Cost optimal scheduling in an agro-food production workshop

Fatma Tangour[*, **], Pierre Borne[**], Mohamed Benrejeb[*]

** LARA Automatique, École Nationale d'Ingénieurs de Tunis, BP 37, Le Belvédère, 1002 Tunis, Tunisie*
***LAGIS, École Centrale de Lille, Cité scientifique, BP 48, 59650 Villeneuve d'Ascq Cedex, France*
fatma.tangour@laposte.net, pierre.borne@ec-lille.fr, mohamed.benrejeb@enit.rnu.tn

Abstract: This paper deals with an original application of the dynamic scheduling problem to a flexible manufacturing system of agro-food production. On the basis of an operations' set, the objective is to find an optimal scheduling which minimizes jointly the cost of the out-of-date products and the cost of distribution discount, a heuristic based on the branch-and-bound method is used. Our goal is achieved while basing on coefficients indicating that the engagement of a production in favour of another is relatively urgent.

Keywords: dynamic scheduling, multi-objective optimization, dominance rules, minimization of the cost, branch-and-bound method.

1. INTRODUCTION

New applicability, like the flexible manufacturing systems or the parallel architectures (Carlier *and al*., 1988), returns the scheduling problems more complex what requires the rigorous resolution of these problems. The passage of a product's type with another is generally done at cost's price and tools adjustment's time (Dusonchet *and al.*, 2004), Thus, the evolution and the dynamic characteristics of the industrial workshops, in particular those of agro-food industries, impose the generation, in real time, of the scheduling process' decision (Gargouri *and al*., 2003).

In general, the scheduling problems are multi-criterion optimization problems or muti-objectives (Gzara, 2001). Solving a problem of optimization consists in finding the best solutions checking a set of constraints and objectives defined by the user (Barichard *and al*., 2003). To determine if a solution is better than another, it is necessary that the problem introduces a comparison criterion. Thus, the best solution, also called optimal solution, is the solution leading to the best evaluation in comparison with the chosen criterion. In our work, we interested in the one-machine scheduling problem. In particular, we study the minimization criteria of the out-of-date products' cost, and that of the discount of distribution; the optimal solution corresponds to that whose cost is minimal.

2. OPTIMIZATION PROBLEMS

2.1 Resolution methods' classes

The optimization problems are in general difficult to solve. Several methods are used in order to find a satisfactory response to these problems. The exact methods and the approximate ones are distinguished (Barichard, 2003).

The exact methods examine, in an implicit way, the totality of the space of research and produce, in theory, an optimal solution. When the computing time necessary to reach this solution is excessive, the approximate methods can produce a quasi-optimal solution at the end of a reasonable computing time.

2.2. Multi-objectives optimization problems

The majority of the real optimization problems are described using several contradictory objectives or criteria to be optimized at the same time; they are known as multi-objectives problems. The sought optimal solution represents a set of points called «the face of Pareto », corresponding to the best possible compromise to solve this type of problem.

A multi-objectives optimization problem is formal-ized, generally, in the following manner:

$$\begin{cases} \text{minimizing} \quad f(x) \ \textit{(k functions to be minimized)} \\ \text{such as} \quad g(x) \le 0 \quad \textit{(m constraints to be satisfied)} \\ x \in R^n,\ f(x):\ R^n \to R^k \ \text{and} \ g(x): R^n \to R^m \end{cases}$$

When k is equal to 1, the optimization problem is known as mono-objective.

To optimize a function of a given problem, it is necessary to determine the set of solutions. There exist two types of sets: the values' set which can be taken by the variables, said search space, and the values' set of the variables satisfying the constraints, known as realizable space.

2.3 Dominance concept

The optimal solution of the multi-objectives optimization problem, constituting a set of points, proves that is necessary to define an order relation between these elements known as a dominance relation, to identify the best compromises. The dominance rule is a constraint that can be added to the initial problem without changing the value of the optimum (Jouglet, 2002).

The most used is defined in the Pareto sense.

- Dominance (within the meaning of Pareto)

Definition 1

For a minimization problem and two vectors u and v,

u dominate v, $u \prec v$, if and only if $f(u) < f(v)$

u dominate slightly v, $u \preceq v$, if and only if $f(u) \leq f(v)$

u is incomparable with v , $u \sim v$,if and only if $f(u) \not\leq f(v)$ and $f(v) \not\leq f(u)$.

- Global Optimality (in the Pareto sense)

Definition 2

A decision vector is known as overall optimal if and only if: $\nexists y \in c$ such as $y < x$; c represents the set of the problem's potential solutions; $f(x)$ is called, in this case, effective solution.

- Local Optimality

Definition 3

A decision vector is known as locally optimal if and only if, for $d > 0$ fixed, $\nexists y \in c$ such as:

$f(y) \in b(f(x), d)$ and $x < y$, where $b(f(x), d)$ is a ball of centre $f(x)$ and of radius $d > 0$.

3. PROBLEMATIC

The problem is how to build a multi-criterion scheduling adapted to agro alimentary industries. Among the constraints and the criteria specific to agro-food industry, one can distinguish the out-of- date of the products and the discount of distribution.

The objective is then to select among the set of candidates operations the one which presents the best reducing compromise between the various criteria and by filtering the initial search space.

The decision to eliminate or to maintain an operation making it possible to avoid the time limitation of certain components, involves the costs' reduction of these out-of-date components.

4. THE OPERATIONS' DOMINANCE'S RULES

Notations

t_i^x : effective starting time of operation manufacture O_i on post x

r_i : earliest starting time of the operation O_i

g_i : effective completion time of the operation O_i

p_i : processing time of the operation O_i

P_i : finished product of the operation O_i

c_{ik} : kth component of the components set of the operation O_i

v_{ik} : validity limit date of the component c_{ik}

C_{P_i} : completion time of the product P_i

$d_{P_i}^{liv}$: delivery date of the product P_i

d_{fP} : completion time of the sequence P

DV_{P_i} : lifespan of the product P_i

DR_{P_i} : return delay of the product P_i

P_{ik}^{rev} : cost price of the component c_{ik} of the product P_i

$P_{P_i}^{ven}$: unit selling price of the product P_i

$C_{P_i}^{stk}$: cost of storage per unit of time of a unit of the product P_i .

4.1. Problem formulation

Let E a set of n operations to be scheduled between two sequences P and A of already sequenced operations. For scheduling a couple of operations O_i and O_j of the set E of the candidates operations, the problem is to determine which of these operations is to be sequenced at first, i.e. the dominant operation, in order to minimize:

- the cost of the out-of-date products,
- the cost of the distribution discount.

So, it is able to filter the search space to build an optimal scheduling.

4.2. Branch-and- bound approach

The heuristic used is based on the branch-and-bound approach (Baptiste *and al*., 1996) (Bratcu *and al*., 1996) (Aggoune, 2004). It is one of the methods of constraints propagation to solve the problems of one-machine scheduling. It consists in using the initial constraints of the problem to develop and deduce new more strict constraints (Baptiste *and al*., 2001) (Le Pape, 1995): Detection of a situation to making a decision. This procedure is based on the technique of Edge Finding. It is a question of applying the technique of "branching" which consists in scheduling a set of operations which use the same machine. The "Bounding" makes it possible to deduce for some operations not belonging to the set E from the operations if they must, can or cannot be carried out before (or afterwards) the elements of E.

These deductions make it possible to generate new relations of order and new bounds of time (Gargouri *and al*., 2003).

The branch-and-bound algorithm is carried out by dynamically building a search tree. The root of the tree is a node at level 1 indicating an initial scheduling S. For this node there are as many nodes child of level 2 as the possible permutations to have corresponding schedulings. If the dominance relation is satisfied, search continues in this branch. On the other hand, if it is not satisfied, search is fallen through with this branch. In general, the dominance relations improve the efficiency of a branch-and-bound algorithm by constraining the search space (Allahverdi *and al*., 2005).

The proposed algorithm is applied to obtain an initial total cost (upper bound). During the branch-and-bound search, this upper bound is updated whenever a feasible scheduling is obtained which has a lower total cost. During the search, any incomplete branch which has a cost which is higher than the current upper limit is fathomed, i.e., this case is eliminated from the search space.

The proof of optimality is then performed "on the fly", by dynamically reducing the cost after that the first optimal solution has been found and continuing the exploration of the same search tree (Caseau *and al.*, 1995).

Algorithm

Beginning

1. Initialization $t, S = \{O_i, i = 1...n\}, O_i = \{r_i, p_i\}, K(S), A = \{r_A, p_A\}$
2. while $j < k$ do
3. For $i = 1...n$ do
4. Permute O_i to find S_j
5. Calculate t_i et C_i
6. End of For
7. Calculate $K_j(S)$
8. If $K_j(S) < K(S)$ then
9. S ß S_j
10. Else S ß S
11. End of If
12. $j = j + 1$
13. End of while

End

4.3. Formulation of the costs in the general case

In the general case, the cost of the out-of-date products $K_1(S)$ and that of the distribution discount $K_2(S)$ are written as following :

$$K_1(S) = \sum_i a_i \sum_k P_{ik}^{rev} \left(\frac{\max\left(0, t_i^x - v_{ik}\right)}{\left(t_i^x - v_{ik}\right)} \right) \quad (1)$$

$$K_2(S) = \sum_i b_i \max\left(0, d_{P_i}^{liv} - C_{P_i}\right) \times \left(\frac{P_{P_i}^{ven}}{DV_{P_i} - DR_{P_i}} + C_{P_i}^{stk} \right) \quad (2)$$

For the multi-objective evaluation, the objective function $K_{tot}(s)$ is reduced to the minimization of the balanced sum of the criteria relating to the use of the aggregation operator OWA (Yager, 1988).

$$K_{tot}(s) = \sum_{i=1}^{n_c} K_i(s) \quad (3)$$

where n_c represented the number of criteria.

Remarks:

- The variables, for a scheduling S_i and a scheduling S_j, are the effective starting time t_i^x and the completion time of the product C_{P_i} .
- The coefficients a_i and b_i favour, from cost point of view, a product compared to another.

4.4 Exploration and study of the various cases of scheduling

To have an optimal scheduling, noted "S_{op}" which optimizes the quoted criteria, all these cases which can occur, in this part, are going to be studied then compared. Best scheduling is then going to be given in the sight of its exploitation.
For two operations O_i and O_j, the following cases are feasible fig.1:

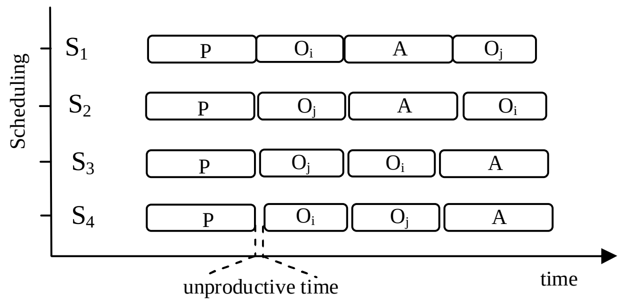

Fig.1. Cases of scheduling

Case1: Scheduling S_1 at an instant t
Case2: Scheduling S_2, relating to the exchange between the operation O_i and the operation O_j
Case3: Scheduling S_3, relating to the insertion of operation O_j just after the sequence P
Case4: Scheduling S_4, relating to the permutation between the operation O_i and the operation O_j of case3
Remark: when the number of operations becomes significant, the cases of scheduling vary exponentially and the computation becomes complex.

4.4.1 Calculation of the costs for the various cases

∗ *Case1*
At a given instant t, for the scheduling S_1, where the sequences A and P are already scheduled; the cost of the out-of-date products $K_1(S)$ is formulated by the expression (1), and the cost of the distribution discount $K_2(S)$ is formulated by the expression (2).
∗ *Case2*
The exchange between operation O_i and operation O_j, while keeping the sequence A, leads to scheduling S_2. In this case the dates of beginning and the end of the operations are updated as follow:

t_{2j}^{x} : effective starting time of the operation O_j,

$$t_{2j}^{x} = \max\left(d_{fP}, r_j\right)$$

g_{2j} : completion time of the operation O_j,

$$g_{2j} = \max\left(d_{fP}, r_j\right) + p_j$$

t_{2A}^{x} : effective starting time of sequence A,

$$t_{2A}^{x} = \max\left(r_A, g_{2j}\right)$$

g_{2A} : effective completion time of sequence A,

$$g_{2A} = \max\left(r_A, g_{2j}\right) + p_A$$

t_{2i}^{x} : effective starting time of O_i,

$$t_{2i}^{x} = \max\left(r_i, g_{2A}\right)$$

g_{2i} : completion time of O_i, $g_{2i} = \max\left(r_i, g_{2A}\right) + p_i$

* *Case3*

Insertion of operation O_j just after the sequence P, the scheduling S_3 is obtained and the dates are updated as follow:

t_{3j}^{x} : effective starting time of the operation O_j,

$$t_{3j}^{x} = \max\left(d_{fP}, r_j\right)$$

g_{3j} : completion time of the operation O_j,

$$g_{3j} = \max\left(d_{fP}, r_j\right) + p_j$$

t_{3i}^{x} : effective starting time of operation O_i, $t_{3i}^{x} = \max\left(r_i, g_{3j}\right)$

g_{3i} : completion time of the operation O_i,

$$g_{3i} = \max\left(r_i, g_{3j}\right) + p_i$$

t_{3A}^{x} : effective starting time of sequence A,

$$t_{3A}^{x} = \max\left(r_A, g_{3i}\right)$$

g_{3A} : effective completion time of sequence A, $g_{3A} = \max\left(r_A, g_{3i}\right) + p_A$

* *Case4*

Insertion of operation O_j just after the operation O_i, one will have the scheduling S_4 and the following dates:

t_{4i}^{x} : effective starting time of operation O_i,

$$t_{4i}^{x} = \max\left(d_{fP}, r_i\right)$$

g_{4i} : completion time of operation O_i,

$$g_{4i} = \max\left(d_{fP}, r_i\right) + p_i$$

t_{4j}^{x} : effective starting time of operation O_j, $t_{4j}^{x} = \max\left(r_j, g_{4i}\right)$

g_{4j} : completion time of operation O_j, $g_{4j} = \max\left(r_j, g_{4i}\right) + p_j$

t_{4A}^{x} : effective starting time of sequence A,

$$t_{4A}^{x} = \max\left(r_A, g_{4j}\right)$$

g_{4A}: effective completion time of sequence A,

$$g_{4A} = \max\left(r_A, g_{4j}\right) + p_A$$

4.4.2 *Example*

Let us consider the following example:

- an operation O_1 is made up of two components c_{11} and c_{12} ; $O_1 = \{c_{11}, c_{12}\}$;
- an operation O_2 is made up of three components c_{21} , c_{22} et c_{23} ; $O_2 = \{c_{21}, c_{22}, c_{23}\}$;
- an operation O_3 is made up of two components c_{31} and c_{32} ; $O_3 = \{c_{31}, c_{32}\}$;
- the sequence A is made up of two operations O_4 and O_5, such as $O_4 = \{c_{41}\}$ and $O_5 = \{c_{51}, c_{52}\}$;
- $a_i = b_i = 1$

For this example, the cost of the out-of-date products is equal to:

$$K_1(S) = \sum_{k=1}^{2} P_{1k}^{rev}\left(\frac{\max\left(0, t_1^x - v_{1k}\right)}{\left(t_1^x - v_{1k}\right)}\right) + \sum_{k=1}^{3} P_{2k}^{rev}\left(\frac{\max\left(0, t_2^x - v_{2k}\right)}{\left(t_2^x - v_{2k}\right)}\right) + \sum_{k=1}^{2} P_{3k}^{rev}\left(\frac{\max\left(0, t_3^x - v_{3k}\right)}{\left(t_3^x - v_{3k}\right)}\right) + \sum_{k=1}^{1} P_{4k}^{rev}\left(\frac{\max\left(0, t_4^x - v_{4k}\right)}{\left(t_4^x - v_{4k}\right)}\right) + \sum_{k=1}^{2} P_{5k}^{rev}\left(\frac{\max\left(0, t_5^x - v_{5k}\right)}{\left(t_5^x - v_{5k}\right)}\right) \quad (4)$$

The cost of the distribution discount is equal to:

$$K_2(S) = \sum_{i=1}^{5} \max\left(0, d_{P_i}^{liv} - C_{P_i}\right) \times \left(\frac{P_{P_i}^{ven}}{DV_{P_i} - DR_{P_i}} + C_{P_i}^{stk}\right) \quad (5)$$

The data relating to this example are defined in the table1.

Table1. The data relating to the example

	O_1	O_2	O_3	O_4	O_5
r_k	1	2	1	4	4
p_k	2	3	2	6	6
v_{i1}	12	11	11	12	15
v_{i2}	8	2	12	-	6
v_{i3}	-	13	-	-	-
P_{i1}^{rev}	1	2	1	1	1
P_{i2}^{rev}	1	2	3	-	2
P_{i3}^{rev}	-	1	-	-	-
$P_{P_k}^{ven}$	3	3	5	6	6
DV_{P_k}	18	12	16	11	11
DR_{P_k}	2	4	6	5	5
$d_{P_k}^{liv}$	8	9	7	10	10
$C_{P_k}^{stk}$	1	2	2	1	1

By application of our heuristics, the following experimental results are obtained, table 2. The unfeasible scheduling are eliminated.

Table2. Experimental results

Sequences				K_1	K_2	K_{tot}
O_1	O_2	O_3	A	5	15.4	20.4
O_1	O_2	A	O_3	5	15.4	20.4
O_1	O_3	O_2	A	7	18.1	25.1
O_1	O_3	A	O_2	7	13.4	20.4
O_1	A	O_2	O_3	7	5.9	12.9
O_1	A	O_3	O_2	7	5.9	12.9
O_2	O_1	O_3	A	5	13	18
O_2	O_1	A	O_3	5	13	18
O_2	O_3	O_1	A	5	15.6	20.6
O_2	O_3	A	O_1	6	14.5	20.5
O_2	A	O_1	O_3	6	9.5	15.5
O_2	A	O_3	O_1	6	9.5	15.5
O_3	O_1	O_2	A	7	19.5	26.5
O_3	O_1	A	O_2	7	14.7	21.7
O_3	O_2	O_1	A	5	20.6	25.6
O_3	O_2	A	O_1	6	19.5	25.5
O_3	A	O_1	O_2	7	10	17
O_3	A	O_2	O_1	8	10	18

The obtained results show a great disparity between the minimum cost and the maximum cost, this disparity is due mainly to the cost of the distribution discount taking into account its importance. It is noted that a good profit was obtained. This approach gives us the optimal solution, but, due to computation time the practical application is limited to small or medium size problems.

5. CONCLUSION

The approach developed in this work provides the possibility to determine an optimal scheduling among several realizable ones; this optimal solution generates the minimization of the cost of the out-of-date products and of the cost of the discount of distribution. Indeed, realizing rules of dominance of the operations and the parameters necessary for the calculation of the costs as well as the data of stock, we can avoid the lapsing of certain components. To achieve this goal, one maintains in the search space the operations whose components have the shortest validity limit dates. On the contrary, the operations of which the lifespan is sufficiently long and generating the manufacture delay of other operations are eliminated from the search space.

The disadvantage of this approach lies in the computation complexity when the number of operations becomes significant because the number of scheduling cases vary exponentially. To cure this difficulty, the future work is directed towards the application of the approximate methods such as genetic algorithms.

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