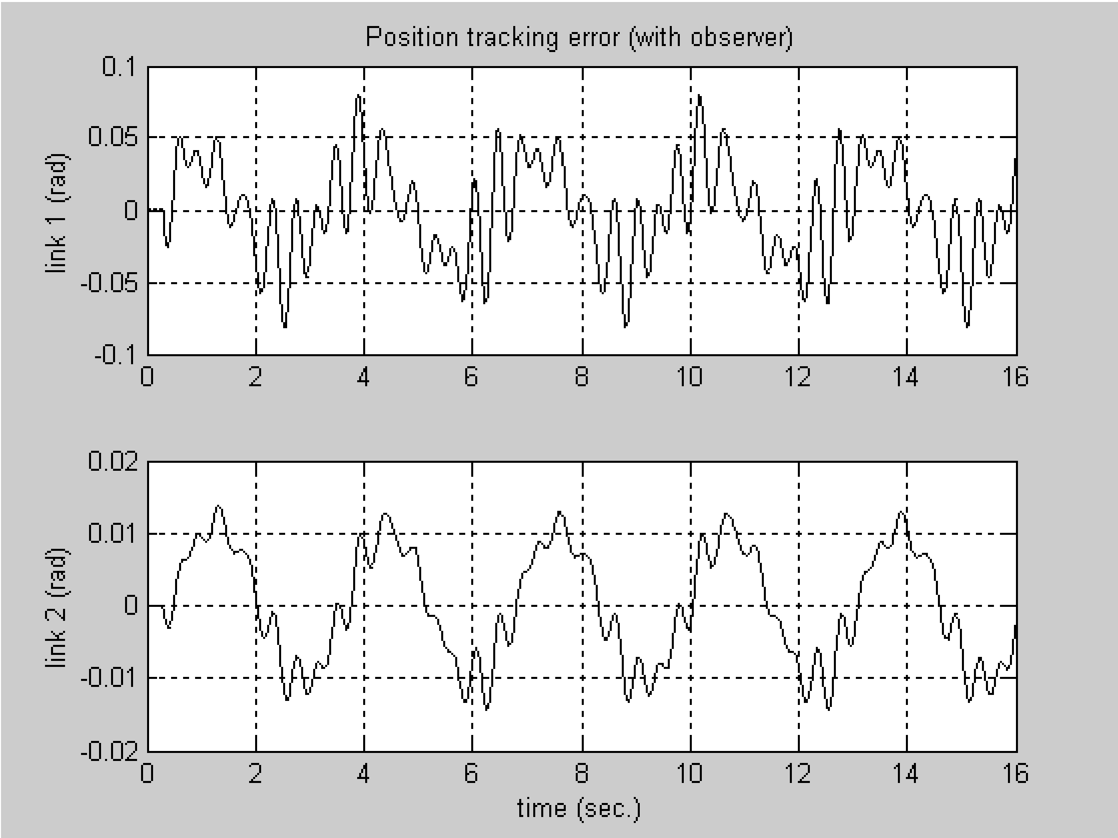

Fig. 2. Position tracking errors (proposed adaptive controller with observer).

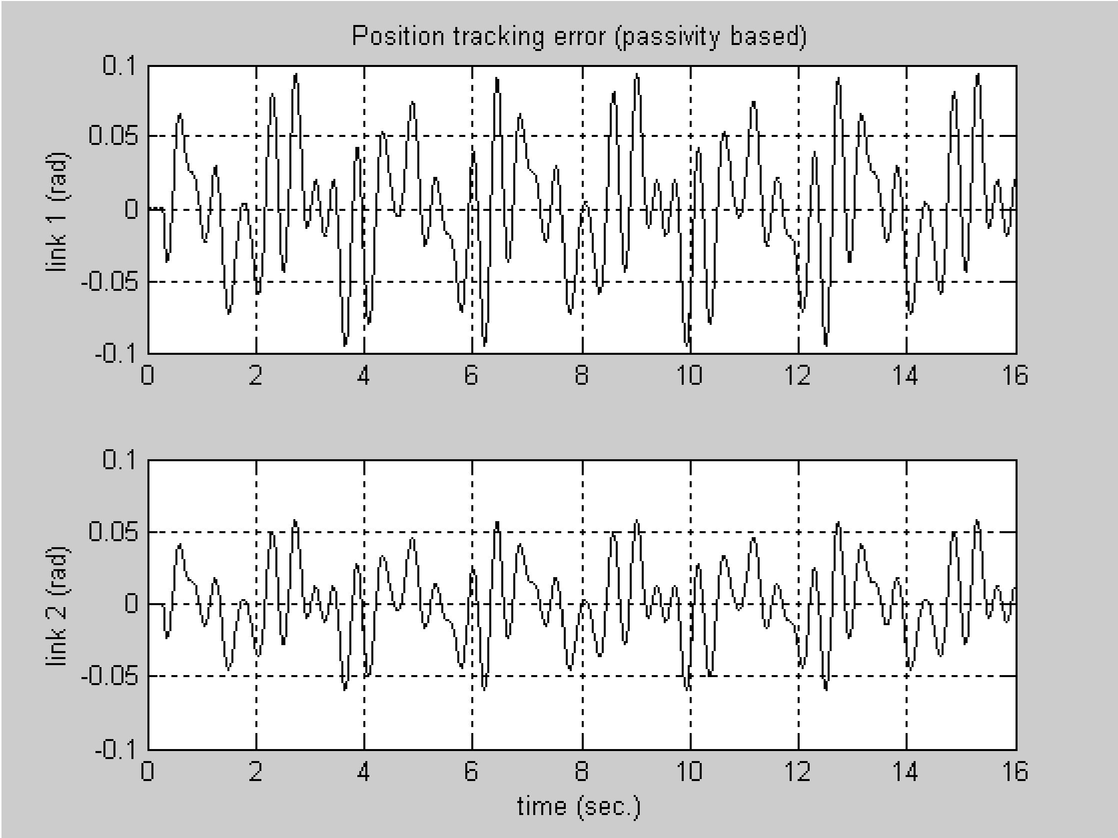

Fig. 3. Position tracking errors (passivity based scheme).

7. REFERENCES

[1] Berghuis, H. and Nijmeijer, H. - "A passivity approach to controller-observer design for robots", *IEEE Transactions on Robotics and Automation*, 9(6), pp. **740-754**, 1993.

[2] Berghuis, H. and Nijmeijer, H. - "Robust control of robots via linear estimated state feed-back". *IEEE Transactions on Robotics and Automation*, 39(10), pp. **2159-2162**, 1994.

[3] Canudas de Wit,C. and Fixot, N. – "Trajectory tracking in robot manipulators via nonlinear estimated state feed-back". *IEEE Transactions on Robotics and Automation*, 8(1), pp. **138-142**, 1992.

[4] Kaneko, K and Horowitz, R. – "Repetitive and adaptive control of robot manipulators with velocity estimation", *IEEE Transactions on Robotics and Automation*, 13(2), pp. **204-217**, 1997.

[5] Ortega, R. and Spong, M.W. – "Adaptive motion control of rigid robots: A tutorial", *Automatica*, 25(6), pp. **877-888**, 1989.

[6] Pagilla, P.R. and Tomizuka, M. – "An adaptive output feedback controller for robot arms: stability and experiments", *Automatica*, 37(7), pp. **983-995**, 2001.

[7] Sadegh, N. and Horowitz, R. – "Stability and robustness analysis of a class of adaptive controllers for robotic manipulators", *The International Journal of Robotics Research*, 9(3), pp. **74-92**, 1990.