Hybrid modelling and simulation approaches for a class of mechatronic systems

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***Abstract**: Starting from a generic model of a magnetically actuated mass spring damper with linear continuous dynamics and nonlinear input constraints, this paper proposes a hybrid approximating model, based on a partitioning of the evolution interval of the mass position variable. A piecewise-constant control law is deduced from the state space partition and the behaviours of the resulting hybrid control system and of the initial nonlinear constrained control are comparatively analysed within MATLAB simulation experiments. A Zeno path avoidance scheme, based on fixed-step discrete time conversion of the local differential equations is applied, and a second hybrid model of the generic mechatronic system is built. Simulation experiments show an adequate dynamic behaviour of this second hybrid model, compared to the hybrid model with continuous state equations.*

Keywords: Discrete event systems, Hybrid control systems, Mechatronic systems, Hybridization, Sampled system, Zeno behaviour.

1. INTRODUCTION

Mechatronic automotive systems, in which *smart* electronic devices monitor and control mechanical subsystems, are in general characterized by tight operating requirements - such as fast transient response, among others - combined with significant nonlinearities and the possibility to capture their dynamics by low-order models.

These contrasting features have stimulated, in recent years, an increasing interest for developing hybrid models, combining event-driven with time-driven dynamics and consisting, in general, of piecewise linear models together with a switching policy, which approximate the original nonlinear models [1], [2], [3]. Making *a compromise between simplicity and representativeness of the process* [5], the approximating hybrid models are well suited for implementation of adequate optimal-control schemes, like model predictive control.

Among these hybrid modelling approaches, there are two major directions, reported in the automotive systems literature. The first one, generally encountered in control engineering, is represented by models that combine finite state machines with differential equations and describing control systems with continuous plants driven by switching control laws [7], [8] or plants with intrinsic hybrid nature – such as the engine and power train dynamics of an automobile [1], [2]. The second modelling approach concerns a hybrid system formalism with both continuous and logical state variables in a single mixed state vector and linear, sampled time state equations, submitted to linear inequality constraints, known as *mixed logical dynamical* (MLD) system [3], [9].

Starting from the variant, discussed in [7], of the *hybrid control systems* (HCS) framework introduced by P.J. Antsaklis and his co-workers in [8], this paper proposes a hybrid approximation of a magnetically actuated mass-spring amper, considered here as a generic subsystem of a large class of models arising in automotive applications.

The hybridization technique is based on a partitioning of the evolution interval of the mass position variable, similar to the approach proposed in [5], but the resulting hybrid model of the valve is different. This re-modelling effort is motivated by the necessity to reduce complexity of model-based control schemes. The design of the control part is not subject of present work.

Zeno behaviour, characterized by execution of infinite many transitions in a finite time interval, is a special problem arising in hybrid models simulation, which may get stuck or imprecise. Although real systems are not Zeno, a hybrid model of a real system may be Zeno, due to the modelling abstraction. In an attempt to classify conditions driving to Zeno behaviour and to deduce possible model transformations, which permit extension beyond the so-called Zeno time, some general properties of these systems have been intensively investigated in past decade [10].

Motivated by these considerations, a Zeno paths avoidance scheme, discussed in [6] and [7] and based on fixed-step discrete time discretization of the differential equations, is applied to the hybrid model of the controlled valve, giving birth to a second hybrid model of the mechatronic system. It has to be noted that, due to same precautions, also the MLD formalism proposed in [3], [4] uses only difference equations to describe the time-driven dynamics of the hybrid simulation model.

The paper is organized as follows. Section 2 reviews in brief the differential model of the actuated mass-spring damper and the hybridization philosophy in [5]. Section 3 presents the basic elements of the HCS framework and introduces the HCS model of the mass-spring damper with partitioned state space. Finally, a Zeno path avoidance scheme is applied to the HCS valve model, and the resulting second hybrid simulation model is compared to the previous one, within simulation experiments.

2. MAGNETICALLY ACTUATED MASS-SPRING DAMPER – GENERIC MODEL AND HYBRIDIZATION TECHNIQUE OVERVIEW

Electromagnetic valves can be considered mechatronic, heterogeneous systems and they are generically composed of a mechanic subsystem – mass and resort – and an electromagnetic subsystem, interacting with each other. Consider the system discussed in [5] with the differential equation

$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x + \frac{F}{m}, \tag{1}$$

which describes the dynamics of the position of the mass centre x[m] under the effect of a controlled magnetic force F [N], for a spring with stiffness k [N/m] and a damper with coefficient c [N·s/m]. The force F, which depends nonlinearly on the current i[A] and on the position x, has also to satisfy, for each value x, the constraint related to the maximum available current, given by

$$F \le \frac{k_a i_{\max}^2}{(d - x + k_b)^2}. \tag{2}$$

In (2), k_a and k_b are positive constants and d[m] is the distance between the contact position and the spring neutral position. The additional constraint

$$-d \le x \le d \tag{3}$$

prevents the moving mass from unwanted collisions (with the coil or the symmetric stop position, respectively), thus avoiding its undesirable bouncing.

Assuming that the dynamics of the electrical subsystems are much faster than the mechanical ones, the authors propose to decouple the control problem and to consider a *model predictive controller* (MPC) for the valve state model deduced only from the mechanical dynamical equations (1), with the state vector $\boldsymbol{x} = [x \quad \dot{x}]^T$, where x satisfies (3) and the nonlinear control input $u = F/m$, with F subject to (2).

In order to reduce the model complexity and to create a MLD model of the mechatronic system, the authors propose a hybridization scheme. In brief, the initial model (1) with *transformed nonlinear constraints* is processed in HYSDEL [9] and for the equivalent MLD model a hybrid MPC optimization problem is formulated and solved, based on the general results in [3].

The basic idea of the hybridization procedure is to approximate the nonlinear function in the right-hand side of the constraint (2) by a set of piecewise affine functions, as reviewed in brief below, in the general case.

Consider the affine approximation of the function $f:\mathbf{R} \to \mathbf{R}$, given by $f(w) = a_i w + b_i$, if $w \in [w_i, w_{i+1})$, $i = 0 : l-1$, where $w_i < w_{i+1}$, such that the points $\{w_i\}_{i=1}^{l-1}$ define the borders between regions in which f has different affine terms. Consider also $l-1$ logical variables $\delta_i \in \{0,1\}$, $i = 1 : l-1$, defined by the logical conditions

$$[\delta_i = 1] \leftrightarrow [w \le w_i],\ i = 1 : l-1, \tag{4}$$

together with the $l-1$ real-valued continuous functions $z_i : \mathbf{R} \to \mathbf{R}$, $i = 1 : l-1$, defined by

$$z_i(w) = \begin{cases} (a_{i-1} - a_i)w + (b_{i-1} - b_i), \text{if } \delta_i = 1 \\ 0, \text{ else} \end{cases} \tag{5.1}$$

$$i = 1 : l-2,$$

$$z_{l-1}(w) = \begin{cases} a_{l-2}w + b_{l-2}, \text{if } \delta_{l-1} = 1 \\ a_{l-1}w + b_{l-1}, \text{else.} \end{cases} \tag{5.2}$$

The piecewise affine approximation of f is then

$$f = \sum_{i=1}^{l-1} z_i. \tag{6}$$

Returning to the model of the valve, the inequality (2) is divided in both sides by the mass value m, thus resulting the constraint for the control $u = F/m$. For the non-linear function in the right-hand side this new constraint, representing the upper control limit,, a piecewise affine approximation with three segments ($l = 3$) is introduced. Two δ and two z auxiliary variables result in this transformation, which can be written

$$u(x) \le z_1(x) + z_2(x), \tag{7}$$

where z_1 and z_2 are defined in (5.1) and (5.2), respectively, with $w = x$ and $l = 3$.

3. HCS MODELLING APPROACH FOR THE MAGNETICALLY ACTUATED MASS-SPRING DAMPER

In the HCS structure, the continuous plant is controlled, through an interface, by a discrete event controller (Fig.1). The continuous state-space is partitioned and the plant coupled to the interface is abstracted to a *discrete event system* (DES), called DES-plant. The controller is built within the DES theory and forces transitions in the DES-plant, thus acting like a supervisor.

In the sequel, the evolution interval of the valve mass position is partitioned and the resulting DES-plant model of the mechanical subsystem is deduced. The nonlinear continuous control is approximated by a switching control law with piecewise constant values and the resulting closed loop system is modelled as a HCS. Finally a discrete-time approach is used to deduce a simulation model of the valve without Zeno behaviour.

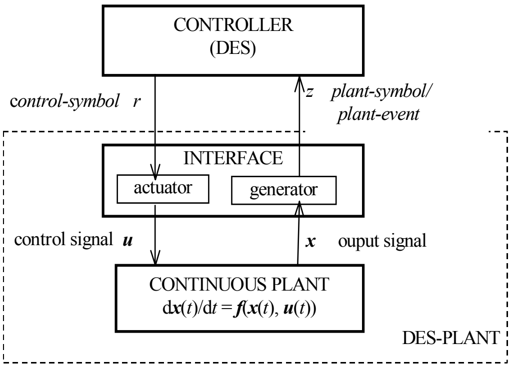


Fig. 1. The architecture of a hybrid control system.

3.1 The structure of a HCS

The *continuous plant* is modeled by the differential system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t),\boldsymbol{u}(t)), \tag{8}$$

where $\boldsymbol{x}(t) \in X \subseteq \mathbf{R}^n$ and $\boldsymbol{u}(t) \in U \subseteq \mathbf{R}^p$ are the state and control vector respectively, at the time $t \in \mathbf{R}$. X is the continuous state space. The set of admissible control values $U = \{\boldsymbol{u}_1, \dots, \boldsymbol{u}_M\}$, with $M \geq 1$, is bijectively mapped to the *alphabet of control-symbols*

$$\widetilde{R} = \{r_1, \dots, r_M\}. \tag{9}$$

The interface converts signals between the plant and the controller and it comprises *the actuator* and *the event generator*. Consider the bijective function relating control values and control-symbols,

$$\gamma : \widetilde{R} \to U,\ \gamma(r_m) = \boldsymbol{u}_m,\ m = 1:M. \tag{10}$$

Denote k the logical time, i.e. the ordering index for events occurrence. The *actuator* converts a string of control-symbols

$$\omega_r = r(0), r(1), \dots, r(k), \dots,\ \omega_r \in \widetilde{R}^*$$

to a vector of piecewise constant control-signals for the plant

$$\boldsymbol{u}(t) = \sum_{k \geq 0} \gamma(r(k)) \cdot I(t, t_c(k), t_c(k+1)) \tag{11}$$

where $t_c(k) \in \mathbf{R}$ is the moment when $r(k)$ is received from the DES controller, so that $t_c(k) < t_c(k+1)$ for all $k > 0$ and $I : \mathbf{R} \times \mathbf{R} \times \mathbf{R} \to \{0,\ 1\}$ is a characteristic function with $I(t, \alpha, \beta) = 1$, if $\alpha \leq t < \beta$ and $I(t, \alpha, \beta) = 0$, if else.

The event generator converts the state trajectory of the plant $\boldsymbol{x}(\cdot)$, evolving in the partitioned state space, into a string of plant-symbols. Consider $N \geq 1$ a natural number and a set of N indexed smooth functions

$$S_h^N = \{h_i : X \to \mathbf{R} \mid h_i \in C^1, i = 1:N\}, \tag{12}$$

which defines the *partition* of the state space X. For any $i \in 1:N$, the hypersurface $\ker(h_i) = \{\boldsymbol{x} \in X \mid h_i(\boldsymbol{x}) = 0\}$ is nonsingular, and separates X into two disjoint half-spaces:

$H_i^+ = \{\boldsymbol{x} \in X \mid h_i(\boldsymbol{x}) > 0\}$, and

$H_i^- = \{\boldsymbol{x} \in X \mid h_i(\boldsymbol{x}) < 0\}$.

The hypersurfaces $\ker(h_i)$, $i = 1:N$, partition the state space X into $\widehat{Q} \leq 2^N$ disjoint open cells, each of which bijectively labelled with a symbol from the *alphabet of the plant discrete states*

$$\widetilde{P} = \{p_1, \dots, p_{\widehat{Q}}\}. \tag{13}$$

A *plant-event*, denoted (i+) or (i–), $i \in 1:N$, occurs whenever the continuous trajectory $\boldsymbol{x}(\cdot)$ crosses the hypersurface $\ker(h_i)$, $h_i \in S_h^N$, in the positive or negative direction, respectively. A *sufficient condition* for the occurrence of a plant-event (i+) at the time $t_e \in \mathbf{R}$ is given by

$$h_i(\boldsymbol{x}(t_e)) = 0 \wedge \dot{h}_i(\boldsymbol{x}(t_e)) > 0. \tag{14}$$

A similar condition is formulated for the plant-event (i–). The *alphabet of plant-symbols* is

$$\widetilde{Z} = \{z_{1+}, z_{1-}, \dots, z_{N+}, z_{N-}\} \cup \{\varepsilon\}, \tag{15}$$

where ε is the *silent event* and the plant-symbol z_{i+}/z_{i-}, $i \in 1:N$, is generated whenever the associated plant-event (i+)/(i–) occurs.

The DES-plant model is the automaton $G_p = \{\widetilde{P}, \widetilde{R}, f_p, \widetilde{Z}, g_p\}$, where $\widetilde{P}$ is the set of discrete states, $\widetilde{R}$ is the *input* alphabet of control-symbols, $\widetilde{Z}$ is the output alphabet of plant-symbols, $f_p : \widetilde{P} \times \widetilde{R} \to 2^{\widetilde{P}}$ is the state transition function, and $g_p : \widetilde{P} \times \widetilde{P} \to \widetilde{Z}$ is the output function. The dynamical equations are

$$p(k+1) \in f_p(p(k), r(k)),$$
$$g_p(p(k), p(k+1)) = z(k+1), \tag{16}$$

where $p(k), p(k+1) \in \widetilde{P}$, $z(k+1) \in \widetilde{Z}$ and $r(k) \in \widetilde{R}$, $\forall k \geq 0$

The DES controller is a deterministic Moore machine $G_c = \{\widetilde{S}, \widetilde{Z}, f_c, s_0, \widetilde{R}, g_c\}$, where $\widetilde{S}$ is the set of discrete states, $s_0 \in \widetilde{S}$ is the initial state, $\widetilde{Z}$ is the input alphabet, $\widetilde{R}$ is the output alphabet, $f_c : \widetilde{S} \times \widetilde{Z} \to \widetilde{S}$ is the state transition function and $g_c : \widetilde{S} \to \widetilde{R}$ is the output function. The dynamical equations are

$$f_c(s(k), z(k+1)) = s(k+1)\,,\ s(0) = s_0\,,$$
$$g_c(s(k+1)) = r(k+1)\,,\ g_c(s(0)) = r(0) \tag{17}$$

with $s(k), s(k+1) \in \widetilde{S}$, $z(k+1) \in \widetilde{Z}$, $r(k) \in \widetilde{R}$, $\forall k \geq 0$.

The DES controller coupled to the DES-plant is a logical supervision system (Fig. 2a), while the DES controller coupled to the interface behaves like a switching control law (Fig. 2b) [7], which, in absence of hysterezis, can be written as

$$\boldsymbol{u}^*(\boldsymbol{x}) = \varphi(\mathrm{sgn}(h_1(\boldsymbol{x})), \ldots, \mathrm{sgn}(h_N(\boldsymbol{x})))\,, \tag{18}$$

with $\varphi : \mathbf{R}^N \rightarrow \mathbf{R}^p$.

3.2 A partition-based hybrid model of the magnetically actuated valve

In order to build a HCS approximation of the valve model (1) with constraints (2) and (3), the function in the right hand side in (2), representing the upper limit of the control signal, is divided by *m* and approximated by a piecewise constant function. The approximation is based on a partition of the *x*-axis into disjoint intervals. For each constant control value, the hybrid system evolves within a discrete state, until the separating point between two adjacent intervals is reached by the position of the mass centre, and then the control signal switches to a new value, generated by the DES controller.

Without loss of generality, consider the particular system deduced from (1), given by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2 + u\,, \tag{19}$$

with $\boldsymbol{x} = [x_1 \quad x_2]^T$ the state, $x_1 = x$, together with the particular form of the constraint for the control $u = F/m$, deduced from (2),

$$u \leq \frac{1}{(d + 0.45 - x_1)^2}\,. \tag{20}$$

Recall that *d* has the significance of the physical motion limit in (3).

Consider now the case of a nonlinear continuous control law analytically described by

$$u(x_1) = \frac{1}{(d + 0.45 - x_1)^2}\,, \tag{21}$$

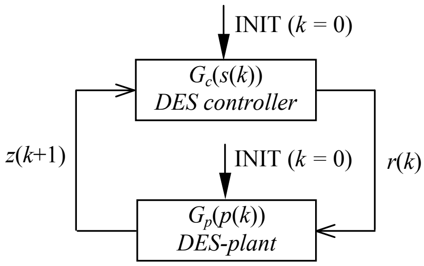
with the shape depicted in Fig.3a. A state space partitioning is obtained by dividing the evolution interval (3) of the $x_1 = x$ variable into three disjoint subintervals,

$$I_1 : -d \leq x_1 < d_1\,,$$
$$I_2 : -d_1 \leq x_1 < d_1\,, \tag{22}$$
$$I_3 : d_1 \leq x_1 < d\,,$$

as illustrated in Fig.3b. In the sequel, for simulation experiments, the numerical values are $d = 1.5$ units and $d_1 = 1$ unit.

The corresponding functions of the state space partition are

$$h_1(\boldsymbol{x}) = x_1 + d_1\,,\ h_2(\boldsymbol{x}) = x_1 - d_1\,. \tag{23}$$



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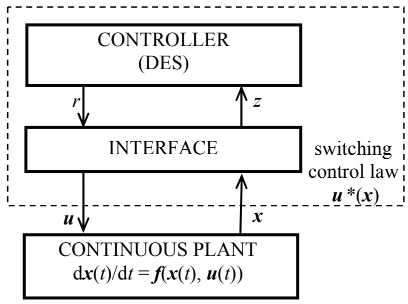

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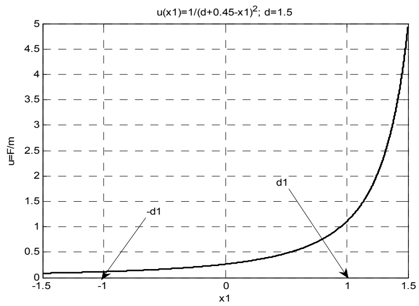
Fig. 2. Modelling approaches of the HCS: a) logical supervision and b) nonlinear control.



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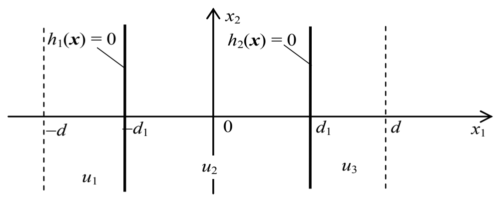


b)

Fig. 3. a) Simulation of the nonlinear control (21) and limits of the partition on the *x*-axis; b) the state space partition for the system (19).

If within each interval in (22) the control takes a constant value, then the switching control law (18) can be expressed as

$$u^*(\boldsymbol{x}) = a_1 \cdot \mathrm{sgn}(h_1(\boldsymbol{x})) + a_2 \cdot \mathrm{sgn}(h_2(\boldsymbol{x})) + b, \qquad (24)$$

where the coefficients a_1, a_2 and b result as the solution of a linear algebraic system, as detailed below. Consider, for simplicity that the hybrid control will take, within each interval in Fig.3b, the mean values:

$$u_1 = (u(-d) + u(-d_1))/2 = 0.0994\,,$$
$$u_2 = (u(-d_1) + u(d_1))/2 = 0.6103\,, \qquad (25)$$
$$u_3 = (u(d_1) + u(d))/2 = 3.0111\,,$$

respectively (Table 1), with u given by (21). Rewriting (24) as

$$u^*(\boldsymbol{x}) = \begin{cases} u_1, & x_1 \in I_1 \\ u_2, & x_1 \in I_2 \\ u_3, & x_1 \in I_3 \end{cases} \qquad (26)$$

the coefficients in (24) can be computed as the solution $\boldsymbol{y}$ of the algebraic system

$$\boldsymbol{A}_c \boldsymbol{y} = \boldsymbol{B}_c\,, \qquad (27)$$

where $\boldsymbol{A}_c$ is a full rank matrix containing the signs of the functions (23) in the intervals (22)

Table 1. The values of the control law (24).

x_1	$-d$		$-d_1$		d_1		d
$u^*(x_1)$		u_1		u_2		u_3	
$h_1(\boldsymbol{x})$		<0	0	>0		>0	
$h_2(\boldsymbol{x})$		<0		<0	0	>0	

$$\boldsymbol{A}_c = \begin{bmatrix} -1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad (28)$$

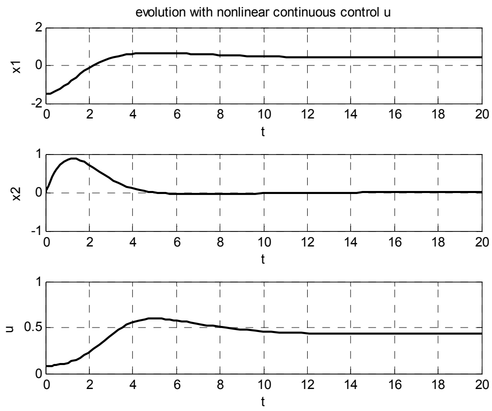
and $\boldsymbol{B}_c = [u_1 \quad u_2 \quad u_3]^T$. The solution of (27) is

$$\boldsymbol{y} = [a_1 \quad a_2 \quad b]^T = [0.255 \quad 1.200 \quad 1.555]^T \qquad (29)$$

Simulation results of the system (19) driven by the feedback law (21) and, comparatively, by the approximating switching law (24), with the numerical values of the coefficients specified in (29), are presented in Fig.4, Fig.5 and Fig.6, respectively.

In order to deduce the HCS model of the controlled valve, consider again the state space partition depicted in Fig.3b, with the discrete state symbols and plant-symbols, as specified in Fig.7. The alphabets of discrete states $\widetilde{P}$ and of plant-symbols $\widetilde{Z}$ are respectively

$$\widetilde{P} = \{p_1, p_2, p_3\}\,, \qquad (30)$$
$$\widetilde{Z} = \{z_{1+}, z_{2+}\}\,. \qquad (31)$$



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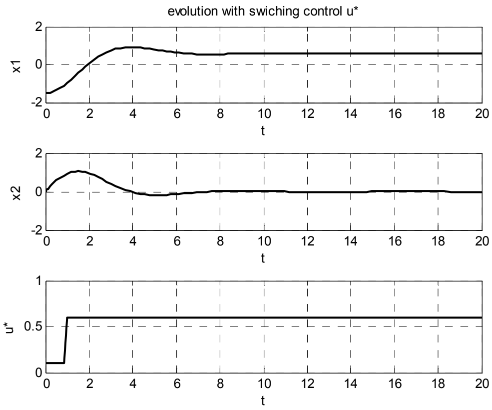
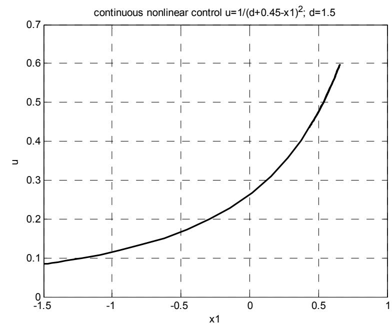


b)

Fig. 4. Time-evolution of the position, speed and control signal for the system (19): a) with control (21); b) with switching control (24), (25), (29).



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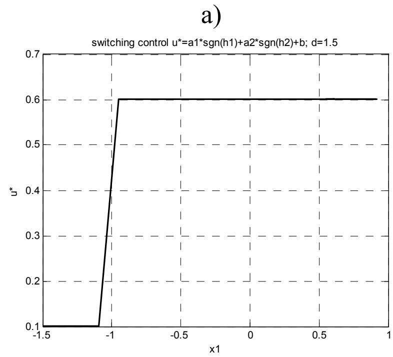

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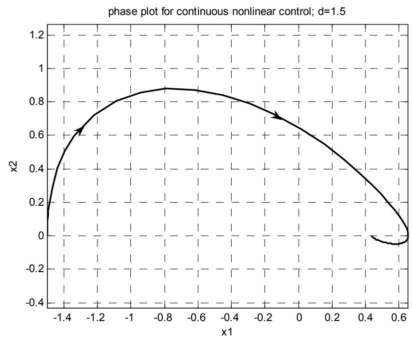
Fig.5. Graphical representation of the control-position dependence: a) with the control law (21); b) with switching control (24), (25), (29).



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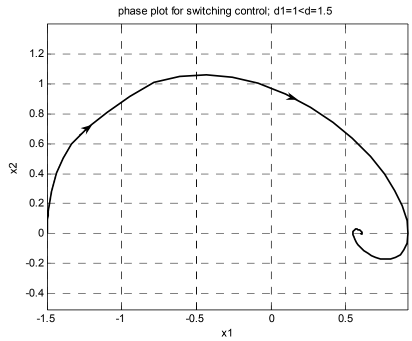


b)

Fig.6. Phase plot of the system (19) driven by a) the nonlinear control (21) and b) the switching control (24), (25), (29).

For example, the significance of the plant-symbol z_{1+} is

$$[z_{1+}] \leftrightarrow [h_1(\boldsymbol{x}(t)) = 0 \wedge \dot{h}_1(\boldsymbol{x}(t)) > 0], \qquad (32)$$

with h_1 as in (23). The alphabet of control-symbols $\widetilde{R}$ is defined by the equivalence with the set $U = \{u_1, u_2, u_3\}$ of constant control values (25) taken by the control law (24),

$$\widetilde{R} = \{r_1, r_2, r_3\}. \qquad (33)$$

The correspondence is defined according to (10), and the switching $u^* \to u_i \in U$ takes place when the DES controller G_c enters the state $s_i \in S$, and instantly outputs the control-symbol $r_i \in \widetilde{R}$.

According to the simulation results depicted in Fig.4b and Fig.5b, in the controlled motion the switching law takes in fact only two values, u_1 and u_2, so the DES controller will enter only the corresponding two discrete states: the mass m starts from a position $x_{10} \in (-d, -d_1) \sim p_1$ and, under control u^* (24), it reaches an equilibrium $x_1^* \in (0, d_1) \sim p_2$.

The DES controller and the controlled DES-plant automaton, representing the discrete-event abstraction of the valve coupled to the actuators and sensors with discontinuous action and controlled by the law (24), are depicted in Fig.8.

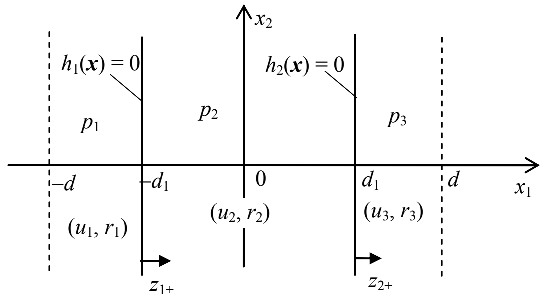
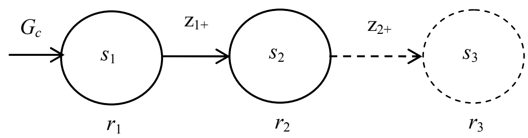


Fig.7. Significance of the symbols of the alphabets (30), (31), and (33) in the state space partition.



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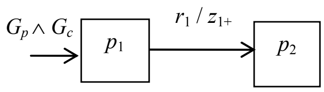

b)

Fig. 8. DES controller and the controlled DES-plant automaton ; in a) the state s_3 is not entered, in the closed loop dynamics, because system reaches an equilibrium in p_2.

The DES-controller implementing the switching law (24) is depicted in Fig.8a. The controlled DES-plant automaton, abstracting the continuous plant (19) with the state space partition in Fig.7 and switching law (24) is presented in Fig.8b. The discrete state p_2 labels the interval I_2 in (22), which contains the equilibrium point.

3.3 Zeno path avoidance by fixed-step time discretization

In order to avoid, in the process of numerical integration, the possible Zeno behaviour of the hybrid simulation model with continuous state equations (19) and switching control law (24), a discrete time approach, proposed in [6] and [7], is used. The technique is based on the linearity of equations (19).

Recall that given a linear continuous system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \qquad (34)$$

and the sampling time $h > 0$, the corresponding discrete-time linear system is

$$\boldsymbol{x}_d(k+1) = \boldsymbol{A}_d \boldsymbol{x}_d(k) + \boldsymbol{B}_d \boldsymbol{u}_d(k), \qquad (35)$$

with k the integer time variable (iteration), $\boldsymbol{x}_d(k) = \boldsymbol{x}(kh)$, $\boldsymbol{u}_d(k) = \boldsymbol{u}(kh)$ and the matrices

$$\boldsymbol{A}_d = \exp(h\boldsymbol{A}), \ \boldsymbol{B}_d = \int_0^h \exp(\theta \boldsymbol{A}) d\theta \cdot \boldsymbol{B}. \qquad (36)$$

For the valve with the dynamic model (19), the matrices of the continuous state model are

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{37}$$

and the analytic computation of the matrices (35) of the corresponding discrete time model gives

$$\boldsymbol{A}_d = \begin{bmatrix} e^{-h} & he^{-h} \\ -he^{-h} & e^{-h}-he^{-h} \end{bmatrix},$$

$$\boldsymbol{B}_d = \begin{bmatrix} 1-e^{-h}-he^{-h} \\ he^{-h} \end{bmatrix} \tag{38}$$

Using MATLAB, the system (19) is converted to a discrete time one (with the *c2d* routine, for example) and the matrices (36) of the resultant system, with sample time $h = 0.1\,\text{sec}$, become

$$\boldsymbol{A}_d = \begin{bmatrix} 0.9952 & 0.950 \\ -0.950 & 0.9002 \end{bmatrix}, \ \boldsymbol{B}_d = \begin{bmatrix} 0.0048 \\ 0.0950 \end{bmatrix}. \tag{39}$$

The simulated step responses of the mass position of the valve, for the continuous time mechatronic model and for the discrete time system respectively, are comparatively depicted in Fig.9, showing an adequate behaviour of the discrete time model (35) with matrices (39), compared to the evolution of the continuous state equations (19).

If the control input is replaced by the discrete time signal resulted from the fixed-step sampling of the piecewise constant feedback signal (24), then a second hybrid model of the controlled valve is obtained, with local state equations given by (35) and (38)-(39). In the dynamic equations of the closed-loop discrete time simulation model, the transition condition for the control u_d can be implemented by simple decision loops such as:

if $x_{1d}(k+1) \le -d_1$ then $u_d(k+1) = u_1$
else
if $-d_1 < x_{1d}(k+1) \le d_1$ then $u_d(k+1) = u_2$
else $u_d(k+1) = u_3$ (40)

Recall that, due to the fact that the simulated motion reaches an equilibrium, with the position variable residing in the second interval I_2, the discrete control signal takes, similarly to the continuous one, only the first two values u_1 and u_2 specified in (25).

Simulation results for the hybrid model with discrete time state equations are shown in Fig.10 and, compared to the evolution of the hybrid valve model, in Fig.4b, they show an adequate behaviour, despite the effects of the sampling process.

For the generation of the discrete time state equations in the hybrid model was implemented, in MATLAB, using the routines *ss*, *cd2* - as already mentioned - and *ssdata*.

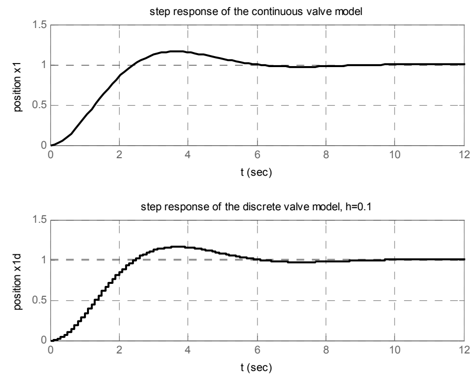

Fig. 9**.** Step responses of the mass position of the mechatronic system in case of the continuous state equations (19) (up) and of the discrete time model given by (35) and (39).

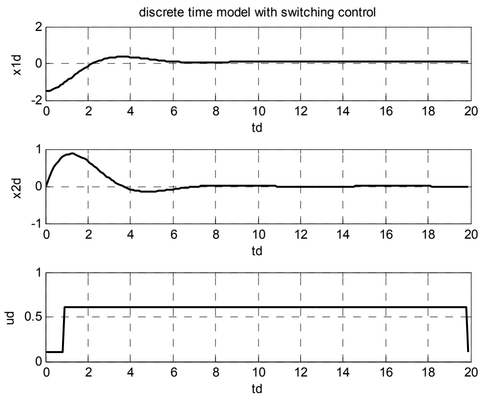

Fig. 10. Time-evolution of the position, speed and control signal for the hybrid system with discrete state equations (35) and (39) and the discrete switching control (40).

4. CONCLUSIONS

Starting from a generic model of a magnetically actuated mass spring damper, with linear continuous dynamics but strongly nonlinear input constraints, this paper introduces a HCS approximating model, based on a partitioning of the evolution interval of the mass position variable. The continuous nonlinear control, obeying the nonlinear constraint, is replaced by a piecewise constant control law, whose formula is deduced. The approximating constant values are computed, for simplicity, as mean values of the nonlinear control. Also, the logical controlled model is deduced. In this particular application, the hybrid character is due only to the switching control law.

Simulation experiments performed in MATLAB and presented in subsection 3.3 show that the hybrid approximating model behaves similarly to the original nonlinear closed loop model. Only the motion of the mass approaching the electromagnetic subsystem was considered, and simulation results suggest that the system reaches an equilibrium.

In view of possible Zeno behaviour avoidance of the proposed HCS model, the technique described in [6] and detailed in [7], based on discrete time conversion of the linear continuous time evolution laws, is applied to the hybrid model based on the continuous state equations of the electromagnetically actuated valve. This approach has a similar philosophy but it is different from the solution adopted in HYSDEL [3], which directly starts from discrete time equations for the time-driven part of the mixed model.

Simulation results, performed in MATLAB and presented in subsection 3.3, show an adequate behaviour of the hybrid model with discrete time local evolution, compared to the behaviour of the initial HCS model with continuous time plant equations.

More sophisticated techniques for the computation of the control values in the switching control law of the HCS, as well as the analysis of the impact of other physical constraints, with influence on the partitioning strategy for hybridization, are subject of future research, together with the study of adequate control algorithms, such as model predictive schemes.

The proposed hybrid approach for the mechatronic system is different from and simpler than the MLD model in [5]. An important advantage is that our study is performed within MATLAB, so no dedicated simulation framework was used. In this regard, a comparison to the valve model generated in HYSDEL [4] may also drive to consistent conclusions.

This opens the direction of a new framework for hybrid modelling of automotive systems and also for development of specific analysis and simulation techniques, with emphasis on the study of stability and reachability, on one side, and of efficient numerical implementations, on the other side.

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