Stabilizing the Flexible Convergence of Swarm Robot by Fuzzy Attraction and Repulsion

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Abstract: Controlling a swarm robot is a way of handling the behaviour of individual robots inspired by the organization of animal societies. Through local rules and interactions, swarm robotics aims to build sustainable collective behaviour, with an open and flexible mechanism to coordinate with large numbers of individuals. This paper proposes a solution for attractive/repulsive functions with clear mathematical descriptions with Mamdani fuzzy method, which is more flexible and closer to natural logic. Conditions for stabilizing the convergence process are introduced based on Lyapunov theory with swarm model and its interaction coefficients. Simulation results verify the important characteristics of the system through Matlab software.

Keywords: Swarm robot; converging process; fuzzy logic; attraction/repulsion forces; stability analysis.

1. INTRODUCTION

Characteristics of the swarm robot are using a large number of relative simple robots to perform tasks that a single robot cannot do or do not perform effectively. Swarm behavior is performed based on the mechanism of the collaboration between individuals. Typical behaviors of swarm robots are divided into four major groups: aggregation, search, transport, and obstacle avoidance.

Swarms of animals and insects tend to aggregate in the swarm, which will help them avoid enemies and deal with the changes of nature. One of the early applications of robot aggregation behaviors was proposed in 1994 (Kube and Zhang). In this research, a mechanism, used to invoke the group behavior, allow the system of robots to perform tasks without centralized control or explicit communication. (Bruemmer et al., 2002) addressed issues surrounding deployment and tasking of a real-world collective of cost-effective, small mobile robots. This research mentions on the use of social potential fields as a means to coordinate group behavior and promote the emergence of swarm intelligence as seen in a colony of ants or swarm of bees. (Correll et al., 2009) considered a case study concerned with the inspection of a jet turbine engine by a swarm of miniature robots. The research summarizes efforts that include multi-robot path planning, modeling of self-organized robotic systems, and implementation of proof-of-concept experiments with real miniature robots. Moreover, the emphasis of the work is on explicitly incorporating the potential limitations of the individual robotic platform in terms of sensor and actuator noise into the modeling and design process of collaborative inspection systems. Another contribution proposed a decentralized method for controlling a homogeneous swarm of autonomous mobile robots that collectively transport a single palletized load. The small tank-like robots have no advanced sensory or communication capabilities as well as information on the position or number of other robots transporting the small pallet. Instead, all information needed by the robots is derived from the dynamics inherent when the system of robots is contacting a common rigid body (Stilwell and Bay, 1993). Parker described the design issues of key importance in these real-world cooperative robotics applications - fault tolerance, reliability, adaptivity and coherence. The work presents a general architecture addressing these design issues - called ALLIANCE - that facilitates multi-robot cooperation of small- to medium-sized teams in dynamic environments, performing missions composed of loosely coupled subtasks (Parker, 1996). Another robotic implementation of the cooperation to retrieve large prey of ants is described in (Kube and Bonabeau, 2000). This is an example of decentralized problem-solving by a group of robots, and it provides the first formalized model of cooperative transport in ants.

Balch et al., described the design and implementation of these reactive trash-collecting robots as a multi-robot cooperating team. The multi-agent cooperation includes color vision for the detection of perceptual object classes, temporal sequencing of behaviors for task completion, and a language for specifying motor schema-based robot behaviors (Balch et al., 1995). When researching on the swarm robot, it is necessary to clarify the mechanism of cooperation and the stability of the swarm. The stability here is the ability to ensure the formation, the distance between individuals while moving. Specifically, individuals in the swarm move together toward a certain target and they would converge around the target at a certain distance.

The stability of the robot swarm plays a role in ensuring that the algorithm is efficient, applicable, and moreover optimal in cooperation. There are many studies on the stability of swarm robots presented in (Gazi and Passino, 2003; Wang and Fang, 2010; Chen et al., 2006; Yang et al., 2008). This stability is mainly based on the attraction/repulsion mechanism between individuals in the swarm. Each work presents a different attraction/repulsion function with explicit mathematical equations. They have specific characteristics, including: magnitude depends on the distance between pairs of individuals in the swarm; the longer the distance between the pairs of individuals, the greater the attraction force, the closer the distance, the greater the repulsion force and vice versa.

As for the use of fuzzy logic for complex systems, there have been many studies applied at different levels. (Hanchevici et al., 2012) proposed one networked control strategy for linear SISO systems affected by variant communication delays. The purpose of the research is to adjust, by using the fuzzy logic, the command provided by the PID controller. The input for the fuzzy logic controller is represented by the delay and the variation of delay, and the output is used to adjust the PID controller's command to the new value of the communication delay which occurs in the network. Another contribution proposes a novel artificial intelligence based Evolved Bat Algorithm controller with machine learning matched membership functions in a complex nonlinear system (Chen et al., 2020). The proposed transformed membership functions are adopted and stabilized. As a result, closed-loop performance criteria TS fuzzy systems are obtained through a new parametric linear matrix inequality technology rearranged by a capacity function member that fits with machine learning. (Pozna et al., 2022) presented a hybrid metaheuristic optimization algorithm that combines Particle Filter (PF) and Particle Swarm Optimization (PSO) algorithms. The Particle Filter-Particle Swarm Optimization (PF-PSO) algorithm consists of two stages, generating randomly the particle population, and zooming the search domain. An application of this algorithm to the optimal tuning of Proportional-Integral-fuzzy controllers for the position control of a family of integral-type servo systems. One contribution related to fuzzy logic to swarm robots is mentioned by (Mısır et al., 2020). This approach utilizes fuzzy logic controllers to evaluate limited sensor data. Experimental results were obtained on different number of swarm robots with different detection areas in arenas of different sizes. However, the stability properties for the swarm robot's operation have not been clearly considered.

As can be seen, the above studies indicate that herd behavior is based on the interactions between individuals in the swarm, as well as between them and the environment. These interacting forces are often well-defined mathematical functions, but without convincing arguments. In addition, due to the complexity and variability of the swarm robot's behavior, mathematical models are often difficult to satisfy.

In order to make the robot's operation closer to natural reality, this paper proposes a new swarm model, in which the interaction force between individual robots is built on the basis of fuzzy logic. Fuzzy logic has flexibility in input/output selection, fuzzy rules, defuzzification, etc. Therefore, using fuzzy logic to determine attraction/repulsion can be more generalized than the use of explicit attraction/repulsion. Simulation results using Matlab software are presented to verify the operating properties of the robots based on the proposed method.

In part 2, the attraction/repulsion function for the convergence problem of the swarm robot is built based on the fuzzy logic approach. The stability of this approach is proven in section 3. The results of verification through simulation are presented in section 4.

2. BUILDING ATTRACTION/REPULSION FUNCTION

Consider a swarm including N individuals in n dimensional Euclidean space (n≤3). The position of the individual i in the swarm is represented by

$$\mathrm{p^i} = \begin{bmatrix} p_1^{\mathrm{i}} \\ p_2^{\mathrm{i}} \\ \vdots \\ p_{\mathrm{n}}^{\mathrm{i}} \end{bmatrix} \in \mathrm{R^n} \tag{1}$$

The movement of individuals in an identical environment would depend on the interaction between each individual and the others in the swarm. An identical environment is one that is free of obstacles, with no external disturbance affecting the swarm. When pairs of individuals are far apart, they need to move toward each other by attraction force to maintain the swarm, and conversely, when close, they need to move away from each other by repulsion force to avoid collisions. This means that the force of interaction between individuals in the swarm would depend on the distance between pairs of individuals. Let f be the force of interaction between pairs of i and j

$$f = f\left(\left\|p^{\mathrm{j}} - p^{\mathrm{i}}\right\|\right) \tag{2}$$

where $\left\|p^{\mathrm{j}} - p^{\mathrm{i}}\right\|$ being the distance between two individuals i and j. Call σ_{s} is actual distance between the pair of individuals, so:

$$\sigma_{\mathrm{s}} = \left\|p^{\mathrm{j}} - p^{\mathrm{i}}\right\| = \sqrt{(p_1^{\mathrm{j}} - p_1^{\mathrm{i}})^2 + (p_2^{\mathrm{j}} - p_2^{\mathrm{i}})^2 + \cdots + (p_{\mathrm{n}}^{\mathrm{j}} - p_{\mathrm{n}}^{\mathrm{i}})^2} \tag{3}$$

Let:

$$g\left(\left\|p^{\mathrm{j}} - p^{\mathrm{i}}\right\|\right) = \frac{f\left(\left\|p^{\mathrm{j}} - p^{\mathrm{i}}\right\|\right)}{\left\|p^{\mathrm{j}} - p^{\mathrm{i}}\right\|} = \frac{f(\sigma_{\mathrm{s}})}{\sigma_{\mathrm{s}}} = g_{\mathrm{a}}(\sigma_{\mathrm{s}}) - g_{\mathrm{r}}(\sigma_{\mathrm{s}}) \tag{4}$$

where $g(.)$ being attraction/repulsion function between individuals i and j, $g_{\mathrm{a}}(.)$ being repulsion force, $g_{\mathrm{r}}(.)$ being attraction force.

Let $\sigma_{\mathrm{s}}^* \in \mathrm{R}$ be the distance between two objects i and j with the force of attraction and repulsion between these two objects in equilibrium. This force is represented by

$$f(\sigma_{\mathrm{s}}) \begin{cases} = 0 \text{ if } \sigma_{\mathrm{s}} = \sigma_{\mathrm{s}}^* \\ < 0 \text{ if } 0 < \sigma_{\mathrm{s}} < \sigma_{\mathrm{s}}^* \\ > 0 \text{ if } 0 < \sigma_{\mathrm{s}}^* < \sigma_{\mathrm{s}} \end{cases} \tag{5}$$

Call $\tilde{\sigma}_{\mathrm{s}}$ the difference between the actual distance and the desired distance:

$$\tilde{\sigma}_{\mathrm{s}} = \sigma_{\mathrm{s}} - \sigma_{\mathrm{s}}^* \tag{6}$$

The interaction force between individuals $f(\sigma_{\mathrm{s}})$ is a nonlinear function that depends on the distance between pairs of individuals (i, j), so that function $f(\sigma_{\mathrm{s}})$ can be built using the Mamdani fuzzy system with SISO structure as follows:

- *Step 1:*

The input signal is

$$u = \widetilde{\sigma}_s = \sigma_s - \sigma_s^* \tag{7}$$

Assuming that u has the value range of $[\alpha_b, \beta_b] \in R$. Divide u into $2N_f+1$ membership functions, denoted by B^k with k = 1,2,…, $2N_f+1$ as shown in Figure 1.

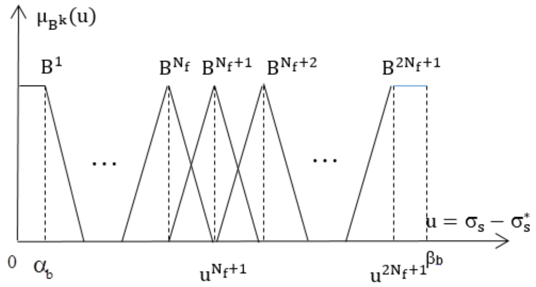


Fig. 1. Fuzzification of input signal of the fuzzy controller.

The output signal is $A = f(\sigma_s - \sigma_s^*) = f(u)$ with its value range of $[\alpha_a, \beta_a]$ (Figure 2). Divide A into $2N_f+1$ membership function, denoted by A^k with k = 1, 2, …, $2N_f+1$. Center of gravity a^k of A^k is determine by

$$a^k \begin{cases} < 0 \text{ if } k = 1, 2, \dots, N_f \\ = 0 \text{ if } k = N_f + 1 \\ > 0 \text{ if } k = N_f + 2, \dots, 2N_f + 1 \end{cases} \tag{8}$$

- *Step 2:* Build $2N_f+1$ rules IF… THEN… with form:

IF $u = B^k$ THEN $A = A^k$

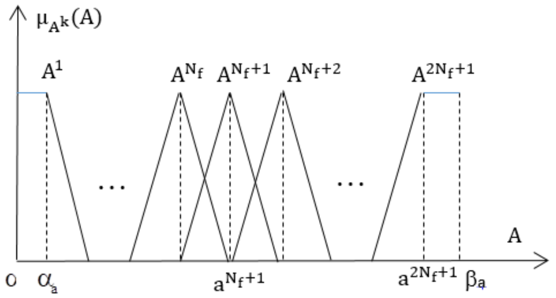


Fig. 2. Defuzzification of output signal of the fuzzy controller.

- *Step 3:* Select fuzzy laws and defuzzificate by the weighted average method, according to (Wang, 1997), output control value is determined by

$$f(u) = \frac{\sum_{k=1}^{2N_f+1} a^k \mu_{B^k}(u)}{\sum_{k=1}^{2N_f+1} \mu_{B^k}(u)} \tag{9}$$

With the above three-step fuzzy set design, it can be seen that the input signal is the distance of the individuals and the output signal is the interaction between the individuals. This relationship has the following properties

$$f(u) \begin{cases} > 0, \text{ if } \sigma_s > \sigma_s^* \\ < 0, \text{ if } 0 < \sigma_s < \sigma_s^* \\ = 0, \text{ if } \sigma_s = \sigma_s^* \end{cases} \tag{10}$$

The fuzzy function $f(u)$ is a continuous function that satisfies the conditions:

- Upper and lower limit:

$$A_{\min} \le f(u) \le A_{\max} \tag{11}$$

where $A_{\min} = a^1,\ A_{\max} = a^{2N_f+1}$

- Applying linearization equation:

$$f(u) = \frac{(a^{k+1} - a^k)u + a^k u^{k+1} - a^{k+1} u^k}{u^{k+1} - u^k} \tag{12}$$

where $u \in [u^k, u^{k+1}]$, with $k \in \{1, 2, \dots, 2N_f\}$

Let G_{amin} and G_{amax} be the smallest and largest value of the attraction function, G_{rmin} and G_{rmax} be smallest and largest value of the repulsion function, respectively. From (12), the limits of the function $g(\sigma_s)$ are determined as follows

$$\begin{cases} 0 \le G_{\text{amin}} \le g(\sigma_s) \le G_{\text{amax}} \text{ nếu } \sigma_s > \sigma_s^* \\ -G_{\text{rmin}} \le g(\sigma_s) \le G_{\text{rmax}} < 0 \text{ nếu } 0 < \sigma_s < \sigma_s^* \end{cases} \tag{13}$$

with

$$\begin{cases} G_{\text{amax}} = \max\limits_{N_f+2 \le k \le 2N_f+1} \left[\frac{a^{k+1} - a^k}{u^{k+1} - u^k}\right] \\ G_{\text{amin}} = \min\limits_{N_f+2 \le k \le 2N_f+1} \left[\frac{a^{k+1} - a^k}{u^{k+1} - u^k}\right] \\ G_{\text{rmax}} = \max\limits_{1 \le k \le N_f} \left[\frac{a^{k+1} - i^k}{u^{k+1} - u^k}\right] \\ G_{\text{rmin}} = \min\limits_{1 \le k \le N_f} \left[\frac{a^{k+1} - a^k}{u^{k+1} - u^k}\right] \end{cases} \tag{14}$$

The properties of (10) can be represented by different nonlinear functions as shown in Table 1. According to the data in this table, three types of membership functions for input and output (middle column) generate attractive and repulsive function relations (right column), respectively. Depending on the actual use, these functions can be fully adjusted to meet different requirements.

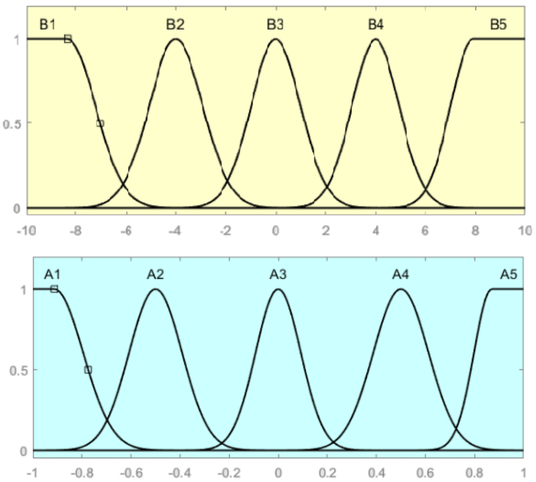
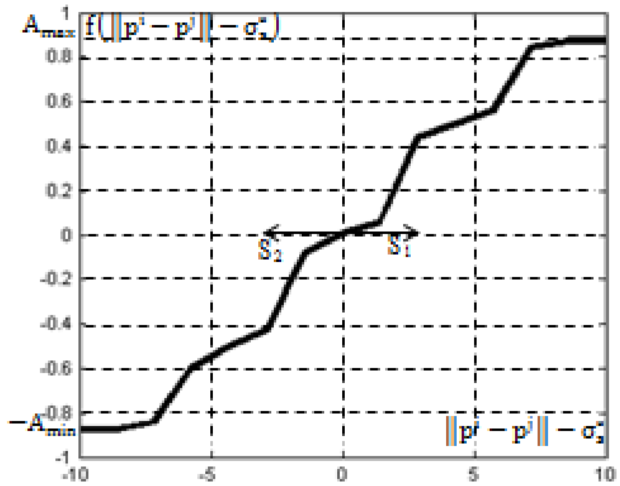
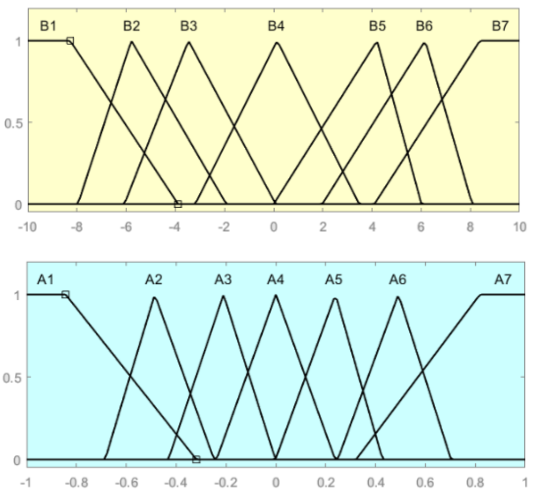
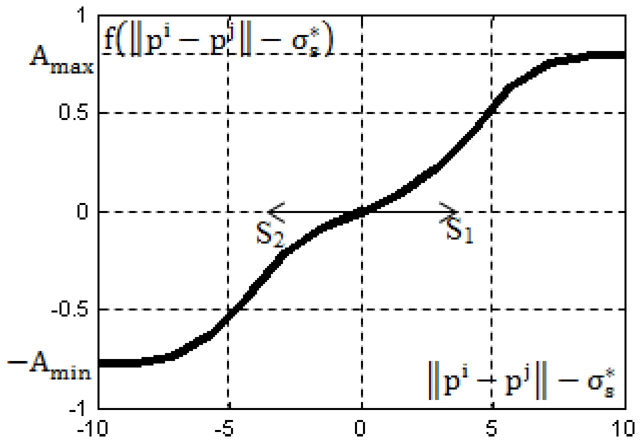
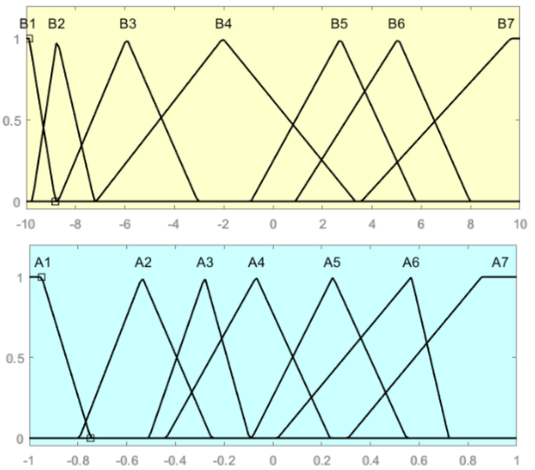
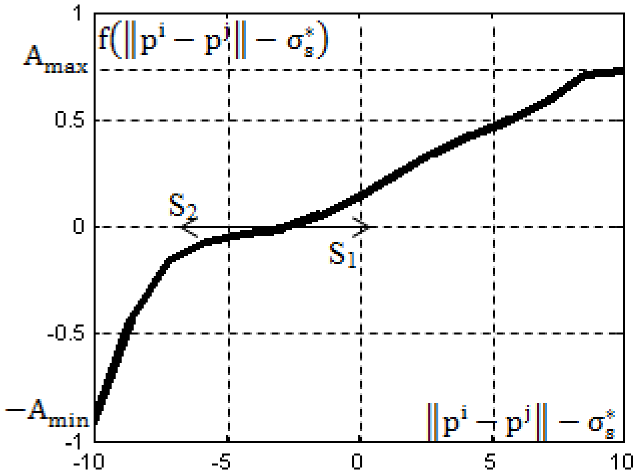
As can be seen from Table 1, the graph of the nonlinear functions is similar to the graph of the explicit functions described in (Gazi and Passino, 2003; Wang and Fang, 2010; Chen et al., 2006; Yang et al., 2008). This means that the explicit functional forms are special cases of the fuzzy function constructed by (9) satisfying the condition (10).

3. STABILITY ANALYSIS STABILITY ANALYSIS OF CONVERGING PROCESS

Stability of converging process is considered as the ability to ensure the formation and distance between individuals while moving. Specifically, the individuals in the herd always move together towards a certain goal and they will converge around the target with a definite distance. In our previous study (Le and Le, 2013), the swarm robot model was only considered to have the same interaction ability between individuals. However, for a real bio-swarm, the mobility of each individual is finite and determined by the neighboring individuals, scope of observation and the communication ability of the individual in the operating environment. Assume that individual robots are considered as points and interactions among individuals in the swarm are the same. Thus, the velocity of each individual in the flock is determined as follows:

$$\dot{p}^i = \sum_{j=1, j\neq i}^{N} w_{ij} f\left(\left\|p^j - p^i\right\|\right) \frac{(p^j - p^i)}{\left\|p^j - p^i\right\|} \tag{15}$$

Table 1. Types of interaction forces between individual *i* and *j* based on fuzzy logic.

No.	Fuzzy set parameters for input and output	Input/output relationship
1		
2		
3		

where $w_{ij} \in R$ being the quantity that characterizes the interaction between the pairs of individuals, with $w_{ij} = w_{ji} \geq 0$.

Let $W = [w_{ij}] \in R^{N\times N}$ be the interaction matrix between individuals. Assuming that $w_{ii} = 0$, if $w_{ij} = 0$. This means that the interaction between the pair of individuals does not exist, and $w_{ij} \neq 0$ means that there exists a interaction between individual *i* and individual *j*. For (15), to simplify the calculation, it is possible to approximate $w_{ij} = 1$ with *j*=1, 2,...., *N*.

Let $L=[l_{ij}]\in R^{N\times N}$ be the Laplace representation of the interaction matrix W, in which

$$l_{ij} = \begin{cases} -w_{ij} \text{ if } i \neq j \\ \sum_{j=1,j\neq i}^{N} w_{ij} \text{ if } i = j \end{cases} \tag{16}$$

L is the Laplace matrix whose sum of elements on a row or on a column is always 0.

The center p_w^c of the swarm is defined by:

$$p_w^c = \frac{1}{N}\sum_{i=1}^{N} p^i \tag{17}$$

Take the derivative of p_w^c with respect to time

$$\dot{p}_w^c = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} w_{ij} g(\|p^j - p^i\|)(p^j - p^i)$$
$$= \frac{1}{N}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{ij}\left[g(\|p^j - p^i\|)(p^j - p^i) + g(\|p^i - p^j\|)(p^i - p^j)\right] = 0 \tag{18}$$

Formula (18) shows that p_w^c of a swarm described by the model (15) with attraction/repulsion function *g*(.) as given in (9) is always identical with every *t* and does not depend on the interaction between the pairs of individual in the swarm.

Theorem: *The individuals of the swarm are described as (15) with an attraction/repulsion function built according to the control law (9), satisfying condition (10), over time, all individuals of the swarm will converge and maintain in the restricted area*

$$\Omega_{\sigma w} = \left\{\sum\|p^j - p^i\|^2 \leq \sigma_w^2\right\} \tag{19}$$

in which $\sigma_w = \sigma_s^* \sqrt{\frac{A_{min}\lambda_n}{\alpha_w \lambda_2}}$ *, with* λ_2, λ_n *being smallest and largest individual values of L, respectively.*

Prove theorem:

Let e_w^i be the difference between the position of the robot i and the center:

$$e_w^i = p^i - p_w^c \tag{20}$$

Take the derivative of the position deviation e_w^i in (20) over time:

$$\dot{e}_w^i = \dot{p}^i - \dot{p}_w^c = \dot{p}^i \tag{21}$$

Select Lyapunov function for individual *i*:

$$V_{iw} = \frac{1}{2}\|e_w^i\|^2 = \frac{1}{2} e_w^{iT} e_w^i \tag{22}$$

Take the derivative of V_{iw} function over time, the result is

$$\dot{V}_{iw} = \dot{e}_w^{iT} e_w^i = \dot{p}^{iT} e_w^i = \sum_{j=1}^{N} w_{ij} g(\|p^j - p^i\|)(p^j - p^i)^T e_w^i \tag{23}$$

Definition of total potential function Lyapunov:

$$V_w = \sum_{i=1}^{N} V_{iw} = \frac{1}{2}\sum_{i=1}^{N} e_w^{iT} e_w^i \tag{24}$$

Taking the derivative of V_w with respect to time t

$$\dot{V}_w = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} g(\|p^j - p^i\|)(p^j - p^i)^T e_w^i$$
$$= \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{ij}\left[g(\|p^j - p^i\|)(p^j - p^i)^T e_w^i + g(\|p^i - p^j\|)(p^i - p^j)^T e_w^j\right]$$
$$= -\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2$$
$$= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2$$
$$= -\frac{1}{2}\sum_{S_1} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 - \frac{1}{2}\sum_{S_2} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2$$
$$= -\frac{1}{2}\left[\sum_{S_1} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 + \sum_{S_2} -w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|\|p^j - p^i\|^2\right] - \frac{1}{2}\left[\sum_{S_2} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 - \sum_{S_2} -w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|\|p^j - p^i\|^2\right]$$
$$= -\frac{1}{2}\left[\sum_{S_1} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 + \sum_{S_2} -w_{ij}\mu(\|p^j - p^i\|)\|p^j - p^i\|^2\right] - \frac{1}{2}\left[\sum_{S_2} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 - \sum_{S_2} -w_{ij}\mu(\|p^j - p^i\|)\|p^j - p^i\|^2\right] \tag{25}$$

Set:

$$V_1 = -\frac{1}{2}\left[\sum_{S_1} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2\right] - \frac{1}{2}\left[\sum_{S_2} -w_{ij} f(\|p^j - p^i\|)\|p^j - p^i\|^2\right] \tag{26}$$

$$V_2 = \frac{1}{2}\left[\sum_{S_2} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2\right] + \frac{1}{2}\left[-\sum_{S_2} -w_{ij} f(\|p^j - p^i\|)\|p^j - p^i\|^2\right] \tag{27}$$

Combine (26), (27) to (25):

$$\dot{V}_w = V_1 - V_2 \tag{28}$$

The result is

$$\begin{cases} \sum_{S_2} -w_{ij} f(\|p^j - p^i\|)\|p^j - p^i\|^2 \leq A_{min}\sum_{S_2} w_{ij}\|p^j - p^i\|^2 \\ \sum_{S_1} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 \geq G_{amin}\sum_{S_1} w_{ij}\|p^j - p^i\|^2 \end{cases} \tag{29}$$

Hence from (27), the following relation is determined

$$V_2 \geq \frac{1}{2}\sum_{S_2} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 - \frac{1}{2}\sum_{S_2} w_{ij} A_{min}\|p^j - p^i\|^2 \quad (30)$$

The right-hand side of the inequality (30) can be rewritten as follows:

$$\frac{1}{2}\sum_{S_2} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 - \frac{1}{2}\sum_{S_2} w_{ij} A_{min}\|p^j - p^i\|^2$$

$$= \frac{1}{2}\sum_{S_2} w_{ij} \frac{f(\|p^j-p^i\|) - A_{min}\|p^j - p^i\|}{\|p^j - p^i\|}\|p^j - p^i\|^2$$

$$= -\frac{1}{2}\sum_{S_2}\{-f(\|p^j - p^i\|) + A_{min}\|p^j - p^i\|\}\frac{(p^j-p^i)^T}{\|p^j-p^i\|} w_{ij}\|p^j - p^i\|\frac{(p^j-p^i)}{\|p^j-p^i\|}$$

$$= -\frac{1}{2}\sum_{S_2}\{-f(\|p^j - p^i\|) + A_{min}\|p^j - p^i\|\}1^T w_{ij}\|p^j - p^i\| 1 \quad (31)$$

Note that:

$$\beta_w = \max_{S_2}\{-f(\|p^j - p^i\|) + A_{min}\|p^j - p^i\|\} = A_{min}\sigma_s^* \quad (32)$$

Combine (31) and (32) infer that (30) is equivalent to:

$$V_2 \geq -\frac{1}{2} A_{min}\sigma_s^{*2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 1^T . w_{ij} . 1 \quad (33)$$

From (33) infer that:

$$-V_2 \leq \frac{1}{2} A_{min}\sigma_s^{*2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 1^T . w_{ij} . 1 \quad (34)$$

Return to formula (26), the result is

$$V_1 \leq -\frac{1}{2}\sum_{S_1} w_{ij} g(\|p^j - p^i\|)\|p^j - p^i\|^2 - \frac{1}{2}\sum_{S_2} w_{ij} A_{min}\|p^j - p^i\|^2 \quad (35)$$

Set: $\alpha_w = \min\{G_{amin}, A_{min}\}$

So:

$$V_1 \leq -\frac{1}{2}\alpha_w \sum_{S_1 \cup S_2} w_{ij}\|p^j - p^i\|^2 = \frac{1}{2}\alpha_w \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(p^j - p^i)^T w_{ij}(p^j - p^i) \quad (36)$$

From (16), the following equation is determined

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(p^j - p^i)^T w_{ij}(p^j - p^i) = e_w^T(L\otimes I_n)e_w \quad (37)$$

and

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 1^T w_{ij} 1 = 1^T(L\otimes I_n)1 \quad (38)$$

in which

$$e_w = p^j - p^i = (p^j - \dot{p}_w^c) - (p^i - \dot{p}_w^c) = e_w^j - e_w^i \quad (39)$$

In (37) and (38), the notation ⊗ is Kronecker product, $L\otimes I_n$ is the matrix of *NN*×*NN* dimension created from *N*×*N* block, in which block (*i*, *j*) is the matrix $l_{ij}I_n$, specifically as follows:

$$L\otimes I_n = \begin{bmatrix} l_{11}I_n & l_{12}I_n & \dots & l_{1N}I_n \\ l_{21}I_n & l_{22}I_n & \dots & l_{2N}I_n \\ \dots & \dots & \dots & \dots \\ l_{N1}I_n & l_{N2}I_n & \dots & l_{NN}I_n \end{bmatrix} \in R^{NN\times NN} \quad (40)$$

Combine (36), (37) and (38), the result is

$$V_1 \leq -\alpha_w e_w^T(L\otimes I_n)e_w \quad (41)$$

When interaction matrix is symmetric and there exists a link between the robots, all the values of the matrix L are denoted by λ_h, *h*=1, 2, 3,…, *n* and satisfy:

$$0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_n \quad (42)$$

$\lambda_1, \lambda_2, \dots, \lambda_n$ is the solution of the system of equations: $\det(L - \lambda I) = 0$, λ_2 is the smallest of the non-zero values of the matrix L, I is the unit matrix.

Hence, from (41), following relationship is formulated

$$V_1 \leq -\alpha_w e_w^T(L\otimes I_n)e_w \leq -\alpha_w\lambda_2\|e_w\|^2 \quad (43)$$

Combined (34) and (43), the result is

$$\dot{V}_w = V_1 - V_2 \leq -\alpha_w\lambda_2\|e_w\|^2 + A_{min}\sigma_s^{*2}\lambda_n\|1\| \quad (44)$$

From (44) infer that to make $\dot{V}_w < 0$ then:

$$\|e_w\|^2 > \frac{A_{min}\sigma_s^{*2}\lambda_n}{\alpha_w\lambda_2}$$

Set:

$$\sigma_w = \sigma_s^*\sqrt{\frac{A_{min}\lambda_n}{\alpha_w\lambda_2}} \quad (45)$$

then:

$$\|p^j - p^i\|^2 > \sigma_w^2 \quad (46)$$

Theorem is proven □

From (46) draws the influence of the parameters to the convergent limit of $\Omega_{\sigma w}$ of the swarm:

- When increasing A_{min} means increasing the repulsive force, which will increase the convergence limit of the swarm.
- On the contrary, if increasing α_w, the convergence limit of the swarm will be decreased.

4. SIMULATION RESULTS

To simulate the moving process of swarm robots, the algorithm is implemented through the following steps:

Step 1:

- Determining the number of robots in the swarm *N*.
- Creating the initial position of the robots in the simulation space.
- Entering the interaction matrix W between the individual robots in the swarm.
- Providing the safe distance between the robots.
- Enter the number of calculating steps *K* and the calculation interval Δ*t* and the total time for moving being calculated by *t* = *K* * Δ*t*.

Step 2:

- Calculating the distance between the robots (*i*, *j*) according to (3) with *j*≠*i* and *j*=1÷N.
- Determining the fuzzy attractive/repulsive force *f*(.) according to the law (9) and satisfying condition (10).

Step 3:

- Determining the moving speed of individual i at step k (with k=1÷K)

$$v^i[k] = w_{\mathrm{ij}} * f[k] * \frac{(p^{\mathrm{j}}[\mathrm{k}]-p^{\mathrm{i}}[\mathrm{k}])}{\|p^{\mathrm{j}}[\mathrm{k}]-p^{\mathrm{i}}[\mathrm{k}]\|} \tag{47}$$

- The distance traveled corresponding to a calculated step is described by:

$$\Delta p^i[k+1] = \Delta p^i[k] + v^i[k] * \Delta t \tag{48}$$

- The new coordinates of the i-th instance after (k+1) moves are updated as follows:

$$p^i[k+1] = p^i[k] + \Delta p^i[k+1] * \Delta t \tag{49}$$

The loop consisting of steps 2 and 3 is executed until K steps are completed. The operating process of robots is simulated through fuzzy logic features presented as follows.

4.1 Fuzzy attraction/repulsion function simulation

As discussed above, the f(.) is a nonlinear function depending on the distance between the robots, so it is possible to calculate approximately of the f(.) based on the Mamdani fuzzy model by the structure SISO as shown in Figure 3:

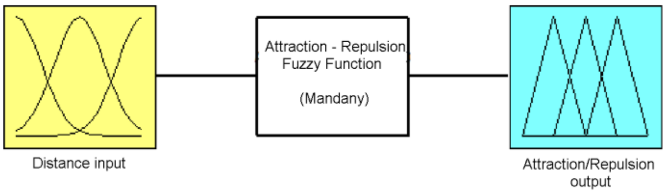


Fig. 3. The fuzzy structure of the attraction/repulsion calculations between robot individuals.

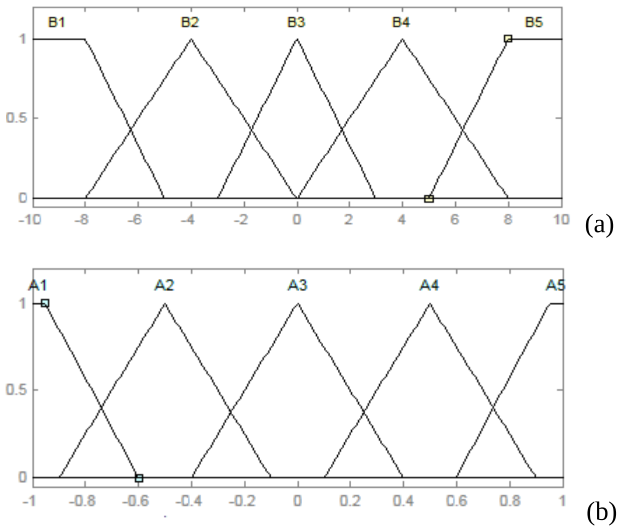


Fig. 4. Function of input signal (a) and of output signal (b) of fuzzy set f(u).

With the steps presented in section 2, the fuzzy set is designed flexibly and needs to satisfy conditions (8), (9) and (10). From there, the input/output values of the simulated fuzzy set are set as follows:

- Input signal is $u = \sigma_s - \sigma_s^*$, u = [-10, 10] ∈R, divide u into 5 spaces B^{k} as figure 4a.
- Output signal is $A = f(\sigma_s - \sigma_s^*)$ with the value range of [-1, 1], divided into 5 spaces A^{k} as figure 4b.
- Build 5 laws with form:

  If $\mathrm{u} = \mathrm{B}^{\mathrm{k}}$ Then $\mathrm{A} = \mathrm{A}^{\mathrm{k}}$ with k=1, 2, …, 5.

- Select law MAX-MIN, defuzzification by the weighted average method.

With the solution of the design of fuzzy sets to calculate interactive force between the individual robots as the above steps, the relation between the input/output signals of the fuzzy set f(.) is obtained as shown in Figure 5.

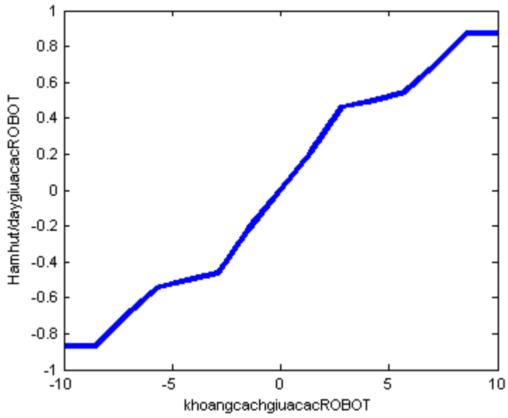


Fig. 5. The relationship between the input and output signals of the fuzzy set f(.)

4.2 Simulation of the converging process of the swarm robot

In the simulations: the symbols o, ♦, — represent the initial position, the final position, and the path of the robot, respectively (Figure 6). Matrix W is symmetric with $w_{\mathrm{ij}} = w_{\mathrm{ji}} \in [0,1]$ if j≠i, $w_{\mathrm{ij}} = 0$ if j=i. Assuming that the interaction matrix W has the following specific values:

$$\mathrm{W} = [w_{\mathrm{ij}}] = \begin{bmatrix} 0 & 0.70 & 0.51 & 0.63 & 0.14 & 0.12 & 0.40 & 0.41 & 0.28 & 0.59 \\ 0.70 & 0 & 0.32 & 0.43 & 0.98 & 0.18 & 0.24 & 0.76 & 0.75 & 0.99 \\ 0.51 & 0.32 & 0 & 0.83 & 0.27 & 0.98 & 0.41 & 0.82 & 0.75 & 0.74 \\ 0.63 & 0.43 & 0.83 & 0 & 0.70 & 0.69 & 0.21 & 0.59 & 0.89 & 0.86 \\ 0.14 & 0.98 & 0.27 & 0.70 & 0 & 0.31 & 0.90 & 0.72 & 0.62 & 0.49 \\ 0.12 & 0.18 & 0.98 & 0.69 & 0.31 & 0 & 0.18 & 0.90 & 0.16 & 0.52 \\ 0.40 & 0.24 & 0.41 & 0.21 & 0.90 & 0.18 & 0 & 0.15 & 0.93 & 0.60 \\ 0.41 & 0.76 & 0.82 & 0.59 & 0.72 & 0.90 & 0.15 & 0 & 0.82 & 0.34 \\ 0.28 & 0.75 & 0.75 & 0.89 & 0.62 & 0.16 & 0.93 & 0.82 & 0 & 0.77 \\ 0.59 & 0.99 & 0.74 & 0.86 & 0.49 & 0.52 & 0.60 & 0.34 & 0.77 & 0 \end{bmatrix} \tag{50}$$

The converging process of the swarm robot is shown in Figure 6.

Figure 7 shows the converging process of the swarm including 5 robots with interaction ability w_{ij} between the different pairs of individuals (i, j).

$$\mathrm{W}_{15} = [w_{\mathrm{ij}}] = \begin{bmatrix} 0.00 & 0.75 & 0.35 & 0.57 & 0.15 \\ 0.75 & 0.00 & 0.52 & 0.84 & 0.91 \\ 0.35 & 0.52 & 0.00 & 1.00 & 0.21 \\ 0.57 & 0.84 & 1.00 & 0.00 & 0.34 \\ 0.15 & 0.91 & 0.21 & 0.34 & 0.00 \end{bmatrix} \tag{51}$$

$$W_{25} = [w_{ij}] = \begin{bmatrix} 0.00 & 0.32 & 0.56 & 0.66 & 0.13 \\ 0.32 & 0.00 & 0.50 & 0.32 & 0.30 \\ 0.56 & 0.50 & 0.00 & 0.71 & 0.83 \\ 0.66 & 0.32 & 0.71 & 0.00 & 0.25 \\ 0.13 & 0.30 & 0.83 & 0.25 & 0.00 \end{bmatrix} \quad (52)$$

$$W_{35} = [w_{ij}] = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (53)$$

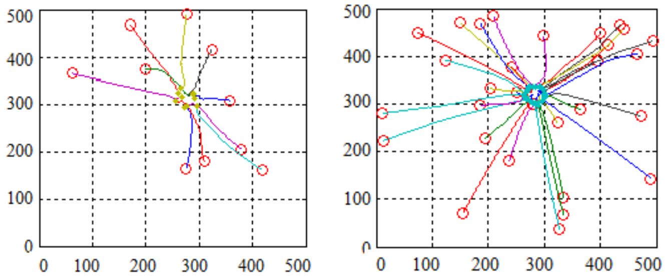
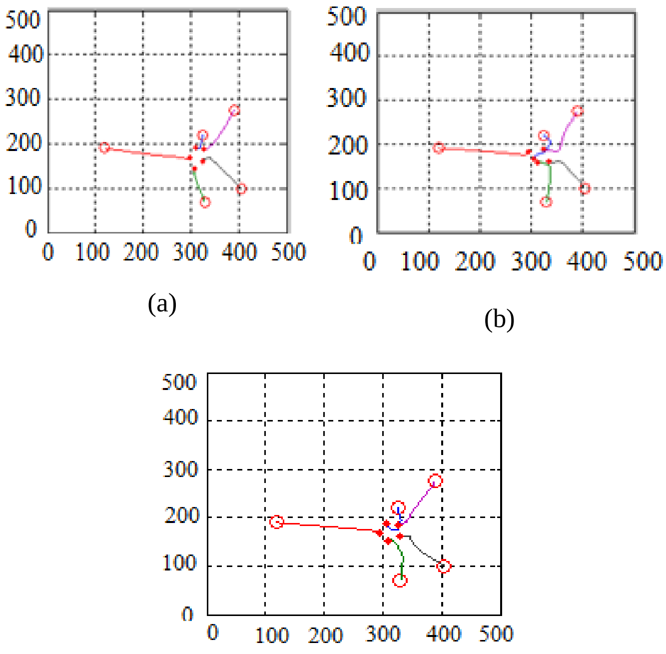


Fig. 6. The converging process of the swarm robot with the communication model when the number of N robots in the swarm changes.

a) N=10, $\sigma_s^* = 30$

b) N=30, $\sigma_s^* = 30$

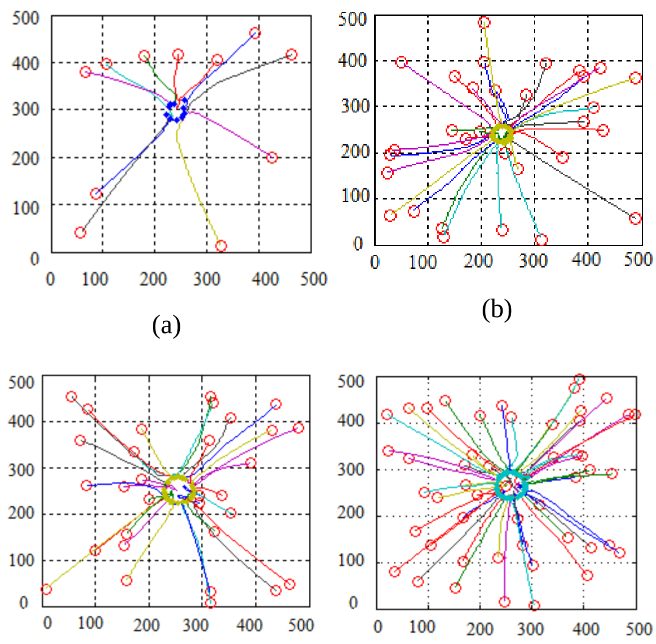
The simulation results of Figure 8 show that: the converging process of the swarm robot based on interaction between individuals.



(a) (b)

(c)

Fig. 7. The converging process of the swarm robot with the communication corresponding to the interaction matrix W_{15} (a), W_{25} (b) and W_{35} (c).



(a) (b)

(c) (d)

Fig. 8. The converging process of the swarm robot corresponding to the case of that the number of robots in the swarm and the ability to communicate between individual change.

a) N=11, $\sigma_s^* = 30, \lambda_2 = 3.97, \lambda_n = 8.29, \sigma_w = 23.8$, R=14.79

b) N=31, $\sigma_s^* = 30, \lambda_2 = 13.4, \lambda_n = 21.05, \sigma_w = 15.93$, R=11.31

c) N=31, $\sigma_s^* = 50, \lambda_2 = 12.6, \lambda_n = 19.93, \sigma_w = 26.6$, R=20.95

d) N=51, $\sigma_s^* = 50, \lambda_2 = 21.98, \lambda_n = 2.28, \sigma_w = 22.67$, R=17.69

From Figure 8, commented:

- The actual radius of convergence R is always smaller than the satisfied calculated radius σ_w
- As the number of robots in the swarm increases, the radius of convergence decreases.
- Safe distance σ_s^* increases, the radius of convergence increases.

The simulation results of the swarm robot show that the solution of building up the attraction/repulsion force between the robot individuals in the fuzzy logic was achieved: after a large enough time of moving, the robots in the swarm have converged around an area with a definite radius. Radius of convergence depends on the following factors:

- The observation of each individual,
- Number of individuals in the swarm,
- Safe distance between individuals,
- The ability to interact between pairs of individuals.

The above conclusions are consistent with the content of the theorem stated in section 3.

5. CONCLUSIONS

This paper develops an algorithm to calculate the attraction/ repulsion force between individual robots in a swarm based on Mamdani fuzzy model with SISO structure. The theorem on convergent stability of the system was stated and proven, which contributed to the theoretical control of nonlinear dynamics. Matlab simulation results confirm that the proposed algorithms ensure reliability, improve controllability and stability of the swarm robot. This result promotes the application of this type of robot in practice, improves the ability to solve complex problems, and further enhances the system's ability to respond to various random external influences.

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