A Fast Method for Classification of Emergent Dynamics in Cellular Automata Based on Uncertainty Profiles

Radu Dogaru

Dept. of Applied Electronics and Information Engineering, University "Politehnica" of Bucharest, Bvd. Iuliu Maniu 1-3, Romania (Tel: +40-21- 4024685; e-mail: radu_d@ ieee.org).

Abstract: A new and efficient method to classify cellular automata is presented and exemplified here for the case of elementary cellular automata (ECA) with 3 cells neighborhood. The approach has an important advantages over other methods: It is extremely fast since it does not require the simulation of the cellular automaton dynamics, instead all classification process is based on a uncertainty profile computed rapidly for a given cell logic and neighborhood. The method may be easily generalized to more complex cellular neighborhoods. A comparison with another recent ECA classification method (based on a completely different approach, namely on the iterated maps theory in nonlinear dynamics) reveals a strong overlap between results.

Keywords: nonlinear systems, cellular neural networks, information theory, cellular logic, uncertainty.

1. INTRODUCTION

Cellular automata (CA), were introduced by Ulam and von Neumann in the '50s to demonstrate the production of complex behaviors such as self-reproduction and universal computation, in artificial systems. Studied today in the wider framework of **cellular nonlinear/neural networks** (CNN) they gained lot of interest as tools for modeling and simulating various natural, social, and other complex behaviors. A collection of similar cells is updated in synchrony, each cell changing its state based on a ***CA rule*** which tells how to change the actual state based on the values of all neighbor cell states in the previous iteration. The structure of a CA is rather simple, yet the underlying dynamics may be extremely complex and intriguing. Such a surprising effect in a CA (that cannot be predicted unless simulating the CA dynamics) was called by different authors ***emergent behavior*** (Dogaru, 2008).

Although many CA simulators exist and various theoretical studies investigate the relationship between CA rules and their emergent dynamics, such theoretic results still lack generality and quite often refer mostly to linear CA rules, a tiny subset of all possible rules. Complex behaviors in CA were inventoried and so far there are large lists with interesting CA and their corresponding emergent behaviors with a diverse range of applications such as: models of various processes, parallel image processing, aids for computer graphics and special effects, music generators, signal processing and many others. The potential of CA stands in the fact that they are a natural computing model, related with even more accurate models proposed recently such as the *Small Worlds networks*, capable to model a wide range of natural phenomena and possessing very interesting computing properties. For instance, in a simplified view the functioning of the human cortex can be regarded as a massive cellular automaton operating in emergent mode. Some authors even suggest that CA may be regarded as models of everything claiming that "a new kind of science" (Wolfram, 2002) shall be constructed on this basis.

Still, the following **inverse problem** remains an open issue in the theory and practice of CA: ***Given a desired behavior (often specified ambiguously, e.g. a complex behavior or one exhibiting chaos, or one exhibiting periodic oscillations, etc.), specify the CA rule to produce such a behavior***. In order to solve this problem one needs to first define certain features describing the dynamic behavior and relate these features to the CA rule with a well defined mathematical function. Often this process is called a ***classification of CA rules***. Solving the inverse problem is possible if an inverse of the classification function may be defined. Quite often such an inverse problem is ambiguously defined as it results in a set of many possible mappings to the desired behavior, i.e. multiple rules. Further investigation among this limited set of rules may reveal the rules that best match the desired behavior. The problem is important because the number of possible rules is extremely high and grows super-exponentially with the size of the neighborhood. For instance, in a CA with 9 neighbor cells there are $2^{2^9} = 2^{512}$ possible rules. A good classification method that maps the rule space to the behavior space allows a faster location of the rules associated with a desired behavior, even in such large spaces.

The ***first classification of CA rules*** into several categories with specific behaviors (Class I,II, III and IV) dates from the 1980's and belongs to Wolfram, but it has the main disadvantage that *is based on visual observation and subjectivity of a human observing the consecutive states of the CA*. It is neither a precise classification (but rather a taxonomy) nor practical to use in an automatic system to classify CA. Still, its application for the limited set of 256 elementary cellular automata (ECA) with 3 cells neighborhood (cells are disposed on a ring and each cell state depends only on its center, right and left neighbors) raised a lot of open questions in studying CA.

Since Wolfram's classification, many other methods to locate and classify CA rules were proposed. In (Dogaru, 2008a) a brief overview of such methods is given. The mostly accepted solution so far is the use evolutionary search techniques in conjunction with classification techniques to locate desired behaviors in huge CA rule spaces. ***Such classification techniques require the simulation of the CA given the rule*** and the calculation of certain numerical features, this process becoming part of the evolutionary algorithm. Consequently *these methods are computationally intensive.* Instead, an ***entirely new method is proposed in (Dogaru, 2008a) based on an information theoretic approach***. In this paper, this method is applied to a widely studied CA category, namely the elementary cellular automata. The purpose is twofold: a) to calibrate the new method; b) to compare with the most advanced classification methods proposed in the literature. **Section 2** gives the principle of the information theoretic approach showing that one may assume an initial state with *m* uncertain states in the middle of the array of cells. Given the neighborhood and the cell ID (identification of cell Boolean logic) it is shown in **Section 3** that a well specified mapping function is defined to project the structural CA information in an uncertainty profile space which gives a good characterization of the emergent CA dynamics without the need to simulate it. Consequently, given this mapping, one can easily solve the abovementioned inverse problem, not only for the elementary CA but also for any other kind of CA with different neighborhood and grid topology. The main advantage of our classification method, called next the ***"uncertainty profile method" (UPM)***, is its computational simplicity since it does not require simulation of the CA dynamics as most of the existent methods do.

In **Section 4** some results published recently in series of papers (Chua *et al*., 2002-2008) are reminded. They perform the first systematic analysis of ECA using nonlinear theory tools and propose another CA classification method. As shown in **Section 5** their predictions overlap substantially to ours, although the approaches were quite different. For a finer description of the emergent behaviors the use of both methods in conjunction gives a better description of the ECA space. Still, as shown in **Conclusions**, the UPM method has the important advantage that it can be easily generalized to any neighborhood size and CA grid type while remaining computationally simple.

2. ELEMENTARY CA RULES, NEIGHBORHOODS, AND THE INFORMATION THEORETIC APPROACH TO EMERGENCE

The discrete-time dynamics of an elementary cellular automaton (ECA) with *n* cells is given by the next equation, which applies synchronously to all cells (a cell is identified by a spatial index $i \in \{1,2,..n\}$):

$$x_i^T(t+1) = Cell\left(x_{i-1}^T(t), x_i^T(t), x_{i+1}^T(t)\right), ID\Big) \qquad (1)$$

where $y_j = Cell(u1,u2,u3,ID)$ is a Boolean function with 3 (in general *m*) binary inputs (*u1,u2, and u3*) specified by a decimal ID number called a *CA rule*. In its binary representation, the most significant bit of ID corresponds to the cell output y_{N-1} when $[u3,u2,u1]=[1,1,1]$, the next bit y_{N-2} is the cell output for $[u3,u2,u1]=[1,1,0]$, and finally its least significant bit y_0 corresponds to $[u3,u2,u1]=[0,0,0]$. The **identifier ID corresponds to the *cellular automaton rule* according to Wolfram's notation** (Wolfram, 2002) and it is the decimal representation of a vector $\mathbf{Y}=\left[y_{N-1}, y_{N-2},.., y_0\right]$ collecting the outputs for all $N=2^m$ possible cell inputs. Consequently the "*j*" index is the decimal value associated to the input vector $\left[u_3,u_2,u_1\right]$. A periodic boundary condition is assumed i.e. if $i=1$, then $i-1 \to n$ and if $i=n$ then $i+1 \to 1$. An initial state for the entire cellular automata is defined as $\mathbf{x(0)}=[x_1(0), x_2(0),..., x_n(0)]$.

In running the CA from an initial state where ***only some cells in the middle are randomly assigned*** to 1 or 0 (with maximum uncertainty, or probability ½) one can observe a decline or "implosive" behavior when after a few iterations all cells will enter a quiescent state (Fig.1a), a stable (or preserving) behavior as seen in Fig. 1b, or a growth (explosion) when uncertainty in the initial cell expands in one or more directions with smaller or larger speed, as seen in Fig.1c for the universal computing CA rule ID=110. In (Dogaru, 2008a), it is concluded that the most interesting ***(complex) CA behaviors require slow growth***. In fact, one of the most complex behavior in CA, the self-reproductive one, clearly belongs to the *slow-growth* type as defined above.

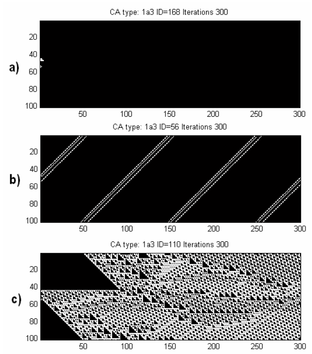

Fig.1. Several typical CA behaviors in elementary cellular automata: a) imploding; b) preserving; c) exploding. Black color represents state 0 while the white state 1. Time axis is horizontal.

As seen in Fig.2, in (Dogaru, 2008a) it is proposed that each cell is assigned a probability (to be in "1" state) and using a relatively simple probabilistic inference formulae one can compute the probability of each cell in the next CA iteration. Given the probability p_k that a cell "k" is in state "1" i, its ***uncertainty*** is defined (other than in Shannon's information theory) as: $u_k = 1 - |2p_k - 1|$ such that in either cased when cells are surely in “0” or “1” states, their uncertainty is 0. Based on experiments it was assumed that ***spreading of uncertainty*** is strong related to the CA dynamic behavior. As seen in the last section this assumption is validated for all ECA. As a consequence, **classifying CA behaviors requires only the computation of an *uncertainty profile*** defined as a vector U of *2m-1* uncertainties (in the case *m=3* there are 5 elements of U). Therefore our *uncertainty profile method* is much faster than simulating the entire CA.

The piecewise-linear approach to uncertainty was used instead of Shannon’s entropy because it leads to less computational efforts while we are essentially interested in its spread through the spatial coordinates of the CA.

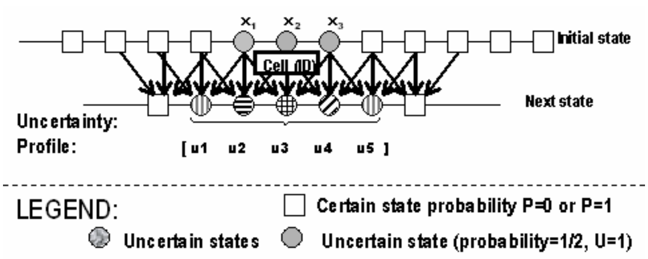

Fig.2. Uncertainty and its spreading in the CA neighborhood. For a given ID and neighborhood geometry an *uncertainty profile* vector may be easily computed knowing the output probabilities of cells.

3. UNCERTAINTY PROFILES AND THEIR CALCULATION USING REPRESENTATIVE PROBABILITY TABLES

The uncertainty profile is computed entirely based on the ID of the CA rule (Y is the binary representation of ID) and on the neighborhood connectivity (as reflected in the *representative probability table* or RPT). As shown in (Dogaru *et al.*, 2008d) there are 2 distinct RPTs, R_0, R_1 depending on whether the quiescent[1] starting state of CA cells is “0” or “1”. Consequently, two different uncertainty profiles may be calculated for each of the abovementioned cases: $\mathbf{U}_0, \mathbf{U}_1$. For reasons mentioned in (Dogaru, 2008a) the calculation of the final uncertainty profile combine both depending on the configurations of the least and the most significant bits y_0, y_{N-1} of Y. The formulae computes two terms only for part of the odd rules, while for all even rules $U=U_0$. For odd rules with most significant bit “1” $U=U_1$. The following formulae summarize the calculation of uncertainties for any CA rule:

$$\mathbf{P_0} = \mathbf{R}_0\mathbf{Y} \text{ and } \mathbf{P}_1 = \mathbf{R}_1\mathbf{Y} \tag{2}$$

$$\mathbf{U}_0 = 1 - |2\mathbf{P_0} - 1| \text{ and } \mathbf{U}_1 = 1 - |2\mathbf{P}_1 - 1| \tag{3}$$

$$\mathbf{U} = 0.5\left[(2 - y_o - y_o y_{N-1})\mathbf{U}_0 + (y_o + y_o y_{N-1})\mathbf{U}_1\right] \tag{4}$$

where P_0 and P_1 are output probability profiles (m-sized line vectors), and the absolute value operator applied to a vector means its application element-wise. Y is a line matrix with 2^m columns and R matrices are predefined depending on the particular CA topology and neighborhood (Dogaru *et al*, 2008d). In particular for the one dimensional elementary cellular automata (ECA) with *m=3* cells neighborhood, the RPTs are given by:

$$\mathbf{R}_0 = \frac{1}{8}\begin{bmatrix} 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 2 & 0 & 2 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 4 \end{bmatrix} \quad \text{and}$$

$$\mathbf{R}_1 = \frac{1}{8}\begin{bmatrix} 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Consequently it turns out that no multiplications are necessary (coefficients of matrices and other multipliers are powers of 2) and in the worst case the main computational burden is about $(2m-1)2^{m+1}$ operations. In the case of *m=3* (elementary CA) the uncertainty profile of any given ID rule requires about 90 basic arithmetic operations (additions and shifts mainly). It is a much lower computational effort than for any CA simulation followed by an evaluation of certain features to characterize the CA dynamics.

4. CLASSIFICATION OF ECA DYNAMICS BASED ON NONLINEAR ANALYSIS

By virtue of the three *global equivalence* transformations derived in (Chua et al., 2004) it is shown that instead of all 256 CA rules it suffices to conduct an in-depth analysis of *only* the 88 local rules listed in Table 3 of (Chua et al., 2007a). Therefore in the next we will consider only those 88 rules, listed in the following:

0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,18,19,22,23,24,25,26,27,28,29,30,32,33,34,35,36,37,38,40,41,42,43,44,45,46,50,51,54,56,57,58,60,62,72,73,74,76,77,78,90,94,104,105,106,108,110,122,126,128,130,132,134,136,138,140,142,146,150,152,154,156,160,162,164,168,170,172,178,184,200,204,232.

If a certain rule of interest for the reader is not among them, its associated “parent”, with identical dynamic behavior, can be easily found among the 88 listed above by applying to that rule one of the next two transforms or both of them consecutively:

a) “left-right” swap transform – meaning that a new Boolean function is defined preserving the same input output relations

[1] All other cell except the randomly initialized one that are set to either 1 or 0 state.

but where the leftmost input is swapped with the rightmost input ($u_1 \leftrightarrow u_3$);

b) "negate inputs/output" transform: In this case the new Boolean function is obtain by inverting all inputs ($u_1 \to \bar{u}_1, u_2 \to \bar{u}_2, u_3 \to \bar{u}_3$) and the output $\left(\mathbf{Y} \to \overline{\mathbf{Y}}\right)$.

Roughly speaking, Chua's analysis detailed in (Chua et al., 2002-2008) results in classification of all ECA rules into 6 categories (classes). The method used was to consider the CA as a nonlinear map (discrete time system) characterized by a nonlinear function, which is directly determined by the ID and the total number of cells *n* in the CA. **The main feature to discriminate among classes is the length (period) of the most likely attractor (given a random initial state) in a CA with a given ID rule and the dependency of this period on the number *n* of CA cells**. Classes 1,2,3 (with 26, 13 and 1 members) are similar in that for all of them there is a *constant* (1,2, or 3) length of the attractor while the difference among these classes is given by the period of the attractor (1, 2 or 3). Class 4, (or Bernoulli-shift, with 30 members) includes all CA rules leading to a period that depends *linearly* by the number of cells *n* with a coefficient that can be accurately predicted. Because periods of attractors for these CA rules are larger, the corresponding cellular automata are more complex than those from the Class 1-3 categories. Nevertheless a complete description for each rule is possible, allowing prediction of the attractor period. Finally Classes 5 and 6 (with 10 and 8 members respectively) include the most complex CA rules, where a prediction of the attractor length is not possible without effectively running the CA. All rules in Class 5 are bilateral (or they have a symmetry, i.e. the Boolean function is not changed by the left-right swap transform) while those in Class 6 are not symmetric at all. **Quite notably, those CA rules found so far by other researchers as possessing interesting properties fall in Chua's Class 6**. Recently it was found that ID=45 posses the interesting property of "binary chaos synchronization", with many potential applications in information theory and communications (Dogaru et al., 2008,2008b, 2009). Note that Chua's classification method for classes 5 and 6 is difficult to generalize in more complex CA, because a detailed analytic and numerical analysis shall be done for each particular rule. In CA with large neighborhoods such analysis becomes prohibitive if not impossible.

5. INFORMATION THEORETIC CLASSIFICATION BASED ON UNCERTAINTY PROFILES

While comparing the dynamics of all 88 elementary CA rules to their uncertainty profiles it turned out that several computable features allow a proper classification of all CA. These features, their significance and computation methods are next given:

a) Cumulated uncertainty: It is the sum of all uncertainties in the profile and gives information about the spreading of uncertainty. Therefore it is defined as $C(\mathbf{U}) = \sum_{i=1}^{2m-1} u_i$. In conjunction with this feature, an "***edge of complexity***" boundary may be defined in the region $C \in [m - 1/2, m + 1/2]$ with both "**exploding" and "preserving" profiles** to be detailed next. The name "edge of complexity" is somehow related to the "edge of chaos" widely used in the complex systems literature as a frontier between ordered and disordered (chaotic) dynamics. In our case it defines a frontier between CA behaviors where the eventual uncertainty in the initial state is conserved within a fixed finite number of cells (preserving profiles or rules) and those interesting behaviors where a global behavior emerges out of the local CA interaction (exploding profiles or rules). Observations of the dynamic evolution of CA with rules within the edge of complexity (Dogaru, 2008a) led to the conclusion that most **complex behaviors** such as those found in various CA systems demonstrated to be universal computing machines, self-replicating systems and in general leading to glider structures **are strong related with asymmetric exploding rules in the edge of complexity**.

b) Symmetric versus asymmetric profiles: For any given uncertainty profile one may compute a *symmetry index*: $s(\mathbf{U}) = \frac{L(\mathbf{U})}{l(\mathbf{U})}$, where $L(\mathbf{U})$ is the sum of all members to the right or to the left (except the center member) of the uncertainty profile. $l(\mathbf{U})$ is the sum of the members (except direction) in the opposite direction, such that $l(\mathbf{U}) \le L(\mathbf{U})$. **If $s(\mathbf{U}) = 1$ the profile is *symmetric*, else it is asymmetric**. The value of asymmetry may give some additional hints about the predicted dynamic behavior. *Note that in the case of odd rules, since the profile U is the average of two uncertainty profiles U_0 and U_1 an additional condition that both U_0 and U_1 are symmetric is necessary to declare the rule a symmetric one.* For instance, rule ID=25, with a symmetric U=[1/2 3/4 3/4 3/4 1] is in fact asymmetric because at least one U_0=[0 1 ¾ 1 1] or U_1=[1 ½ ¾ ½ 0] are asymmetric.

c) Exploding, preserving and imploding profiles:
An exploding profile corresponds to a growth in uncertainty from an initial state where only *m* cells have maximum uncertainty to a situation where all cells are in an uncertain state. A heuristic rule to compute whether a profile is exploding is the following: A profile is "exploding" if there are at least two members (elements) of the profile with maximum value($u_k = u_l = 1$) and they are distant at more than *m* cells (i.e. $|k - l| \ge m$). Also, a profile is "exploding" if the above condition is not fulfilled but if the sum of uncertainties is larger than $m + 1/2$. For instance, in the case of ECA with m=3, the CA with ID=78 has U=[0 1 1 1/2 1] and satisfies the above criterion since $u_2 = u_5 = 1$ and $|2 - 5| = m = 3$. The same definition can begeneralized to

any other CA configuration. For multi-dimensional grids the above conditions should be satisfied for at least one particular expanding direction. For such exploding profiles a complex dynamics is expected (characterized by long period attractors or very long transients). As seen in Fig.3, not surprisingly most of these exploding rules overlap with Class 5 and Class 6 categories, while the distinction between Class 5 and 6 is simply made by the asymmetry index. Though, some exploding rules can be found into less complex classes according to Chua's classification, for instance rule 13 with U=[1 3/4 3/4 1/4 2/4] which was found to be a period-1 (Class 1) rule. Still we might not consider this rule as a "simple" one because there is a relatively large transient. This may be explained by the fact that rule 13 falls within the edge of complexity boundary while it is an asymmetric one and therefore we may consider it as a complex one. Therefore, unlike Chua's classification focusing on the attractors and their periods, the UPM approach reveals aspects that might be important for information processing. For instance, **a complex computation behaves much like a CA with a long transient ending in a low period attractor (i.e. a fixed point representing the end of the computation)**. **In such cases, the period of the attractor is not as relevant as the transient length to qualify the behavior as a complex**. This is also the reason why non-expanding rules but with large $C(\mathbf{U}) > m + 0.5$ were included in the edge of complexity.

The degree of asymmetry as well as the sum of uncertainty plays an important role and can differentiate among such exploding rules. Such computable features may be used to induce an order within the roughly defined qualitative categories.

A preserving profile is a non-exploding one with at least m non-zero elements. Such a profile guarantees that uncertainty is neither spreading nor implode, it will be always preserved in a finite number of cells. A preserving profile indicates that there is no major global effect of the CA coupling. Still quite complex oscillatory behaviors may be associated with such categories. For asymmetric preserving profiles, as seen in Figs 10-11, since uncertainty circulates with a certain speed (cells to the right or left per iteration) it is obvious that the underlying nonlinear dynamics will have a periodic attractor with a length proportional to the number *n* of cells. Not surprisingly the asymmetric preserving category almost overlaps with Chua's Class 4 (Bernoulli-shift rules). The dynamics of the symmetric preserving rules is expected to be simpler, and not surprisingly most pf these rules overlap with rules in Chua's Class 2 and Class 1.

An imploding profile is a non-exploding one but with less than m non-zero elements. Such profiles are associated with a dynamics of the CA such that after a few (usually less than *n*) iterations all cells have a sure state (either 1 or 0). From the point of view of nonlinear dynamics this evolution corresponds to either Period 1 or Period 2 attractors.

The table in Fig.3 reveals several interesting aspects about our classification scheme: For most complex rules (with large period attractors, depending on the number of CA cells, i.e. Classes 4-6) there is a strong overlap between our categories and those obtained using the nonlinear theory classification. While the methods to classify these rules are entirely different both capture the essential qualitative differences in CA behaviors. Small differences may be explained in that UPM method does not focus on attractors and their lengths but rather on uncertainty spreading. In fact, each method brings something new to the other. For instance, Chua's method cannot predict spatial explosions in rules such as 62,28,156,78 while our method cannot predict that some of the asymmetric exploding rules correspond to a low period attractor.

	Asymmetric Exploding (13 rules)	Symmetric Exploding (14 rules)	Asymmetric Preserving (35 rules)	Symmetric Preserving (18 rules)	Imploding (8 rules)
Class 6	**110**,41,45, 60,26,30,154		**106**		
Class 5		**18**,**126**,146,73,22, 122,54,90,105,150			
Class 4	57,58		2,3,34,130,7,162, 10,24,**11**,**14**, **35**,42,**56**,138,**152**, **6**,**9**,15,27,**43**, 46, **142**,170,**184**,38, **25**, **74**,**134**		
Class 3	62				
Class 2	28,156	50,178	29	1,23,19,5,33, 51,108,**37**	
Class 1	78	94,77	12,44,140,172,13	4,36,72,132, 104,164,200, 232,76,204	0,32,128,160, 8,40,136,168

Fig. 3. Both Chua's and UPM classification of ECA rules: A row corresponds to a class according to Chua, a column corresponds to a UPM class. Inside each box, rule IDs are listed in an increasing order of the cumulated uncertainty and the rules falling within the edge of complexity are represented underlined. Those that were effectively found interesting (large lengths of transients or gliders) by computer simulations are represented with bold characters.

The *uncertainty profile method* allows a finer subdivision inside each class. For instance observing CA simulations it turns out that more complex behaviors are expected to happen within the edge of complexity also depending on the symmetry index. As long as the class is asymmetric, the smallest the symmetry index the larger the complexity of the CA behavior. This observation explains why not all "underlined" rules in the table (having the potential to generate complexity) are also "bold" (observed to effectively generate complex behaviors in simulations). At a closer look it turns out that "bold" rules in the table have a symmetry index lower than 2.5. Comparisons of uncertainty profiles with CA simulations for all elementary CA reveal that the **less complex rules are the symmetric ones**.

In addition to the above, Chua's classification based on nonlinear dynamics reveals another interesting aspect: If a rule belongs to a low-period category, even if it falls within the complexity edge, it has a smaller chance to behave as a complex rule in computer simulations. Such cases are rules 78,76 and 204. But as observed from the table, most effectively complex rules (underlined and bold) are to be found within the "asymmetric preserving" category. In our opinion such rules deserve further investigation. In fact, rule 184 was already extensively investigated due to its traffic modeling capabilities (Fuks and Boccara, 1998). In conjunction with a new phenomena described recently as emerging in CA, that of binary synchronization (Dogaru et al, 2009), another predictive conjecture emerged comparing simulations of CA where this phenomena is present, with ours and Chua's classification:

Most asymmetric rules in Classes 3 to 6 do have attractors that synchronize binary. There is a single exception, namely ID=60. Also it was found that **in the case of symmetric rules there is no binary synchronization**.

In the next figures, representative examples from each category mentioned in Fig.3 are considered. For each example two plots are displayed. The upper one corresponds to 100 different runs, each starting with a different random initial state with a few random cells in the middle. Its evolution represents the uncertainty evolution of all cells (maximum uncertainty is represented here in black). The lower plot in each figure represents a particular simulation example where the time evolution corresponds to the dynamics of the CA state. In the next, the examples are grouped into categories according to the UPM classification.

Asymmetric exploding rules:

First let us consider 2 examples from the "Asymmetric Exploding & Class 6" rules:

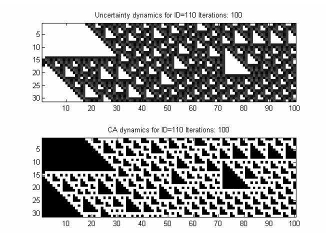

Fig.4 CA simulations for ID=110

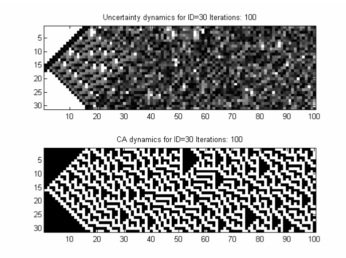

Fig.5 CA simulations for ID=110

The first example (ID=110, quite famous since it was demonstrated in (Wolfram, 2002) to represent the simplest CA capable of universal computation) was underlined since its uncertainty profile enters the "edge of complexity". Indeed, compared to the second example (ID=30), out of the edge of complexity, regular interacting structures called "gliders" emerge for ID=110 (better seen in Fig.1). For both ID=110 and 30 the periods of the attractors are obviously quite large, in any case larger than the simulation interval.

Let now compare the above examples with the one in Fig. 6 obtained for ID=78 from the same UPT category but in Class 1 (Chua's classification).

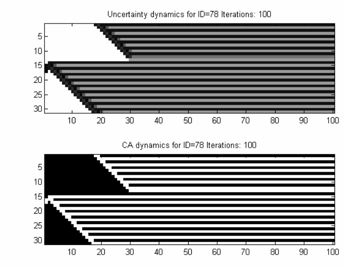

Fig. 6. CA simulations for ID=78

While the exploding type of behavior was correctly predicted for all above examples only Chua's classification predicted correctly the attractor period that is indeed small (period 1) in case of ID=78. Since this rule falls within the *edge of complexity*, it is expected to provide some computationally meaningful effects. Indeed, for all random initial states, this CA converges towards a much smaller number of attractors. This behavior may be eventually exploited as an associative memory. Note that Chua's method could not predict from the ID number and without CA simulation the exploding character and the complexity of the spatial behavior for ID=78.

Symmetric exploding rules:

As seen in both Fig.7 and 8 the symmetry of uncertainty profile determines the symmetrical pattern evolution of such rules. Both rules give less complex (more predictable) behaviors than asymmetric rules discussed before. Besides, rule 90 in Fig. 8 may be regarded as being "simpler" than rule 126 in Fig. 7 because the uncertainty profile has only 3 gray shades for rule 90 and therefore all future states are quite predictable, while it has more gray shades for rule 126 indicating that the pattern evolution depends stronger on the initial state choice.

Chua's method makes no particular distinction between these 2 rules, placing both of them in class 5. From the example in Fig. 7 it appears that ID=126 must fall in class 1 (period 1 attractor) but this is not the case if all cells are in a random initial state. Of course the uncertainty probability method could not predict the most likely attractor period but allows predicting many other features.

As expected, the exploding and symmetric character of the rule ID=77 is predicted by UPM theory but the periodicity of 1 for the attractor can be predicted only by Chua's approach.

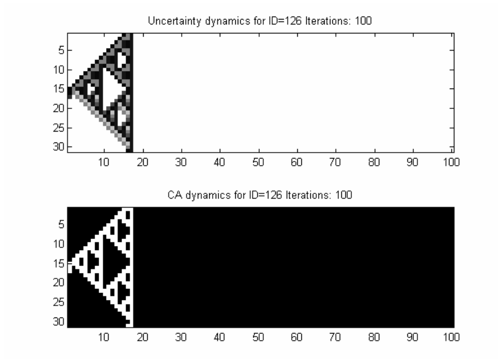

Fig. 7 CA simulations for ID=126

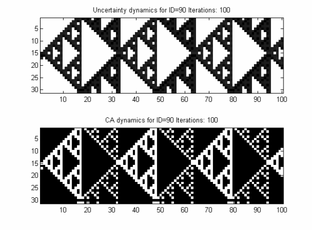

Fig. 8. CA simulations for ID=90

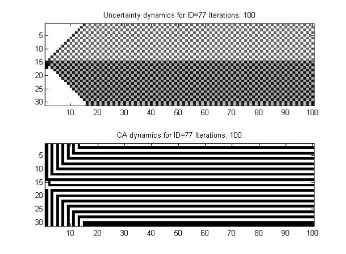

Fig.9. CA simulation for ID=77

Asymmetric preserving rules:

As predicted by the uncertainty profiles, both above rules give an expected behavior, an asymmetric spreading of uncertainty but limited such that it will never affect all cells in the array. But since the cumulated uncertainty is larger in the case of ID=106, the number of cells affected by uncertainty is even larger. For ID=184 investigated in (Fuks et al. 1998) the behavior is apparently not so complex but the situation changes if the initial state cells are all random. But in any case there is less complexity than for ID=106 as correctly predicts our *uncertainty profile method*. Quite interestingly in this case, the theory of uncertainty can explain why the period of the attractors is a multiple of *n* (number of cells). Since there is a lateral propagation due to an asymmetric profile, uncertainty repeats periodically at certain cell sites and the period is obviously proportional to *n*. Not surprisingly most rules in this UPM category falls also in Class 4 (attractor period proportional to *n*). The exceptions in Class 1 have a weak asymmetry (*S*=1.3) that is not enough to allow a lateral propagation, as seen in Fig. 12, and consequently the attractor period becomes small and not dependent on the number of cells. This behavior is also predicted by the UPM via the symmetry index, very close to 1 in such cases.

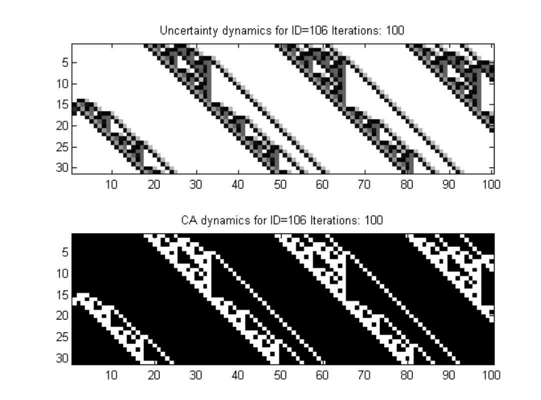

Fig. 10. CA simulation for ID=106

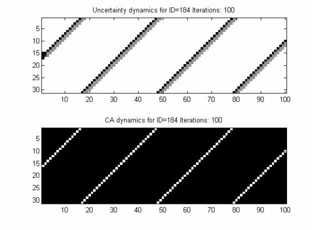

Fig. 11. CA simulation for ID=184

Since this rule falls in the *edge of complexity* there is no surprise that for all cells in random initial states this CA behaves as an associative memory and therefore it has some computational potential.

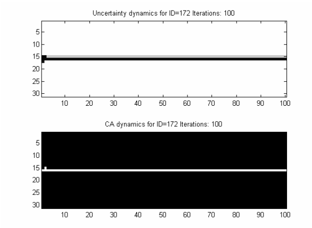

Fig. 12. CA simulation for ID=172

Symmetric preserving rules:

The typical evolution of such CA rules is shown in Fig. 13. It is somehow similar to that in Fig.12 but now the uncertainty profile is symmetric and there is no lateral propagation. As expected, the behavior is even less complex than in the previous cases, only certain cells including the initially uncertain ones and some very few around them propagate the uncertainty, but always on the same sites. Also in this case it is easy for UPM to explain why the period of the attractor cannot be in general very large (it is expected that attractors should be small and indeed they belong mostly to Chua's classes 2 and 1), and in any case is not dependent of the number of cells *n*.

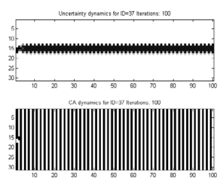

Fig.13 CA simulations for ID=37

Imploding behaviors: In these cases the CA behavior is rather dull as seen in Fig. 14, since uncertainty vanishes after a few iterations. There is, of course, no computational meaning for such CA systems.

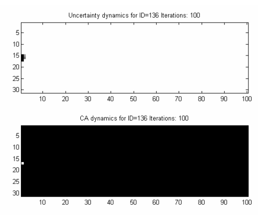

Fig. 14. CA simulation for ID=136.

6. CONCLUSIONS

A novel classification method of arbitrary cellular automata was proposed. It computes uncertainty profiles as *(2m-1)* sized vectors of all affected cell in the next CA iteration while the starting state is defined with a number of *m* cells in a completely uncertain state. Since uncertainty spreading was found to be strongly related to the dynamic behavior of the CA it can be used as a feature to classify CA rules according to several simple algorithms. Moreover, in addition to classification of a given rule, numerical indicators such as *asymmetry index* and *cumulated uncertainty* allow to define a finer hierarchy within each category.

Our method was compared to the most recent ECA classification results based on nonlinear theory and a large overlap was found between classifications provided by both methods. This result validates our method as one capturing correctly the complexity aspects in cellular automata. Moreover, unlike Chua's method, ours can be easily generalized since concepts like uncertainty spreading remain unchanged for larger CA neighborhoods and topologies. For instance, in (Dogaru, 2008c,d) we show that CA rules providing complex behaviors (e.g. gliders) can be identified quite easily, without the need of a genetic or evolutionary algorithm, by simply inspecting the representative probability table and allowing bits in Y to be set or reset in such a way to create a desired uncertainty profile. In the abovementioned case, such rules were found in the relatively large space of 2^{32} possible rules of CA with 5 neighbors. The algorithm to classify CA rules as imploding, preserving or exploding is the same for any other kind of CA, as long as one provides the RPT (representative probability matrices) for the particular kind of neighborhood in that CA.

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