A fault tolerant control for nonlinear systems with simultaneous actuator and sensor faults. Application to a CSTR

Sana BZIOUI * **Rafik CHANNA** **

* *EASISI, Department of Physics, Faculty of Sciences Semlalia, Cadi Ayyad University. B.P. 2390,40000. Marrakech, Morocco.(e-mail: bzioui.sana@gmail.com)*
** *EASISI, Department of Physics, Faculty of Sciences Semlalia, Cadi Ayyad University. B.P. 2390,40000. Marrakech, Morocco.(e-mail: channa@uca.ac.ma)*

Abstract: In this paper, a novel active fault tolerant tracking control is proposed for nonlinear systems described by Takagi-Sugeno model. The considered systems are affected by simultaneous actuator and sensor faults and have unmeasurable premise variables and depending on the acting faults. Firstly, a mathematical transformation is used to transform the faulty system to an augmented system where all the faults appear as actuator faults. Secondly, an H-infinity control is conceived to ensure the trajectory tracking in the fault free situation. The system state and the faults are estimated from a proportional integral unknown input (PIUI) observer or a proportional multiple integral unknown input (PMIUI) observer. The proposed fault tolerant control law is based on the use of the robust tracking control developed for the system in fault-free case and an additional term related to the estimated faults. The objective is to minimize the deviation of the faulty system compared to the healthy one, even in the presence of actuator and sensor faults. Thus, sufficient conditions are studied with the Lyapunov theory and L2 optimization, and presented in terms of LMIs. Finally, three case studies are provided to show the advantages of the proposed approach. The numerical simulation is carried out on a Continuous Stirred Tank Reactor (CSTR), and a comparison with the existing results is made.

Keywords: Fault estimation, Active fault tolerant control, Proportional integral unknown input observer, Takagi-Sugeno fuzzy models, Actuator fault and sensor fault, CSTR.

1. INTRODUCTION

In many industrial applications, processes are usually subject to different faults or loss of effectiveness in actuators and sensors. Hence, if a fault occurs, it can interrupt the normal behavior of systems, which leads to deteriorating process performance and providing harmful effects in the closed loop stability. Indeed, the performances and the stability of the process cannot be ensured with classical control. This problem has motivated a strategy commonly called fault tolerant control (FTC). It consists in preserving the system's stability and maintaining the current performance even if the various faults occurs. This technique computes a new control law by taking into account the faults affecting the system.

In general, FTCs are divided into two classes: passive and active methods. The first one is based on robust controller design technique. This class is designed to be sufficiently reliable for a predefined fault, so that no modification in the control law is needed. This technique requires no online faults detection that makes it very restricted due to its disadvantages. This kind of control is studied in (Z Qu et al (2003); F Liao et al (2002); X Yu and Y Zhang (2015); M Blanke et al (2003); H Tohidi et al (2017)). As opposed by the passive approach, the active methods consist of redesigning controllers online or selecting predesigned controllers. This strategy is more interesting due to its variable structure. It requires a fault detection and diagnosis (FDD) scheme in order to provide online information on faults that eventually occurs in the system. So the reliability and acceptable fault-tolerant performance of the process could be ensured. Many ideas of active fault tolerant control (AFTC) are developed essentially for linear systems (**?**; M Staroswiecki (2005); M Mufeed et al (2003); B Marx et al (2004); S de Oca et al (2012); X J Li and G H Yang (2012)) and descriptor systems (B Marx et al (2004)). However, most physical systems have nonlinear behaviors. Nevertheless, from the mathematical point of view, a control based on nonlinear models is very complex. An efficient way to deal with the complex nonlinear behaviors is the Takagi-Sugeno (TS) approach (T Takagi and M Sugeno (1985)). The idea is to rewrite the nonlinear model as an interpolation of linear sub-models using nonlinear functions satisfying the convex sum property, which can describe the global behavior of the system in a large operating zone. Accordingly, different ideas of control approach are developed based on TS representation, in (M Bouakou and R Channa (2018)) a stabilization of a TS system based parallel distributed compensation controller (PDC) in fault free case is presented, with application to fault detection using a Luenberger observer. In (S Bzioui and R Channa (2017)) an observer based tracking control is constructed for a TS system without faults. In FTC framework, a passive FTC based H-infinity control for TS system is presented in (S Bzioui and R Channa (2020);S Bzioui and R Channa (2018)). An interesting works in active FTC for

TS multimodel have been addressed in (Ali Bakhshi and Prof Alireza Alfi (2020); M Bouattour et al (2011); A Chamsedine et al (2015); M Sami Shaker and R J Patton (2014); Tong et al (2008); Zhang et al (2017)).

The FTC law requires the knowledge of system states and faults affecting it. For that purpose, an observer is necessary to estimate simultaneously these signals. In the observer design framework, several studies tried to reconstruct the system states with unknown input. This technique consists on the elimination of the unknown inputs (Jian Han et al (2015)). On the other hand, some works choose to estimate the system states and the unknown input simultaneously (Guan and Saif (1991); Akhenak et al (2009); Khedher et al (2010)). Consequently, unknown input observers can be implemented in FTC strategy to estimate the actuator fault by considering it as an unknown input. Many researchers have used this kind of observer to develop a FTC controller for nonlinear systems described by Takagi-Sugeno fuzzy models and affected by actuator fault (Samir Abdelmalek et al (2018); Dalil ichalal et al (2012); Abdelmalek et al (2017)) or to develop a FTC coped with sensor faults without reference to actuator faults (Ben Zina et al (2016); Wafa Jamel et al (2017); Boukhari et al (2016)). In practice, actuator and sensor faults may occur simultaneously. Indeed, if a fault affect a sensor, it can lead to a damaged in the actuator due to the wrong measurement. For that, there are urgent needs of FTC for system with actuator and sensor faults simultaneously. However, a few works have interested to this problem. In (Atef Khedher et al (2011)) a proportional integral observer has been designed to estimate system states, actuator and sensor faults simultaneously in order to develop a FTC for TS fuzzy systems with weighting functions depending on the FTC, but the trajectory tracking is not considered in this work. In (Li et al (2018)) a sliding mode observer is designed to construct a FTC for TS system against simultaneous actuator and sensor faults, however, the premise variables are assumed measurable in this work, so this method are less applicable. In (Samira Asadi et al (2020)) a robust sliding mode observer design for simultaneous fault reconstruction in perturbed Takagi-Sugeno fuzzy systems using non-quadratic stability analysis, but the trajectory tracking is not considered. In (Aouaoudaa et al (2012)) a proportional integral observer based FTC is synthesized for TS models with unmeasurable premise variables. However, this work based on the assumption that actuator fault and sensor fault are of the same form.

In this article, motivated by the adaptive observer constructed in (Habib Hamdi et al (2012)) for descriptor system and proposed recently in (S Bzioui and R Channa (2021)) for a TS system affected by constant faults and disturbances in order to estimate states and faults, a proportional integral unknown inputs (PIUI) observer is introduced to estimate simultaneously states, actuator and sensor faults. Once the fault is estimated, an active FTC based on H-infinity controller is implemented as a state feedback controller in order to guarantee a satisfactory performance and preserve stability conditions in the presence of actuator and sensor faults. In this paper, the PIUI observer and the controller are designed independently in order to avoid the coupling problem, and their gains are calculated separately by a set of Linear Matrix Inequalities (LMIs). Finally, based on state and fault estimation errors and the error between the faulty system state and a reference system state, stability and tracking analysis properties are analyzed with Lyapunov theory and L2 optimization, which are formulated in terms of LMIs. The PIUI observer gains are computed by solving the proposed LMIs stability conditions.

The objective of this paper is to present a new approach of trajectory tracking AFTC for nonlinear systems described by Takagi-Sugeno models with simultaneously acting actuator and sensor faults. The weighting functions of the considered system are unmeasurable and affected by faults. First, a mathematical transformation is introduced to transform the considered system to an augmented one where all faults occurs appear as actuator faults. Then, an H-infinity controller is constructed for the augmented system in fault-free case to ensure the trajectory tracking. The idea is to reuse this controller to conceive an active fault tolerant control by taking into account the faults affecting the system in order to maintain the system stability and to provide an acceptable system trajectory in the faulty situations. Finally, three case studies are provided to illustrate the effectiveness of the proposed FTC in the presence of different types of acting faults and disturbance. An application to a Continuous Stirred Tank Reactor (CSTR) and a comparison with the existing results is made in order to improve the efficiency of the proposed strategy.

The main contributions of this work consist on extending the adaptive observer proposed by (Habib Hamdi et al (2012)) to construct a tracking AFTC for TS systems with faulty and unmeasurable premise variables. The considered system is affected by simultaneous actuator and sensor faults and the trajectory tracking is ensured even in the presence of actuator and sensor faults. The major difference between this method and the existing approaches, where the proposed observer and the controller are synthesized separately which can avoid the coupling problem and simplify the design. The proposed FTC strategy can improve efficiently performances in presence of constant and slowly time-varying faults. The applicability of the method is extended to the case which the nonlinear system is affected by both actuator and sensor time-varying faults and exposed to unknown disturbance and measurement noise. A new stability conditions are expressed in terms of LMIs which can be solved easily with Matlab. Finally, a comparative study with the existing results is made.

The remainder of this paper is organized as follows: Section 2 presents a short introduction to the Takagi-Sugeno approach. Section 3 focuses on the design of the proposed strategy. In section 4, the application to a CSTR is given in order to show the effectiveness of the suggested method. Finally, conclusions are drawn in section 5.

2. TAKAGI-SUGENO APPROACH

The objective of TS multi-model approach is to represent the nonlinear system as an interpolation of simple linear models. Each sub model represents the behavior of the system on a limited part of the operating space. The validity of each local model is defined via a weighting function which provides a smooth transition between the sub models. The TS approach has been largely popularized in the modeling framework, and it is able to approximate a large class of complex nonlinear systems to a high degree of accuracy (Tanaka and Wang (2001)).

We assume that the local models are defined by their state space representation and the global system behavior is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{N} \mu_i(\xi(t)) C_i x(t) \end{cases} \quad (1)$$

with $u(t) \in \mathbb{R}^m$, $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$. $A_i \in \mathbb{R}^{n \times n}$ is the state matrix, $B_i \in \mathbb{R}^{n \times m}$ is the input matrix and $C_i \in \mathbb{R}^{p \times n}$ is the matrix the output. $\mu_i(\xi(t))$ are the activation functions which define the activation degree of a local model, and $\xi(t)$ is the premise variable. These functions satisfy the following convexity property:

$$\begin{cases} \sum_{i=1}^{N} \mu_i(\xi(t)) = 1 \\ 0 \leq \mu_i(\xi(t)) \leq 1, \quad \forall i = 1, \ldots N \end{cases} \quad (2)$$

3. ACTIVE FAULT TOLERANT CONTROL DESIGN

3.1 Problem statement

The objective is to design an active fault tolerant tracking control for TS systems with simultaneous acting actuator and sensor faults.

The Takagi-Sugeno model without faults is described as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (3)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output, $u(t) \in \mathbb{R}^m$ is the control input. N is the number of local models, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$ are constant matrices of appropriate dimensions.
Let us consider the following TS model with actuator and sensor faults:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{N} \mu_i(\xi_f(t))(A_i x_f(t) + B_i u_f(t) + E_i f_a(t)) \\ y_f(t) = Cx_f(t) + S f_s(t) \end{cases} \quad (4)$$

$u_f(t) \in \mathbb{R}^m$ is the active fault tolerant control which will be conceived, $x_f(t)$ and $y_f(t)$ are respectively the state vector and the output of the faulty system. $f_a(t)$ and $f_s(t)$ are respectively the actuator and sensor faults. E_i and S are the faults distribution matrices which are supposed to be known.

A system transformation is considered in order to rewrite sensor faults as actuator faults. Let us define the new following states which present the filtered versions of the outputs $y(t)$ and $y_f(t)$:

$$\dot{q}(t) = \sum_{i=1}^{N} \mu_i(\xi(t))(-Dq(t) + Dy(t)) \quad (5)$$

$$\dot{q}_f(t) = \sum_{i=1}^{N} \mu_i(\xi_f(t))(-Dq_f(t) + Dy_f(t)) \quad (6)$$

where $-D$ is an arbitrary matrix with stable eigenvalues.
We consider the integral of the tracking error for ensuring the trajectory tracking.

$$e_{TI} = \int (y_r(t) - y(t))dt \quad (7)$$

where $y_r(t)$ is the reference signal of the system. Let us consider the augmented states $X_r(t)$ and $X_f(t)$:

$$X_r(t) = \begin{bmatrix} x(t) \\ q(t) \\ e_{TI}(t) \end{bmatrix}; \quad X_f(t) = \begin{bmatrix} x_f(t) \\ q_f(t) \\ e_{TI}(t) \end{bmatrix}$$

The augmented systems can be modeled as:

$$\begin{cases} \dot{X}_r(t) = \sum_{i=1}^{N} \mu_i(\xi(t))(\bar{A}_i X_r(t) + \bar{B}_i u(t) + \bar{D} y_r(t)) \\ Y(t) = \bar{C} X_r(t) \end{cases} \quad (8)$$

$$\begin{cases} \dot{X}_f(t) = \sum_{i=1}^{N} \mu_i(\xi_f(t))(\bar{A}_i X_f(t) + \bar{B}_i u_f(t) + \bar{E}_i f(t) + \bar{D} y_r(t)) \\ Y_f(t) = \bar{C} X_f(t) \end{cases} \quad (9)$$

with: $\bar{A}_i = \begin{bmatrix} A_i & 0 & 0 \\ DC & -D & 0 \\ -C & 0 & 0 \end{bmatrix}$; $\bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}$; $\bar{D} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$;

$\bar{C} = \begin{bmatrix} C & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$; $f(t) = \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}$; $\bar{E}_i(t) = \begin{bmatrix} E_i & 0 \\ 0 & DS \\ 0 & 0 \end{bmatrix}$

The proposed FTC has the following structure for the control law:

$$u_f(t) = u(t) - \theta \hat{f}(t) \quad (10)$$

where $u(t)$ is an H-infinity controller and $\hat{f}(t)$ represents the estimated faults.

The matrix θ is chosen so that $\bar{B}_i \theta = \bar{E}_i$. The matrix θ permits to the FTC to compensate both actuator and sensor faults.

To conceive the proposed FTC control law, an H-infinity controller will be firstly designed for the healthy augmented system to ensure tracking trajectory. Secondly, an adaptive observer based FTC will be introduced to stabilize the faulty augmented system.

3.2 Synthesis of the tracking H-infinity controller

The goal of this part is to design an H-infinity controller $u(t)$ in order to ensure stability and trajectory tracking of the system in fault-free case represented by (8). This robust controller is carried out via the parallel distributed compensation PDC controller (Benzaouia and El Hajjaji (2014)) with H_∞ criteria optimization to guarantee good performance requirements and high robustness (Wang L et al (2018)).

The H-infinity controller law is defined as follows:

$$u(t) = \sum_{i=1}^{N} \mu_i(\xi(t)) K_i X_r(t) \quad (11)$$

where K_i are the gain matrices with appropriate dimension.

By combining the control law (11) and the system (8), the closed-loop system becomes:

$$\begin{cases} \dot{X}_r(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \mu_i(\xi(t)) \mu_j(\xi(t)) G_{ij} X_r(t) + \bar{D} y_r(t) \\ Y(t) = \bar{C} X_r(t) \end{cases} \quad (12)$$

where $G_{ij} = \bar{A}_i + \bar{B}_i K_j$

In order to calculate the gain matrices K_j that ensure the asymptotic stability of the closed-loop system (12) and guarantee the H-infinity tracking performance, the following H-infinity criterion is used.

$$\int_0^\infty X_r^T(t)QX_r(t)dt < \rho^2 \int_0^\infty \bar{\Phi}^T(t)\bar{\Phi}(t)dt \tag{13}$$

with Q is a positive definite matrix, ρ is a scalar performance level to be minimized and $\bar{\Phi} = y_r(t)$.

We consider the following Lyapunov function:

$$V(t) = X_r^T(t)PX_r(t) \tag{14}$$

with $P = P^{-1} > 0$

The exponential convergence of the system (12) is verified if:

$$\exists \alpha \quad \text{such that} \quad \dot{V}(t) + 2\alpha V(t) < 0 \tag{15}$$

By replacing V(t) by its definition, the inequality (15) becomes:

$$\sum_{i=1}^N \sum_{j=1}^N \mu_i(\xi(t))\mu_j(\xi(t))(X_r^T(t)P(\bar{A}_i + \bar{B}_iK_j) + (\bar{A}_i + \bar{B}_iK_j)^T PX_r(t) + X_r^T(t)P\bar{D}\bar{\Phi} + \bar{\Phi}^T\bar{D}^T PX_r(t) + X_r^T(t)2\alpha PX_r(t)) < 0 \tag{16}$$

Lemma 1. (Zhou K and Khargonedkar P (1988)) For real matrices $\bar{R}$ and $\bar{S}$ with appropriate dimensions and a positive constant η, the following inequalities hold:

$$\bar{R}^T\bar{S} + \bar{S}^T\bar{R} \leq \eta \bar{R}^T\bar{R} + \eta^{-1}\bar{S}^T\bar{S} \tag{17}$$

By applying the Lemma 1 for the inequality (16) we obtain:

$$\sum_{i=1}^N \sum_{j=1}^N \mu_i(\xi(t))\mu_j(\xi(t))X_r^T(t)(\Psi_{ij} + \eta^{-1}P\bar{D}\bar{D}^T P + 2\alpha P)X_r(t) + \eta\bar{\Phi}^T\bar{\Phi} < 0 \tag{18}$$

where $\Psi_{ij} = (\bar{A}_i + \bar{B}_iK_j)^T P + P(\bar{A}_i + \bar{B}_iK_j)$

By considering the H-infinity criterion (13), the stability conditions are verified if:

$$\sum_{i=1}^N \sum_{j=1}^N \mu_i(\xi(t))\mu_j(\xi(t))X_r^T(t)(\Psi_{ij} + \eta^{-1}P\bar{D}\bar{D}^T P + 2\alpha P + Q)X_r(t) < 0 \tag{19}$$

with $\eta = \rho^2$

The inequality (19) can be rewritten as:

$$\sum_{i=1}^N \sum_{j=1}^N \mu_i^2(\xi(t))(\Psi_{ii} + \eta^{-1}P\bar{D}\bar{D}^T P + 2\alpha P + Q) + \sum_{i=1}^N \sum_{j=1}^N \mu_i(\xi(t))\mu_j(\xi(t))(\frac{\Psi_{ij} + \Psi_{ji}}{2} + \eta^{-1}P\bar{D}\bar{D}^T P + 2\alpha P + Q) < 0 \tag{20}$$

Therefore, the stability conditions become:

$$\Psi_{ii} + \eta^{-1}P\bar{D}\bar{D}^T P + 2\alpha P + Q < 0, \quad i = 1,2..N \tag{21}$$

$$\Psi_{ij} + \Psi_{ji} + 2\eta^{-1}P(\bar{D}\bar{D}^T)P + 4\alpha P + 2Q < 0, \quad i < j \tag{22}$$

The obtained solutions are not LMIs. In order to resolve this problem, some transformations are necessary. For this purpose, we consider a new variable $\bar{P} = P^{-1}$ and using the convenient bijective change of variable $M_i = K_i\bar{P}$. Then, the above inequalities can be rewritten as an LMIs feasibility problem in the following theorem.

Theorem 1. The exponential convergence of (12) is verified if there exist a positive definite symmetric matrix $\bar{P}$, a positive constant η, a positive matrices M_i and Q that satisfy the following conditions:

$$\begin{bmatrix} \bar{\Psi}_{ii} + \eta^{-1}\bar{D}\bar{D}^T + 2\alpha\bar{P} & \bar{P} \\ \bar{P} & -Q^{-1} \end{bmatrix} < 0, \quad i = 1,2..N \tag{23}$$

$$\begin{bmatrix} \bar{\Psi}_{ij} + \bar{\Psi}_{ji} + 2\eta^{-1}\bar{D}\bar{D}^T + 4\alpha\bar{P} & \bar{P} \\ \bar{P} & -\frac{1}{2}Q^{-1} \end{bmatrix} < 0, \quad i < j \tag{24}$$

where $\bar{\Psi}_{ij} = P\bar{A}_i^T + \bar{A}_iP + M_j^T\bar{B}_i^T + \bar{B}_iM_j$

The scalar α is called the decay rate, and $K_i = M_i\bar{P}^{-1}$

3.3 The proportional integral unknown input observer

In this section, a proportional integral observer with unknown inputs (PIUI) is proposed to estimate simultaneously the states of the faulty augmented system (9) and the faults affecting the system. The proposed (PIUI) is implemented to design an active fault tolerant tracking control for a TS system with simultaneously acting actuator and sensor faults. This observer has the following structure:

$$\begin{cases} \dot{Z}_f(t) = \sum_{i=1}^N \mu_i(\hat{\xi}_f(t))(R_iZ_f(t) + T\bar{B}_iu_f(t) + T\bar{E}_i\hat{f}(t) + K_{Pi}Y_f(t) \\ \hat{X}_f(t) = Z_f(t) + HY_f(t) \\ \dot{\hat{f}}(t) = \sum_{i=1}^N \mu_i(\hat{\xi}_f(t))K_{Ii}(Y_f(t) - \bar{C}\hat{X}_f(t)) \end{cases} \tag{25}$$

where $\hat{X}_f(t)$ is the estimated system state, $Z_f(t)$ is the intermediate variable and $\hat{f}(t)$ the estimated faults. The variables R_i, T, K_{Pi}, K_{Ii} and H are the observer gains.

3.4 Method of resolution

Let us define the error $e_r(t)$ between the healthy system state $X_r(t)$ and the faulty system state $X_f(t)$, the state estimation error $e_{xf}(t)$ and the faults estimation error $e_f(t)$.

$$e_r(t) = X_r(t) - X_f(t) \tag{26}$$

$$e_{xf}(t) = X_f(t) - \hat{X}_f(t) \tag{27}$$

$$e_f(t) = f(t) - \hat{f}(t) \tag{28}$$

If we assume that: $\dot{f}(t) = 0$, the dynamics of $e_r(t)$ and $e_{xf}(t)$ are respectively given by:

$$\dot{e}_r(t) = \sum_{i=1}^N \mu_i(\xi_f(t))(\bar{A}_ie_r(t) - \bar{E}_ie_f(t)) + \Delta_1(t) \tag{29}$$

with $\Delta_1(t) = \sum_{i=1}^N (\mu_i(\xi(t)) - \mu_i(\xi_f(t)))(\bar{A}_iX_r(t) + \bar{B}_iu(t))$

The dynamic of $e_{xf}(t)$ is written by:

$$\dot{e}_{xf}(t) = \Omega\dot{X}_f - \dot{Z}_f(t) = \sum_{i=1}^N \mu_i(\hat{\xi}_f(t))((\Omega\bar{A}_i - R_i - \bar{F}_i\bar{C})X_f(t) + (\Omega\bar{B}_i - T\bar{B}_i)u_f(t) + (\Omega\bar{E}_i - T\bar{E}_i)f(t) + T\bar{E}_ie_f(t) + \Omega\bar{D}y_r(t) + R_ie_{xf}(t)) + \Delta_2(t) \tag{30}$$

where $\Delta_2(t) = \sum_{i=1}^N (\mu_i(\xi_f(t)) - \mu_i(\hat{\xi}_f(t)))\Omega(\bar{A}_iX_f(t) + \bar{B}_iu_f(t) + \bar{E}_if(t))$

$$\Omega = I - H\bar{C} \tag{31}$$

$$\bar{F}_i = K_{Pi} - R_iH \tag{32}$$

If the following conditions are verified

$$\Omega = T \tag{33}$$

$$R_i = \Omega\bar{A}_i - \bar{F}_i\bar{C} \tag{34}$$

The estimation error of state can be reduced to:

$$\dot{e}_{xf}(t) = \sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))(R_i e_{xf}(t) + T\bar{E}_i e_f(t) + \Omega\bar{D}y_r(t) + \Delta_2(t)) \tag{35}$$

The dynamics of the fault estimation error is expressed as:

$$\begin{aligned}\dot{e}_f(t) &= -\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}(Y_f(t) - \bar{C}\hat{X}_f(t)) \\ &= -\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}\bar{C}e_{xf}(t)\end{aligned} \tag{36}$$

The estimation errors $e_{xf}(t)$ and $e_f(t)$ can be rewritten in augmented form:

$$\dot{e}_a(t) = \sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))(\bar{A}_{ai} - \bar{K}_i\bar{C}_a)e_a(t) + \omega\phi(t) \tag{37}$$

where: $e_a(t) = \begin{bmatrix} e_{xf}(t) \\ e_f(t) \end{bmatrix}$; $\bar{A}_{ai} = \begin{bmatrix} \Omega\bar{A}_i & T\bar{E}_i \\ 0 & 0 \end{bmatrix}$; $\bar{K}_i = \begin{bmatrix} \bar{F}_i \\ K_{Ii} \end{bmatrix}$; $\bar{C}_a = \begin{bmatrix} \bar{C} & 0 \end{bmatrix}$; $\omega = \begin{bmatrix} \Omega\bar{D} & I \\ 0 & 0 \end{bmatrix}$ $\phi(t) = \begin{bmatrix} y_r(t) \\ \Delta_2(t) \end{bmatrix}$

Therefore, the errors $e_r(t)$, $e_{xf}(t)$ and $e_f(t)$ evolves according to the following equation:

$$\dot{\bar{e}}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\xi_f(t))\mu_j(\hat{\xi}_f(t))\tilde{A}_{ij}\bar{e}(t) + \psi\tilde{\phi}(t) \tag{38}$$

where: $\bar{e}(t) = \begin{bmatrix} e_r(t) \\ e_a(t) \end{bmatrix}$; $\tilde{A}_{ij} = \begin{bmatrix} \bar{A}_i & \tilde{E}_i \\ 0 & \bar{A}_{aj} - \bar{K}_j\bar{C}_a \end{bmatrix}$; $\psi = \begin{bmatrix} I & 0 \\ 0 & \omega \end{bmatrix}$; $\tilde{\phi}(t) = \begin{bmatrix} \Delta_1(t) \\ \phi(t) \end{bmatrix}$ with $\tilde{E}_i = \begin{bmatrix} 0 & -\bar{E}_i \end{bmatrix}$

The gains of the proposed (PIUI) observer are computed by solving a minimization problem under LMIs constraints, presented by the following theorem.

Theorem 2. The exponential convergence of the generated error $\bar{e}(t)$ describing the evolution of the errors $e_r(t)$, $e_{xf}(t)$ and $e_f(t)$ is verified and the L2-gain of the transfer from $\tilde{\phi}(t)$ to the error $\bar{e}(t)$ is bounded if there exist symmetric and positive definite matrices X_1 and X_2, matrix $\bar{V}_j$ and positive scalars $\bar{\gamma}$ solution to the following optimization problem:

$$\min_{X_1, X_2, \bar{V}_j, \bar{\gamma}} \bar{\gamma}$$

for a prescribed scalar $\alpha > 0$, the following conditions hold:

$$\begin{bmatrix} \Xi_i & X_1\tilde{E}_i & X_1 & 0 \\ \tilde{E}_i^T X_1 & \Upsilon_j & 0 & X_2\omega \\ X_1 & 0 & -\bar{\gamma}I_1 & 0 \\ 0 & \omega^T X_2 & 0 & -\bar{\gamma}I_3 \end{bmatrix} < 0 \quad i, j = 1, ..., N \tag{39}$$

$$\Xi_i = \bar{A}_i^T X_1 + X_1\bar{A}_i + I_1 + 2\alpha X_1 \tag{40}$$

$$\Upsilon_j = X_2\bar{A}_{aj} - \bar{V}_j\bar{C}_a + \bar{A}_{aj}^T X_2 - \bar{C}_a^T\bar{V}_j^T + I_2 + 2\alpha X_2 \tag{41}$$

The PIUI observer gains are computed from:

$$\bar{K}_j = \begin{bmatrix} \bar{F}_j \\ K_{Ij} \end{bmatrix} = X_2^{-1}\bar{V}_j \tag{42}$$

$$R_j = \Omega\bar{A}_j - \bar{F}_j\bar{C} \tag{43}$$

$$K_{Pj} = \bar{F}_j + R_j H \tag{44}$$

where $[\Omega \;\; H] = \begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}^+$ and $T = \Omega$

and the L2-gain of the transfer from $\tilde{\phi}$ to the error $\bar{e}(t)$ is given by: $\gamma = \sqrt{\bar{\gamma}}$

Proof. According to (31) and (33), we can write:

$$I = T + H\bar{C} = [T \;\; H]\begin{bmatrix} I_n \\ \bar{C} \end{bmatrix} \tag{45}$$

Then, the matrices T and H are obtained by the following expression:

$$[T \;\; H] = \begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}^+ \tag{46}$$

where $\begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}^+$ presents the pseudo-inverse of $\begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}$

Consider the following quadratic Lyapunov function:

$$V(t) = \dot{\bar{e}}^T(t)X\dot{\bar{e}}(t) \tag{47}$$

X is defined as :

$$\begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \tag{48}$$

The generalized error vector $\bar{e}(t)$ converge to zero if:

$$\exists X = X^T > 0, \quad \alpha > 0 : \quad \dot{V}(t) + 2\alpha V(t) < 0 \tag{49}$$

Assume that $\tilde{\phi}(t)$ is bounded, the objective is to minimize the L2-gain of the transfer from $\tilde{\phi}(t)$ to the error $\bar{e}(t)$, this is given by:

$$\frac{\|\bar{e}(t)\|_2}{\|\tilde{\phi}(t)\|_2} < \gamma, \quad \|\tilde{\phi}(t)\|_2 \neq 0 \tag{50}$$

Then, the goal is to guarantee an asymptotic convergence toward zero if $\tilde{\phi}(t) = 0$ and to ensure a bounded L2-gain if $\tilde{\phi}(t) \neq 0$. This is formulated by:

$$\dot{V}(t) + \bar{e}^T(t)\bar{e}(t) - \gamma^2\tilde{\phi}(t)^T\tilde{\phi}(t) + 2\alpha V(t) < 0 \tag{51}$$

By replacing the expression of $V(t)$ we obtain the following inequality:

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\xi_f(t))\mu_j(\hat{\xi}_f(t))(\bar{e}^T(t)(\tilde{A}_{ij}^T X + X\tilde{A}_{ij})\bar{e}(t) \\ &+\bar{e}^T(t)X\psi\tilde{\phi}(t) + \tilde{\phi}^T(t)\psi^T X\bar{e}(t) + \bar{e}^T(t)2\alpha X\bar{e}(t) \\ &+\bar{e}^T(t)\bar{e}(t) - \gamma^2\tilde{\phi}(t)^T\tilde{\phi}(t)) < 0\end{aligned} \tag{52}$$

That can be rewritten by the following way:

$$\begin{bmatrix} \tilde{A}_{ij}^T X + X\tilde{A}_{ij} + I + 2\alpha X & X\psi \\ \psi^T X & -\gamma^2 I \end{bmatrix} < 0 \quad \forall i, j = 1, ...N \tag{53}$$

By replacing $\tilde{A}_{ij}$, ψ and X by their definitions, and after some calculations, the exponential stability conditions satisfying the attenuation level of the L2-gain are given in the theorem 2.

3.5 The proportional multiple integral unknown input observer

A. Time varying faults

The assumption that the faults are constant or slowly variable over the time is restrictive, but in many practical situations, the faults are time-varying signals. In this problem, the PIUI observer can be replaced by a proportional multiple integral unknown input (PMIUI) observer. Such an observer is used to estimate a large class of time-varying signals which have the following assumption:

$$f^{(k)} = 0 \tag{54}$$

The (PMIUI) observer is based on the estimation of the $(k-1)^{th}$ first derivative of the fault $f(t)$, and can be extended to the case where $f^{(k)}$ is bounded. Its structure is described as follows:

$$
\begin{cases}
\dot{\hat{Z}}_f(t)=\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))(R_iZ_f(t)+T\bar{B}_iu_f(t)+T\bar{E}_i\hat{f}(t)+K_{Pi}Y_f(t)\\
\hat{X}_f(t)=Z_f(t)+HY_f(t)\\
\dot{\hat{f}}(t)=\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}^{0}(Y_f(t)-\bar{C}\hat{X}_f(t))+\hat{f}_1(t)\\
\quad\vdots\\
\dot{\hat{f}}_{k-2}(t)=\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}^{k-2}(Y_f(t)-\bar{C}\hat{X}_f(t))+\hat{f}_{k-1}(t)\\
\dot{\hat{f}}_{k-1}(t)=\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}^{k-1}(Y_f(t)-\bar{C}\hat{X}_f(t))
\end{cases}
\tag{55}
$$

where $\dot{f}(t)=f_1(t)$; $\dot{f}_1(t)=f_2(t)$; ...;$\dot{f}_{k-1}(t)=f_k(t)$ and $f_k(t)=0$. R_i,T,K_{Pi},H and $K_{Ii}^{j}(j=1,...,k-1)$ are the gains of the PMIUI observer.

Therefore, The dynamics of the faults estimation errors become:

$$
\begin{cases}
\dot{e}_0(t)=-\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}^{0}\bar{C}e_{xf}+e_1(t)\\
\quad\vdots\\
\dot{e}_{k-2}(t)=-\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}^{k-2}\bar{C}e_{xf}+e_{k-1}(t)\\
\dot{e}_{k-1}(t)=-\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))K_{Ii}^{k-1}\bar{C}e_{xf}
\end{cases}
\tag{56}
$$

and the augmented form of the states and faults errors is rewritten as:

$$
\dot{\tilde{e}}_a(t)=\sum_{i=1}^{N}\mu_i(\hat{\xi}_f(t))(\tilde{A}_{ai}-\tilde{K}_i\tilde{C}_a)\tilde{e}_a(t)+\tilde{\omega}\phi(t) \tag{57}
$$

where: $\tilde{e}_a(t)=\begin{bmatrix}e_{xf}(t)\\e_0(t)\\\vdots\\e_{k-2}(t)\\e_{k-1}(t)\end{bmatrix}$; $\tilde{A}_{ai}=\begin{bmatrix}\Omega\bar{A}_i & T\bar{E}_i & 0 & \dots & 0\\0&0&I&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&0&I\\0&0&0&0&0\end{bmatrix}$;

$\tilde{K}_i=\begin{bmatrix}\bar{F}_i\\K_{Ii}^{0}\\\vdots\\K_{Ii}^{k-2}\\K_{Ii}^{k-1}\end{bmatrix}$; $\tilde{C}_a=\begin{bmatrix}\bar{C} & 0 & \dots & 0 & 0\end{bmatrix}$; $\tilde{\omega}=\begin{bmatrix}\Omega\bar{D} & I\\0&0\\\vdots&\vdots\\0&0\\0&0\end{bmatrix}$

Therefore, the augmented form of the errors $e_r(t)$ and $\tilde{e}_a(t)$ becomes:

$$
\dot{\tilde{\tilde{e}}}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\xi_f(t))\mu_j(\hat{\xi}_f(t))\Lambda_{ij}\tilde{\tilde{e}}(t)+\Psi\tilde{\phi}(t) \tag{58}
$$

where: $\tilde{\tilde{e}}(t)=\begin{bmatrix}e_r(t)\\\tilde{e}_a(t)\end{bmatrix}$; $\Lambda_{ij}=\begin{bmatrix}\bar{A}_i & \tilde{\tilde{E}}_i\\0 & \tilde{A}_{aj}-\tilde{K}_j\tilde{C}_a\end{bmatrix}$; $\Psi=\begin{bmatrix}I&0\\0&\tilde{\omega}\end{bmatrix}$ with $\tilde{\tilde{E}}_i=\begin{bmatrix}0 & -\bar{E}_i & 0 & \dots & 0 & 0\end{bmatrix}$

Thus, the structure of equations (58) is similar to that presented in the equations (38). Therefore, the synthesis of the gains can be obtained by solving the LMIs given in the theorem 2.

B. Time varying faults with unknown disturbance and measurement noise

The considered system in this situation is affected by simultaneous actuator and sensor time-varying faults, and exposed to unknown disturbance and measurement noise. It is described by the following equations.

$$
\begin{cases}
\dot{x}_f(t)=\sum_{i=1}^{N}\mu_i(\xi_f(t))(A_ix_f(t)+B_iu_f(t)+E_if_a(t)+W_i\nu(t))\\
y_f(t)=Cx_f(t)+Sf_s(t)+G\varsigma(t)
\end{cases}
\tag{59}
$$

where W_i and G are respectively the distribution matrices of the unknown disturbance $\nu(t)$ and the measurement noise $\varsigma(t)$.

After the system transformation, equations described in (8) and (9) become:

$$
\begin{cases}
\dot{X}_r(t)=\sum_{i=1}^{N}\mu_i(\xi(t))(\bar{A}_iX_r(t)+\bar{B}_iu(t)+\mathscr{D}_i\tilde{\Phi}(t))\\
Y(t)=\bar{C}X_r(t)
\end{cases}
\tag{60}
$$

$$
\begin{cases}
\dot{X}_f(t)=\sum_{i=1}^{N}\mu_i(\xi_f(t))(\bar{A}_iX_f(t)+\bar{B}_iu_f(t)+\bar{E}_if(t)+\mathscr{D}_i\tilde{\Phi}(t))\\
Y_f(t)=\bar{C}X_f(t)
\end{cases}
\tag{61}
$$

where $\mathscr{D}_i=\begin{bmatrix}W_i&0&0\\0&DG&0\\0&0&I\end{bmatrix}$ and $\tilde{\Phi}(t)=\begin{bmatrix}\nu(t)\\\varsigma(t)\\y_r\end{bmatrix}$

In order to ensure the trajectory tracking despite the presence of unknown disturbance and noise, $\tilde{\Phi}(t)$ is minimized using the H_∞ criterion and the stability condition given in the theorem 1 by replacing $\bar{D}\bar{D}^T$ in (23) by $\mathscr{D}_i\mathscr{D}_i^T$ and in (24) by $(\mathscr{D}_i\mathscr{D}_j^T+\mathscr{D}_j\mathscr{D}_i^T/2)$

Then, the PMIUI observer can estimate the acting faults against the unknown disturbance and the measurement noise by replacing $\bar{D}$ by $\mathscr{D}_i$ and $\phi(t)$ by $\tilde{\Phi}(t)$ in (57). Therefore, the augmented form of the states and the faults errors become:

$$
\dot{\tilde{\mathscr{E}}}_a(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\xi_f(t))\mu_j(\hat{\xi}_f(t))(\Lambda_{ij}\tilde{\mathscr{E}}_a(t)+\mathscr{W}_i\tilde{\Phi}(t)) \tag{62}
$$

where: $\mathscr{W}_i=\begin{bmatrix}\Omega\mathscr{D}_i & I\\0&0\\\vdots&\vdots\\0&0\\0&0\end{bmatrix}$

Thus, the augmented form of the errors $e_r(t)$ and $\tilde{\mathscr{E}}_a$ can be rewritten as follows:

$$
\dot{\tilde{\tilde{\mathscr{E}}}}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\xi_f(t))\mu_j(\hat{\xi}_f(t))\Lambda_{ij}\tilde{\tilde{\mathscr{E}}}(t)+\tilde{\Psi}_i\tilde{\tilde{\phi}}(t) \tag{63}
$$

where: $\tilde{\tilde{\mathscr{E}}}(t)=\begin{bmatrix}e_r(t)\\\tilde{\mathscr{E}}_a(t)\end{bmatrix}$; $\tilde{\Psi}_i=\begin{bmatrix}I&0\\0&\mathscr{W}_i\end{bmatrix}$; $\tilde{\tilde{\phi}}(t)=\begin{bmatrix}\Delta_1(t)\\\tilde{\Phi}(t)\end{bmatrix}$

Finally, the gains of the PMIUI observer can be calculated using theorem 2.

4. APPLICATION TO THE CSTR

Continuous stirred tank reactor (CSTR) plays a vital role in chemical and pharmaceutical industry due to their appropriate mixing property. In practice, chemical systems are exposed to several faults such as failures or changes in actuators/sensors operations, providing an undesirable control performances and so deteriorating process behavior. The objective of this section is the application of the proposed FTC to a CSTR in order to ensure good output reference tracking in the presence of actuator and sensor faults.

4.1 CSTR system description

A continuous stirred tank reactor is represented in Fig. 1. Her isothermal series-parallel reaction (Van der Vusse reaction) is given by the following reactions (M Nagarajan et al (2016)):

$$A \overset{k1}{\rightarrow} B \overset{k2}{\rightarrow} C$$

$$2A \overset{k3}{\rightarrow} D$$

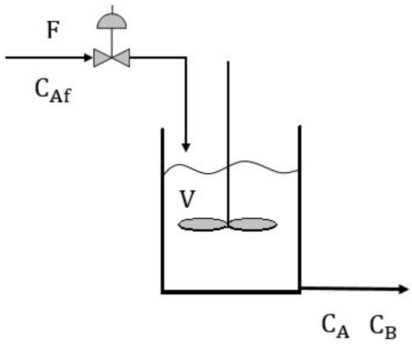

Fig. 1. Schematic diagram of continuous stirred tank reactor.

The mathematical model of this reactor is described by the following differential equations:

$$\begin{cases} \dot{C}_A = \dfrac{F}{V}(C_{Af} - C_A) - k_1 C_A - k_3 C_A^2 \\ \dot{C}_B = -\dfrac{F}{V} C_B - k_1 C_A - k_2 C_B \end{cases} \quad (64)$$

where F is the flow rate, V is the volume, k_1, k_2 and k_3 are the reaction rate constants, C_{Af} is the inlet concentration of component A, C_A is the concentration of component A. The control input is represented by the dilution rate $\frac{F}{V}$, and the controlled output is the product concentration C_B. The equations for C_C and C_D are neglected because C_B is independent of them. The numerical values of the parameters model are:
$k_1 = \dfrac{5}{6}\ min^{-1}$; $\quad k_2 = \dfrac{5}{3}\ min^{-1}$; $\quad k_3 = \dfrac{1}{6}\ mol/l.min$;
$C_{Af} = 10\ mol/l$; $\quad V = 1l$.

4.2 Takagi-Sugeno model design

The CSTR is a chemical system with complex nonlinear dynamic characteristics. To deal with this non-linearity, the Takagi-Sugeno approach is introduced to represent the nonlinear model of the CSTR by an interpolation of simple linear submodels. In the considered system, we have two nonlinear terms or premise variables: $\xi_1(t) = C_A, \xi_2(t) = C_B$.

with

$$min(\xi_i(t)) < \xi_i(t) < max(\xi_i(t)) \quad \forall i = 1,2 \quad (65)$$

The premise variables can be represented by the membership functions F_i, f_i:

$$\xi_i(t) = F_i(\xi_i(t)).max(\xi_i(t)) + f_i(\xi_i(t)).min(\xi_i(t)) \quad \forall i = 1,2 \quad (66)$$

According to (2), the membership functions can be given as follows:

$$\begin{cases} F_i(\xi_i(t)) = \dfrac{\xi_i(t) - min(\xi_i(t))}{max(\xi_i(t)) - min(\xi_i(t))} \\ f_i(\xi_i(t)) = \dfrac{max(\xi_i(t)) - \xi_i(t)}{max(\xi_i(t)) - min(\xi_i(t))} \end{cases} \forall i = 1,2 \quad (67)$$

The activation functions are obtained by:

$$\begin{cases} \mu_1(\xi(t)) = F_1(\xi_1(t))F_2(\xi_2(t)) \\ \mu_2(\xi(t)) = f_1(\xi_1(t))F_2(\xi_2(t)) \\ \mu_3(\xi(t)) = F_1(\xi_1(t))f_2(\xi_2(t)) \\ \mu_4(\xi(t)) = f_1(\xi_1(t))f_2(\xi_2(t)) \end{cases} \quad (68)$$

Then, the aggregated TS model of the CSTR can be described by the following structure:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (69)$$

where: $C = [0\ \ 1]$; $\quad A_1 = A_3 = \begin{bmatrix} -1.3500 & 0 \\ 0.8333 & -1.6667 \end{bmatrix}$;
$A_2 = A_4 = \begin{bmatrix} -1.2500 & 0 \\ 0.8333 & -1.6667 \end{bmatrix}$; $\quad B_1 = \begin{bmatrix} 6.9000 \\ -1.3000 \end{bmatrix}$
$B_2 = \begin{bmatrix} 7.5000 \\ -1.3000 \end{bmatrix}$; $\quad B_3 = \begin{bmatrix} 6.9000 \\ 0 \end{bmatrix}$; $\quad B_4 = \begin{bmatrix} 7.5000 \\ 0 \end{bmatrix}$.

4.3 Simulations results and remarks

Considering a CSTR affected by actuator and sensor faults:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{4} \mu_i(\xi_f(t))(A_i x_f(t) + B_i u_f(t) + E_i f_a(t)) \\ y_f(t) = Cx_f(t) + Sf_s(t) \end{cases} \quad (70)$$

Three case studies are considered in this work.

Case 1: in this case the acting faults are chosen as bias which appears and disappears during an time interval as follows:

$$f_a(t) = \begin{cases} 0,11 & 312.5 < t < 800 \\ 0 & elsewhere \end{cases}$$

$$f_s(t) = \begin{cases} 0,165 & 625 < t < 937.5 \\ 0 & elsewhere \end{cases}$$

The objective is to show the effectiveness of the proposed FTC. For that purpose, the LMIs elaborated in section 3 are solved using Matlab's Yalmip toolbox. Then, several simulations are implemented on the CSTR.

The actuator and sensor faults with their estimations are depicted in Fig. 2. In Fig. 3, we present a comparison between the reference model output (without faults), the output of the faulty system without FTC, and the output of the faulty system with the proposed FTC. Fig. 4 compares the nominal control input and the proposed FTC.

From the simulations in Fig. 2, we can notice that the proposed PIUI observer provides the estimation of simultaneous actuator and sensor faults and the system state. In Fig. 3, one can see that in the presence of the actuator and sensor faults, the

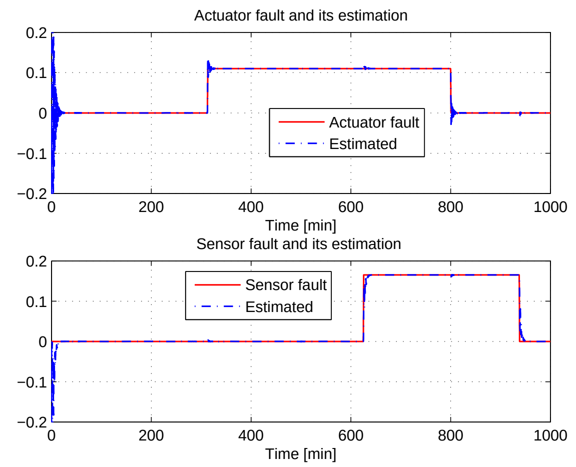

Fig. 2. Actuator and sensor faults with their estimates (Case 1).

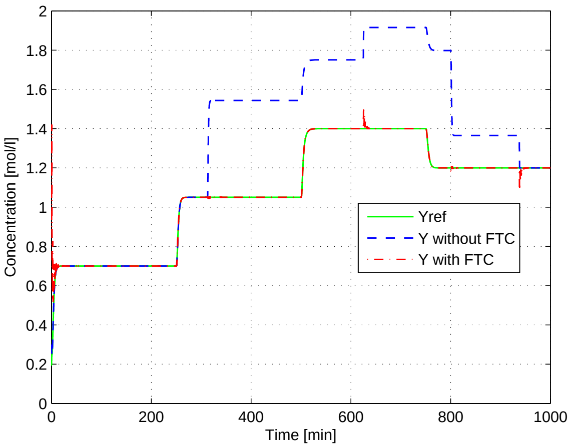

Fig. 3. Reference model, system output without FTC, system output with FTC (Case 1).

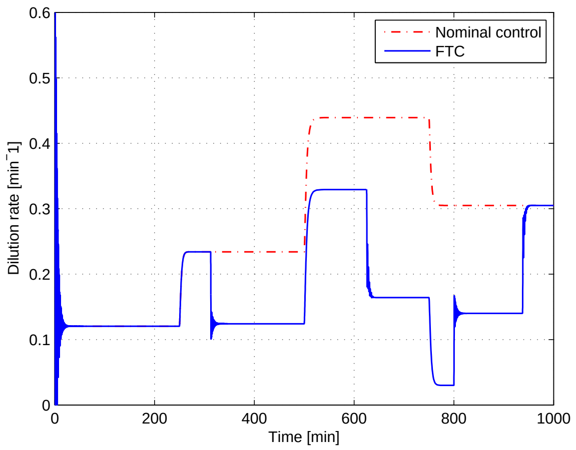

Fig. 4. Nominal control and FTC control (Case 1).

output of the faulty system without the proposed FTC lost its performance and diverge from the output of the reference healthy system. On the other hand, Fig. 4 shows that the faulty system output with the proposed FTC law converge from the reference trajectory even in the presence of the acting faults.

Case 2: in this case, the considered faults are slowly time-varying signals and modeled as follows:

$$f_a(t) = \begin{cases} 0.07sin(0.05\pi t) & 150 < t < 400 \\ 0 & elsewhere \end{cases}$$

$$f_s(t) = \begin{cases} 0,0002t & 300 < t < 700 \\ 0 & elsewhere \end{cases}$$

The simulation results are illustrated in the following figures. According to the Fig. 5 we note that the proposed observer is able to reconstruct time-varying signals with slow variation. Fig. 6 shows a comparison between reference model, the output of the system without FTC and the output with FTC in the presence of slowly time-varying faults. We can remark that the output with the proposed FTC converges to the reference trajectory even in the presence of both slowly time-varying actuator and sensor faults. From Fig. 7, we can see that the proposed FTC compensates the fault and allows normal functioning of the system in spite of the occurrence of time-varying faults with a bounded norm of first time derivative.

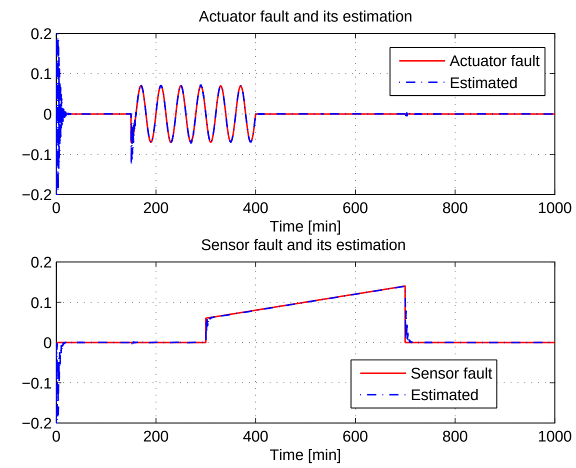

Fig. 5. Actuator and sensor faults with their estimates (Case 2).

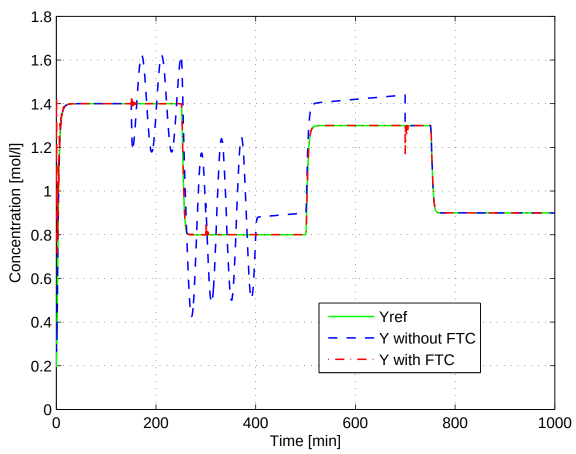

Fig. 6. Reference model, system output without FTC, system output with FTC (Case 2).

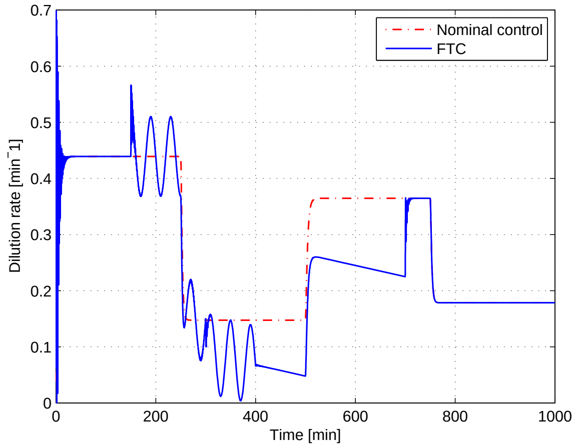

Fig. 7. Nominal control and FTC control (Case 2).

Case 3: the system is exposed to actuator and sensor time-varying faults with unknown disturbance and measurement noise:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{4} \mu_i(\xi_f(t))(A_i x_f(t) + B_i u_f(t) + E_i f_a(t) + W_i \nu(t)) \\ y_f(t) = C x_f(t) + S f_s(t) + G\xi(t) \end{cases} \tag{71}$$

The actuator and the sensor faults are time-varying as follows:

$$f_a(t) = \begin{cases} 2.10^{-4} sin(0.08\pi t) \times t \quad 200 < t < 500 \\ 0.19 sin(0.15\pi t) \quad 500 < t < 700 \\ 0 \quad elsewhere \end{cases}$$

$$f_s(t) = \begin{cases} -3.10^{-4}(t-200)^2 + 6.10^{-8}(t-200)^3 \quad 100 < t < 400 \\ 2.10^{-5}(t-400) - 2.10^{-5}(t-400)^2 \quad 400 < t < 500 \\ 0.08 sin(0.08\pi t) \quad 500 < t < 620 \\ 0 \quad elsewhere \end{cases}$$

The unknown disturbance is presented by a Sawtooth signal of period 30 and amplitude 0.03, the measurement noise is presented by zero mean noise with standard deviations equal to 0.05. The simulation results are given in the following figures.

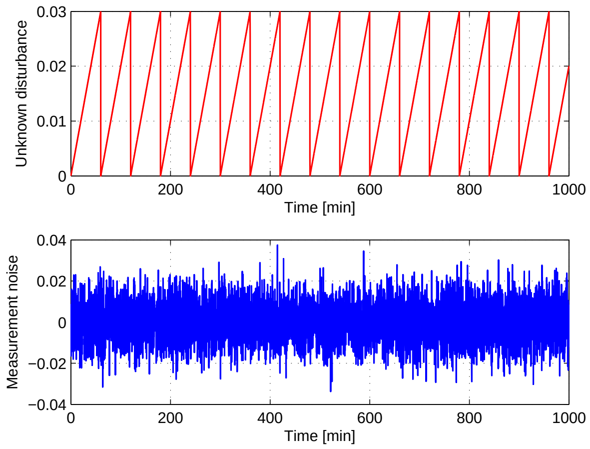

Fig. 8. Unknown disturbance and measurement noise signals.

Fig.9 shows that the proposed PMIUI observer estimate simultaneously the actuator and the sensor varying faults under

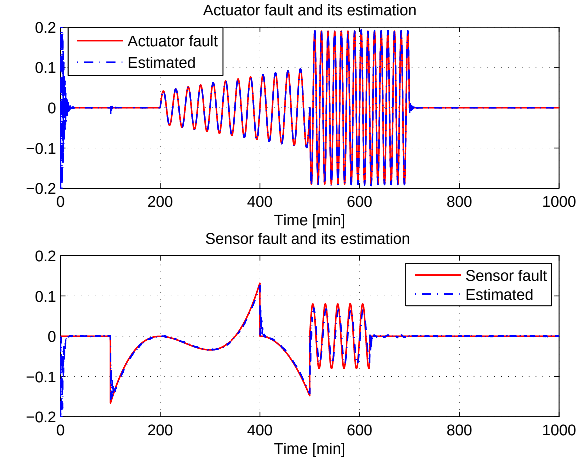

Fig. 9. Actuator and sensor faults with their estimates (Case 3).

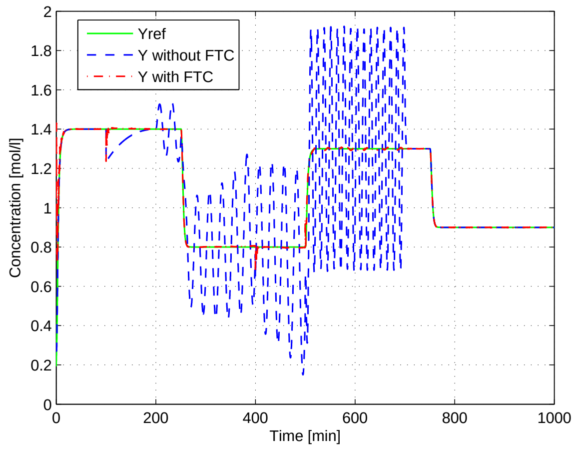

Fig. 10. Reference model, system output without FTC, system output with FTC (Case 3).

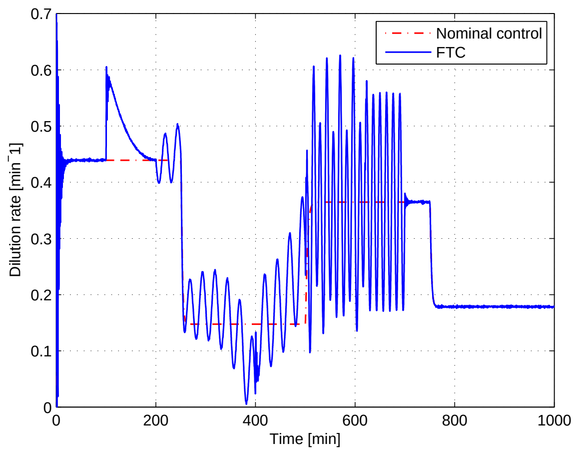

Fig. 11. Nominal control and FTC control (Case 3).

unknown disturbance and measurement noise. In Fig.10, the system output with the proposed FCT track asymptotically the reference trajectory even in the case of time-varying faults, and the effect of the measurement noise and the unknown disturbance is minimized. In Fig.11, one can also see that the proposed FTC strategy accommodate effectively the time-varying actuator and sensor faults.

4.4 Comparison

In order to compare the proposed FTC strategy with the existing methods, we choose the technique used in (Ichalal et al (2010); El Youssfi et al (2019)) which based on the same objective, the authors present the synthesis of an active fault tolerant control for nonlinear Takagi-Sugeno fuzzy systems in order to guarantee the convergence of the system to the reference model even the presence of faults. The control law uses the nominal control input of the fault-free case and two additional terms related to the trajectory tracking error and the estimated fault. The estimation is based on a proportional integral observer and the stability is studied with L2 optimization and the Lyapunov function.

Remark 1. In our work, the observer and the controller are designed independently, and their gains are calculated separately, which permits to avoid the coupling problem. As opposed by (Ichalal et al (2010); El Youssfi et al (2019)) the control and the observer gains are calculated simultaneously. In addition, the H_∞ controller used in our strategy permits to the faulty system output to track a desired trajectory.

A comparative study is provided to show that our result lead to less conservative LMIs in comparison with the method given in (Ichalal et al (2010); El Youssfi et al (2019)). The result of comparison is depicted in Fig. 12.

Fig. 12. Feasibility of LMIs in theorem 2 indicated with o and in (Ichalal et al (2010); El Youssfi et al (2019)) with +.

The considered system in (70) is slightly modified in order to test the feasibility of the LMIs in the proposed strategy and the reference methods for several system models. The matrices A_i and C are modified by two free parameters a and b because the LMIs of theorem 2 and the one described in the reference methods depend on A_i and C.

$$A_1 = A_3 = \begin{bmatrix} -1.3500 & a \\ 0.8333 & -1.6667 \end{bmatrix};$$

$$A_2 = A_4 = \begin{bmatrix} -1.2500 & a \\ 0.8333 & -1.6667 \end{bmatrix}; \quad C = [0 \ b];$$

The feasibility of LMIs in theorem 2 and (Ichalal et al (2010); El Youssfi et al (2019)) is tested for several values of pairs (a,b), $a \in [-5, 5]$ and $b \in [-1, 3]$ by a step of 0.1.

Remark 2. We can clearly see that the feasibility of the proposed LMIs are more expanded than the one obtained in (Ichalal et al (2010); El Youssfi et al (2019)). This result shows that the proposed strategy provides a larger stabilization regions and a much feasible set of solutions than the method in (Ichalal et al (2010); El Youssfi et al (2019)).

Remark 3. The attenuation level of the L2-gain obtained by the proposed method in this result is $0.3 < \gamma < 0.5$, it is less than the one obtained in the reference methods which have a value greater than 0.9. Therefore, the proposed method provides a better performance of disturbance rejection.

4.5 Computer process interfacing

The proposed strategy can be implemented in real-time using Matlab/Simulink. After solving the LMIs of theorem 1 and 2 by LMIs toolbox in Matlab, the FTC gains can be obtained and the controller application procedure is shown in Fig. 13.

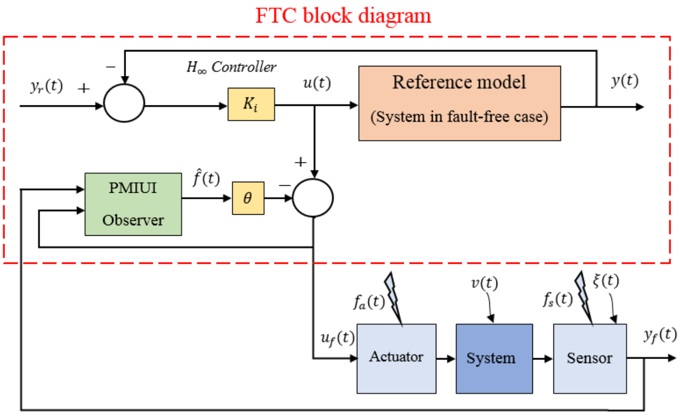

Fig. 13. Sheme of FTC application procedure

The FTC program is written by a Matlab Script on a digital computer, this latter can be connected to the CSTR system using an interface which is a collection of hardware and software modules. The following figure shows the parts of a real time implementation.

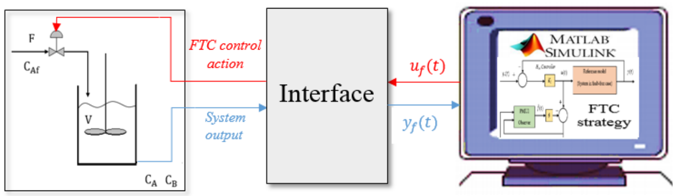

Fig. 14. Parts of a real time implementation

As shown in Fig.13, the proposed FTC is performed without a fault detection and identification (FDI) because it is able to estimate directely the fault value, which reduce the time of computing and avoid the false alarms, non-detection and delay due to FDI. Moreover, this approach consider several types of faults and disturbances in the system in addition to normal modes of operation. In real-time, this strategy can provide a robust performance to engineering systems, especially embedded control systems which requires good reliability and high security such that they can operate in safety-critical applications.

5. CONCLUSION

The main contribution of this work is the design of a trajectory tracking fault tolerant control for nonlinear systems described by TS models. The considered systems are affected by simultaneous actuator and sensor faults and have unmeasurable premise variables and depending on the faults occur. The first time, a mathematical transformation is introduced to construct an augmented system in which all the faults affecting the initial system appear as actuator faults. Secondly, a nominal control based on H_∞ strategy is developed for the augmented system in fault-free case in order to ensure trajectory tracking. The objective is to minimize the deviation between the healthy reference system and the eventually faulty system, and to track a reference trajectory. This scheme requires the knowledge of the system states and the simultaneous acting actuator and sensor faults. These signals are estimated from a PIUI observer in the cases of constant and slowly time-varying faults, or a PMIUI observer in the case of time varying faults. The stability conditions are studied with Lyapunov theory and L2 optimization, and formulated in terms of LMIs. The performances of the proposed method are illustrated through the application to a CSTR. The results clearly show that the proposed FTC guarantee a good tracking performances despite on the occurrence of actuator and sensor faults simultaneously, and attenuate the unknown disturbance and the measurement noise.

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