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Stabilization of Linear Networked Control Systems: A Switched Scheduling Method ⋆

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Abstract: This paper considers the stabilization of networked linear systems with spatially deployed sensors and actuators. Each local agent of sensor/actuator suffers from lack of observability/controllability, but the collective measurements of all sensors are observable and the joint effects of all actuators render controllability of the plant. To compensate observability and controllability inadequacies of individual agent, a recent effort is the distributed scheme by using consensus-based communication with neighboring agents. Considering the fact that inter-agent communications are usually more expensive than local computations by agents, this paper abandons the communication based scheme and adopts another strategy where the plant at each time selects exactly one sensor/actuator in a switching way. Unlike existing sophisticated sensor/actuator scheduling algorithms, the switched scheduling design used in this paper exhibits remarkable simplification since it allows each senso/actuator to be arbitrarily picked up, with only one requirement that there exists contiguous time intervals not exceeding a certain bound such that on each time interval all sensors/actuators are selected without missing, leaving the switching instants and the dwell times of each sensor/actuator totally random. The explicit calculation of this bound is presented by utilizing an averaging method. The method is illustrated with detailed numerical experiments.

Keywords: Switching observers, switching controllers, linear systems, local observability, local controllability, averaging.

1. INTRODUCTION

The progress of digital computation and communication has enabled the development of networked control systems (NCS) in which multiple sensors and actuators are connected to a plant via a shared communication network. This architecture can date back to as early as 1980s under the name of "integrated communication and control networks" (Halevi and Ray, 1988). Compared with conventional point-to-point control in which maintenance and upgrades had become increasingly difficult, the NCS can offer much capability of building a large control system with increased flexibility and reliability, together with convenience for diagnosis and maintenance. With the advent of networking technologies, NCS can now be found in a variety of settings, including automobiles, surveillance, data gathering, aircraft, and autonomous vehicles, to name a few. However, these bring new challenges such as network-induced delays, packet dropouts, data disordering, date rate limitation and quantization effects, plant model inaccuracies and sensor/actuator noise etc. These issues can greatly degrade the system performance or even destabilize the system at certain conditions, and therefore considerable works have been done for a better understanding of them; see the reviews (Zhang et al., 2001; Walsh and Ye, 2001; Yang, 2006; Gupta and Chow, 2010; Tipsuwan and Chow, 2003) for details.

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Among active research fields in NCS, in view of sensing and actuating being two key ingredients of NCS (Gupta and Chow, 2010; Hespanha et al., 2007), the sensor/actuator scheduling is one of the most challenge problem. To see this clearly, the author notes that there are a large amount of sensors and actuators connecting to a plant via a shared communication network and usually the network cannot accommodate them simultaneously at one time. This enforces that only a limited number of sensors and actuators can get access to the network at each time step, unfortunately with the system's observability/controllability being destroyed since a subset of sensors/actuators are inadequate to estimate/stabilize the systems. To compensate the lack of observability/controllability of individual sensor/actuator, a recent effort is the distributed scheme by using consensus-based communication with neighboring agents (Kim et al., 2019; Wang and Morse, 2017). However, considering the fact that inter-agent communications are usually more expensive than local computations by agents, the sensor/actuator scheduling remains a parallel research line. In sensor/actuator scheduling, the plant selects exactly one sensor/actuator at each time according to a selection rule among them. The design of selecting rule achieving desired performance stays as one of the central tasks in NCS theory (Zhang and Hristu-Varsakelis, 2006; Millán et al., 2013; Dacic and Nesic, 2008; Gupta et al., 2006).

The sensor/actuator scheduling is not an easy task since a rule has to be designed according to which one or multiple sensors/actuators can be selected at each time out of many sensors/actuators. The schedulability conditions are presented in (Hristu-Varsakelis, 2005; Dai et al., 2010; Branicky et al., 2002) for a group of continuous-time linear systems by a common Lyapunov function. Also, various protocols, including the round-robin protocols (Walsh et al., 2001), the gossip protocols (Shin et al., 2020), the try-once-discard protocols(Walsh et al., 2002) and the proximal algorithms based protocols (Zare et al., 2019) have been proposed in the literature, where each sensor/actuator is connected to the plant for a certain amount of time and the activation interval for each sensor/actuator is called the transmission interval. Following (Walsh et al., 2001, 2002), (Heemels et al., 2010) and (Carnevale et al., 2007) compute the maximum allowable transmission interval (MATI) such that stability of the NCS is preserved if the actual transmission intervals are smaller than the MATI. Also, a sensor scheduling algorithm preserving observability and an actuator schedulability condition preserving controllability are respectively proposed in (Zhang and Hristu-Varsakelis, 2006) and (Xie et al., 2002). The concept of communication sequence describing the network-access assignment is presented in (Rehbinder and Sanfridson, 2004) as a scheduling policy. An optimal dynamic scheduler determining the network allocation that optimizes performance of multi-loop control systems has been presented in (Ma et al., 2019; Walsh and Ye, 2001) by solving a nonlinear integer programming problem. Some stochastic scheduling polices have been developed such as (Long et al., 2017) for the controller/scheduler co-design, (Gupta et al., 2006) for the estimation problem over a shared communication network, and (Zhang et al., 2020) for the output feedback stabilization of networked control systems. Other approaches includes the rate-monotonic scheduling algorithm (Branicky et al., 2002), the time-division based scheduling policy (Lin et al., 2005), and the scheduling-and-feedback-control co-design procedure (Dai et al., 2010).

However, the scheduling methods reviewed above suffer from intrinsic computational complexity and thus face limited applications. For example, the method in (Guo, 2010, Theorem 1) requires solving nonlinear matrix inequalities whose solution is generally hard to obtain. To seek a more numerically tractable algorithm, the authors in (Guo et al., 2012, Theorems 1, 2) present an LMIs based design approach. Nonetheless, the number of these LMIs in (Guo et al., 2012, Theorems 1, 2) is $2^r \cdot 2^s$ and therefore this method is obviously impractical when the dimensions r and s for the input and output are large. The more recent work (Kundu and Quevedo, 2021) considers periodic scheduling and control design for NCS, where the existence of scheduling policy is characterized via the feasibility of a large set of complex matrix equations (for example, feasibility of equations (5)-(8) in Theorem 1 and feasibility of equations (11)-(13) in Theorem 2), to which further conditions are not provided. Furthermore, these characterizations are based on the fact that the gain matrix is given in advance; when the gain matrix is also viewed as an unknown variable, the above matrix equations become nonlinear and thus are difficult to solve.

To reduce the complication in scheduling design, this paper offers a switched scheduling approach. Note that, by modeling the access of sensors (actuators) to the plant via a switching mechanism, the resulting NCS is investigated by (Donkers et al., 2011) in terms of protocol modelings of Round-Robin, Try-Once-Discard, and Random-Access, by (Dacic and Nesic, 2008) in terms of switching observer design, and by (Zhang and Hristu-Varsakelis, 2006; Kundu and Quevedo, 2021) in terms of controller and scheduling co-design. However, the switching sequences in these work are restricted to be periodic ones. This strong assumption is not used in the present paper. Instead, it is assumed that the sensors (actuators) are classified into several groups and, at each different time instant, only one group of sensors can send its measure to the plant and again exactly one group of actuators is active. This scenario arises in, for example, the telemetry-data aerospace system, the tracking and discrimination problem and the socioeconomic problem (see for example (Athans, 1972)). The switched scheduling method in the current paper is similar to but different from the Round-Robin algorithm since, in each round, the order for choosing the communication group is randomly selected rather than sequently. To show this distinction more clear, let us mention that our method is scheduling free since it imposes no selection order on sensors and actuators, with only one requirement that there exists contiguous time intervals not exceeding a certain bound such that on each time interval all sensors (actuators) are selected without missing, leaving the switching instants and the dwell times of each sensor (actuator) totally random. The explicit calculation of this bound is presented by utilizing an averaging method and the scheduling of sensors and actuators for stabilization is avoided since one only requires that all sensors (or actuators) are activated in each interval without missing, leaving the switching instants and the dwell times of each sensor (actuator) totally random. This feature makes our random switching scheduling strategy different from some existing Markov sensor/acutor assignment. For example, the work (Zhu et al., 2020; Guo, 2010; Zhang and Guo, 2019; Guo et al., 2012) use a Markov chain to specify actuator assignment and a control synthesis is established by solving matrix inequality depending on the probability transition matrix of the modelled Markov chain. An underlying difficult is how to obtain the probability transition matrix and this difficulty is avoided in the current paper. Furthermore, in terms of algorithm complexity, the computational cost of our method is largely reduce compared with (Guo, 2010; Kundu and Quevedo, 2021) which requires solving nonlinear matrix equations and (Guo et al., 2012) where the number of designed LMIs scales exponentially with the dimensions of input and output.

While the synthesis of switched observer or controller is not new in the literature of switched systems (see (Alessandri and Coletta, 2001)), these results are based on some restrictive assumptions that all the modes of the switched systems are observable or controllable and that all mode of the closed-loop system share a common Lyapunov function which is obviously over-restrictive. In this paper, a less limited assumption is made that none of mode is observable and controllable (i.e., the local observability and local controllability are not assumed), but the system with all sensors is collectively observable and the system with all actuators is collectively controllable. As a consequence of this assumption, the traditional design by using switching observer mentioned above is not applicable here. Some other lines of designing observer under switching measurement equations are also proposed. For example, the authors in (Babaali et al., 2004) transform algebraically a randomly switched linear measurement equation into a nonlinear yet deterministic equation and construct an asymptotic observer. However, the transformed measurement equation is nonlinear and the resulting convergence analysis become more difficult. To tackle the challenge problem of switched observe and controller design without local observability and controllability, the author suggests an averaging method whose main idea is included in Lemma 1. This averaging principle yields an easy-to-implement design procedure (refer to Algorithms 1-3) and simple-to-prove convergence analysis (refer to Theorems 1-2) for linear systems without local controllability and observability. Under our framework, both observer-based and static-output-based feedback control for networked linear systems are investigated.

The author also mentions the merits and advantages of our method method from the numerical aspect. The sequential architecture of unobservable local sensors (as well as uncontrollable local actuators) connecting to the plant entails that a kind of fast switching mechanism should be exploited to compensate potential divergence due to unobservable local sensors and uncontrollable local actuators. That is, fast switching plays a decisive role if unobservable local sensors and uncontrollable local actuators are connected to the plant in a sequential way. This is well illustrated via simulations: system with a stabilizing switching may lead to divergence if one slows down the speed of switching among sensors or actuators. Furthermore, it has been also observed in our simulation that faster switchings renders system convergence in less time. Aside from the advantage of speed-up convergence inherent in fast switching, our simulation additionally discovers that the undesirable overshoot phenomena can also be combated via fast switching. While it is good to see that fast switching has a couple of advantages, fast switching leads to chattering and thus imposes severe requirement on hardware to overcome it so that energy dealing with chattering can be saved. Keeping this caution in mind and retaining the advantages of fast switching, one can adopt fast switching in earlier stage of sensor/actuator selection to fight with overshoot, continued with a moderate switching during the subsequent times to achieve a fast convergence, and followed by a slow switching to save energy during the remaining state when the deviation of the system response from the origin is relatively small.

The rest of paper is organized as follows. With the problem formulation being presented in Section 2, the main result is proposed in Section 3, where Section 3.1 elaborates on the existence of scheduling switching for stabilization, Section 3.2 presents an averaging lemma on which our theoretic results build, Section 3.3 is devoted to the observer-based feedback control and Section 3.4 deals with the static output feedback control. Detailed numerical experiments are provided in Section 4.

2. PROBLEM FORMULATION

Consider a linear control system

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx \end{cases}, \tag{2.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^r$ and $y \in \mathbb{R}^s$ are respectively the state, input and output of the system, $A \in \mathbb{R}^{n\times n}, B = [b_1, \cdots, b_r] \in \mathbb{R}^{n\times r}, C = [c_1^T, \cdots, c_s^T]^T \in \mathbb{R}^{s\times n}$ are system matrices. There are r actuators corresponding to $\{b_1, \cdots, b_r\}$ and s sensors corresponding to $\{c_1, \cdots, c_s\}$. The traditional control system configuration assumes that at each time the r actuators or the s sensors work simultaneously. That is, the plant at each time is affected by $b_1u_1 + \cdots + b_ru_r$, and the s sensed information $\{c_1x, \cdots, c_sx\}$ are collectively taken as input to the observer.

However in practice, it is usually the case that the actuators or sensors are spatially distributed and only a subset of the actuators or sensors work at each time. More specifically, by classifying the actuators into κ groups so that $B = [B_1, \cdots, B_\kappa]$ with $B_i \in \mathbb{R}^{n\times r_i}$ and $r_1 + \cdots + r_\kappa = r$, and by dividing the sensor into ℓ groups so that $C = [C_1^T, \cdots, C_\ell^T]^T$ with $C_i \in \mathbb{R}^{s_i\times n}$ and $s_1 + \cdots + s_\ell = s$, a more realistic situation is that only one B_i or one C_j is connected to the plant at each instant. To this end, a selecting rule $\sigma : [0, \infty) \to \{1, \cdots, \kappa\}$ among actuators $\{B_1, \cdots, B_\kappa\}$ and another selecting rule $\theta : [0, \infty) \to \{1, \cdots, \ell\}$ among sensors $\{C_1, \cdots, C_\ell\}$, which are both continuous-time Markov chains adapted to the filtration $\{\mathcal{F}_t | t \geq 0\}$, must be specified so that the actual input to the plant is $B_{\sigma(t)}u_{\sigma(t)}$ and the actual output of system is $C_{\theta(t)}x(t)$; see also Figure 1 for illustration. This scenario is widely discussed in the literature (Dacic and Nesic, 2008; Lee et al., 2001; Wu et al., 2014). To avoid the triviality, the following assumptions are made.

Assumption 1. None of $(A, B_i), i = 1, \cdots, \kappa$, is controllable, while (A, B) is controllable. None of the pair $(A, C_i), i = 1, \cdots, \ell$ is observable, while (A, C) is observable.

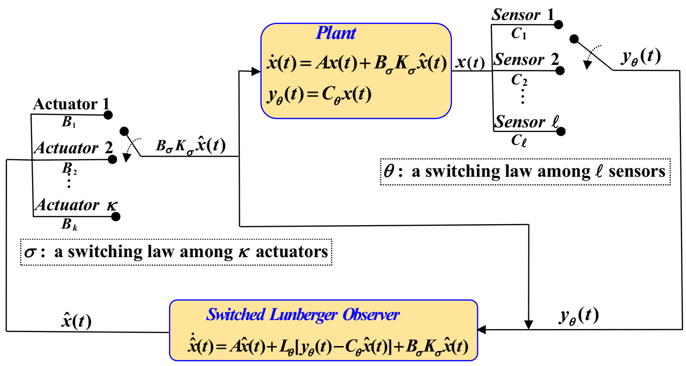

Fig. 1. Observer-based feedback of NCS: the state $x(t)$ is sensed by a switching sensor C_θ to give the sensed signal $C_\theta x(t)$, which is taken as an input to an observer B_σ, giving rise to an estimation $\hat{x}(t)$ of the state $x(t)$. The estimated state $\hat{x}(t)$ is feedback into the plant via a switching actuator B_σ with the feedback gain matrix K_σ.

Remark 1. If for some mode, say mode i, the pair (A, B_i) is controllable, then the stabilization problem becomes trivial because the system can be stabilized by letting the switching law be dwelled on mode i; similar remark holds for the assumption on the observability.

This paper will consider the stabilization of linear networked control systems under two scenarios: observer-based stabilization and static output based stabilization. They are formulated in Subsection 2.1 and Subsection 2.2, respectively.

2.1 Observer design and observer-based stabilization.

Under the above framework, this section designs switched linear observers and observer-based switched feedback controllers to stabilize the system, with the closed-loop dynamics being given as follows,

$$\dot{\hat{x}} = A\hat{x} + L_{\theta(t)}[C_{\theta(t)}\hat{x}(t) - y_{\theta(t)}] + B_{\sigma(t)}K_{\sigma(t)}\hat{x}(t), \quad (2.2)$$

$$\dot{x} = Ax + B_{\sigma(t)}K_{\sigma(t)}\hat{x}(t), \quad (2.3)$$

where $L_i \in \mathbb{R}^{n\times s_i}, i = 1, \cdots, \ell$ and $K_i \in \mathbb{R}^{r_i\times n}, i = 1, \cdots, \kappa$.

Since the scheduling policies σ and θ for stabilization are hard to design, then it arises a nature question: how to reduce the complexity in the design of scheduling policies or can one only design gain matrices K_i, L_i such that the closed-loop system is stable for arbitrary switchings σ and θ? Unfortunately, the answer is no. It has been established in (Lin et al., 1996; Mancilla-Aguilar and Garcia, 2000) that the verification of asymptotic stability of switched linear system for arbitrary switchings is equivalent to the existence of a common Lyapunov function. Due to the lack of local controllability and local observability in Assumptions 1, the existence of a common Lyapunov function is impossible. Therefore, the observer-based stabilization of NCS becomes solving the following problem.

Problem A: (Observer-based feedback control of NCS) For the networked control system (2.1), design the scheduling rules $\sigma(t), \theta(t)$ and observer feedback gain matrices $L_j, j = 1, \cdots, \ell$ and control feedback matrices $K_i, i = 1, \cdots, \kappa$, such that the closed-loop system (2.2)-(2.3) is asymptotically stable.

Since $A_i^c \triangleq A+B_iK_i$ is unstable for any K_i, it then follows from the lemma in (Vidyasagar, 1978, p.204, Eq. (70)) that there exists $T > 0$ such that for any switching signal with dwell time greater than or equal to T, the resulting switching system is unstable. Therefore, a necessary condition for the stabilizing switchings is that these switchings should have dwell time smaller than a positive number τ. It will be shown in what follows that the above condition is also sufficient if the positive number τ to be chosen small appropriately. More specifically, there exists a small positive number T^* such that for any switching with dwell time smaller than T^*, the closed-loop system is asymptotically stable. This result brings us great advantage in reducing the complexity of designing the scheduling policies which are only required to switch with transmission times smaller than T^*.

2.2 Static output feedback for NCS

To formulate the static output feedback for NCS, some definitions and notations are presented as preparation.

Definition 1. **(Activation functions for sensors and actuators)**

(1) At any time t, one uses a binary function $\alpha_i(t) : \mathbb{R} \to \{0, 1\}$ to denote the action state of the actuator $i \in \{1, \cdots, \kappa\}$ in the following sense: $\alpha_i(t) = 1$ if the actuator i is connected to the plant at time t and $\alpha_i(t) = 0$ otherwise.

(2) At any time t, one uses a binary function $\beta_i(t) : \mathbb{R} \to \{0, 1\}$ to denote the action state of the sensor $i \in \{1, \cdots, \ell\}$ in the following sense: $\beta_i(t) = 1$ if the sensor i is connected to the plant at time t and $\beta_i(t) = 0$ otherwise.

Then the control system can be rewritten in the following way

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{\kappa} B_i\alpha_i(t)u_i(t),$$

where $u_i(t)$ is the control signal generated by actuator i at time t. The problem now becomes designing a pair of activation functions for sensors and actuators and a set of controller gain matrices so that desired control objectives (e.g., stability) can be achieved. Define

$$\bar{u}_i(t) = \alpha_i(t)u_i(t), i = 1, \cdots, \kappa,$$
$$\bar{y}_j(t) = \beta_j(t)y_j(t), j = 1, \cdots, \ell$$

to be the input and output signals actually used by the plant at time t respectively. Define the following stacked vectors $\bar{u}(t) = [\bar{u}_1^T(t), \cdots, \bar{u}_\kappa^T(t)]^T$ and $\bar{y}(t) = [\bar{y}_1^T(t), \cdots, \bar{y}_\ell^T(t)]^T$. By introducing the matrices

$$M_I(t) = \mathrm{diag}([\alpha_1(t), \cdots, \alpha_\kappa(t)]),$$
$$M_O(t) = \mathrm{diag}([\beta_1(t), \cdots, \beta_\ell(t)]),$$

one has

$$\bar{u}(t) = M_I(t)u(t),$$
$$\bar{y}(t) = M_O(t)y(t).$$

That is, the sensors (actuators) which are not actively connect to the plant are ignored and whose values are set to zero. With these preparation, the original control system can be rewritten in the following form

$$\dot{x}(t) = Ax(t) + BM_I(t)\bar{u}(t), \quad (2.4)$$
$$\bar{y}(t) = M_O(t)Cx(t). \quad (2.5)$$

For the system (2.4)-(2.5), let the output feedback matrix be $F \in \mathbb{R}^{r\times s}$. Therefore, the closed-loop system is

$$\dot{x}(t) = [A + BM_I(t)FM_O(t)C]x(t). \quad (2.6)$$

According to the partitions $B = [B_1, \cdots, B_\kappa]$ and $C = [C_1^T, \cdots, C_\ell^T]^T$, one can partition the matrix F as follows

$$F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1\ell} \\ F_{21} & F_{22} & \cdots & F_{2\ell} \\ \cdots & \cdots & \cdots & \cdots \\ F_{\kappa 1} & F_{\kappa 2} & \cdots & F_{\kappa\ell} \end{bmatrix}$$

with block matrices F_{ij}, $i = 1, \cdots, \kappa, j = 1, \cdots, \ell$ being of appropriate dimensions.

Since at each time t exactly one element of the matrix $M_I(t)$ (and the matrix $M_O(t)$) can be nonzero, the time-varying matrix $M_I(t)$ has κ possible values and $M_O(t)$ has ℓ possible values. At any time t, denoting the activation mode of the sensor to be $\theta(t) \in \{1, \cdots, \ell\}$ and the actuator to be $\sigma(t) \in \{1, \cdots, \kappa\}$, then the matrix $BM_I(t)FM_O(t)C$ in the closed-loop system (2.6) can be calculated based on the partitions of B, C, F as

$$BM_I(t)FM_O(t)C = B_{\sigma(t)}F_{\sigma(t),\theta(t)}C_{\theta(t)}.$$

Therefore, the system (2.6) can be rewritten as

$$\dot{x}(t) = \left[A + B_{\sigma(t)}F_{\sigma(t),\theta(t)}C_{\theta(t)}\right]x(t). \qquad (2.7)$$

The diagram illustration of this system is shown in Figure 2. Now the task of control of NCS becomes solving the following problem.

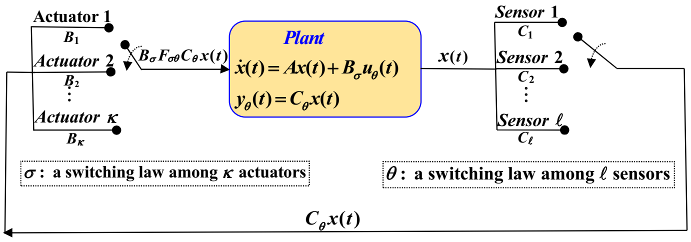

Fig. 2. Static output feedback of NCS: the state $x(t)$ is sensed by a switched sensor C_θ to give the sensed signal $C_\theta x(t)$, which is feedback to the plant via a switching actuator B_σ with the feedback gain matrix $F_{\sigma,\theta}$, with the feedback signal being as $B_\sigma F_{\sigma,\theta} C_\theta x(t)$.

Problem B: (Static output feedback control of NCS) For the networked control system (2.1), design scheduling rules $\sigma(t), \theta(t)$ and static output feedback gain matrices F_{ij}, $i = 1, \cdots, \kappa$, $j = 1, \cdots, \ell$, such that the closed-loop system (2.6) or (2.7) is asymptotically stable.

Due to the difficult in designing the scheduling policies σ and θ, this paper will also follow a convenient line to design the scheduling policies. More specifically, as will be shown, there exists a small positive number T^* such that for any switching σ and θ with dwell time smaller than T^*, the closed-loop system is asymptotically stable.

3. STABILIZATION OF LINEAR NCS

Before addressing the stabilization of the NCS (2.1) under our framework, this section first elaborates on the existence of scheduling switchings that preserve controllability and observability. To tackle the stabilization problem of the NCS (2.1) under our setup, a technique lemma on averaging is presented in Section 3.2. Then Sections 3.3 and 3.4 tackle the stabilization problem, where Section 3.3 deals with the observer based feedback and Section 3.4 considers the static output feedback.

3.1 Existence of scheduling switchings for stabilization

This section begins with the existence of scheduling switchings that preserve controllability and observability. While it has been assumed that the system is collectively controllable (i.e., (A, B) is controllable), the setup in this paper allows only one actuator at each time t to be connected to the plant and the choice of an actuator at time t is specified by a switching signal $\sigma(t)$. This leads to the following switched system

$$\dot{x}(t) = Ax(t) + B_{\sigma(t)}u_{\sigma(t)}(t). \qquad (3.1)$$

It can be shown that there exists a scheduling switching σ that preserve the system controllability.

To clarify this, a result on the controllability of a general switched linear system is cited.

$$\dot{\mathbb{x}}(t) = \mathbb{A}_{\sigma(t)}\mathbb{x}(t) + \mathbb{B}_{\sigma(t)}\mathbb{u}_{\sigma(t)}(t). \qquad (3.2)$$

Recall that, for the switched linear system (3.2), a nonzero state $\mathbb{x}$ is said to be controllable, if there exist a switching $\sigma(t)$ and a piecewise continuous input $\mathbb{u}(t)$ during $[0, T]$ such that the closed-loop system (3.2) satisfies $\mathbb{x}(0) = \mathbb{x}$ and $\mathbb{x}(T) = 0$. The switched system (3.2) is said to be controllable if its any nonzero state is controllable. For the multi-input switched linear system (3.2) with N pairs $(\mathbb{A}_1, \mathbb{B}_1), \ldots, (\mathbb{A}_N, \mathbb{B}_N)$, one partitions $\mathbb{B}_i$ as $\mathbb{B}_i = \left[\mathbb{b}_i^1, \cdots, \mathbb{b}_i^{p_i}\right]$ with $i = 1, \cdots, N$. Now construct another switched linear system with pairs $(\mathbb{A}_1, \mathbb{b}_1^1), \cdots, (\mathbb{A}_1, \mathbb{b}_1^{p_1})$, $(\mathbb{A}_2, \mathbb{b}_2^1), \cdots, (\mathbb{A}_2, \mathbb{b}_2^{p_2}), \cdots, (\mathbb{A}_N, \mathbb{b}_N^1), \cdots, (\mathbb{A}_N, \mathbb{b}_N^{p_N})$. Denote this new switched linear system as Σ_{new}, and the original linear switched system (3.2) as Σ_{original}. Then it has been established in (Xie et al., 2002, Theorem 2) that the controllability of the system Σ_{original} is equivalent to that of the system Σ_{new}.

To use the above result to show the existence of scheduling switchings for stability, consider a special case that there is a single pair in the system Σ_{original}, denoted as $(\mathbb{A}, \mathbb{B})$ which is assumed to be controllability in Assumption 1. Then the corresponding new system Σ_{new} has pairs $(\mathbb{A}, \mathbb{B}_1), \cdots, (\mathbb{A}, \mathbb{B}_\kappa)$ according to the partition of $\mathbb{B}$. The equivalence of the controllability of Σ_{original} and Σ_{new}, together with the controllability of Σ_{original} (i.e., $(\mathbb{A}, \mathbb{B})$), implies the controllability of Σ_{new}. Therefore, there exists stabilizing switchings σ for the system (3.1). Arguing in a similar manner, it can be shown, for the switched system with pair $(\mathbb{A}, \mathbb{C}_1), \cdots, (\mathbb{A}, \mathbb{C}_\ell)$, there exists a scheduling switching θ such that the resulting switched system has the same observability as that of $(\mathbb{A}, \mathbb{C})$. Due to the observability of $(\mathbb{A}, \mathbb{C})$ in Assumption 1, one shows the existence of scheduling switching θ that preserves the system's observability.

The above result extends Theorems 1 and 2 in (Zhang and Hristu-Varsakelis, 2006) where the system matrix $\mathbb{A}$ is required to be invertible. Next, a technique lemma on averaging is presented as preparation for the stabilization of linear NCS.

3.2 An averaging lemma for linear systems

A technique tool regarding the averaging for ordinary differential equation with stochastic disturbance is firstly developed. The result obtained in this section is vital to the controller and observer synthesis and the corresponding convergence analysis in next section.

The following result presents an averaging method for stability of fast time-varying linear systems. The general result for nonlinear systems can be found in (Aeyels and Peuteman, 1999; Kosut et al., 1987; Bellman et al., 1985). For convenient of later use, the result is rewritten in the following form.

Lemma 1. Consider a linear time-varying systems $\dot{x}(t) = A(t)x(t)$ with $A(\cdot) : \mathbb{R} \to \mathbb{R}^{n\times n}$. Suppose there exists an increasing sequence of times $t_k, k \in \mathbb{Z}$, with $t_k \to +\infty$ as $k \to +\infty$, $t_k \to -\infty$ as $k \to -\infty$, and $t_{k+1} - t_k \le T$ for some $T > 0$, such that each of the following average systems $(k = 0, 1, \cdots)$

$$\dot{\bar{x}}(t) = \bar{A}_k\bar{x}(t), \bar{A}_k = \left(\int_{t_k}^{t_{k+1}} A(t)dt\right) / (t_{k+1} - t_k) \qquad (3.3)$$

is asymptotically stable in sense of $P\bar{A}_k+\bar{A}_k^TP \leq -vI, k = 0, 1, \cdots$ with P a symmetric and positive definite $n \times n$ matrix and v a positive number. Then the following results hold.

(1) There exists an $\alpha^* > 0$ such that the following fast time-varying system

$$\dot{x}(t) = A(\alpha t)x(t) \tag{3.4}$$

is asymptotically stable for all $\alpha > \alpha^*$;

(2) The value α^* can be uniquely determined from T by solving the equation

$$e^{\frac{MT}{\alpha}}\frac{T}{\alpha} = \frac{1}{M}\left(-1+\sqrt{1+\frac{v}{M_vMT}}\right) \tag{3.5}$$

for α, where $M = \sup_{t\geq} \|A(t)\|$ and $M_v = 2\|P\|$.

(3) $\lim\limits_{T\to 0^+} \alpha^* = 0$.

Proof: The results of (1) and (2) are direct consequence of (Aeyels and Peuteman, 1999, Theorem 2); see also (Aeyels and Peuteman, 1999, Remark 4). It remains to prove (3). For any $\varepsilon > 0$, consider the following function

$$h(T) = \frac{1}{M}\left(-1+\sqrt{1+\frac{v}{M_vMT}}\right) - \frac{T}{\varepsilon}e^{\frac{MT}{\varepsilon}},$$

which is obviously monotonely decreasing and satisfies $\lim_{T\to 0+} h(T) = +\infty$. Therefore, there exists $T_1 > 0$ such that $T > T_1$ implies $h(T) > 0$. This, together with equation (3.5), gives rise to

$$\frac{T}{\alpha}e^{\frac{MT}{\alpha}} > \frac{T}{\varepsilon}e^{\frac{MT}{\varepsilon}}.$$

Noting that the function$\alpha \to \frac{T}{\alpha}e^{\frac{MT}{\alpha}}$ is monotonely decreasing, one has $\alpha < \varepsilon$. In conclusion, for any $\varepsilon > 0$, there exists a $T_1 > 0$ such that $T \in (0, T_1)$ implies $\alpha < \varepsilon$. This completes the proof. ∎

Remark 2. Closely related to the above averaging approach for stability analysis of fast time-varying linear systems is the exponential splitting method. The basic idea is to approximate the product of exponentials $e^{A_1}e^{A_2}$ by $e^{\bar{A}}$ for some $\bar{A}$, where $A_1, A_2, \bar{A}$ are square matrices. Motivated by this, the transfer matrix of a switched linear system at given time interval $[t_k, t_{k+1})$ is usually given in the form $e^{A_1}\cdots e^{A_m}$, which can be approximated by $e^{\bar{A}_k}$ for some matrix $\bar{A}_k$. It is expected that the stability of averaged systems $\dot{x} = \bar{A}_kx$, $k = 0, 1, \cdots$, give hints to stability of the switched systems. The exponential splitting method leads to a similar result as in Lemma 1. For detailed elaboration, the interested readers are referred to (Kosut et al., 1987), (Porfiri et al., 2008).

Remark 3. By the third result of this lemma, there exists $T^* > 0$ such that $\alpha^* < 1$. Since the time-varying systems (3.4) is stable for any $\alpha > \alpha^*$, one chose $\alpha = 1$ and thus the system $\dot{x} = A(t)x$ is asymptotically stable. In fact, such an T^* can be obtained by solving the following inequality. Let $\alpha^* < 1$, which implies that the left hand side of the equation (3.5) (with α and T being replaced with α^* and T^* respectively) satisfies $\frac{T^*}{\alpha^*}e^{\frac{MT^*}{\alpha^*}} > T^*e^{MT^*}$ due to the monotonicity of the function $\alpha \to \frac{T}{\alpha}e^{\frac{MT}{\alpha}}$. Therefore, the right hand side satisfies

$$\frac{1}{M}\left(-1+\sqrt{1+\frac{v}{M_vMT^*}}\right) > T^*e^{MT^*}. \tag{3.6}$$

Since the switched system is stable for any $\alpha > \alpha^*$, one can chose $\alpha = 1$. The case of $\alpha = 1$ corresponds to ordinary switching.

To ensure that the switchings used in this paper is ordinary switchings, consider the following set of scheduling switchings.

Definition 2. **(Scheduling switchings with upper-bounded dwell time)** Let T^*_{actuator} and T^*_{sensor} be determined from Remark 3. Define the following admissible switchings with upper bound

$$\begin{aligned}\mathcal{S}_{\text{actuator}} = \{\sigma | &\text{the switching instants } t_k \text{ of } \sigma \\ &\text{satisfies } t_{k+1}-t_k < T^*_{\text{actuator}}, k = 0, 1, \cdots\},\\ \mathcal{S}_{\text{sensor}} = \{\theta | &\text{the switching instants } t_k \text{ of } \theta \\ &\text{satisfies } t_{k+1}-t_k < T^*_{\text{sensor}}, k = 0, 1, \cdots\}.\end{aligned}$$

Definition 3. **(Jointly connected scheduling policies)** For a given switching signal θ with switching times $\{t_0^\theta, t_1^\theta, t_2^\theta, \cdots\}$ with $t_0^\theta = 0$, if there exists a subset $\{t_{i_0}^\theta, t_{i_1}^\theta, t_{i_2}^\theta, \cdots\}$ of switching times with $t_{i_0}^\theta = 0$ such that during each interval $[t_{i_j}^\theta, t_{i_{j+1}}^\theta), j = 0, 1, \cdots$, all sensors are actuated, then θ is said to be a jointly connected sensor scheduling policy. Similarly, for a given switching signal σ with switching times $\{t_0^\sigma, t_1^\sigma, t_2^\sigma, \cdots\}$ with $t_0^\sigma = 0$, if there exists a subset $\{t_{i_0}^\sigma, t_{i_1}^\sigma, t_{i_2}^\sigma, \cdots\}$ of switching times with $t_{i_0}^\sigma = 0$ such that during each interval $[t_{i_j}^\sigma, t_{i_{j+1}}^\sigma), j = 0, 1, \cdots$, all actuators are actuated, then σ is said to be a jointly connected actuator scheduling policy.

The Jointly connected scheduling policies with upper-bounded dwell time defined in Definitions 2-3 are similar in spirit to the graph switching investigated in (Ni et al., 2013) for consensus controllability and observability problems. With Definitions 2 and 3 in hand, the following assumption is made.

Assumption 2. Let the sensor and actuator scheduling switchings σ and θ be respectively in the class $\mathcal{S}_{\text{sensor}}$ and $\mathcal{S}_{\text{actuator}}$ in Definition 2 and satisfy the joint connectivity condition in Definition 3.

According to this definition, during each interval $[t_{i_j}^\theta, t_{i_{j+1}}^\theta)$, $j = 0, 1, \cdots$, there is a unit vector $(\pi_1^{(i_j)}, \cdots, \pi_\ell^{(i_j)}) \in \Delta$ with Δ a simplex, such that the sensor $s \in \{1, \cdots, \ell\}$ is activated for a fraction $\pi_s^{(i_j)}$ of the total time $t_{i_{j+1}}^\theta - t_{i_j}^\theta$. Similarity, during each interval $[t_{i_j}^\sigma, t_{i_{j+1}}^\sigma)$, $j = 0, 1, \cdots$, there is a unit vector $(\chi_1^{(i_j)}, \cdots, \chi_\kappa^{(i_j)}) \in \Delta$, such that the sensor $k \in \{1, \cdots, \ell\}$ is activated for a fraction $\chi_k^{(i_j)}$ of the total time $t_{i_{j+1}}^\sigma - t_{i_j}^\sigma$.

3.3 Observer-based feedback control of linear NCS

This subsection is devoted to solving Problem A. With the averaging tool developed in last subsection, this section now concentrates on the problem of designing gain matrices $\{L_1, \cdots, L_\ell\}$ and $\{K_, \cdots, K_\kappa\}$ and scheduling policies σ and θ, together with the issue of convergence analysis. The stabilizing scheduling as well as the gain matrices for actuators and sensors are respectively presented in Algorithm 1 and Algorithm 2, with the corresponding convergence analysis being shown in Theorem 1.

Algorithm 1 : The stabilizing scheduling and gain matrices for actuators

Initialize

1: Give a positive number $v > 0$;

2: Solve $XA^T + AX + BY + Y^T B^T < -vI$ with variables $X > 0, Y$. Let $P = X$ and $K = YX^{-1}$;

3: Let $M = \max\{\|A + B_1K_1\|, \cdots, \|A + B_\kappa K_\kappa\|\}$ and $M_v = 2\|P\|$. Chose a T^*_{actuator} satisfying (3.6).

Do for $i = 0, 1, \cdots$

4: Randomly generate a unit vector $(\chi_1^{(i)}, \cdots, \chi_\kappa^{(i)}) \in \Delta$;

5: Solve the linear matrix equation $\chi_1^{(i)} B_1 K_1^{(i)} + \cdots + \chi_\kappa^{(i)} B_\kappa K_\kappa^{(i)} = BK$ to obtain $K_1^{(i)}, \cdots, K_\kappa^{(i)}$.

6: The scheduling policy for actuators is run as follows:

(A_1) Select an actuator $K_{j_1}^{(i)}$ randomly from $\{K_1^{(i)}, \cdots, K_\kappa^{(i)}\}$ and connect it to the plant for a period time $\chi_1^{(i)} T^*_{\text{actuator}}$;

(A_2) Select an actuator $K_{j_2}^{(i)}$ randomly from the remaining set $\{K_1^{(i)}, \cdots, K_\kappa^{(i)}\} - \{K_{j_1}^{(i)}\}$ and connect it to the plant for a period time $\chi_2^{(i)} T^*_{\text{actuator}}$;

$\vdots$

(A_κ) Select the last actuator K_{j_κ} and connect it to the plant for a period time $\chi_\kappa^{(i)} T^*_{\text{actuator}}$;

7: Stop if $\|x((i+1)T^*_{\text{actuator}})\| \leq e_{\text{thredhold}}$ for a preassigned error $e_{\text{thredhold}}$.

Algorithm 2 : The stabilizing scheduling and gain matrices for sensors

Initialize

1: Give a positive number $v > 0$;

2: Solve $XA + A^T X + YC + C^T Y^T < -vI$ with variables $X > 0, Y$. Let $P = X$ and $L = X^{-1}Y$;

3: Let $M = \max\{\|A + L_1C_1\|, \cdots, \|A + L_\ell C_\ell\|\}$ and $M_v = 2\|P\|$. Chose a T^*_{sensor} satisfying (3.6).

Do for $i = 0, 1, \cdots$

4: Randomly generate a unit vector $(\pi_1^{(i)}, \cdots, \pi_\ell^{(i)}) \in \Delta$;

5: Solve the linear matrix equation $\pi_1^{(i)} L_1 C_1^{(i)} + \cdots + \pi_\ell^{(i)} L_\ell C_\ell^{(i)} = LC$ to obtain $L_1^{(i)}, \cdots, L_\ell^{(i)}$.

6: The scheduling policy for actuators is run as follows:

(A_1) Select an actuator $L_{j_1}^{(i)}$ randomly from $\{L_1^{(i)}, \cdots, L_\ell^{(i)}\}$ and connect it to the plant for a period time $\pi_1^{(i)} T^*_{\text{sensor}}$;

(A_2) Select an actuator $L_{j_2}^{(i)}$ randomly from the remaining set $\{L_1^{(i)}, \cdots, L_\ell^{(i)}\} - \{L_{j_1}^{(i)}\}$ and connect it to the plant for a period time $\pi_2^{(i)} T^*_{\text{sensor}}$;

$\vdots$

(A_ℓ) Select the last actuator L_{j_ℓ} and connect it to the plant for a period time $\pi_\ell^{(i)} T^*_{\text{sensor}}$;

7: Stop if $\|x((i+1)T^*_{\text{actuator}})\| \leq e_{\text{thredhold}}$ for a preassigned error $e_{\text{thredhold}}$.

Theorem 1. For the linear system (2.1) with $B = [B_1, \cdots, B_\kappa]$, $C = [C_1^T, \cdots, C_\ell^T]^T$ and $u = [u_1^T, \cdots, u_\kappa^T]^T$, let Assumptions 1 and 2 hold. Suppose that the system is actuated by switching among κ actuators so that a switched actuation $B_\sigma u_\sigma(t)$ is exerted to the system. Also suppose that the output of the system is measured by switching among ℓ sensors, giving rise to a switched measurement $y_\theta(t) = C_\theta x(t)$. Consider the observer-based linear feedback $u_i = K_i \hat{x}(t), i = 1, \cdots, \kappa$ with $\hat{x}$ being the state of the switched observer (2.2) and consider the corresponding closed-loop system (2.3). Let the gain matrices $\{K_1, \cdots, K_\kappa\}$ and the scheduling policy σ for actuators be obtained via Algorithm 1 and let the gain matrices $\{L_1, \cdots, L_\ell\}$ and the scheduling policy θ for sensors be obtained via Algorithm 2. Then the closed-loop system (2.3) via the switched observer (2.2) is asymptotically stable.

Proof: Obviously, the scheduling switchings σ and θ designed in Algorithms 1-2 satisfy Assumptions 1-2. Letting $e(t) = \hat{x}(t) - x(t)$, then the dynamics for $[e(t), x(t)]$ can be obtained by referring to equations (2.2)-(2.3) as

$$\dot{e}(t) = [A + L_{\theta(t)} C_{\theta(t)}] e(t), \tag{3.7}$$

$$\dot{x}(t) = [A + B_{\sigma(t)} K_{\sigma(t)}] x(t) + B_{\sigma(t)} K_{\sigma(t)} e(t). \tag{3.8}$$

The averaging method presented in Lemma 1 is now used to prove the stability of above systems.

1^o The first step is to show that the dynamics for $e(t)$ is asymptotically stable. In view of Assumption 2, it can be seen that, during each interval $[t^\theta_{i_j}, t^\theta_{i_{j+1}})$ for $j = 0, 1, 2, \cdots$,

$$\begin{aligned}\frac{1}{t^\theta_{i_{j+1}} - t^\theta_{i_j}} \int_{t^\theta_{i_j}}^{t^\theta_{i_{j+1}}} [A + L_{\theta(s)} C_{\theta(t)}] e dt &= [A + \sum_{s=1}^{\ell} \pi_s^{(i_j)} L_s C_s] e \\ &= [A + LC] e.\end{aligned}$$

That is, the average system for (3.7) is $\dot{\bar{e}} = [A + LC]\bar{e}$. Notice that the LMI in Step 2 of Algorithm 2 can be rewritten in terms of P, L as $P(A + LC) + (A + LC)^T P < -vI$, one sees that the average system is asymptotically stable. Application Lemma 1 to the equation (3.7) shows that $\lim_{t\to\infty} e(t) = 0$.

2^o The second step is to show that the dynamics for $x(t)$ is asymptotically stable. Similarly, the average system for (3.8) can be computed as $\dot{\bar{x}} = [A + BK]\bar{x} + BK\bar{e}(t)$. Also notice that the LMI in Step 2 of Algorithm 1 can be rewritten in terms of P, K as $P(A + BK)^T + (A + BK)P < -vI$, one sees that the matrix $A + BK$ is Hurwitz, or equivalently the matrix $(A + BK)^T$ is Hurwitz. Due to the fact that $\lim_{t\to\infty} \bar{e}(t) = 0$ and the matrix $A + BK$ is Hurwitz, it then follows from (Ni et al., 2012, Lemma 2.7) that $\lim_{r\to\infty} \bar{x}(t) = 0$. This means that the average system for (3.8) is asymptotically stable. Also, a direct consequence of Lemma 1 which is applied to the equation (3.7) is that $\lim_{t\to\infty} x(t) = 0$. This completes the proof. ■

Remark 4. A comparison of our result with (Guo et al., 2012, Theorems 1, 2) and (Guo, 2010, Theorem 1) is made. The method in (Guo, 2010, Theorem 1) requires solving nonlinear matrix inequalities whose solution is generally hard to obtain. To seek a more numerically tractable algorithm, the authors in (Guo et al., 2012, Theorems 1, 2) present an LMIs based design approach. However, the number of these LMIs in (Guo et al., 2012, Theorems 1, 2) is $2^r \cdot 2^s$ and therefore this method is obviously impractical when the dimensions r and s for the input and output are large.

Remark 5. The work in (Dacic and Nesic, 2008) also considers the state estimation for the system (2.1) by using switching observers. The switching σ is treated as a control input and the periodic switching is consider in that paper with periodicity T. A switching observer is designed in (Dacic and Nesic, 2008, Eq (4)) and it can be rephrased according to the remarks blow (Dacic and Nesic, 2008, Assumption 2) as $\dot{\hat{x}} = A\hat{x} + L_{\theta(t)}[C_{\theta(t)}\hat{x}(t) - y_{\theta(t)}]$ which is similar to (2.2). The switching signal σ in (Dacic and Nesic, 2008) is designed as

$$\sigma(t_i) = \arg\max_{j\in\{1,\cdots,l\}} [C_j x(t_i^-) - C_j\hat{x}(t_i^-)]^T Q_j [C_j x(t_i^-) - C_j\hat{x}(t_i^-)],$$

with the matrix $Q = Q^{\mathrm{T}} \triangleq \mathrm{diag}\{Q_1, \ldots, Q_l\} > 0$ to be determined. Aside from the design parameter σ, the gain matrices $L_i, i = 1, \cdots, l$ and the positive definite matrices $Q_i > 0, i = 1, \cdots, l$ for stabilization are obtained in (Dacic and Nesic, 2008, Theorem 1) by solving the following matrix inequalities

$$\begin{bmatrix} P - H - \sum_{k=1}^{l} \tau_{jk}\left(C_j^{\mathrm{T}} Q_j C_j - C_k^{\mathrm{T}} Q_k C_k\right), & * \\ Pe^{AT} + M_j C_j, & P \end{bmatrix} > 0$$

for $j = 1, \cdots, s$ with unknowns $P > 0, Q_i > 0, H > 0, M_i, \tau_{jk} \geq 0$. Obviously, sufficient conditions to guarantee the existence of solutions are difficult to obtain. Also, the design procedure and the corresponding convergence analysis are more complex than ours. It has also been proved in (Dacic and Nesic, 2008, Theorem 2) that the feasibility of above matrix inequality is ensured if the periodicity T of the switching is sufficiently small. This result is consistent with result since it is assumed in Assumption 2 that the dwell time for the switching σ is up bounded by T^*_{actuator} which can be made sufficiently small by choosing α small enough according to Remark 3.

3.4 Static output based feedback control of linear NCS

This subsection contributes to Problem B, i.e., the static output feedback control of linear NCS. The scheduling rules σ, θ and the static output feedback gain matrices F_{ij}, $i = 1, \cdots, \kappa$, $j = 1, \cdots, \ell$, are designed according to Algorithm 3. The convergence analysis of the algorithm is provided in provided in Theorem 2 whose proof is similar as that for Theorem 1 and thus omitted.

Algorithm 3 The Stabilizing Schedulings and static output gain matrices

Initialize

1: Given a positive number $v > 0$;
2: Chose a feedback gain matrix F such that $A + BFC$ is Hurwitz.
3: Solve the LMI $P(A + BFC) + (A + BFC)^T P \leq -vP$ with variables $P > 0$.
4: Let $M = \max\{\|A + B_i F_{ij} C_j\| | i = 1, \cdots, \kappa, j = 1, \cdots, \ell\}$ and $M_v = 2\|P\|$. Chose a T^*_{actuator} satisfying the inequality (3.6).

Do for $i = 0, 1, \cdots$

5: Randomly gnerate a unit vector with elements $\pi_{kl}^{(i)}$, $k = 1, \cdots, \kappa$, $l = 1, \cdots, \ell$;
6: Solve the linear matrix equation $\sum_{k=1}^{\kappa}\sum_{l=1}^{\ell} \pi_{kl}^{(i)} B_k F_{kl} C_l = BFC$ to obtain F_{kl}, $k = 1, \cdots, \kappa$, $l = 1, \cdots, \ell$;
7: The scheduling policy for sensors and actuators are run as follows:

(AC_{11}) Select a sensor-actuator $F^{(i)}_{k_{i_1}, l_{i_1}}$ randomly from $\{F_{kl} | k = 1, \cdots, \kappa, l = 1, \cdots, \ell\}$ and connect it to the plant for a period time $\pi^{(i)}_{k_1,l_1} T^*_{\text{actuator}}$;

(AC_{12}) Select a sensor-actuator $F^{(i)}_{k_{i_2}, l_{i_2}}$ randomly from the remaining set $\{F_{kl} | k = 1, \cdots, \kappa, l = 1, \cdots, \ell\} - \{F^{(i)}_{k_{i_1}, l_{i_1}}\}$ and connect it to the plant for a period time $\pi^{(i)}_{k_2,l_2} T^*_{\text{actuator}}$;

$\vdots$

($AC_{\kappa\ell}$) Select the last sensor-actuator $F^{(i)}_{k_{i_\kappa}, l_{i_\ell}}$ and connect it to the plant for a period time $\pi^{(i)}_{k_\kappa,l_\ell} T^*_{\text{actuator}}$;

8: Stop if $\|x((i+1)T^*_{\text{actuator}})\| \leq e_{\text{thredhold}}$ for a preassigned error $e_{\text{thredhold}}$.

Theorem 2. For the linear system (2.1) with $B = [B_1, \cdots, B_\kappa]$, $C = [C_1^T, \cdots, C_\ell^T]^T$ and $u = [u_1^T, \cdots, u_\kappa^T]^T$, let Assumptions 1 and 2 hold. Suppose that the system is actuated by switching among κ actuators so that a switched actuation $B_\sigma u_\sigma(t)$ is exerted to the system. Also suppose that the output of the system is measured by switching among ℓ sensors, giving rise to a switched measurement $y_\theta(t) = C_\theta x(t)$. Consider the static output feedback so that the closed-loop system is 2.7. Let the gain matrices $\{F_{ij} | i = \cdots, \kappa, j = 1, \cdots, \ell\}$ and the scheduling policies σ and θ for actuators and sensors be obtained via Algorithm 3. Then the closed-loop system (2.7) is asymptotically stable.

3.5 A refined conclusion of the algorithms

A. Co-design of actuator scheduling and controller gains

- Chose a dwell time bound T^*_{actuator} satisfying (3.6) via the system matrices A and $\{B_1, \cdots, B_\kappa\}$.
- Divide the time interval $[0, +\infty)$ into disjoint union of contiguous intervals $[t_i^{\text{act}}, t_{i+1}^{\text{act}})$ with $t_{i+1}^{\text{act}} - t_i^{\text{act}} \leq T^*_{\text{actuator}}$, where $i = 0, 1, \cdots$ and $t_0^{\text{act}} = 0$.
- In each $[t_i^{\text{act}}, t_{i+1}^{\text{act}})$, all κ actuators are activated one by one without order with each for only one time, and the proportions of activation times for them are arbitrarily given as $(\chi_1^i, \cdots, \chi_\kappa^i) \in \Delta$ and the controller matrices $K_1, \cdots, K_\kappa$ are designed such that $\chi_1^i B_1 K_1^i + \cdots + \chi_\kappa^i B_\kappa K_\kappa^i$ is Hurwitz.

B. Co-design of sensor scheduling and sensor gains

- Chose a dwell time bound T^*_{sensor} satisfying (3.6) via the system matrices A and $\{C_1, \cdots, C_\ell\}$.
- Divide the time interval $[0, +\infty)$ into disjoint union of contiguous intervals $[t_i^{\text{sens}}, t_{i+1}^{\text{sens}})$ with $t_{i+1}^{\text{sens}} - t_i^{\text{sens}} \leq T^*_{\text{sensor}}$, where $i = 0, 1, \cdots$ and $t_0^{\text{sens}} = 0$.
- In each $[t_i^{\text{sens}}, t_{i+1}^{\text{sens}})$, all ℓ sensors are activated one by one with each for only one time, and the proportions of activation times for them are arbitrary given as $(\pi_1^i, \cdots, \pi_\ell^i) \in \Delta$, and the observer gain matrices $L_1, \cdots, L_\ell$ are designed such that $\pi_1^i L_1^i C_1 + \cdots + \pi_\ell^i L_\ell^i C_\ell$ is Hurwitz.

C. Co-design of actuator/sensor scheduling and static output gains

- Chose a dwell time bound T^*_{static} satisfying (3.6) via the system matrices A, $\{B_1, \cdots, B_\kappa\}$ and $\{C_1, \cdots, C_\ell\}$.
- Divide the time interval $[0, +\infty)$ into disjoint union of contiguous intervals $[t_i^{\text{static}}, t_{i+1}^{\text{static}})$ with $t_{i+1}^{\text{static}} - t_i^{\text{static}} \leq T^*_{\text{static}}$, where $i = 0, 1, \cdots$ and $t_0^{\text{static}} = 0$.
- In each $[t_i^{\text{static}}, t_{i+1}^{\text{static}})$, all κ actuators are activated one by one and all ℓ sensors are activated one by one, with each for only one time, and the proportions of activation time for the actuator-sensor pairs $\{(k,l)|k = 1, \cdots, \kappa, l = 1, \cdots, \ell\}$ are arbitrary given as a $\{\pi_{kl}^i | k = 1, \cdots, \kappa, l = 1, \cdots, \ell\} \in \Delta$, and the static output gain matrices $\{F_{kl}^i | k = 1, \cdot, \kappa, l = 1, \cdots, \ell\}$ is designed such that $\sum_{k=1}^{\kappa} \sum_{l=1}^{\ell} \pi_{kl}^i B_k F_{kl}^i C_l$ is Hurwitz (so that $A + BF^iC$ with $F = [F_{kl}^i]$ is Hurwitz).

Two factors are crucial for the success of the switched scheduling algorithms. One is the dwell time bound T^* and another is ergodicity of all actuators (and sensors) in each interval $[t_i, t_{i+1})$, $i = 0, 1, \cdots$. Big T^* would rend the system to lost stability. Also, if in some interval $[t_i, t_{i+1})$, only a subset of actuators/sensors are activated, then the resulting switched system would suffer from instability.

4. SIMULATION EXPERIMENTS: THE EFFECTS OF SENSOR/ACTUATOR SWITCHINGS ON STABILIZATION, CONVERGENCE SPEED AND OVERSHOOT

Consider a 4-th order linear system with 3 sensors and 4 actuators, where the system matrices $A \in \mathbb{R}^{4\times 4}$, $B = [B_1, B_2, B_3, B_4]$, $C = [C_1^T, C_2^T, C_3^T]^T$ are given as

$$A = \begin{pmatrix} 1.2882 & 2.0803 & 1.4563 & 1.1285 \\ -6.2066 & 6.0919 & 3.1267 & -1.3068 \\ 4.2985 & 0.1036 & 1.7971 & 3.2580 \\ 2.8037 & -3.8848 & -2.5326 & 0.8228 \end{pmatrix}$$

and

$$\begin{aligned} C_1 &= (-6.8273 \quad 5.6011 \quad 3.4394 \quad -1.4374), \\ C_2 &= (5.1582 \quad 0.1243 \quad -2.6435 \quad 3.9096), \\ C_3 &= (3.6448 \quad -5.0503 \quad -3.2924 \quad -4.1304), \\ B_1 &= (-2.9830 \quad -6.8273 \quad 4.7284 \quad 3.0841)^T, \\ B_2 &= (2.4964 \quad 2.5103 \quad 0.1243 \quad -4.6618)^T, \\ B_3 &= (1.8932 \quad 4.0647 \quad -0.2638 \quad -3.2924)^T, \\ B_4 &= (1.5798 \quad -1.8295 \quad 4.5612 \quad -1.6481)^T. \end{aligned}$$

It can be checked that each (A, C_i) is unobservable for $i = 1, 2, 3$ and each (A, B_i) is uncontrollable for $i = 1, 2, 3, 4$. Therefore, only one sensor does not suffice to construct an observer to estimate the system state; likewise, only one controller is incapable of stabilizing the system. Due to resource constraints, these 3 sensors (or 4 actuators) are not allowed to be connected to the plant simultaneously, instead at each time exact one mender of the sensors (or the actuators) is connected to the plant. The sequential order of sensor (or actuator) connection and corresponding connection time play vital role of ensuring system stability. Therefore, aside from the computation of observer and feedback gain matrices, the sensor and actuator scheduling is a main focus of the networked linear systems design and most schedule schemes are more or less complex. In sharp contrast, there is no requirement in our algorithm on the order of the sensors' (or actuators') connecting to the system; the selection of sensors (or actuators) can be totally random with only a mild restriction: each of the 3 sensors (or 4 actuators) should be link to the plant at least once during every $[(k-1)T^*, kT^*)$, $K = 0, 1, \cdots$ for some interval length T^* which can be computed from the system matrices. Therefore, our method is schedule free and avoids the difficult task of selection design for sensors and actors.

Chose $T^*_{actuator} = 1/3$ in our initial simulation. With reference to Figure 3, one lets the 4 actuators connect to the plant at random during each time interval $[(k-1)/3, k/3)$, $k = 0, 1, \cdots$. These intervals of length 1/3 are separated by vertical blue lines and each of these intervals, say interval $[(k-1)/3, k/3)$, is randomly divided red vertical lines into 4 subintervals with random random dividing points and correspondingly random lengths $p_1^k, p_2^k, p_3^k, p_4^k$, $\sum_{i=1}^4 p_i^k = 1/3$, during which each sensor is activated. The switching instants among different sensors are highlighted with red vertical lines. With the gain matrices in Algorithm 1 being computed by solving corresponding linear matrix inequalities (explicit computation results are not presented here to save place), the trajectory of the closed-loop system with initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (4, 5, -5, 4)$ is plotted in Figure 3, from which one sees that all the components of the states converge to the zero. Similar simulation analysis hold for error evolution between the observer state and the system state.

The effect of the switching speed of sensors (or actuators) on the convergence performance is also analyzed. It is obvious that the smaller the T^*, the fast speed the switch of the sensors (or actuators). In the second round of simulation, a smaller $T^*_{actuator} = 1/9$ is used and it is found that the system states converge to the origin with less time (approximate 3.5 seconds by using $T^*_{actuator} = 1/3$ in Figure 3 and 2 seconds by using $T^*_{actuator} = 1/9$ in Figure 4). The result shows that fast switching can speed up converge time.

Aside from the advantage of speed-up convergence inherent in fast switching, it is observed in simulation that overshoot can also be combated via fast switching. To show this, take the evolution of the error between the observer state and the system system as examples. To see the overshoot phenomenon clearly, a relatively "larger" initial condition $(e_1(0), e_2(0), e_3(0), e_4(0)) = (6, -6, 5, 6)$ is chosen. In Figure 5, one choses $T^*_{sensor} = 1/3$ and $T^*_{sensor} = 1/9$, respectively. As the results indicate, the maximum value of the transient response climbs approximately to 500 if one uses a relatively slower switching with $T^*_{sensor} = 1/3$. In contrast, by utilizing a relatively fast switching with $T^*_{sensor} = 1/9$, the corresponding maximum value does not exceed 100. These simulation results show that fast switching can effectively alleviate the overshoot.

While it good to see that fast switching has a couple of advantages observed above, fast switching imposes severe requirement on hardware to overcome switching-induce chattering which is disadvantageous in energy saving. Keeping this caution in mind and retaining the advantages of fast switching, one can adopt fast switching in earlier stage of sensor/actuator selection to fight with overshoot, continued with a moderate switching during the subsequent times to achieve a fast convergence, and followed by a slow switching to save energy during the remaining state

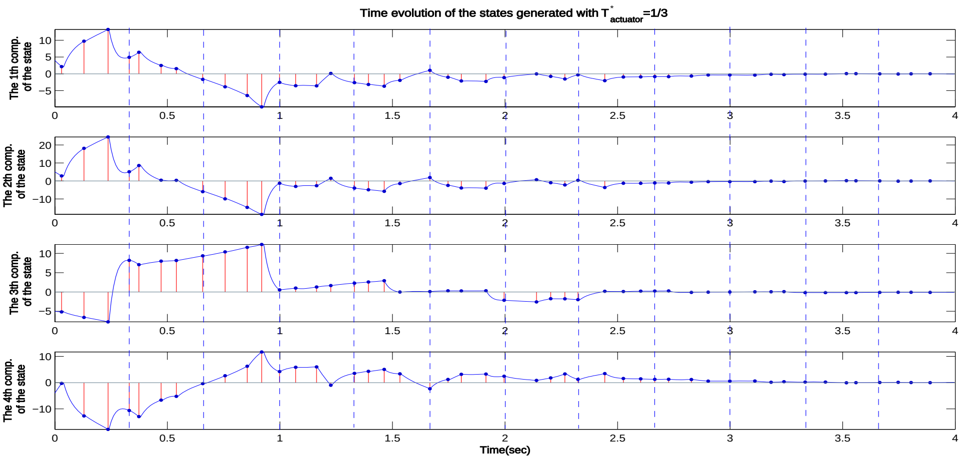

Fig. 3. Time evolution of the states under switching actuators with switching speed controlled by $T^*_{actuator} = 1/3$. On each interval of length 1/3 divided by vertical blue and dash lines, the controller switches randomly among the four actuators without missing, with the switching instants being indicated by vertical red and solid lines.

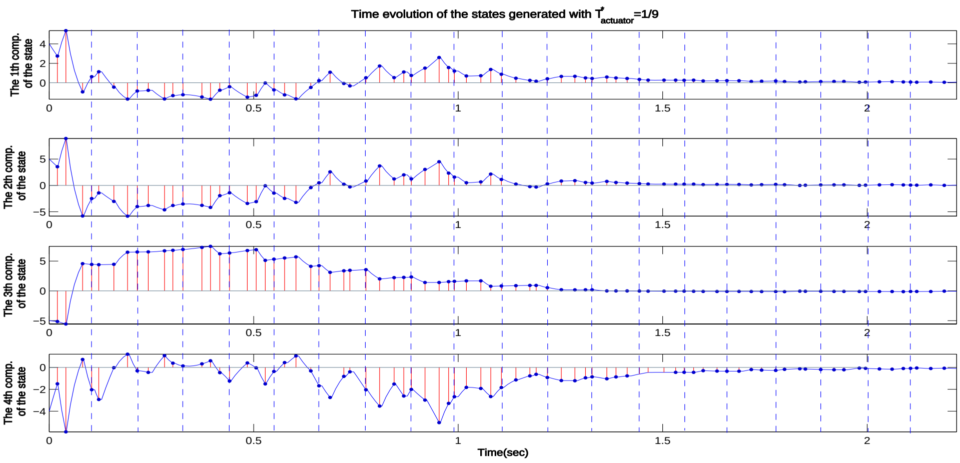

Fig. 4. Time evolution of the states under switching actuators with a faster switching speed controlled by $T^*_{actuator} = 1/9$. Faster convergence in 2s is observed, being less than 3.5s in Figure 3 under a slower switching controlled by $T^*_{actuator} = 1/9$.

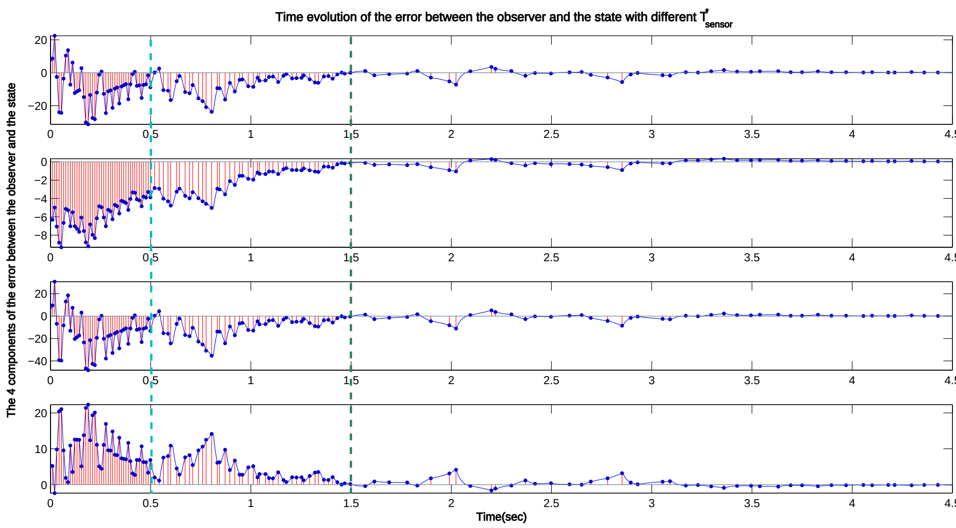

Fig. 6. Time evolution of the error between the observer state and the system state under switching sensors, with a fast switching speed ($T^*_{sensor} = 1/30$) being used in the earlier stage $[0, 0.5)$ to combat overshoot, a middle switching speed ($T^*_{sensor} = 1/15$) in the subsequent stage $[0.5, 1.5)$ to achieve fast convergence, and slow switching ($T^*_{sensor} = 1/5$) in the in the remaining stage $[1.5, 4.5]$ to avoid chattering.

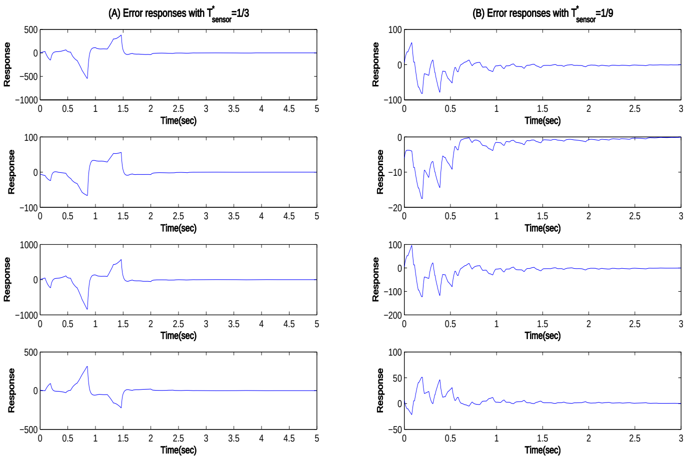

Fig. 5. Time evolution of the error between the observer state and the system state under switching sensors with switching speeds respectively controlled by $T^*_{sensor} = 1/3$ and $T^*_{sensor} = 1/9$. Overshoot is weakened with increased switching speed, where $T^*_{sensor} = 1/3$ corresponds to a smaller speed and $T^*_{sensor} = 1/3$ corresponds to a faster speed.

when the deviation of the system response from the origin is relatively small. Following this strategy, the simulation is run in Figure 6, where in the earlier stage $[0, 0.5)$ a fast switching scheme with $T^*_{sensor} = 1/30$ is used so that overshoot is avoided and the system response does not exceed $[-50, 20]$, in the middle stage $[0.5, 1.5)$ a moderate switching rule with $T^*_{sensor} = 1/15$ is adopted to rendering a rapid decrease of the system response toward zero, and in the remaining stage $[1.5, 4.5]$ where the system response is already close to zero, a slow switching with $T^*_{sensor} = 1/5$ is employed, with the spare task of energy saving being realized. These three stages are separated by two vertical green lines.

As an ending remark, it is clear that simple switching mechanisms bring low local computations, but that should be in balance with the number of switches and the time for stabilizing the system.

5. CONCLUSIONS

A switching method for actuator/sensor scheduling of NCS is presented in this paper. Unlike existing switched system methods which require designing the complex mechanism of stabilizing switching law, the method in the current paper avoids this difficult since it imposes no selection order on sensors and actuators, with only an easy-to-satisfy requirement that there exists contiguous time intervals not exceeding a certain bound such that on each time interval all sensors (actuators) are selected one by one without missing, leaving the switching instants and the dwell times of each sensor (actuator) totally random. By resorting to an averaging method proposed in this paper, the design of switched actuator and sensor gain matrices is simplified to that under the centralized scenario where all actors (sensors) are available to the plants simultaneously. These are also true for switched actuator/sensor scheduling for NCS under static output. The approach allows the computation burden to be reduced in comparison with existing results.

The main merit of the switched scheduling method is that an explicitly dwell time bound T^* satisfying the inequality (3.6) can be calculated via the system matrices and it can grantee the stability for almost arbitrary scheduling policies of actuators/sensors. Note that other topics in NCS such as data packet dropout, network-induced delays, and signal sampling can also be addressed by adopting a switched system framework, with the stabilizing upper bound T^* encoding the maximum of the data dropout rate, the delay interval, and the sampling frequency for preserving stability. Therefore, it is hope that our method could simplify corresponding analysis and design. Due to space limitation, these issues are put for future research.

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