Extension of the clustering identification by extending the Density Based Spatial Clustering of Applications with Noise approach to Multi-Input Multi-Output Piece Wise Affine systems: Application to an industrial robot

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Abstract: In this paper the problem of clustering based identification of a Multi-Input Multi-Output (MIMO) PieceWise Affine systems (PWA) is considered. This approach, originally designed for systems with a Multiple-Input Single-Output (MISO) structure, is carried out by three main steps which are data clustering, parameters vectors estimation and regions computing. Data clustering is the most important step because the two other steps depend on the results given by the used clustering algorithm. In case of MIMO PWA systems, we should cluster matrices of parameters which are considered as high dimensional data. However, most of the conventional clustering algorithms do not work well in terms of effectiveness and efficiency since the similarity assessment which is based on the distances between objects is fruitless in high dimension space. Therefore, we propose an extension of the DBSCAN (Density Based Spatial Clustering of Applications with Noise) clustering approach for the identification of MIMO PWA systems. The simulation results presented in this paper prove the performance of the suggested approach. An application of the proposed approach to an industrial robot manipulator is then presented in order to validate the simulation results.

Keywords: MIMO process, PWA model, DBSCAN clustering algorithm, identification.

1. INTRODUCTION

The PieceWise Affine (PWA) model is the most important class of hybrid systems since it is considered as an universal approximator having the ability to model any nonlinear system with arbitrary accuracy Bako and Yahya (2019), Paoletti et al. (2019), van den Boom et al. (2016), Kim (2020).
The PWA models are based on the fact that any real world system can be considered as a linear system in a local restricted region. The principal of these systems consists in decomposing the regression domain of the system into regions, then, an ARX sub-model is assigned to every one. There exist numerous approaches in the literature for the identification of PWA models Tian et al. (2011), Ferrari-Trecate et al. (2003), Bemporad et al. (2003), Juloski et al. (2005), Bemporad et al. (2005). We have advocated the use of clustering based approach Lassoued and Abderrahim (2014c), which is based on the fact that process has local affine behaviours. So, first of all, close regression vectors are gathered in local datasets. For each one, an ARX model is identified using least squares. After that, similar ARX models are assembled into clusters by the use of a convenient clustering technique Lassoued and Abderrahim (2019), Lassoued and Abderrahim (2014a). Finally, the obtained clusters are delimited with hyperplanes using the Supprot Vector Machine (SVM) approach Wang (2005).

The PWA clustering based identification approach was originally developed for systems having Multiple Inputs and Single Output (MISO). However, in case of identification of MIMO process, one possible solution is to identify for each output of the process a MISO PWA model, then merge them all in a single MIMO PWA model Vasak et al. (2006). For this solution, the submodels' number of the MISO PWA model must be a priori defined while the final number of regions will be determined by merging the identified MISO models into the MIMO one. Added to that, a transformation of the feature vector must be added in order to alleviate the problem of partitioning in arbitrary dimensions of the regressor vectors. In addition, optimal controller synthesis requires systems of MIMO

structure and compel MIMO PWA map identification.
In order to overcome these problems, we propose to extend the clustering based approach to MIMO PWA systems.

Obviously, data classification is the cornerstone for successful clustering based identification of PWA systems. However, the major problem in the identification of MIMO PWA systems is that the data classification step must deal with matrices of parameters instead of vectors of parameters. The dimension of these matrices depends on the order of sub-models and the number of outputs of the system. Therefore, it is difficult to differentiate similar data points from dissimilar ones because the metrics distance used to evaluate the similarity of data in the same group becomes insignificant in high dimensional space.
That's why, almost all conventional clustering algorithms lack of effectiveness and efficiency when data sets are in high dimensional spaces Steinbach et al. (2004).
The proposed identification approach is based on the DBSCAN clustering technique which has proven its performances with the SISO (Single Input Single Output) PWA systems. However, for MIMO PWA systems the dimension of the matrices of parameters to be classified affect the concept of research of similar objects which is based on the mesure of distances. Added to that, the choice of the synthesis parameters of this method becomes more difficult because identifying the same number of objects in a given neighborhood needs a large radius which will influence the resulting clusters. As a solution for these problems, we propose a modification in the computation of distances between objects to make the concept of similar patterns eventually meaningfull as the dimensionality of the data increases.
The main objectif of this paper is to represent robot manipulator with MIMO (PWA) model based on the input-output data.
Establishing a high accurate mathematical model from available system experimental data guaranty to get desired performance for model-based control and correct decision dealing with faults detections. Robot manipulators are characterized by lack of details according to their dynamic parameters in relation with inertia elements or friction parameters. Therefore, robot modeling and identification is a complex task and will need advanced techniques to get the best model.

This paper is organized as follows. In section 2, a detailed description of the MIMO PWA systems is introduced. Section 3 develops the extented clustering based identification approach to MIMO PWA systems. In section 4, we propose a solution to overcome the problems faced with the DBSCAN clustering technique in the case of high dimensional objects. The performance of the proposed approach is evaluated with a simulation example in section 5. Section 6 is devoted to the real application of the proposed approach consisting in an industrial robot manipulator.

2. PWA DESCRIPTION OF MIMO SYSTEMS

The regression domain of a PieceWise Affine system is partitioned into a collection of regions $P = \{H_i\}$, where each region H_i is associated with a system parameter θ_i. The PWA system is defined jointly by its collection of sub-models (θ_i, H_i). Switching from submodel to another depends on the region where the regression vector $\varphi(k)$ is located.

A PWA representation of a MIMO system is defined as:

$$y(k) = \begin{cases} \theta_1^T \varphi(k) + e(k) \quad if \quad \varphi(k) \in H_1 \\ \vdots \\ \theta_s^T \varphi(k) + e(k) \quad if \quad \varphi(k) \in H_s \end{cases} \tag{1}$$

$$\varphi(k) = \begin{bmatrix} \phi_y \\ \phi_u \\ 1 \end{bmatrix}, \tag{2}$$

$$\phi_y = \begin{bmatrix} y_{(1)}(k-1) \\ \vdots \\ y_{(1)}(k-n_a) \\ \vdots \\ y_{(m)}(k-1) \\ \vdots \\ y_{(m)}(k-n_a) \end{bmatrix} \quad \phi_u = \begin{bmatrix} u_{(1)}(k-1) \\ \vdots \\ u_{(1)}(k-n_b) \\ \vdots \\ u_{(r)}(k-1) \\ \vdots \\ u_{(r)}(k-n_b) \end{bmatrix} \tag{3}$$

$$\theta_i = \begin{bmatrix} \theta_{(1),i} \; \theta_{(2),i} \; \cdots \; \theta_{(m),i} \end{bmatrix} \tag{4}$$

where

- $s \in \mathbb{N}$, $r \in \mathbb{N}$ and $m \in \mathbb{N}$ are respectively the number of sub-models, the number of inputs and the number of outputs.

- $u(k) \in \mathbb{R}^r$ and $e(k) \in \mathbb{R}^m$ are respectively the input and the additive noise.

- $y_{(j)}$ is the *j-th* output,
  $y(k) = \left[y_{(1)}(k), y_{(2)}(k), \ldots, y_{(m)}(k)\right]^T \in \mathbb{R}^m$ is the system output.

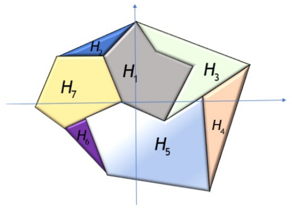
- $\theta_i \in \mathbb{R}^{(1+m.n_a+r.n_b)\times m}$ is the parameter matrix of the i^{th} sub-model having n_a and n_b as orders.

- $\varphi(k) \in \mathbb{R}^d$ is the regressor vector of length $d = 1 + m.n_a + r.n_b$.

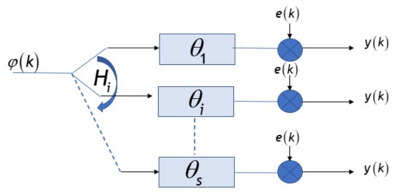
- H_i is the polyhedral partition of the i^{th} sub-model. The polyhedral partitions $H_i, i = 1, .., s$ must verify the following assumptions:

$$\begin{cases} \bigcup_{i=1}^{s} H_i = H \\ H_i \bigcap H_j = \emptyset \quad \forall i \neq j \end{cases} \tag{5}$$

The Figure 1 depicts the structure of a PWA system.



(a) Polyhedral regions



(b) PWA structure

Fig. 1. PieceWise Affine system.

3. EXTENTED CLUSTERING-BASED IDENTIFICATION APPROACH TO MIMO SYSTEMS

The main difficulty of PWA model identification arises in the inseparable steps of identification of regions defining the regressor domain and the identification of parameters of the corresponding affine submodels.
Most of PWA systems identification methods can only deal with single-input, single-output (SISO) systems. While a few identification methods for MIMO PWA systems in the state space form exist Bako et al. (2013) Mejari et al. (2020) Verdult and Verhaegen (2004). These methods, called subspace methods, generally need the restrictive assumption of a minimum dwell time in each discrete state. However, there are few results on MIMO PWA systems described in Input-Output form in the literature Breschi et al. (2016) Hure and Vašak (2017).

Clustering-based identification approach is dedicated to systems having MISO structures. However, the extension of this approach to the case of MIMO PWA systems is possible. Which is the objective of this work.

The main steps of the proposed method can be summarized as follows:

(1) Create the N pairs of data $\{\varphi(k), y(k)\}_{k=1}^{N}$ from the given dataset $\{u(k), y(k)\}_{k=1}^{N}$.

(2) Construct local sets C_k to each pair of data. Every set C_k contains $\{\varphi(k), y(k)\}$ and their $(n_\rho - 1)$ nearest neighbors Ferrari-Trecate et al. (2001). The parameter n_ρ defining the cardinality of datapoints that belongs to C_k is randomly chosen.

(3) Compute the local parameter matrix $\{\theta_k\}_{k=1}^{N}$ for each data in $\{C_k\}_{k=1}^{N}$ using the least square method:

$$\theta_k = \left[\, \theta_{(1),k} \;\; \theta_{(2),k} \;\; \cdots \;\; \theta_{(m),k} \,\right]$$

$$\begin{cases} \theta_{(1),k} = (\phi_k^T \phi_k)^{-1} \phi_k^T Y_{(1),k} \\ \vdots \\ \theta_{(m),k} = (\phi_k^T \phi_k)^{-1} \phi_k^T Y_{(m),k} \end{cases} \tag{6}$$

where

$$\begin{aligned} \phi_k &= \left[\varphi(k_1) ... \varphi(k_{n_\rho})\right]^T, \\ Y_{(j),k} &= \left[y_{(j)}(k_1), ..., y_{(j)}(k_{n_\rho})\right]^T; j = 1, \dots, m \end{aligned} \tag{7}$$

$(k_1, ..., k_{n_\rho})$ are the indexes of the elements belonging in C_k.

(4) Cluster the local parameter matrix $\{\theta_k\}_{k=1}^{N}$ into s disjoint clusters using a suitable classification technique which can cluster data in high dimensional spaces. Notice that each local parameter matrix θ_k is concatenated of m parameter vectors, one for each output, so, it has a dimension of $((m.n_a + r.n_b) \times m)$. It is necessary to choose the convenient classification technique which can classify data of large dimension. In this paper, the proposed technique of clustering is an extension of the DBSCAN algorithm which is detailed in section 4.

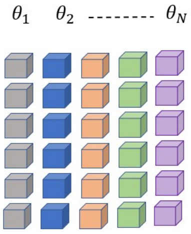
(5) Determine the final sub-models parameters matrices $\{\theta_i\}_{i=1}^{s}$ which represent the centers of clusters already determined by the clustering technique.

(6) Estimate the polyhedral partitions $\{H_i\}_{i=1}^{s}$ *i.e.* estimate the hyperplanes separating H_i from H_j, $i \neq j$. This is a standard pattern recognition/classification problem that can be solved by several established techniques. The most common technique is the Support Vector Machines (SVM) Wang (2005).

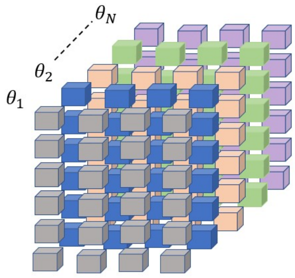
4. DBSCAN CLUSTERING FOR MIMO PWA SYSTEMS

Clustering has been a widely used technique to discover the natural grouping of a set of objects in such a way that objects in the same group (called a cluster) are more similar (in some sense) to each other than to those in other groups. Clustering algorithms are used to partition data into clusters, where the number of clusters can either be prescribed, or can be a result of the clustering procedure. In general, different clustering algorithms Benabdellah et al. (2019) Ezugwu et al. (2022) can be broadly classified into partitioning-based Äyrämö and Kärkkäinen (2006), density-based Bhattacharjee and Mitra (2021), and hierarchical-based algorithms Hassan et al. (2020).
For the density-based clustering methods, DBSCAN (Density Based Spatial Clustering of Applications with Noise) algorithm is considered as the most efficient one Jeffrey Erman (2006) Campello et al. (2020). For this algorithm, data are separated on different groups as their density connection varies. In a given dataset the point are labeled core point if the number of their neighbors within a priory fixed radius (ϵ) is greater than chosen density threshold ($MinPts$). If the data point is a neighbor of a core point

and doesn't satisfy the core point label condition it will be considered a border point. Otherwise, the data point is categorized as a noise point. In the DBSCAN algorithm distances separating one point from all the different points in the dataset are computed for point similarity assessments and labeling. This algorithm is well sweated for the identification of PWA hybrid system because data that are used in one sub-model will have local parameter vectors close to each other (density connected) Lassoued and Abderrahim (2019) Lassoued and Abderrahim (2014b). For SISO system where the local parameter are all vectors (dimension $(1 + n_a + n_b) \times 1$), the Euclidian norm is sufficient to measure distance between different points to confirm their similarity and then data are easily labeled Walters-Williams and Li (2010). But, when dealing with MIMO PWA systems, we have local parameter matrices (dimension $(1 + m.n_a + r.n_b) \times m$) instead of vectors.



(a) SISO system



(b) MIMO system

Fig. 2. Distribution of local parameters for a PWA system.

The local parameter matrices are considered multi dimensional data and this is the major drawback for existing clustering algorithm known as the 'curse of dimensionality'. In high-dimensional spaces, traditional clustering algorithms fail to provide satisfactory result Pavithra and Parvathi (2017). The problem is that the concept of nearest neighbors becomes meaningless for multi-dimensional data because in high dimension space the distance separating two point is not significance of their similarity .
For clustering, this means that in high dimensions, objects are almost equi-distant, and clustering based on any such similarity assessment is meaningless.
One other appearance of the 'curse of dimensionality' that affects the density definition. Thus, to identify the same number of objects in the neighborhood as an assessment of the density, requires dramatically larger radius Aggarwal et al. (2001). As a consequence, different clusters will be merged into one cluster as the larger radius cover the data space. Reducing the radius is also problematic, as the space is sparse and density assessment would be based on few objects, thereby being sensitive to outliers. To overcome this problem many norms are defined instead of the Euclidian norm to measure the distance between two matrices Campello et al. (2020). In our case, clustering the local parameters matrices, we don't need only that the matrices are close one to each other but we need to prove that every element in one matrix parameter is close enough to the corresponding element of the second matrix so we can call them close neighbors. To do so, we define new distance that can compute the similarity of two matrices and confirm if they are close neighbors as below:

$$dis\,(\theta_k, \theta_l) = \|V\theta_k, V\theta_l\|_2; \quad k, l = 1, \dots N \tag{8}$$

$$V\theta_k = \begin{bmatrix} \theta^T_{(1),k} \; \theta^T_{(2),k} \; \cdots \; \theta^T_{(m),k} \quad \Omega \end{bmatrix} \tag{9}$$

$$\Omega = \begin{bmatrix} \theta_{(p),k} - \theta_{(q),k} \end{bmatrix}^T_{\substack{q,p=1\dots m \\ q \neq p}} \tag{10}$$

The vector $V\theta_k$ is a one dimensional representation of the corresponding matrix θ_k. It's computed by the concatenation of the θ_k column's and the column's difference vectors. The $V\theta_k$ structure will allow to compare the similarity between θ_k matrices by evaluating the similarity in one dimensional space. Including the column's difference vectors in the $V\theta_k$ structure requires similar matrices to have corresponding columns placed at similar distances one from each other.

5. SIMULATION EXAMPLE

We propose a simulation example to evaluate the performance of the proposed method. For this aim, we consider the same system used in Hure and Vašak (2017). It consists of a coupled PWA system having two inputs, two outputs, eight first order discrete transfer functions $G_{i,\lambda}$ and eight gains $K_{i,\lambda}$ $(i = 1, 2\ \lambda = 1, \dots, 4)$ as described by figure 3.

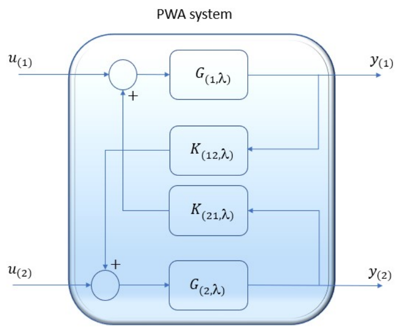


Fig. 3. PieceWise Affine system.

The multivariable system of figure 3 can be described by a MIMO PWA model defined as follows:

$$y(k) = \begin{cases} \begin{bmatrix} 0.9 & 0 & 0.1 & 0 \\ 0 & 0.6 & 0 & 0.4 \end{bmatrix} \varphi_k & if \quad \lambda = 1, \\ \begin{bmatrix} 0.7 & 0 & 0.3 & 0 \\ 0.15 & 0.6 & 0 & 0.4 \end{bmatrix} \varphi_k & if \quad \lambda = 2, \\ \begin{bmatrix} 0.7 & -0.2 & 0.3 & 0 \\ 0.15 & 0.8 & 0 & 0.2 \end{bmatrix} \varphi_k & if \quad \lambda = 3, \\ \begin{bmatrix} 0.9 & -0.2 & 0.1 & 0 \\ 0 & 0.8 & 0 & 0.2 \end{bmatrix} \varphi_k & if \quad \lambda = 4. \end{cases} \tag{11}$$

$$\lambda = \begin{cases} 1, & if \; y_{(1)}(k-1) > 0.5, \quad y_{(2)}(k-1) > 0.5, \\ 2, & if \; y_{(1)}(k-1) \leq 0.5, \quad y_{(2)}(k-1) > 0.5, \\ 3, & if \; y_{(1)}(k-1) \leq 0.5, \quad y_{(2)}(k-1) \leq 0.5, \\ 4, & if \; y_{(1)}(k-1) > 0.5, \quad y_{(2)}(k-1) \leq 0.5, \end{cases} \quad (12)$$

$$\varphi_k = \left[y_{(1)}(k-1) \; y_{(2)}(k-1) \; u_{(1)}(k-1) \; u_{(2)}(k-1) \right]^T \quad (13)$$

The MIMO PWA system is characterized by four operating modes ($s = 4$).
The excitation input *u(k)* is a white noise sequence with uniform distribution in the box $[-4, 6] \times [-2, 4]$ and length $N = 4000$.
The output used in the training phase is corrupted by an additive zero-mean white noise e_k with Gaussian distribution. The effect of measurement noise on the output signal is quantified through the Signal-to-Noise Ratio (SNR=30dB), that is defined for the *i-th* output channel as:

$$SNR_i = 10 \log \frac{\sum_{k=1}^{N} (y_i(k) - e_i(k))^2}{\sum_{k=1}^{N} e_i^2(k)} \quad (14)$$

The proposed clustering identification method is then applied in order to represent the simulated model by a MIMO PWA model with the following synthesis parameters:

- $n_\rho = 18$, the parameter defining the cardinality of the local sets.
- $MinPts = 20$, the minimum number of clusters objects.
- $\epsilon = 0.1$, the distance defining the neighborhood of an object of the clusters.

The objectif of the proposed approach is the determination of the number of sub-models s, the parameters of each sub-model and the regions where each sub-model is defined. The estimated parameters matrices are as follows:

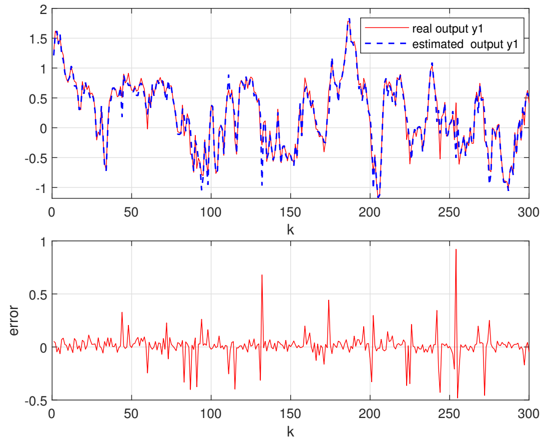
$$\theta_1 = \begin{bmatrix} 0.8843 & -0.0021 & 0.0098 & -0.0117 \\ 0.0078 & 0.6156 & 0.0166 & 0.4118 \end{bmatrix}^T$$

$$\theta_2 = \begin{bmatrix} 0.7073 & -0.0015 & 0.3008 & -0.0033 \\ 0.1645 & 0.6378 & -0.0401 & 0.4123 \end{bmatrix}^T$$

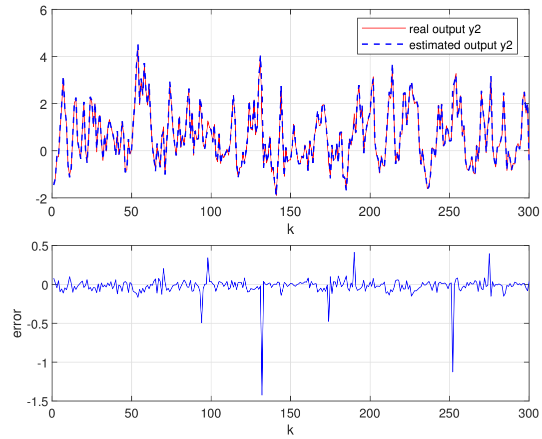
$$\theta_3 = \begin{bmatrix} 0.7194 & -0.1993 & 0.2940 & -0.0110 \\ 0.1463 & 0.8131 & -0.0078 & 0.1974 \end{bmatrix}^T$$

$$\theta_4 = \begin{bmatrix} 0.8990 & -0.1884 & 0.0964 & 0.0123 \\ -0.0070 & 0.8091 & -0.0039 & 0.2226 \end{bmatrix}^T$$

The extented SVM approach is then used to assign each submodel to the corresponding region in order to determine the estimated outputs.
For the sake of visualization, only the samples from time 1 to 300 related to the first and second output are represented in Figure 4 where the estimation error is also presented. In Figure 5 the switching instances related to the active submodels are presented.



(a) Real and estimated output y1



(b) Real and estimated output y2

Fig. 4. Identification results.

The obtained results prove the good performance of the suggested approach. In fact, the evolution of the simulated system output is very close to the output of the identified model. In addition, the four identified sub-models contribute in the evolution of the estimated system's output.

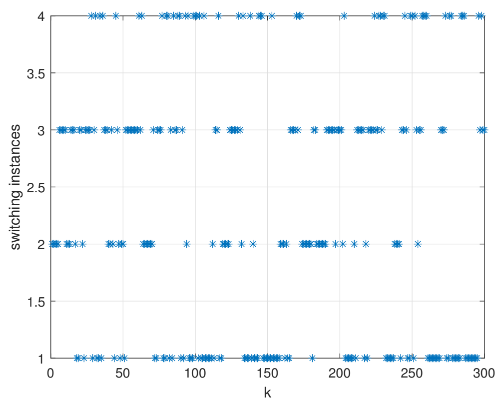


Fig. 5. Switching Instances.

6. EXPERIMENTAL EXAMPLE

Industrial serial robot manipulators have become a necessary automation tool in order to increase productivity and flexibility of production units of modern industry.
Obviously, an appropriate dynamical model is necessary in order to conveniently achieve the desired task.
This model can be defined through a detailed analysis of the phenomena described by the robot system based on the laws of physics and mechanics resulting in a set of differential equations. This approach can lead to very complicated models that cause problems of exploitation and implementation. However, for engineering, a mathematical model must provide a compromise between accuracy and simplicity of operation. A solution to this problem consists in using the identification approach which allows to build a mathematical model from observed input-output data.

The clustering identification approach for MIMO PWA systems is then applied to an industiral robot manipulator. In fact, we have used a benchmark dataset for a full robot movement of a KUKA KR300 R2500 ultra SE industrial robot which is published by the authors of Weigand et al. (2022).
Common applications of the KUKA KR300 R2500 ultra robot include: 3D printing, assembly, dispensing, finishing, material handling, palletizing, and pick and place.

6.1 Process description

The robot studied in this paper is characterized by 6 joints. Each one is modified by an electric servo-motor.
The servo- motors are controlled by servo-drives that generate the necessary current to apply a torque needed to move the joint to the desired direction with a specific angle. The robot encounters backlash in all joints, and it behaves with a great non linearity. Figure 6 represents a photo of the robot.

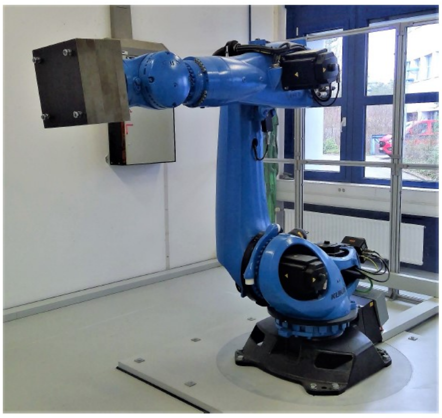


Fig. 6. Real photo of the industrial robot KUKA KR300.

By refering to the literature Ding et al. (2018), a n-DOF (Degrees Of Freedom) serial manipulator can be described by the following dynamical model:

$$\tau = \tau_f + M(q)\ddot{q} + C(q, \dot{q}) + G(q) \tag{15}$$

where τ is the joint torque, q is the joint position, $\dot{q}$ is the joint velocity, and $\ddot{q}$ is the joint acceleration, τ_f represents the n-dimensional joint friction vector, $M(q)$ is a $(n \times n)$ inertial matrix, $C(q, \dot{q})$ is a n-dimensional vector including Coriolis and centrifugal forces, and $G(q)$ is n-dimensional gravity vector.
The design of an identification experiment requires that the excitation trajectory must be sufficient to provide fast and accurate parameters estimation. Moreover, it must be designed carefully to reduce the effect of the disturbance. The choice of parameterization for the excitation trajectory is a very important issue. It directly determines the number of parameters in the optimization problem and the effort needed to calculate velocity and acceleration from the joints positions measurements. Many works were proposed in the litterature on how to find the excitation trajectory for the dynamical identification of manipulator. In this paper, we adopt the excitation trajectories proposed with the benchmark in Weigand et al. (2022).
Each trajectory is formulated as the sum of sine and cosine functions to ensure continuous differentiability. The trajectory for joint n of a manipulator is designed as:

$$q_n(k) = \sum_{v=1}^{V} \left(\frac{A_{v,n}}{wv} \sin(wvk) - \frac{B_{v,n}}{wv} \cos(wvk) \right) \tag{16}$$

where the frequency grid in the design of experiments ranges from the base frequency $w = \frac{1}{60} \cong 0.0166$ Hz to a maximum frequency of 1 Hz and the parameter $V = 10$. Furthermore, the sine and cosine coefficients $A_{v,n}$ and $B_{v,n}$ reduces the number of optimization variables compared to a discrete-time formulation by multiple orders of magnitude.

When the robot joints repeatedly track the excitation trajectories with the PID controllers, motor current and joint positions can be sampled in the time domain. The motor current can be transformed into joint torques with a simple torque constant.
The measured quantities are the joints displacements in degree and the corresponding torques applied by the motors assigned to the joints. The joints displacement are the model outputs and they are directly measured through incremental encoders mounted on the joint. The torques applied by the servo motors are the inputs. As the generated torque is linear dependent to the servo-motor current it will be computed through the measurement of the current by means of amperemeters. The measurement acquisition step is carried out by the authors of Weigand et al. (2022). The published benchmark contains the input-output measurements that we use for identification.

So, the dataset used for the identification of a forward model of the robot consists in considering the 6 motor torques $\{\tau_i\}_{i=1}^{6}$ as inputs and the 6 joint positions $\{q_i\}_{i=1}^{6}$ as outputs. From the given input output measurement, we have picked out 1500 samples as illustrated in figure 7 and figure 8.

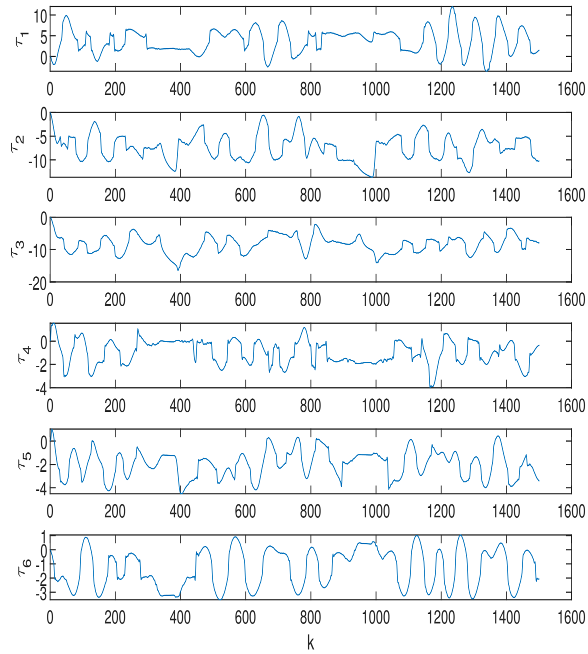


Fig. 7. Inputs signals: motor torques τ_i in (Nm).

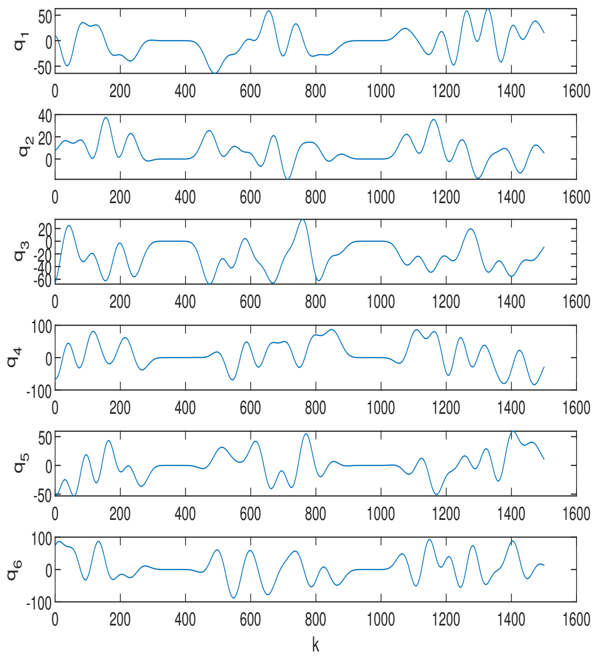


Fig. 8. Outputs signals: joints positions q_i in (deg).

6.2 Identification results

The proposed identification method is applied in order to represent the industrial robot manipulator by a PWA MIMO system.
The used synthesis parameters are:
$n_\rho = 61$, $MinPts = 22$ and $\epsilon = 8$.
The orders of each sub-model n_a and n_b are fixed to the value 2 since this system can be considered as a second-order system around each operating point. The following structure is then adopted:

$$y(k) = \begin{cases} \theta_1^T \varphi(k) + e(k) \;\; if \;\; \varphi(k) \in H_1 \\ \vdots \\ \theta_s^T \varphi(k) + e(k) \;\; if \;\; \varphi(k) \in H_s \end{cases} \tag{17}$$

$$\varphi(k) = \begin{bmatrix} \phi_y \\ \phi_u \end{bmatrix} \tag{18}$$

$$\phi_y = \begin{bmatrix} y_1(k-1) \\ y_1(k-2) \\ \vdots \\ y_6(k-1) \\ y_6(k-2) \end{bmatrix} \quad \phi_u = \begin{bmatrix} u_1(k-1) \\ u_1(k-2) \\ \vdots \\ u_6(k-1) \\ u_6(k-2) \end{bmatrix} \tag{19}$$

where $\{u_i = \tau_i\}_{i=1}^{6}$ are the inputs and $\{y_i = q_i\}_{i=1}^{6}$ are the ouputs of the system.
By applying the proposed clustering identification method, we obtain as results:

- The number of sub-models s is equal to 4.
- The estimated parameters matrices as well as the corresponding regions are also determined. Figure 9 illustrates the estimated outputs and the real outputs.

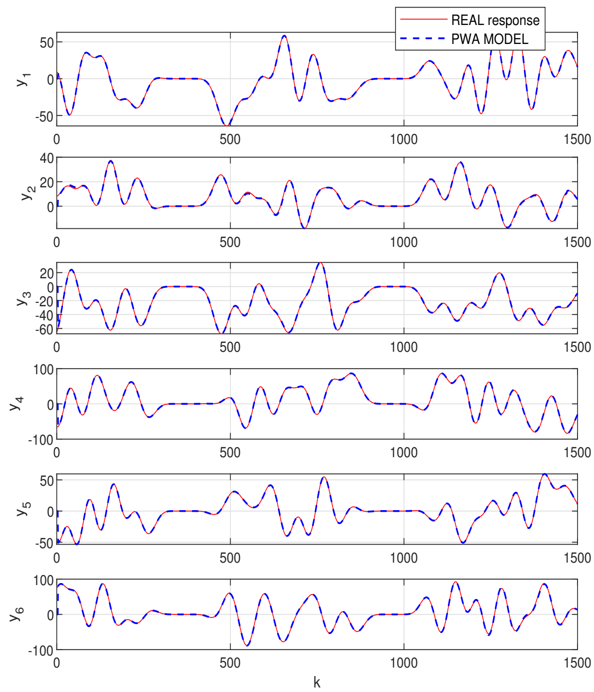


Fig. 9. Estimated and real outputs: PWA model.

The switching instants of the submodels are also determined. They are depicted in Figure 10.

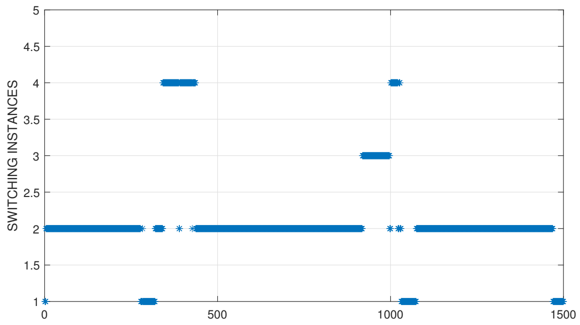


Fig. 10. Switching instances: Robot example.

Observations:
The obtained results reveal the performances of the proposed approach in identifying the robot manipulator. In fact, we remark that:

- The 6 estimated outputs are very close to the real ones.
- All of the 4 sub-models contribute in the evolution of the outputs.

6.3 Comparison study

In Weigand et al. (2022), the robot manipulator model is determined by using the subspace algorithm. Then a linear state-space model as a baseline is identified.
The Linear model is then defined as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \tag{20}$$

Where

$$A \in \mathbb{R}^{12\times 12}; B \in \mathbb{R}^{12\times 6}; K \in \mathbb{R}^{12\times 6}$$
$$C \in \mathbb{R}^{6\times 12}; D \in \mathbb{R}^{6\times 6};$$
$$x \in \mathbb{R}^{12\times 1}; y \in \mathbb{R}^{6\times 1};$$

The model response and the real system output are presented on figure 11. It's clearly shown that the linear model lacks to perform well and can not track real system output with high efficiency contrary to the PWA model.
As for performance metrics, we apply the Normalized Root Mean Squared Error (NRMSE) and the R^2 norm for both identification techniques.

$$NRMSE_i = \sqrt{\frac{1}{N\sigma_{q_i}^2}\sum_{k=1}^{N}(q_{i,k} - y_{i,k})^2} \tag{21}$$

Where σ is the standard deviation for the real measured signal. The estimated outputs are denoted by y_i and the real outputs by q_i $(i = 1, ..., 6)$ and N is the number of measures.

$$R_i^2 = 100 \times \left(1 - \frac{\sum_{k=1}^{N}(q_{i,k} - y_{i,k})^2}{\sum_{k=1}^{N}\left(q_{i,k} - \frac{1}{N}\sum_{k=1}^{N}(y_{i,k})\right)^2}\right) \tag{22}$$

The average value for these metrics over all joints is also computed as below:

$$\overline{NRMSE} = \frac{1}{6}\sum_{i=1}^{6} NRMSE_i \tag{23}$$

$$\overline{R^2} = \frac{1}{6}\sum_{i=1}^{6} R_i^2 \tag{24}$$

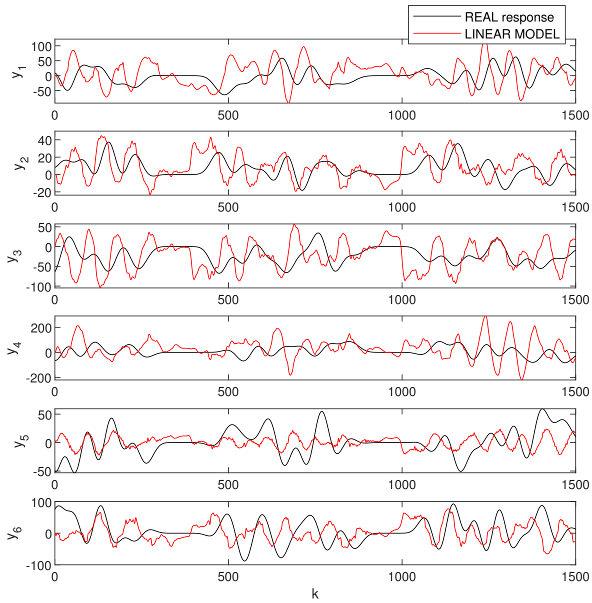


Fig. 11. Estimated and real outputs: Linear model.

Table 1. Comparison of performance.

	SubSpace method		Proposed method	
Mode	$NRMSE_i$	R_i^2	$NRMSE_i$	R_i^2
q_1	1.9577	22.8973	0.0158	99.9751
q_2	1.4793	31.8318	0.0542	99.7061
q_3	1.8025	23.5452	0.1273	98.3796
q_4	2.2767	20.5573	0.0751	99.4355
q_5	1.0192	49.0381	0.0987	99.0250
q_6	1.2277	39.9854	0.0938	99.1204
Average	1.6271	31.3091	0.07748	99.2736

As it's indicated on the above table the identification of the robot manipultor is well performed by the MIMO PWA identification algorithm.

7. CONCLUSION

The representation of the nonlinear system by a PWA model can be developed using the identification approach which allows building a mathematical model from input-output data. It is important to recall that the identification problem of the PWA system consists in simultaneously estimating, from a set of input-output measurements, the orders of sub-models, the number of sub-models, the parameters of the sub-models and the affine hyperplane coefficients defining the regression partitions.
Since most industrial processes have several inputs and/or several outputs (MIMO: Multi Inputs Multi Outputs), it is necessary to extend the identification methods based on PWA models to multivariable systems. The major difficulty in the identification of MIMO PWA systems is

manifested in the data classification step since one must classify parameters matrices instead of parameters vectors. The dimension of these matrices depends on the order of sub-models as well as the number of outputs of the system. The classification of these matrices is a difficult task because the notion of distance used to evaluate the similarity of data from the same group becomes insignificant. Therefore, we propose in this paper an extension of the DBSCAN clustering identification approach.
This proposed solution is successfully applied to a real nonlinear process which is the KUKA KR300 R2500 industrial robot.

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