A PIECEWISE CONTINUOUS APPROACH FOR ON-LINE IDENTIFICATION OF CONTINUOUS-TIME PLANTS

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Abstract: *This paper proposes an original method for recursive on-line plant identification. The method identifies directly a linear continuous time state space model of the plant from full state measurement through a black box approach. The basic principle consists in using a reference model, called clone, which runs in parallel so as to reproduce the behaviour of the plant. To do so, the state error between the plant and its clone is minimised by means of an adaptive algorithm that tunes the clone's varying parameters iteratively. These parameters represent the estimators of those of the plant. The originality lies in the particular structure of the clone. The latter is in fact a Piecewise Continuous System (PCS) characterised by exogenous switchings of its state. The method is appropriate for real time applications, where real plants are controlled by digital calculators. Results from a computer simulation and a real time application are given.*

Keywords: *recursive parameter estimation, on-line identification, adaptive systems, state-space methods, piecewise functioning systems*

1. INTRODUCTION

The problem of system identification has been given much attention for its necessity in plant control [17-19]. Literature shows that the identification problem is often brought to a model's parameter estimation, using a transfer function approach [14-16]. Classically, methods such as least squares parameter estimation [10, 13] have conducted to well known recursive identifiers which allow on-line identification of a model, useful in adaptive control [5].

However, state-space representation of systems being widely used in process control, it is interesting to identify a state-space model of a given plant. The present paper deals with such an identification method, adapted for real time applications. The method realises a *direct* on-line identification of a continuous-time state-space model

(CTSM) by using real-time bounded values of the input $u(t) \in U^r$ of the plant and its state $x(t) \in \Sigma^n$. It allows both open loop identification and adaptive control when combined with a suitable compatible controller.

The present identification strategy is close to the methods in [1, 7, 11], where an adaptive algorithm identifies *continuously* the plant's model. The idea is to establish a reference model, called *clone* in our approach, whose goal is to reproduce the behaviour of the plant in real time. This procedure, called *cloning*, utilises an adaptive procedure that tunes the parameters of the clone so that, at the end, the state error between the plant and its clone is reduced to nil. It is shown that at the same time, the parameters of the clone are bound to be the best estimates of those of the plant's CTSM.

The originality of the present method lies in the choice of a particular structure for the *clone*. The latter is actually a Piecewise Continuous System (PCS) These systems, defined in [4, 9], possess hybrid properties and are comparable to Singularly Impulsive Dynamical Systems (SID) [6, 8]. PCS represent the class of hybrid systems where algebraic equations represent constraints that differential and difference equations need to satisfy. They are characterised by switchings of their state in response to controlled impulses. Particularly, the state of the PCS *clone* undergoes switching at discrete time instants defined by $S = \{t_k, k = 0,1,2...\}$. This switching allows *synchronisation* of the *clone*'s state with that of the plant at each $t_k \in S$, so that the state error is nil at each switching instant. Based on this property, the adaptive algorithm modifies recursively the *clone*'s parameters which are kept constant between switching instants. In contrast with usual methods, the identification procedure is *not* continuous due to the PCS *clone*. However, the synchronisation property leads to fast identification of the plant's CTSM.

The theory proposed in the present paper can be classified as a *black box* approach. The aim is to obtain a representative model of the plant without any *a priori* knowledge of its internal physical equations. In our case, only the order *n* of the state representation is used as a guide to the structure of the *clone*. In addition, the proposed method is directly transposable on real time configurations enabling on-line identification of real plants.

Due to the particular nature of the PCS *clone*, which switches at specific time instants $S = \{t_k, k = 0,1,2...\}$, the functions introduced in this paper can be discontinuous at those time instants. So, for a given time function $f(t)$, we define $f(t_k^-) = f_k^-$ and $f(t_k^+) = f_k^+$. In general, when considering discontinuous function f at switching instants, the notation f_k is often used as a simplified one for f_k^+. Naturally, in case of continuity, $f_k^- = f_k^+ = f_k$. In this context, the time interval $]t_k, t_{k+1}]$, which is the k^{th} piece of time, is referred to as the *k*-piece.

2. PRINCIPLE OF PLANT CLONING

2.1 Mathematical model

Basically, we associate a mathematical n^{th} order continuous time linear state model to the plant that is to be identified:

$$x'(t) = a.x(t) + b.u(t) \tag{1}$$

with $x(t) \in \Sigma^n$ being the state and $u(t) \in U^r$ the plant's input and with matrices $a \in \Re^{n \times n}$ and $b \in \Re^{n \times r}$ representing the model's parameters to be identified.

Assumption 1. *The model is chosen such that the state $x(t)$ is available.*

Assumption 2. *The state $x(t)$ is supposed to be bounded either because of natural stability of the plant or due to a compatible PCS controller* [4, 9].

Let the matrix $\varphi \in \Re^{(n+r) \times n}$ represent the plant's parameter set such that $\varphi^T = [a \quad b]$.

2.2 Identification by cloning

Fig. 1 reflects the principle of the present identification method. Two main operating blocks are essential to realise identification:

- a *clone* that runs in parallel with the plant,

- an adaptive algorithm block that tunes recursively the *clone*'s parameters.

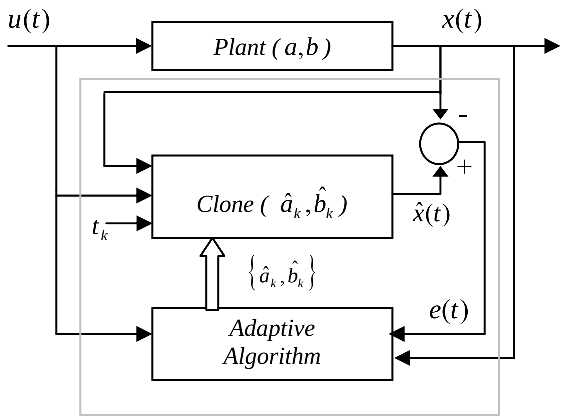

Fig. 1. Identification structure

The clone, whose aim is to reproduce the behaviour of the plant, is of the same order as the latter's model and its state is noted $\hat{x}(t) \in \Sigma^n$. It intakes the same input $u(t)$ as the plant, and runs in parallel to the plant. It is defined by state matrices $\hat{a}_k \in \Re^{n\times n}$ and $\hat{b}_k \in \Re^{n\times r}$, which are constant upon a *k*-piece, but tuned recursively at each t_k by the adaptive block. Let matrix $\hat{\varphi}_k \in \Re^{(n+r)\times n}$ represent the clone's parameter set such that $\hat{\varphi}_k^T = \begin{bmatrix} \hat{a}_k & \hat{b}_k \end{bmatrix}$.

The clone's hybrid nature is such that $\hat{x}(t)$ presents a piecewise continuous time evolution, being continuous upon each k-piece and switched to the plant's state $x(t_k)$ at each t_k.

Denote by $e(t) = \hat{x}(t) - x(t)$ the state error and by $\Phi_k \in \Re^{(n+r)\times n}$ the parametric error matrix such that $\Phi_k^T = \hat{\varphi}_k^T - \varphi^T$.

According to the present approach, cloning is represented by $\hat{x}(t) \xrightarrow[k\to\infty]{} x(t)$ or $e(t) \xrightarrow[k\to\infty]{} 0$ and identification by $\hat{\varphi}_k \xrightarrow[k\to\infty]{} \varphi$.

3. THE CLONE

As a PCS, the clone is a finite-dimensional, strictly causal linear system, which, according to the taxonomy of hybrid systems, admits *autonomous switching* [3] of its state in response to *controlled impulses* [2]. In this way, a PCS possesses two input spaces (continuous/discrete) and refers to two different time scales (continuous/discrete).

The continuous time scale $\Im$ corresponds to the continuous dynamics of the PCS according to a continuous input, whereas the discrete time scale $S \subset \Im$ defines the switching instants at which the state is *forced* by means of a discrete input.

According to the PCS formalism, the clone is defined by the following equation set:

$$\hat{x}'(t) = \hat{a}_k.\hat{x}(t) + \hat{\varphi}_k^T.u_c(t) \qquad \forall t \notin S \qquad (2a)$$

$$\hat{x}(t_k^+) = x(t_k) \qquad \forall t_k \in S \qquad (2b)$$

Equation (2a) represents the continuous dynamics of the clone with a continuous input $u_c(t) = \begin{bmatrix} -e^T(t) & u^T(t) \end{bmatrix}^T$ referring to $\Im$, with $u(t)$ and $e(t)$ as defined previously.

Equation (2b) corresponds to the switching of the clone's state with a discrete input referring to S. The switched value of $\hat{x}(t)$ is determined by sampled values of the plant's state $x(t_k)$

Property 1. *Equation (2b) describes the resetting property of the clone which allows synchronisation between the plant and its clone at each switching instants. Thus,* $e(t_k^+) = 0$ *as shown in Fig. 2.*

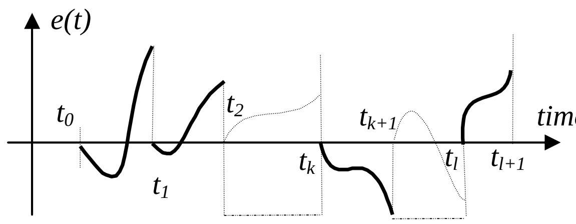

Fig. 2. Resetting property

Replacing $e(t)$ by its expression, the clone's equations can be written as:

$$\hat{x}'(t) = \hat{a}_k.x(t) + \hat{b}_k.u(t) \qquad \forall t \notin S \qquad (3a)$$

$$\hat{x}(t_k^+) = x(t_k) \qquad \forall t_k \in S \qquad (3b)$$

This equation set shows that the plant's state interferes in the evolution of the clone's state:

- continuously upon each k-piece (3a),
- by synchronisation at each $t_k \in S$ (3b).

Fig. 3 illustrates the clone's architecture corresponding to its functioning equations (3). Vertical arrows marked with t_k next to each parameter block indicate that the estimators $\hat{a}_k$ and $\hat{b}_k$ change at each switching instant during the cloning procedure.

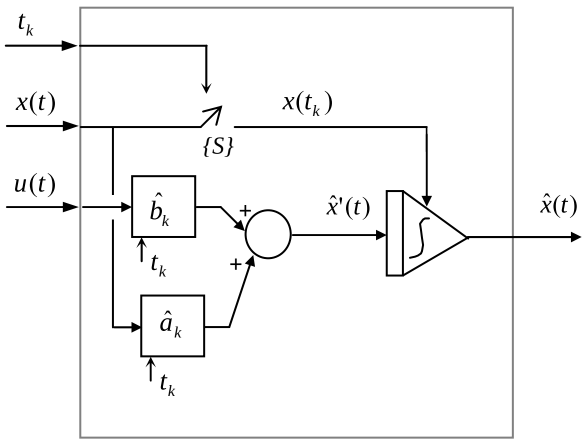

Fig. 3. Architecture of the PCS clone

4. MATHEMATICAL APPROACH

Note that the state of the plant and that of its clone are continuous, bounded and derivable between switching instants, which implies that the state error $e(t)$ is also continuous, bounded and derivable between switching instants.

Theorem 1. *If $u(t)$ shows discontinuities inside a k-piece, cloning and identification are achievable iff $e(t)$ is rendered identically nil $\forall t \to \infty$.*

Proof. *Concerning cloning, it is obvious that if $e(t) \equiv 0 \ \forall t \to \infty$, the behaviour of the plant is totally reproduced by its clone. As for the identification procedure, we proceed as follows to show how it is guaranteed from the stated condition.*

Equations (1) and (3a) yield:

$$e'(t) = [\hat{a}_k - a].x(t) + [\hat{b}_k - b].u(t) \ \forall t \notin S \quad (4)$$

Ensuring that $e(t) \equiv 0 \ \forall t \to \infty$ implies $e'(t) \equiv 0 \ \forall t \to \infty$, thus giving:

$$[\hat{a}_k - a].x(t) + [\hat{b}_k - b].u(t) \equiv 0 \ \forall t \notin S \mid k \to \infty \ (5)$$

The adaptive algorithm tunes parameters $\hat{a}_k$ and $\hat{b}_k$ so as to satisfy (5) *for all pair of values of $u(t)$ and $x(t)$ and such that $\hat{a}_k$ and $\hat{b}_k$ are constant upon each k-piece. Given that $u(t)$ is chosen to be discontinuous upon a k-piece and that $x(t)$ is necessarily continuous according to* (1)*, the only possible constant values of $\hat{a}_k$ and $\hat{b}_k$ that satisfy* (5) *are bound to be those equal to the plant's parameters.*

Hence, in the conditions stated in theorem 1, $e(t) \equiv 0 \ \Leftrightarrow \left\{\hat{a}_k \equiv a \,; \hat{b}_k \equiv b\right\}$*, thus realising the plant's identification.*

Remark 1. *The use of a "rich" input signal, i.e. one showing discontinuities inside each k-piece, is often the case in classical identification methods* [1]. *This can be compared, by analogy, to a driver testing his car on hilly and curved roads rather than on a flat straight one in order to determine its characteristics.*

4.1 Reduction to zero of the state error

Property 1 one already ensures $e_k = 0$. The aim is thus to render the state error nil *inside* a k-piece. To do so, we make use of the adaptive law that tunes the clone's parameters till $e(t)$ becomes identically nil as $t \to \infty$.

Strategy. *In order to bring the state error to zero, we consider cancelling $e(t)$ component by component. In this way, we introduce for each k-piece the scalar quantity*

$$\varepsilon_k^i = \int_{t_k}^{t_{k+1}} \left| e^i(\theta) \right| .d\theta \quad \forall i = 1,...,n,$$

with $e^i(t)$ being the i^{th} component of $e(t)$.

Property 2. *If the integral of a positive and bounded time function $f(t)$ upon a time interval is nil, then this function is nil upon that interval of time.*

In our case, $f = \left| e^i(t) \right|$. So, if we can ensure that $\varepsilon_k^i = 0 \ (k \to \infty) \forall i$, then $e_k^i = 0 \ \forall i, \ \forall t \notin S$ according to property 2. Moreover, $e(t)$ being nil at each switching instant, we have:

$$\varepsilon_k^i = 0 \ \ (k \to \infty) \ \ \forall i \Leftrightarrow e^i(t) \equiv 0 \ \ \forall i, \ \forall t \qquad (6)$$

4.2 Adaptive procedure

According to the cloning principle, we define an adaptive law, similar to that used in [12], that allows cancellation of the aforedefined ε_k^i term, in view of realising *simultaneously* cloning (and identification. In this subsection, we first express ε_k^i in a form that suits the adaptive algorithm before defining the latter.

The i^{th} columns of φ, $\hat{\varphi}_k$ and Φ_k are denoted respectively by φ^i, $\hat{\varphi}_k^i$ and Φ_k^i. Thus, ${\Phi_k}^i = \hat{\varphi}_k^i - \varphi^i$.

4.2.1 Expressing ε_k^i

Let the observation vector $W(t) \in \Re^{(n+r)}$ be such that $W^T(t) = [x^T(t), u^T(t)]$. According to the definitions of φ, $\hat{\varphi}_k$ and Φ_k given in section 2, we can express (4) as such:

$$e'(t) = \Phi_k^T . W(t), \ \forall t \in]t_k, \ t_{k+1}] \qquad (7)$$

Integrating (7) upon a k-piece and taking into account that $e(t_k^+) = 0$ and that Φ_k^T is constant upon a k-piece, $e(t)$ can be expressed as:

$$e(t) = \Phi_k^T . \int_{t_k}^{t} W(\theta).d\theta, \ \forall t \in]t_k, \ t_{k+1}] \qquad (8)$$

Hence, $e(t)$ is of the form:

$$e(t) = \Phi_k^T . \tilde{W}(t), \ \ \forall t \in]t_k, \ t_{k+1}], \qquad (9)$$

with $\tilde{W}(t) = \int_{t_k}^{t} W(\theta).d\theta$, $\forall t \in]t_k, \ t_{k+1}]$ (10)

To evaluate ε_k^i, let's first express $e^i(t)$:

$$e^i(t) = \tilde{W}^T(t).\Phi_k^i, \ \ \forall t \in]t_k, \ t_{k+1}] \qquad (11)$$

Inside the k-piece, ε_k^i can be written as:

$$\varepsilon_k^i = \int_{t_k}^{t_{k+1}} \left\{ sign[e^i(\theta)] \right\} . e^i(\theta).d\theta \ \ \forall i = 1, \ldots, n \qquad (12)$$

with $sign(f) = \begin{cases} 1 & \text{if } f \geq 0 \\ -1 & \text{elsewise} \end{cases}$

Replacing (11) in (12) leads to ($\forall i = 1, \ldots, n$):

$$\varepsilon_k^i = \left\{ \int_{t_k}^{t_{k+1}} \left\{ sign[e^i(\theta)] \right\} . \tilde{W}^T(\theta).d\theta \right\} . \Phi_k^i \qquad (13)$$

Thus, letting $V_k^i = \int_{t_k}^{t_{k+1}} \left\{ sign[e^i(\theta)] \right\} . \tilde{W}^T(\theta).d\theta$ $\forall i = 1, \ldots, n$ allows us to express ε_k^i as such:

$$\varepsilon_k^i = {V_k^i}^T . {\Phi_k}^i \qquad (14)$$

4.2.2 The adaptive recurrence

With the new expression of ε_k^i, we define the adaptive law for each component as follows:

$$\hat{\varphi}_{k+1}^i = \hat{\varphi}_k^i - \frac{g_k^i . V_k^i . \varepsilon_k^i}{1 + g_k^i . V_k^{iT} . V_k^i} \ \text{ for } i = 1, \ldots, n, \qquad (15)$$

where g_k^i is a real, positive gain, which influences the convergence of (15).

Expressions (14) and (15) constitute the iterative procedure used in practice to achieve the cloning process. It is shown thereafter that with an equation in the form of (14), the recurrence in (15) allows:

- $\varepsilon_k^i \xrightarrow[k \to \infty]{} 0$ and
- $\hat{\varphi}_k^i \xrightarrow[k \to \infty]{} \varphi^i$,

ensuring respectively cloning and identification.

Remark 2. *Considering the reduction to zero of vector $e(t)$ component by component brings us to construct n separate recurrence blocks that tune the parameters of n-column matrix $\hat{\varphi}_k$ in the case of black box identification. Naturally, if some columns of φ are known (grey box identification), the cloning structure can be simplified by using fewer recurrence blocks. Moreover, the use of separate recurrence blocks allows parallel computing that helps in accelerating cloning procedure.*

4.2.3. Proof of convergence

The aim is to show that $\varepsilon_k^i \xrightarrow[k\to\infty]{} 0$. To do so, we need to consider the sequence $\Phi_k^{i\,T}.\Phi_k^i$, whose terms are necessarily positive and scalar ($\Phi_k^{i\,T}.\Phi_k^i$ is the Euclidian norm of Φ_k^i).

Let's replace ε_k^i by its expression from (14):

$$\hat{\varphi}_{k+1}^i = \hat{\varphi}_k^i - \frac{g_k^i.V_k^i.V_k^{iT}}{1+g_k^i.V_k^{iT}.V_k^i}\Phi_k^i \quad \text{for } i=1,\ldots,n \qquad (16)$$

Assuming that the plant is invariant (at least during one cloning procedure), we can subtract φ^i from both sides of (16). Thus:

$$\Phi_{k+1}^i = [I - S_k^i].\Phi_k^i, \qquad (17)$$

$$\text{with } S_k^i = \frac{g_k^i.V_k^i.V_k^{i\,T}}{1+g_k^i.V_k^{i\,T}.V_k^i} \quad \text{for } i=1,\ldots,n \qquad (18)$$

Note that $S_k^{i\,T} = S_k^i$. Moreover:

$$S_k^{i\,T}.S_k^i = \mu_k^i.S_k^i, \qquad (19)$$

$$\text{where } \mu_k^i = \frac{g_k^i.V_k^{i\,T}.V_k^i}{1+g_k^i.V_k^{i\,T}.V_k^i}, \text{ with } 0 \le \mu_k^i < 1 \qquad (20)$$

Let's analyse the evolution of $\Phi_k^{i\,T}.\Phi_k^i$:

$$\Phi_{k+1}^{i\,T}.\Phi_{k+1}^i = \Phi_k^{i\,T}.[I-S_k^i]^T.[I-S_k^i].\Phi_{k+1}^i$$

Thus, as from (18) and (19):

$$\Phi_{k+1}^{i\,T}.\Phi_{k+1}^i$$

$$= \Phi_k^{i\,T}.\Phi_k^i - (2-\mu_k^i).\Phi_k^{i\,T}\frac{g_k^i.V_k^i.V_k^{i\,T}}{1+g_k^i.V_k^{i\,T}.V_k^i}\Phi_k^i \qquad (21)$$

Knowing that $\Phi_k^{i\,T}.V_k^i.V_k^{i\,T}.\Phi_k^i = \varepsilon_k^{i\,2}$ from (14):

$$\Phi_{k+1}^{i\,T}.\Phi_{k+1}^i = \Phi_k^{i\,T}.\Phi_k^i - g_k^i.(2-\mu_k^i).(1-\mu_k^i).\varepsilon_k^{i\,2}$$

Hence, the difference between two successive terms is of the form:

$$\Phi_{k+1}^{i\,T}.\Phi_{k+1}^i - \Phi_k^{i\,T}.\Phi_k^i = -\gamma_k^i.\varepsilon_k^{i\,2}, \qquad (22)$$

with $\gamma_k^i = g_k^i.(2-\mu_k^i).(1-\mu_k^i)$.

Since $g_k^i > 0$ by definition and $0 \le \mu_k^i < 1$ (from (20)), it is obvious that:

$$\gamma_k^i = g_k^i.(2-\mu_k^i).(1-\mu_k^i) > 0 \qquad (23)$$

Corollary. *By the above analysis, we have:*

$$\Phi_{k+1}^{i\,T}.\Phi_{k+1}^i - \Phi_k^{i\,T}.\Phi_k^i \le 0 \qquad (24)$$

$$\Phi_k^{i\,T}.\Phi_k^i \xrightarrow[k\to\infty]{} cst \qquad (25)$$

Therefore, given (24) *and that the* $\Phi_k^{i\,T}.\Phi_k^i$ *terms are positive scalars, we can state that* $\Phi_k^{i\,T}.\Phi_k^i$ *is a positive sequence decreasing asymptotically to zero or to a positive limit as* k *tends to infinity.*

When this sequence reaches its limit (end of the cloning procedure), we have:

$$\Phi_{k+1}^{i\,T}.\Phi_{k+1}^i - \Phi_k^{i\,T}.\Phi_k^i \xrightarrow[k\to\infty]{} 0$$

Thus, according to (22):

$$-\gamma_k^i.\varepsilon_k^{i\,2} \xrightarrow[k\to\infty]{} 0$$

Since $\gamma_k^i \ne 0$ (23), *we can claim that:*

$$\varepsilon_k^i \xrightarrow[k\to\infty]{} 0$$

Hence, according to (6), *the state error becomes identically nil. Moreover, according to theorem 1, identification is consequently realised.*

5. ILLUSTRATING EXAMPLES

Computer aided simulations and real-time applications helped in testing the realisation and efficiency of the cloning method. Based on the architecture of Fig. 1 and the method's equations, a Functional Cloning Structure (FCS) was designed in Simulink®. Dealing with bounded signals, the PCS approach allows direct implementation using classical blocks (integrators with reset mode, zero order holders, ...).

Some illustrating examples that extend the theoretical assumptions have been selected in order to take into account practical cases. Results from simulation tests of the FCS are proposed as a weight of comparison with existing methods. Moreover, to illustrate the real-time performances of our method, we show the results obtained while testing the method upon a real-time controlled bench using Matlab®RTW-Simulink-dSpace® technology as the interface between the controlling computer and the real physical system.

Remark 3. *In all the examples, the adaptive gain* g_k^i *has been fixed to a high constant value* $\forall i$ *to ensure fast convergence of the adaptive algorithm:* $g_k^i = g^i = 1\times 10^{20}$ $\forall i$. *Precautions should be taken such that the digital calculator does not overflow while increasing the gain for faster cloning.*

Remark 4. *During experimentation, the switching of the PCS clone was realised at constant period T, such that* $S = \{kT, k = 0,1,2...\}$.

Remark 5. *According to theorem 1, a non-zero scalar input* $u(t)$ *showing discontinuities inside* $]kT,\ (k+1)T]$ *was fed to the plant in each experiment. It is obvious that if* $u(t)$ *remains constant on a k-piece, there may exist solutions for* $\hat{a}_k$ *and* $\hat{b}_k$ *other than* $\hat{a}_k \equiv a$ *and* $\hat{b}_k \equiv b$ *satisfying (5). In practice, a "poor" input signal leads to convergence of estimators to wrong values. On the other hand, the "richer" the signal, the faster the cloning is.*

5.1 Cloning of simulated plants

The FCS has been tested in simulation and it has shown efficiency in realising on-line *black box* identification of linear time invariant stable plants of any order even with far-fetched initial conditions. In this section, we present some practical cases that extend the theoretical assumptions. In most cases, *grey box* identification was possible, thus simplifying the cloning structure.

5.1.1 Second-order time invariant plant

As a first example, we identify a plant whose model (1) is defined by:

$$a = \begin{bmatrix} -3 & 1 \\ 5 & 4 \end{bmatrix} \text{ and } b = \begin{bmatrix} 30 \\ 90 \end{bmatrix}$$

In this case, *black box* identification is necessary to estimate all the components of φ.

During this simulation, the initial condition of the FCS was

$$\hat{\varphi}_0^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and the switching period of the clone was $T = 100$ ms. Note that smaller values of T accelerate the identification procedure, the limitation being related to minimum calculation step size of the simulation software.

Fig. 4 shows how the parameter estimators converge rapidly (in less than 5s) to the actual values of the plant's corresponding parameters. It can be observed that the components of the state error tend to nil.

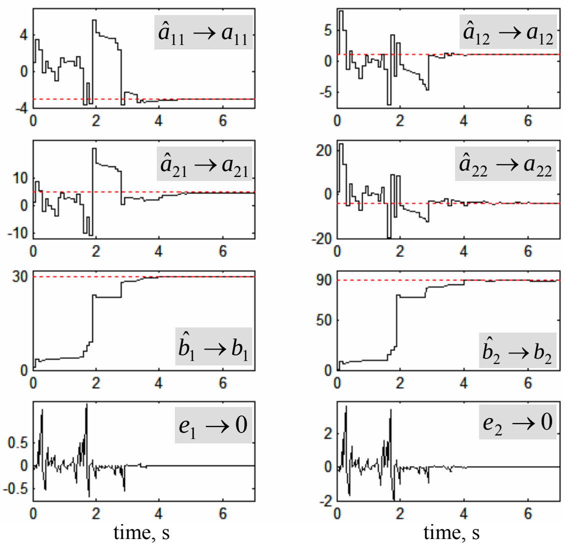

Fig. 4. Identification of a second-order plant

5.1.2 Third-order time invariant plant

In this example, we identify a plant that is considered in [7]. The model (1), which is to be identified, is defined by:

$$a = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4.5 & -10.5 & -3.5 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

The above values were used to simulate the plant in Simulink®, and white noise (proportional to the state) was added to its state before being injected into the identifier to test the robustness of the cloning method against disturbances.

In this case, not all of the components of φ need identification, since we had an *a priori* knowledge of the plant's internal structure. We thus performed *grey box* identification, needing only one adaptive block that tunes parameters of the third line of $\hat{\varphi}_k^T$.

During this simulation, the initial condition of the FCS was

$$\hat{\varphi}_0^{3T} = [1 \quad 1 \quad 1 \quad 1]$$

and the switching period of the clone was $T = 100$ ms.

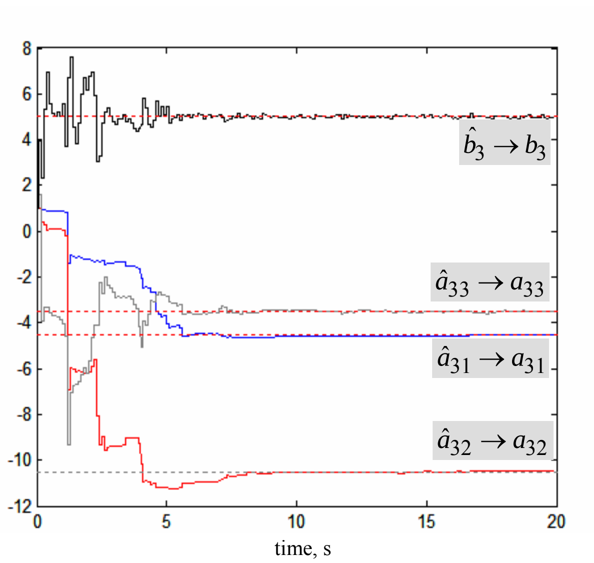

Fig. 5. Identification of a third-order plant

Fig. 5 shows how the parameter estimators converge to the actual values of the plant's corresponding parameters. The results show that our method allows identification in 10s time, while the technique in [7] necessitates more than 150s. In fact, the two methods are rather similar, except that in the present method, the synchronisation property of the clone helps in accelerating identification.

5.1.2 Time Varying Plants (discontinuous case)

As mentioned formerly, the theory requires that plants to be identified need to be invariant at least during the convergence of the algorithm.

However, if the parameters are piecewisely constant, undergoing discontinuous changes, cloning is possible if each constant time piece is large enough for the algorithm to converge to the corresponding constant parameter values. Then, after each discontinuous change, we can consider that a new cloning operation begins with initial conditions being the last identified values.

Let's consider the identification of a second order Time Varying Plant (TVP) in which one parameter of the CTSM shows a discontinuous change: $a_{11}(t) = -3 + f(t)$, $f(t)$ being a square signal (magnitude = 3 and period = 0.5s). Supposing that the five other parameters are known constants, *grey box* identification is possible with only one adaptive block.

For this example, the initial condition was: $(\hat{a}_{11})_0 = -6{,}5$ and the clone's switching period was chosen to be $T = 5$ ms for more "reactive" cloning.

Fig. 6 shows how the estimator follows the discontinuous changes of the plant's parameter. Only a short transition is observed (100ms) after each discontinuity. It can be considered that a new cloning procedure begins at each discontinuity.

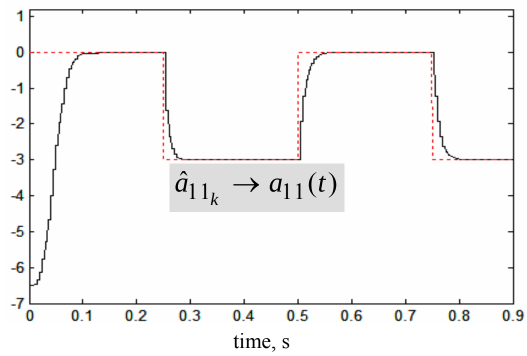

Fig. 6. Identification of a parameter varying discontinuously

5.1.3 Time Varying Plants (continuous case)

In the present example, we show that even continuously varying parameters can be identified. This can be explained by the fact that once the current value of φ is reached by its estimator $\hat{\varphi}_k$ after a transition time, then cloning goes on successively with initial conditions being very close to the current value of φ. However, the dynamics of those changes must be relatively slow so that parameters can be considered as nearly constant inside a k-piece.

The model which is to be identified here is defined by $x'(t) = a(t).x(t) + b.u(t)$ with:

$$a(t) = \begin{bmatrix} -3+\sin(2\pi.t) & 1 \\ 5 & -4+2\sin(4\pi.t) \end{bmatrix},$$

$$b = \begin{bmatrix} 30 \\ 90 \end{bmatrix}.$$

In this case, we applied a *grey box* identification, necessitating two adaptive blocks that identify the time varying parameters a_{11} and a_{22}. The initial conditions of their estimators are $(\hat{a}_{11})_0 = (\hat{a}_{22})_0 = 1$ and the switching period of the clone was $T = 5$ ms.

The results represented by Fig. 7 show how the estimators $(\hat{a}_{11})_k$ and $(\hat{a}_{22})_k$ follow the variation of the model's actual parameters a_{11} and a_{22}. We notice the high performance of the algorithm leading to a short transition phase in this case where $T = 5$ ms.

However, we observed that the use of cloning in such a case of time varying parameters leads to loss of the method's *black box* faculty since only one varying parameter can be identified per line of φ^T for best results. The other parameters' values must be given and fixed correspondingly in $\hat{\varphi}_k$ so that the cloning algorithm doesn't identify them as varying terms which satisfy $\hat{\varphi}_k^i = \varphi^i$.

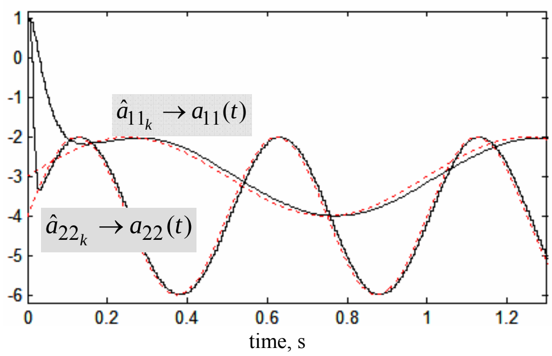

Fig. 7. Identification of a parameter varying continuously

5.2 Cloning of a real plant

The method has been used for online identification of an electrical motor used to move a cart along a line segment. As shown in Fig. 8, the motor is controlled by a computer via an amplifier.

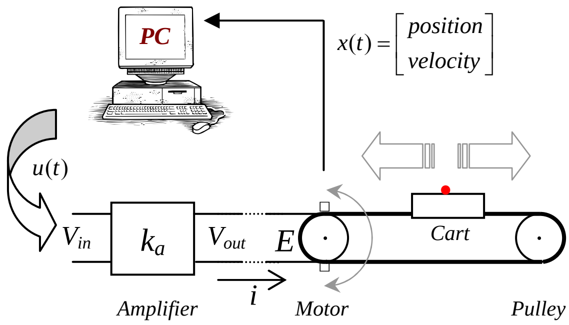

Fig. 8. Real time setup

On the bench, a notched belt is used to allow linear motion of the cart via rotation of the motor's axis. Actually, we have already used common off-line identification techniques (Bode, harmonic analysis) and found out that the system behaves more or less as a linear, second order system with respect to position. The goal being to perform cloning of the amplifier-motor-cart set, we associate to the latter a mathematical model as in (1) defined by:

$$x(t) = \begin{bmatrix} position \\ velocity \end{bmatrix},\ a = \begin{bmatrix} 0 & 1 \\ 0 & -1/\tau \end{bmatrix},\ b = \begin{bmatrix} 0 \\ k/\tau \end{bmatrix},$$

where τ is the time constant and k the gain of the amplifier-motor-cart set.

In this case, not all of the components of φ need identification (here, $\varphi^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1/\tau & k/\tau \end{bmatrix}$), since we had an *a priori* knowledge of the plant's internal structure. We thus performed a *grey box* identification, needing only one adaptive block that tunes parameters of the second line of $\hat{\varphi}_k^T$.

During the real time experiment, the initial condition of the FCS was $\hat{\varphi}_0^{2^T} = [1 \quad 1 \quad 1]$ and the switching period of the clone was $T = 100$ ms.

Fig. 9 illustrates the evolution with time of the parameter estimators, $(\hat{a}_{22})_k$ and $(\hat{b}_2)_k$, represented by a_{22}/b_2 estimators which tend towards $-1/\tau$ and k/τ respectively.

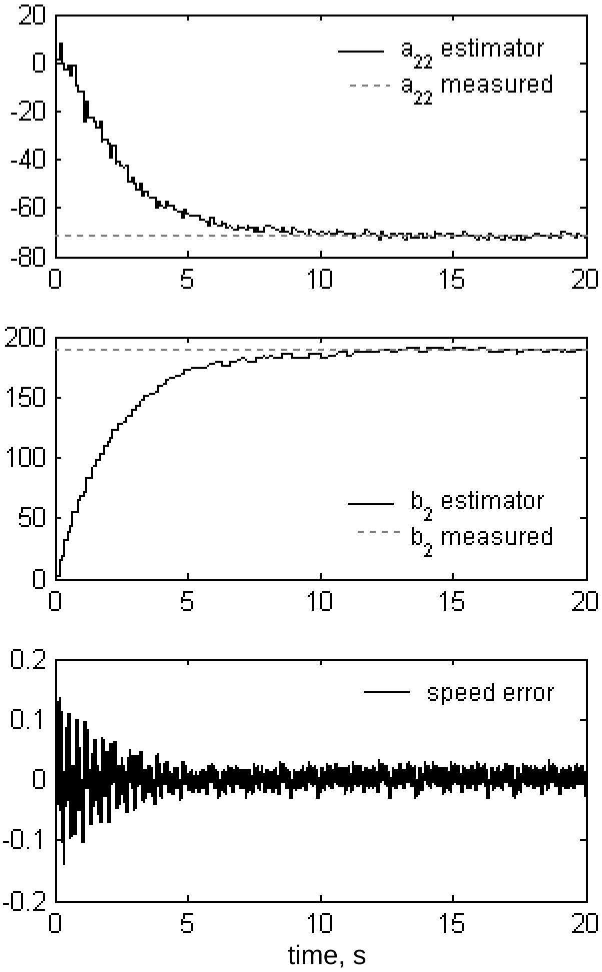

Fig. 9. Identification of a real motor

The values found after cloning correspond to the results obtained by off-line harmonic analysis identification denoted by a_{22}/b_2 measured on the same figure.

In addition, the decreasing speed error is displayed to represent the cloning performance. The noisy signals are due to the poor quality of the sensors – this shows the robustness of our method in the presence of disturbances.

6. CONCLUSION AND PERSPECTIVES

This article deals with a new approach of plant identification based on a PCS model reference. The method, which can easily be implemented on digital calculators, allows fast on-line parameter estimation of a plant's linear CTSM. It shows a high converging power due to the original combination of the particular PCS clone and the proposed adaptive procedure.

Experiments have shown that the identification method is robust against white noise disturbances. This is explained by the fact that integration operations on the observation vector on each k-piece bring a filtering effect. Moreover, identification of time varying parameters is possible, given that the variations are of a slower dynamics than that of the cloning procedure.

Besides, we found out that unstable plants could be identified: they simply need to be controlled during the cloning procedure. However, in order to supply the particular input signal $u(t)$ required by theorem 1, we used a PCS controller (PCC) [9] whose parameters are the estimators $\hat{a}_k$ and $\hat{b}_k$ delivered by the identifier. This idea is illustrated by Fig. 10. Note that the combination of the PCC and the FCS is possible due to the two following characteristics:

- the PCC uses directly state space matrices which is delivered by the FCS,
- the PCC feeds the plant with an input command which is piecewisely continuous, thus satisfying theorem 1.

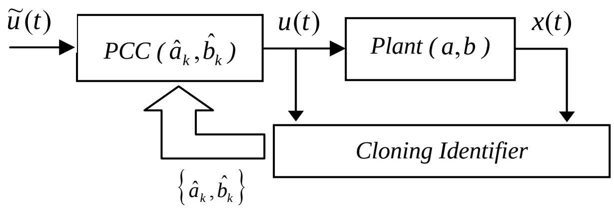

Fig. 10. PCS controller and identifier

In addition, coupling the identifier with a PCS control unit (Fig. 10) extends the use of the cloning method to an application in adaptive control for time varying plants. Furthermore, non linear plants can be considered in some conditions where it is possible to provide a linear approximation. Present studies are undertaken in this context.

It is obvious that identification of plants presenting a known input delay is possible. The clone's input is simply delayed accordingly in this case.

Nevertheless, the method being limited to an available state, present development is considered so as to make use of an input-output strategy.

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