Fault Estimation and Robust Fault-tolerant Control for Singular Markov Switching Systems with Mixed Time-Delays and UAV Applications

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Abstract: In this paper, the robust fault-tolerant control for a class of uncertain Markov switching singular complex systems with mixed time-delays is studied. It is assumed that there are multiple fault conditions such as sensor faults, actuator faults, and external disturbances. Under sensor faults and actuator faults, the estimated states can be achieved by building the adaptive observer, which can detect system states in real time. In order to enable Markov systems to be well controlled in the event of various faults, a robust fault-tolerant controller based on control law compensation and fault signal reconstruction is designed to ensure the stability of Markov switching singular complex systems. Finally, the feasibility of the method designed is proved by a numerical simulation example, and it is applied to the UAV motion system to verify the effectiveness.

Keywords: Fault Estimation, Robust Fault-tolerant Control, Singular Markov System, Mixed Time-delay, Four-rotor UAV

1. INTRODUCTION

Time-delay often appears in the control system. The mixed time-delay is a combination of discrete time-delay and distributed time-delay, and it is also one of the important reasons that lead to system instability. Singular system, also known as generalized system, or description system, or semi-state system, or differential algebra system, is a natural dynamic system. Compared to linear systems, singular systems describe a larger range of systems. Because singular systems are widely used, they have attracted the attention of some scholars (L. Yulianti et al., 2019; Li Li et al., 2016; Ding C et al., 2013; Ma Y et al., 2017).

The fault-tolerant control is a method that can maintain system performance when the system's sensors, actuators and other components fail (Wenhui Liu et al., 2019; Xue Liu et al., 2018; Libing Wu et al., 2018; Chao Huang et al., 2018; Zheng Wang and Yanpeng Pan, 2017; Imen Haj Brahim et al., 2016). In the actual control system, the sensors and actuators may have sudden failures individually or at the same time during operation, and the system has disturbances from the external environment. In order to improve the reliability and safety of the system, a robust fault-tolerant controller needs to be designed to compensate for the adverse effects caused by the fault (Yan Liu et al., 2016; Libing Wu et al., 2016; Huaming Qian et al., 2015; Lili et al., 2019). The problem of sensor fault control is studied in the literature (Imen Haj Brahim et al., 2016). For actuator failure, an observer-based active fault-tolerant controller is proposed in (Xue Liu et al., 2018). For parameter uncertainty, external disturbances and actuator failures, the adaptive H∞ fault-tolerant control for uncertain switched nonlinear time-delay systems has been studied in (Libing Wu et al., 2018 and Libing Wu et al., 2016).

Markov switching systems are hybrid and random systems that can be used to describe systems affected by random mutations and environmental changes, so it can describe a wide range of practical systems, including aerospace systems, manufacturing systems, and network control systems, etc. (Feifei Chen et al., 2019). In recent years, the research on Markov switching systems has become a hot topic. The main research includes stability and control design in (Rathinasamy Sakthivel et al., 2015; Mohsen Bahreini et al., 2018; Kaiyan Cui et al., 2019; Marcos G et al., 2016; Deyin Yao et al.,2018), robust filtering in (Tohidi H et al., 2017; Tianliang Zhang et al., 2019; Mouquan Shen et al., 2017; Lijie Zhu et al., 2019; Huijiao Wang et al.,2017), state and fault estimation in (Lu Dong et al., 2018); Zhang Y et al., 2016; Chunyan Han et al., 2019; Liwei Li et al., 2016), fault detection and fault tolerance control, etc. in [30-36]. In (Kaiyan Cui et al., 2019), the exponential stability of the mean square of a stochastic Markov jump system with mixed time-varying delays and partially unknown transfer rates is discussed. In (Mouquan Shen et al., 2017; Lijie Zhu et al., 2019), the H∞ filtering problem of unknown transition probability of the system is studied. In (Lu Dong et al., 2018), the problem of fault estimation for Markov jump systems based on adaptive observers is discussed. In (Chunyan Han et al., 2019), the minimum mean square error estimation of linear Markovian jump systems with unknown transition probability, multi-channel mode and observation delay, packet loss, etc. is studied. In (Xiaohang Li et al., 2019), the simultaneous estimation and fault-tolerant control problem of actuator failure and sensor failure of Markov jump system are studied. The sensor fault is extended to a part of the state vector, the original system is transformed into a singular system and an adaptive observer is designed.

At present, the research on singular Markov jump systems with both actuator and sensor failures needs to be improved, and the research on simultaneous estimation of system states, actuator failures and sensor failures is rare. In practical application systems, there are uncertainties in system parameters and random time-delays are common. Therefore, it is of great value to study the robust fault-tolerant control for singular Markov jump systems with mixed time-delays.

This paper focuses on a class of singular Markov switching complex systems with mixed time-delays. The problems of comprehensive fault states estimation and fault-tolerant control in the presence of sensor faults, actuator faults, and external disturbances are studied. By designing adaptive observers, the states are estimated. By constructing a robust fault-tolerant controller, the effects of faults and disturbances are eliminated to ensure the stability and reliability of the system. The feasibility of the method is proved by numerical simulation, and it is also applied to the four-rotor UAV motion system to prove the applicability. Compared with the fault-tolerant control strategies in other references (Xiaohang Li et al., 2017; Shidong Xu et al., 2017; Dunke Lu et al., 2017), the FTC in this paper is active fault-tolerant control with more advantages. The active fault-tolerant control is a strong robust control under the fault conditions. When the system fails, the fault-tolerant controller can be solved immediately according to the type of failures to ensure the stability performance of the system. Moreover, in this paper, for the uncertain state transition probability, the fault-tolerant control strategy is provided for a Markov singular jump system with both actuator and sensor failures, which is more practical.

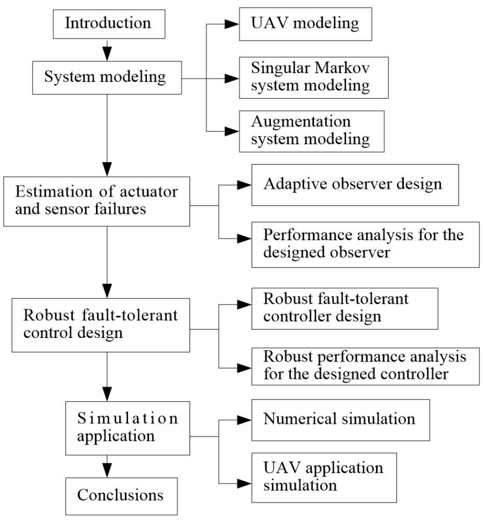

Fig. 1. The process figure.

The process figure is shown in Fig.1. This paper mainly includes 6 sections. In the first section, the current research progress and main conclusions of Markov system and fault-tolerant control are analysed. And the main novelty research work of this article is described. In the second section, system modelling is presented. The four-rotor UAV model and the complex singular Markov switching system model are established. In the third section, in order to estimate the system states under the sensor or actuator failures, a robust adaptive observer design is given. In the fourth section, the active fault-tolerant controller is designed to make the system stable when the sensors and actuators fail. In the fifth section, the research results of this paper are verified by simulation. The feasibility is verified by numerical simulation and application simulation of UAV motion. The sixth section is conclusions.

2. SYSTEM MODELING

2.1 Four-rotor UAV Modeling

By adjusting the speed of the four rotors, the four-rotor UAV can complete the flight attitude conversion. The attitude control mainly includes three parts, namely the roll angle ϕ, the pitch angle θ and the yaw angle ψ. The pitch attitude is achieved by controlling the speed difference between the two rotors in the forward and backward directions of the UAV. The roll angle attitude is determined by controlling the speed of the two rotors in the left and right directions. The yaw angle attitude is achieved by adjusting the speed of the two pairs of rotors.

The carrier coordinate system $oxyz$ is corresponding to the ground coordinate system $OXYZ$, the rotation angle vector is $\Theta = \begin{bmatrix}\phi & \theta & \psi\end{bmatrix}^T$, and the rotation attitude matrix is

$$R(x,\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{1}$$

$$R(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{2}$$

$$R(z,\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The rotation transformation matrix $R(\phi,\theta,\psi)$ is

$$R(\phi,\theta,\psi) = R(x,\phi)R(y,\theta)R(z,\psi) \tag{4}$$

The relationship between the carrier coordinate system and the airflow coordinate system is as follows.

The carrier coordinate system can be rotated by α angle (angle of attack) to obtain a stable coordinate system.

$$S(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{5}$$

Turn the β angle (side slip angle) of the stable coordinate system to get the air current coordinate system

$$S(\beta) = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The transformation matrix $S_{\alpha\beta}$ between the coordinate systems is

$$S_{\alpha\beta}=\begin{bmatrix}\cos\alpha & \sin\beta & \sin\alpha\cos\beta\\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix} \tag{7}$$

According to Euler's kinetic equation and the centroid motion theorem, the dynamic equilibrium equation and moment equilibrium equation for the UAV in the carrier coordinate system can be obtained (Zhao Xingcheng, 2019).

$$F_x=m\left(\frac{du}{dt}+wq-vr\right) \tag{8}$$

$$F_y=m\left(\frac{dv}{dt}+ur-wq\right) \tag{9}$$

$$F_z=m\left(\frac{dw}{dt}+vp-uq\right) \tag{10}$$

$$M_x=I_x\frac{dp}{dt}-\left(I_y-I_z\right)qr \tag{11}$$

$$M_y=I_y\frac{dq}{dt}-\left(I_z-I_x\right)rp \tag{12}$$

$$M_z=I_z\frac{dr}{dt}-\left(I_x-I_y\right)pq \tag{13}$$

where I_x, I_y and I_z are the moments of inertia of the three axes, respectively. p, q and r are the angular velocities of the three axes, respectively. u , v and w are the linear velocities of the three axes. F_x , F_y and F_z are the combined external forces in the triaxial direction. M_x , M_y and M_z are the combined external moments in the triaxial direction.

The angular velocity and the Euler angle in the carrier coordinate system corresponding to the ground coordinate system have the following relationship

$$\begin{bmatrix}p\\ q\\ r\end{bmatrix}=\begin{bmatrix}1 & 0 & -\sin\theta\\ 0 & \cos\theta & \sin\phi\cos\theta\\ 0 & -\sin\theta & \cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \tag{14}$$

Assuming that there is no frictional resistance in the air, ignoring the spiral gyro effect, the center of the rotor is the center of mass of the UAV, and regardless of its inertia, the dynamic model of the four-rotor UAV can be approximated as:

$$\begin{cases}I_x\ddot{\phi}=\dot{\theta}\dot{\psi}\left(I_y-I_z\right)+lu_1\\ I_y\ddot{\theta}=\dot{\phi}\dot{\psi}\left(I_z-I_x\right)+lu_2\\ I_z\ddot{\psi}=\dot{\phi}\dot{\theta}\left(I_x-I_y\right)+lu_3\\ ma_x=\left(\cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi\right)u_4\\ ma_y=\left(\sin\psi\sin\theta\cos\phi-\cos\psi\sin\phi\right)u_4\\ ma_z=mg-\left(\cos\theta\cos\phi\right)u_4\end{cases} \tag{15}$$

where a_x, a_y and a_z are the acceleration of the three axes in the coordinate system, l is the arm length from the rotor to the center of gravity. u_1,u_2,u_3 and u_4 is control input. u_1 is the roll input. u_2 is the pitch input. u_3 is the yaw input. u_4 is the vertical speed change which means the throttle input.

2.2 Singular Markov Switching System Statement

Considering the component failure, the complex external environment and disturbances in the actual system, the motion state model containing sensor and actuator failures is represented as the following singular Markov switching system with mixed time-delays:

$$\begin{cases}E\dot{x}(t)=\left[A(r(t))+\Delta A(r(t))\right]x(t)\\ +\left[A_\tau(r(t))+\Delta A_\tau(r(t))\right]x\left(t-\tau_{1,r(t)}\right)\\ +B(r(t))u(t)+D(r(t))f_a(t)\\ +H(r(t))\int_{t-\tau_{2,r(t)}}^{t}f(x(s))ds+W(r(t))\omega(t)\\ y(t)=C(r(t))x(t)\\ +G(r(t))f_s(t),x(t)=\phi(t),\ t\in[-\tau,0]\end{cases} \tag{16}$$

This is a singular Markov switching system with mixed time-delays in probability space. $\{r(t),t\geq 0\}$ is a right continuous Markov process, which takes values in the finite set $S=\{1,2,\cdots,s\}$, and its state transition probability is as follows

$$P\{r(t+\Delta t)=j:r(t)=i\}=\begin{cases}\delta_{ij}\Delta t+\mathrm{O}(\Delta t), & j\neq i\\ 1+\delta_{ij}\Delta t+\mathrm{O}(\Delta t), & j=i\end{cases}$$

where $\Delta t>0$, $\lim_{\Delta t\to 0}\mathrm{O}(\Delta t)/\Delta t=0$ and $\delta_{ij}\geq 0$ are transition probabilities from state i to state j. If $j\neq i$ then $\delta_{ij}\geq 0$, otherwise $\delta_{ii}=-\sum_{j=1,j\neq i}^{S}\delta_{ij}$. In the system model, $x(t)\in\mathbb{R}^n$ is the state vector. $u(t)\in\mathbb{R}^m$ is the control input. $y(t)$ is the output vector. $f_a(t)\in\mathbb{R}^p$ and $f_s(t)\in\mathbb{R}^q$ are unknown faults of the system actuator and sensor, respectively (Xiaohang Li et al., 2018; Li H Y et al., 2014). $\omega(t)\in L_2[0,\infty)$ indicates the external disturbance of the system. $f(\cdot):\mathbb{R}^n\to\mathbb{R}^n$ is a non-linear function vector. E , $A(r(t))$, $A_\tau(r(t))$, $B(r(t))$, $C(r(t))$, $D(r(t))$, $H(r(t))$, $G(r(t))$ and $W(r(t))$ are constant matrices with appropriate dimensions, and it is assumed that $G(r(t))$ is a column full rank. $\Delta A(r(t))$ and $\Delta A_\tau(r(t))$ are unknown matrices and represent the uncertainty of state parameters.For $r(t)=i,i\in S$, the uncertain term is $\left[\Delta A_i \quad \Delta A_{\tau i}\right]=U_iF_i(t)\left[V_{1i} \quad V_{2i}\right]$, where the matrices U_i, V_{1i} and V_{2i} are known constant matrices with appropriate dimensions. $F_i(t)$ is an unknown time-varying matrix that satisfies $F_i^T(t)F_i(t)\le I$. In the mixed time-delay, $\tau_{1,r(t)}$ represents the mode-dependent discrete time-delay, $\tau_{2,r(t)}$ represents the mode-dependent distribution time-delay, $\tau=\max\ \{\tau_{i,j}\,|i=1,2,j-1,2,\cdots,s\}$, and the function $\phi(t)$ is the initial condition on $[-\tau,0]$.

***Assumption* 1** Let the nonlinear function $f(x)$ satisfies

$$[f(x)-Z_1x]^T[f(x)-Z_2x]\le 0,\ f(0)\equiv 0 \tag{17}$$

where Z_1 and Z_2 are known constant matrices, which can be obtained from the above formula

$$\begin{bmatrix} x(t) & f(x(t))\end{bmatrix}\begin{bmatrix}\breve{Z}_1 & -\breve{Z}_2\\ -\breve{Z}_2 & I\end{bmatrix}\begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix}\le 0 \tag{18}$$

Where $\breve{Z}_1=\left(Z_1^TZ_2+Z_2^TZ_1\right)/2$, $\breve{Z}_2=\left(Z_1^T+Z_2^T\right)/2$.

***Definition* 1** for singular Markov switching systems

$$\begin{cases} E\dot{x}(t)=A(r(t))x(t)+A_\tau(r(t))x\left(t-\tau_{1,r(t)}\right)\\ +B(r(t))u(t)+W(r(t))\omega(t)\\ y(t)=C(r(t))x(t)+D(r(t))u(t)\end{cases} \tag{19}$$

where $\omega(t)$ is the external disturbance. For any $x(0)\in\mathbb{R}^n$ and $r(0)\in S$, there exist constants $M(x(0),r(0))$ and $\gamma>0$, so that the following inequality

$$\lim_{t\to\infty}\varepsilon\left\{\int_0^t y^T(s)y(s)ds\mid x(0),r(0)\right\}^{\frac{1}{2}}$$
$$\le\gamma\left[\|\omega(t)\|_2^2+M(x(0),r(0))\right]^{\frac{1}{2}}$$

holds, then the system (19) is randomly stable and has robust H_∞ performance index γ.

***Lemma* 1** (Schur complement) For the symmetric matrix $S=\begin{bmatrix}S_{11} & S_{12}\\ S_{21} & S_{22}\end{bmatrix}$, where S_{11} is $r\times r$ -dimensional, the following three conditions are equivalent:

$S<0;$

$S_{11}<0,\ S_{22}-S^T{}_{12}S^{-1}{}_{11}S_{12}<0;$

$S_{22}<0,\ S_{11}-S_{12}S^{-1}{}_{22}S^T{}_{12}<0.$

***Lemma* 2** For any positive definite symmetric matrix $W\in R^{n\times n}$, $W=W^T$, parameter $0\le\tau\le\tau_M$ and vector function $x:[-\tau_M\quad 0]\to R^n$, the following integral inequality holds

$$-\tau\int_{-\tau}^0\dot{x}^T(t+s)W\dot{x}(t+s)ds$$
$$\le\begin{bmatrix}x^T(t) & x^T(t-\tau)\end{bmatrix}\begin{bmatrix}-W & W\\ * & -W\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-\tau)\end{bmatrix}$$

***Lemma* 3** Given symmetric positive definite matrix $\tilde{Y}>0$, matrices $\tilde{D}$ and $\tilde{E}$ with appropriate dimensions, then $\tilde{Y}+\tilde{D}F(k)\tilde{E}+\tilde{E}^TF^T(k)\tilde{D}^T<0$,for all F satisfying $F^TF\le I$, if and only if there exists a scalar $\varepsilon>0$ such that $\tilde{Y}+\varepsilon\tilde{D}\tilde{D}^T+\varepsilon^{-1}\tilde{E}^T\tilde{E}<0$.

***Lemma* 4** For any positive definite matrix R, the integrable vector function $f:[0,t]\to\mathbb{R}^n$, the following inequality holds

$$\left[\int_0^t f(s)ds\right]^T R\int_0^t f(s)ds\le t\int_0^t f^T(s)Rf(s)ds$$

In order to be able to consider both sensor failure and actuator failure, and treat sensor failure and actuator failure as state vectors, a new augmentation system is constructed as

$$\begin{cases}\bar{E}\dot{\xi}(t)=\bar{A}_i\xi(t)+\Delta A_ix(t)+(A_{\tau i}+\Delta A_{\tau i})x(t-\tau_{1,i})\\ +B_iu(t)+H_i\int_{t-\tau_{2,i}}^t f(x(s))ds+W_i\omega(t)\\ y_g(t)=\bar{C}_i\xi(t)\end{cases} \tag{20}$$

where $\bar{A}_i=\begin{bmatrix}A_i & D_i & 0\end{bmatrix},\bar{C}_i=\begin{bmatrix}C_i & 0 & G_i\end{bmatrix}$

$$\xi(t)=\begin{bmatrix}x(t)\\ f_a(t)\\ f_s(t)\end{bmatrix}\in\mathbb{R}^{n+p+q},\bar{E}=\begin{bmatrix}E & 0 & 0\end{bmatrix}$$

3. ESTIMATION OF ACTUATOR AND SENSOR FAILURES

In order to effectively estimate the system states under sensor or actuator failures, a robust adaptive observer of the following form is constructed

$$\begin{cases}\dot{z}(t)=M_iz(t)+L_iy_g(t)+T_i\hat{A}_{\tau i}\hat{\xi}(t-\tau_{1,i})\\ +T_i\hat{B}_iu(t)+T_i\hat{H}_i\int_{t-\tau_{2,i}}^t f\left(\hat{\xi}(s)\right)ds\\ \hat{\xi}(t)=z(t)+N_iy_g(t)\end{cases} \tag{21}$$

where $z(t)$ is the observer variable, $\hat{\xi}(t)$, $\hat{\xi}(t-\tau_{1,i})$, and $\hat{\xi}(s)$ are the state estimates, $\hat{A}_{\tau i}=\begin{bmatrix}A_{\tau i} & 0 & 0\end{bmatrix}$, $\hat{B}_i=\begin{bmatrix}B_i & 0 & 0\end{bmatrix}$, $\hat{H}_i=\begin{bmatrix}H_i & 0 & 0\end{bmatrix}$ are the estimated parameters, and M_i, L_i, T_i, and N_i are the matrices required by the observer design.

Define the observation error as

$$e(t)=\xi(t)-\hat{\xi}(t)=\left(I-N_i\bar{C}\right)\xi(t)-z(t)$$

and let $\Xi=I-N_i\bar{C}$, then the observation error system is

$$\begin{aligned}\dot{e}&=M_ie+\left(\Xi\bar{A}_i-L_i\bar{C}_i-M_i\Xi\right)\xi-T_iH_i\int_{t-\tau_{2,i}}^{t}f\left(\hat{x}(s)\right)ds\\&+\Xi\Delta A_ix(t)+\Xi\left(A_{\tau i}+\Delta A_{\tau i}\right)x\left(t-\tau_{1,i}\right)+\left(\Xi-T_i\right)B_iu(t)\\&+\Xi H_i\int_{t-\tau_{2,i}}^{t}f\left(x(s)\right)ds+\Xi W_i\omega(t)-T_iA_{\tau i}\hat{x}\left(t-\tau_{1,i}\right)\end{aligned}\tag{22}$$

let $\Xi-T_i=0,\ \Xi\bar{A}_i-L_i\bar{C}_i-M_i\Xi=0$,then

$$I-N_i\bar{C}_i=T_i\bar{E}\tag{23}$$

$$M_i=T_i\bar{A}_i-\left(L_i-M_iN_i\right)\bar{C}_i\tag{24}$$

Because G_i is full rank, a set of special solutions can be obtained according to formula (23)

$$T_i=\begin{bmatrix}\bar{E}\\ \bar{C}_i\end{bmatrix}^{-1}\begin{bmatrix}I_n\\0\end{bmatrix},\ N_i=\begin{bmatrix}\bar{E}\\ \bar{C}_i\end{bmatrix}^{-1}\begin{bmatrix}0\\I_n\end{bmatrix}\tag{25}$$

Assuming $L_i-M_iN_i=K_i$, then

$$M_i=T_i\bar{A}_i-K_i\bar{C}_i\ ,\ L_i=K_i+M_iN_i\tag{26}$$

where K_i is matrix with appropriate dimensions. The observer error system (22) can be simplified as

$$\begin{aligned}\dot{e}&=M_ie+\bar{A}_{\tau i}e\left(t-\tau_{1,i}\right)+\bar{H}_i\int_{t-\tau_{2,i}}^{t}f\left(e(s)\right)ds\\&+T_i\Delta A_ix+T_i\Delta A_{\tau i}x\left(t-\tau_{1,i}\right)+T_iW_i\omega\end{aligned}\tag{27}$$

where $\bar{A}_{\tau i}=T_i\hat{A}_{\tau i},\bar{H}_i=T_i\hat{H}_{\tau i}$.

Theorem1 Defines $\bar{\tau}_i=\max\left\{\tau_{ij},j\in S\right\},\underline{\tau}_i=\min\left\{\tau_{ij},j\in S\right\}$, $\bar{\delta}=\max\left\{\delta_{ii},i\in S\right\}$. If exist the constant $\alpha>0$, $\gamma>0$, positive definite symmetric matrices P_{1i}, P_{2i}, Q_1, Q_2, O_1, O_2, R_1 and R_2, matrices Y_{Pi}, Y_{Oi} and K_i, so that the following linear matrix inequality

$$\Phi=\begin{bmatrix}\Phi_e&0&\Phi_{e\varpi}&\Phi_{\dot{e}}&&\\ *&\Phi_x&\Phi_{x\varpi}&\Phi_{\dot{x}}&\vec{U}_1&\vec{V}_1^T\\ *&*&\Phi_{99}&\Phi_{\dot{\varpi}}&&\\ *&*&*&\Phi_o&&\\ &&*&&-\alpha^{-1}I&0\\ &&&&*&-\alpha I\end{bmatrix}<0\tag{28}$$

holds. Then observer error system (27) is robust asymptotically stable and satisfies H∞ performance γ .

$$\Phi_e=\begin{bmatrix}\Phi_{11}&P_{1i}\bar{A}_{\tau i}&P_{1i}\bar{H}_i&\breve{Z}_2\\ *&-Q_1-O_1&0&0\\ *&*&-\frac{1}{\tau_{2,i}}R_1&0\\ *&*&*&\Phi_{44}\end{bmatrix},$$

$$\Phi_x=\begin{bmatrix}\Phi_{55}&\Phi_{56}&P_{2i}H_i&\breve{Z}_2\\ *&\Phi_{66}&0&0\\ *&*&-\frac{1}{\tau_{2,i}}R_2&0\\ *&*&*&\Phi_{88}\end{bmatrix},\Phi_{e\varpi}=\begin{bmatrix}\Phi_{19}\\0\\0\\0\end{bmatrix}$$

$$\Phi_{x\varpi}=\begin{bmatrix}\Phi_{59}\\0\\0\\0\end{bmatrix},\Phi_{99}=\begin{bmatrix}-\gamma^2I&0&0\\0&-\gamma^2I&0\\0&0&-\gamma^2I\end{bmatrix},$$

$$\Phi_{\dot{e}}=\begin{bmatrix}\Phi_{110}&0\\ \tau_{1,i}\bar{A}_{\tau i}^TO_1&0\\ \tau_{1,i}\bar{H}_i^TO_1&0\\0&0\end{bmatrix},\Phi_{\dot{x}}=\begin{bmatrix}0&\tau_{1,i}A_i^TO_2\\0&\tau_{1,i}A_{\tau i}^TO_2\\0&\tau_{1,i}H_i^TO_2\\0&0\end{bmatrix}$$

$$\Phi_{\dot{\varpi}}=\begin{bmatrix}0&\tau_{1,i}B_i^TO_2\\0&\tau_{1,i}D_i^TO_2\\ \tau_{1,i}\left(T_iW_i\right)^TO_1&\tau_{1,i}W_i^TO_2\end{bmatrix},\Phi_o=\begin{bmatrix}-O_1&0\\ *&-O_2\end{bmatrix}$$

$$\begin{aligned}\Phi_{11}&=P_{1i}T_i\bar{A}_i+\left(T_i\bar{A}_i\right)^TP_{1i}-Y_{Pi}\bar{C}_i-\bar{C}_i^TY_{Pi}^T\\&+\sum_{j=1}^{S}\delta_{ij}P_{1j}+\left(1+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)\right)Q_1-O_1-\breve{Z}_1+I\end{aligned}$$

$$\Phi_{19}=\begin{bmatrix}0&0&P_{1i}T_iW_i\end{bmatrix},\Phi_{110}=\tau_{1,i}\left(T\bar{A}_i\right)^TO_1-\tau_{1,i}\bar{C}_i^TY_{Oi}^T,$$

$$\Phi_{44}=\left[\tau_{2,i}+\tfrac{\bar{\delta}}{2}\left(\bar{\tau}_2^{\,2}-\underline{\tau}_2^{\,2}\right)\right]R_1-I$$

$$\begin{aligned}\Phi_{55}&=P_{2i}A_i+A_i^TP_{2i}+\sum_{j=1}^{S}\delta_{ij}E^TP_{2j}E+\left(1+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)\right)Q_2\\&-E^TO_2E-\breve{Z}_1,\Phi_{56}=P_{2i}A_{\tau i}+E^TO_2E\end{aligned}$$

$$\Phi_{59}=\begin{bmatrix}P_{2i}B_i&P_{2i}D_i&P_{2i}W_i\end{bmatrix},\Phi_{66}=-Q_2-E^TO_2E$$

$$\Phi_{66}=-Q_2-E^TO_2E,\Phi_{88}=\left[\tau_{2,i}+\tfrac{\bar{\delta}}{2}\left(\bar{\tau}_2^{\,2}-\underline{\tau}_2^{\,2}\right)\right]R_2-I$$

Proof: Choose the Lyapunov-Krasovskii function as follows

$$V\left(e(t),x(t),r(t)\right)=V\left(e(t),r(t)\right)+V\left(x(t),r(t)\right)\tag{29}$$

where

$$\begin{aligned}V\left(e(t),r(t)\right)&=e^T(t)P_{1i}e(t)+\int_{t-\tau_{1,i}}^{t}e^T(\theta)Q_1e(\theta)d\theta\\&+\bar{\delta}\int_{\underline{\tau}_1}^{\bar{\tau}_1}\int_{t+s}^{t}e^T(\theta)Q_1e(\theta)d\theta ds\\&+\tau_{1,i}\int_{-\tau_{1,i}}^{0}\int_{t+s}^{t}\dot{e}^T(\theta)O_1\dot{e}(\theta)d\theta ds\\&+\int_{0}^{\tau_{2,i}}\int_{t-s}^{t}f^T\left(e(\theta)\right)R_1f\left(e(\theta)\right)d\theta ds\end{aligned}$$

$$+\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a\int_{t+s}^t f^T\left(e(\theta)\right)R_1 f\left(e(\theta)\right)d\theta ds da$$

$$V\left(x(t),r(t)\right)=x^T(t)P_{2i}x(t)+\int_{t-\tau_{1,i}}^t x^T(\theta)Q_2x(\theta)d\theta$$

$$+\bar{\delta}\int_{\underline{\tau}_1}^{\bar{\tau}_1}\int_{t+s}^t x^T(\theta)Q_2x(\theta)d\theta ds+$$

$$\tau_{1,i}\int_{-\tau_{1,i}}^0\int_{t+s}^t \dot{x}^T(\theta)E^TO_2E\dot{x}(\theta)d\theta ds$$

$$+\int_0^{\tau_{2,i}}\int_{t-s}^t f^T\left(x(\theta)\right)R_2 f\left(x(\theta)\right)d\theta ds$$

$$+\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a\int_{t+s}^t f^T\left(x(\theta)\right)R_2 f\left(x(\theta)\right)d\theta ds da$$

For $r(t)=i, i\in S$, the weak infinitesimal operator ℓ with a Markov process

$$\ell V(e,i)=2e^TP_{1i}\dot{e}+e^T\sum_{j=1}^S\delta_{ij}P_{1j}e+e^TQ_1e-e_{\tau_{1,i}}^TQ_1e_{\tau_{1,i}}$$

$$+\sum_{j=1}^S\delta_{ij}\int_{t-\tau_{1,j}}^t e^T(s)Q_1e(s)ds+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)e^TQ_1e$$

$$-\bar{\delta}\int_{\underline{\tau}_1}^{\bar{\tau}_1}e^T(t-s)Q_1e(t-s)ds+\tau_{1,i}{}^2\dot{e}^TO_1\dot{e}$$

$$-\tau_{1,i}\int_{t-\tau_{1,i}}^t\dot{e}^T(s)O_1\dot{e}(s)ds+\tau_{2,i}f^T\left(e(t)\right)R_1f\left(e(t)\right)$$

$$-\int_0^{\tau_{2,i}}f^T\left(e(t-s)\right)R_1f\left(e(t-s)\right)ds$$

$$+\sum_{j=1}^S\delta_{ij}\int_0^{\tau_{2,j}}\int_{t-s}^t f^T\left(e(\theta)\right)R_1f\left(e(\theta)\right)d\theta ds$$

$$+\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a f^T\left(e(t)\right)R_1f\left(e(t)\right)dsda$$

$$-\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a f^T\left(e(t-s)\right)R_1f\left(e(t-s)\right)dsda$$

$$\ell V(x,i)=2x^TP_{2i}E\dot{x}+x^T\sum_{j=1}^S\delta_{ij}E^TP_{2j}Ex+x^TQ_2x$$

$$-x_{\tau_{1,i}}^TQ_2x_{\tau_{1,i}}+\sum_{j=1}^S\delta_{ij}\int_{t-\tau_{1,j}}^t x^T(s)Q_2x(s)ds$$

$$+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)x^TQ_2x-\bar{\delta}\int_{\underline{\tau}_1}^{\bar{\tau}_1}x^T(t-s)Q_2x(t-s)ds$$

$$+\tau_{1,i}{}^2\dot{x}^TE^TO_2E\dot{x}-\tau_{1,i}\int_{t-\tau_{1,i}}^t\dot{x}^T(s)E^TO_2E\dot{x}(s)ds$$

$$+\tau_{2,i}f^T\left(x(t)\right)R_2f\left(x(t)\right)$$

$$-\int_0^{\tau_{2,i}}f^T\left(x(t-s)\right)R_2f\left(x(t-s)\right)ds$$

$$+\sum_{j=1}^S\delta_{ij}\int_0^{\tau_{2,j}}\int_{t-s}^t f^T\left(x(\theta)\right)R_2f\left(x(\theta)\right)d\theta ds$$

$$+\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a f^T\left(x(t)\right)R_2f\left(x(t)\right)dsda \tag{30}$$

$$-\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a f^T\left(x(t-s)\right)R_2f\left(x(t-s)\right)dsda$$

where

$$\begin{cases}\sum_{j=1}^S\delta_{ij}\int_{t-\tau_{1,j}}^t e^T(s)Q_1e(s)ds\le\bar{\delta}\int_{t-\underline{\tau}_1}^{t-\bar{\tau}_1}e^T(s)Q_1e(s)ds\\ \sum_{j=1}^S\delta_{ij}\int_{t-\tau_{1,j}}^t x^T(s)Q_2x(s)ds\le\bar{\delta}\int_{t-\underline{\tau}_1}^{t-\bar{\tau}_1}x^T(s)Q_2x(s)ds\end{cases} \tag{31}$$

$$\begin{cases}-\bar{\delta}\int_{\underline{\tau}_1}^{\bar{\tau}_1}e^T(t-s)Q_1e(t-s)ds=-\bar{\delta}\int_{t-\underline{\tau}_1}^{t-\bar{\tau}_1}e^T(s)Q_1e(s)ds\\ -\bar{\delta}\int_{\underline{\tau}_1}^{\bar{\tau}_1}x^T(t-s)Q_2x(t-s)ds=-\bar{\delta}\int_{t-\underline{\tau}_1}^{t-\bar{\tau}_1}x^T(s)Q_2x(s)ds\end{cases} \tag{32}$$

$$\begin{cases}\sum_{j=1}^S\delta_{ij}\int_0^{\tau_{2,j}}\int_{t-s}^t f^T\left(e(\theta)\right)R_1f\left(e(\theta)\right)d\theta ds\\ \le\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_{t-s}^t f^T\left(e(\theta)\right)R_1f\left(e(\theta)\right)d\theta da\\ \sum_{j=1}^S\delta_{ij}\int_0^{\tau_{2,j}}\int_{t-s}^t f^T\left(x(\theta)\right)R_1f\left(x(\theta)\right)d\theta ds\\ \le\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_{t-s}^t f^T\left(x(\theta)\right)R_1f\left(x(\theta)\right)d\theta da\end{cases} \tag{33}$$

$$\begin{cases}-\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a f^T\left(e(t-s)\right)R_1f\left(e(t-s)\right)dsda\\ \quad\le-\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_{t-s}^t f^T\left(e(\theta)\right)R_1f\left(e(\theta)\right)d\theta da\\ -\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_0^a f^T\left(x(t-s)\right)R_2f\left(x(t-s)\right)dsda\\ \quad\le-\bar{\delta}\int_{\underline{\tau}_2}^{\bar{\tau}_2}\int_{t-s}^t f^T\left(x(\theta)\right)R_1f\left(x(\theta)\right)d\theta da\end{cases} \tag{34}$$

From the Lemma 4, we get

$$-\int_0^{\tau_{2,i}}f^T\left(e(t-s)\right)R_1f\left(e(t-s)\right)ds$$
$$\le-\frac{1}{\tau_{2,i}}\left(\int_{t-\tau_{2,i}}^t f\left(e(s)\right)ds\right)^TR_1\int_{t-\tau_{2,i}}^t f\left(e(s)\right)ds$$

$$-\int_0^{\tau_{2,i}}f^T\left(x(t-s)\right)R_2f\left(x(t-s)\right)ds$$
$$\le-\frac{1}{\tau_{2,i}}\left(\int_{t-\tau_{2,i}}^t f\left(x(s)\right)ds\right)^TR_2\int_{t-\tau_{2,i}}^t f\left(x(s)\right)ds \tag{35}$$

From the Lemma 2,we get

$$\begin{cases}-\tau_{1,i}\int_{t-\tau_{1,i}}^t\dot{e}^T(s)O_1\dot{e}(s)ds\\ \le-e^TO_1e+e_{\tau_{1,i}}^TO_1e+e^TO_1e_{\tau_{1,i}}-e_{\tau_{1,i}}^TO_1e_{\tau_{1,i}}\\ -\tau_{1,i}\int_{t-\tau_{1,i}}^t\dot{x}^T(s)E^TO_2E\dot{x}(s)ds\le-x^TE^TO_2Ex\\ +x_{\tau_{1,i}}^TE^TO_2Ex+x^TE^TO_2Ex_{\tau_{1,i}}-x_{\tau_{1,i}}^TE^TO_2Ex_{\tau_{1,i}}\end{cases} \tag{36}$$

Substituting the equation (31)–(36) into the equation (30)

$$\ell V(e,x,i) \le 2e^T P_{1i}\dot{e} + e^T\left[\sum_{j=1}^{S}\delta_{ij}P_{1j} + \left(1+\bar{\delta}\left(\bar{\tau}_1 - \underline{\tau}_1\right)\right)Q_1 - O_1\right]e$$
$$-e_{\tau_{1,i}}^T\left(Q_1+O_1\right)e_{\tau_{1,i}} + \tau_{1,i}{}^2\dot{e}^T O_1\dot{e} + e_{\tau_{1,i}}^T O_1 e$$
$$+e^T O_1 e_{\tau_{1,i}} + \left[\tau_{2,i} + \frac{\bar{\delta}}{2}\left(\bar{\tau}_2{}^2 - \underline{\tau}_2{}^2\right)\right]f^T\left(e(t)\right)R_1 f\left(e(t)\right)$$
$$-\frac{1}{\tau_{2,i}}\left(\int_{t-\tau_{2,i}}^{t} f\left(e(s)\right)ds\right)^T R_1\int_{t-\tau_{2,i}}^{t} f\left(e(s)\right)ds$$
$$+x^T\left[\sum_{j=1}^{S}\delta_{ij}E^T P_{2j}E + \left(1+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)\right)Q_2 - E^T O_2 E\right]x$$
$$+2x^T P_{2i}E\dot{x} - x_{\tau_{1,i}}^T\left(Q_2 + E^T O_2 E\right)x_{\tau_{1,i}} + \tau_{1,i}{}^2\dot{x}^T E^T O_2 E\dot{x}$$
$$+x_{\tau_{1,i}}^T E^T O_2 E x + x^T E^T O_2 E x_{\tau_{1,i}}$$
$$+\left[\tau_{2,i} + \frac{\bar{\delta}}{2}\left(\bar{\tau}_2{}^2-\underline{\tau}_2{}^2\right)\right]f^T\left(x(t)\right)R_2 f\left(x(t)\right) \tag{37}$$
$$-\frac{1}{\tau_{2,i}}\left(\int_{t-\tau_{2,i}}^{t} f\left(x(s)\right)ds\right)^T R_2\int_{t-\tau_{2,i}}^{t} f\left(x(s)\right)ds$$

then

$$\ell V(e,x,i) + e^T(t)e(t) - \gamma^2\varpi^T(t)\varpi(t)$$
$$\le \eta^T(t)\left[\tilde{\Phi} + \tau_{1,i}{}^2\varphi_e O_1\varphi_e + \tau_{1,i}{}^2\varphi_x O_2\varphi_x\right]\eta(t) \tag{38}$$

Let $\dot{e}^T(t) = \varphi_e\eta(t)$ and $E\dot{x} = \varphi_x\eta(t)$, the inputs, disturbances and faults are expressed as

$$\varpi = \begin{bmatrix} u(t) & f_a(t) & \omega(t)\end{bmatrix}^T,$$
$$\phi_e = \begin{bmatrix} M_i & \bar{A}_{\tau i} & \bar{H}_i & 0 & T_i\Delta A_i & T_i\Delta A_{\tau i} & 0 & 0 & 0 & 0 & T_i W_i\end{bmatrix}$$
$$\phi_x = \begin{bmatrix} 0 & 0 & 0 & 0 & A_i+\Delta A_i & A_{\tau i}+\Delta A_{\tau i} & H_i & 0 & B_i & D_i & W_i\end{bmatrix}$$

$$\eta(t) = \begin{bmatrix} e(t) & \eta_{1e} & \eta_1 & f\left(e(t)\right) & x(t) & \eta_{1x} & \eta_2 & f\left(x(t)\right) & \varpi\end{bmatrix}^T$$
$$\eta_1 = \int_{t-\tau_{2,i}}^{t} f\left(e(s)\right)ds,\ \eta_2 = \int_{t-\tau_{2,i}}^{t} f\left(x(s)\right)ds,$$
$$\eta_{1e} = e\left(t-\tau_{1,i}\right), \eta_{1x} = x\left(t-\tau_{1,i}\right)$$

where

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_e & \tilde{\Phi}_{ex} & \Phi_{e\varpi} \\ * & \tilde{\Phi}_x & \Phi_{x\varpi} \\ * & * & \Phi_{99}\end{bmatrix} < 0 \tag{39}$$

where

$$\tilde{\Phi}_e = \begin{bmatrix} \tilde{\Phi}_{11} & P_{1i}\bar{A}_{\tau i} & P_{1i}\bar{H}_i & \breve{Z}_2 \\ * & -Q_1 - O_1 & 0 & 0 \\ * & * & -\frac{1}{\tau_{2,i}}R_1 & 0 \\ * & * & * & \Phi_{44}\end{bmatrix},$$

$$\tilde{\Phi}_x = \begin{bmatrix} \tilde{\Phi}_{55} & \tilde{\Phi}_{56} & P_{2i}H_i & \breve{Z}_2 \\ * & \Phi_{66} & 0 & 0 \\ * & * & -\frac{1}{\tau_{2,i}}R_1 & 0 \\ * & * & * & \Phi_{88}\end{bmatrix}$$

$$\tilde{\Phi}_{ex} = \begin{bmatrix} P_{1i}T_i\Delta A & P_{1i}T_i\Delta A_{\tau i} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\tilde{\Phi}_{11} = P_{1i}M_i + M_i^T P_{1i} + \sum_{j=1}^{S}\delta_{ij}P_{1j} + \left(1+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)\right)Q_1 - O_1 - \breve{Z}_1 + I$$
$$\tilde{\Phi}_{56} = P_{2i}\left(A_{\tau i} + \Delta A_{\tau i}\right) + E^T O_2 E\ ,$$
$$\tilde{\Phi}_{55} = P_{2i}\left(A_i + \Delta A_i\right) + \left(A_i + \Delta A_i\right)^T P_{2i} + \sum_{j=1}^{S}\delta_{ij}E^T P_{2j}E$$
$$+\left(1+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)\right)Q_2 - E^T O_2 E - \breve{Z}_1$$

let $\Phi = \tilde{\Phi} + \tau_{1,i}{}^2\phi_e{}^T O_1\phi_e + \tau_{1,i}{}^2\phi_x{}^T O_2\phi_x$, according to the Lemma 3, the uncertain terms in the matrix are eliminated. Combining the formula (26), and using the schur complement, it can get the $\Phi = \hat{\Phi} + \alpha\vec{U}_1\vec{U}_1{}^T + \alpha^{-1}\vec{V}_1{}^T\vec{V}_1$, where the $\vec{V} = \begin{bmatrix} 0 & 0 & 0 & 0 & V_1 & V_2 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$, $\vec{U}_1{}^T =$

$$\begin{bmatrix} U^T P_{1i}T_i & 0 & 0 & 0 & U^T P_{2i} & 0 & 0 & 0 & 0 & \tau_{1,i}U^T O_1 & \tau_{1,i}U^T O_2\end{bmatrix}$$

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_e & 0 & \Phi_{e\varpi} & \hat{\Phi}_{\dot{e}} \\ & \Phi_x & \Phi_{x\varpi} & \Phi_{\dot{x}} \\ & & \Phi_{99} & \Phi_{\dot{\varpi}} \\ * & & & \Phi_o\end{bmatrix} < 0 \tag{40}$$

$$\hat{\Phi}_e = \begin{bmatrix} \hat{\Phi}_{11} & P_{1i}\bar{A}_{\tau i} & P_{1i}\bar{H}_i & \breve{Z}_2 \\ * & -Q_1 - O_1 & 0 & 0 \\ * & * & -\frac{1}{\tau_{2,i}}R_1 & 0 \\ * & * & * & \Phi_{44}\end{bmatrix}, \Phi_{\dot{e}} = \begin{bmatrix} \hat{\Phi}_{110} & 0 \\ \tau_{1,i}\bar{A}_{\tau i}{}^T O_1 & 0 \\ \tau_{1,i}\bar{H}_i{}^T O_1 & 0 \\ 0 & 0\end{bmatrix}$$

$$\hat{\Phi}_{11} = P_{1i}T_i\bar{A}_i + \left(T_i\bar{A}_i\right)^T P_{1i} - P_{1i}K_i\bar{C}_i - \left(K_i\bar{C}_i\right)^T P_{1i}$$
$$+\sum_{j=1}^{S}\delta_{ij}P_{1j} + \left(1+\bar{\delta}\left(\bar{\tau}_1-\underline{\tau}_1\right)\right)Q_1 - O_1 - \breve{Z}_1 + I$$
$$\Phi_{110} = \tau_{1,i}\left(T\bar{A}_i\right)^T O_1 - \tau_{1,i}\bar{C}_i^T K_i^T O_1$$

let $Y_{Pi} = P_{1i}K_i$, $Y_{Oi} = O_1 K_i$. Then

$$\ell V(e,x,i) + e^T(t)e(t) - \gamma^2\varpi^T(t)\varpi(t) \le \eta^T(t)\Phi\eta(t) < 0$$

According to the Dynkin formula, there is

$$\left\{V\left(e,x,i\right)\right\}-\left\{V\left(e\left(0\right),x\left(0\right),r\left(0\right)\right)\right\}$$
$$<-\left\{\int_0^\infty e^T\left(s\right)e\left(s\right)ds\right\}+\gamma^2\left\{\int_0^\infty \varpi^T\left(s\right)\varpi\left(s\right)ds\right\}<0$$

The following inequality can be get

$$\left\{\int_0^\infty e^T\left(s\right)e\left(s\right)ds\right\}-\gamma^2\left\{\int_0^\infty \varpi^T\left(s\right)\varpi\left(s\right)ds\right\}$$
$$<\left\{V\left(e\left(0\right),x\left(0\right),r\left(0\right)\right)\right\}$$

By the definition 1

$$\begin{aligned}&\lim_{t\to\infty}\left\{\int_0^\infty e^T\left(s\right)e\left(s\right)ds\right\}^{\frac{1}{2}}\\&\le\left[\gamma^2\left\|\varpi\left(s\right)\right\|_2^2+V\left(e\left(0\right),x\left(0\right),r\left(0\right)\right)\right]^{\frac{1}{2}}\end{aligned} \qquad (41)$$

The proof is completed.

4. ROBUST FAULT-TOLERANT CONTROL DESIGN

For the control strategy of the system, the output feedback controller is used as the basic controller. The output feedback controller is highly adaptable and not sensitive to the presence of faults and disturbances in the system. In order to make the system still have good stability in the event of sensor or actuator failure, a fault tolerance algorithm based on control law compensation and fault sensor signal reconstruction is designed to achieve fault tolerant control. Suppose that the output feedback controller gain is K_c and the control input is

$$u\left(t\right)=K_c y\left(t\right) \qquad (42)$$

4.1 Fault-tolerant Control for Simultaneous Actuator and Sensor Failure

In the case of actuator failure, assuming $rank(B_i,D_i)=rank(B_i)$, the compensation controller based on the output feedback controller is designed as follows

$$u_a=-B_i^{-1}D_i f_a \qquad (43)$$

When there is an actuator failure in the system, the fault-tolerant control is

$$u_c=u+u_a \qquad (44)$$

When a sensor failure occurs in the system, it is assumed that the output signal is y_s, and the output signal can be reconstructed as

$$y_r=y_s-G_i f_s \qquad (45)$$

Then in the output feedback control, the signal reconstruction fault-tolerant control is

$$u\left(t\right)=K_{ci}\left(y_s-G_i f_s\right) \qquad (46)$$

Considering both sensor and actuator failure conditions, the fault-tolerant controller is designed as

$$u_c=K_{ci}\left(y_s-G_i f_s\right)-B_i^{-1}D_i f_a \qquad (47)$$

4.2 Robust fault-tolerant Controller Design

The closed-loop control system with the fault-tolerant control is

$$\begin{cases}E\dot{x}\left(t\right)=\left(A_i+\Delta A_i+B_iK_{ci}C_i\right)x\left(t\right)+\left(A_{\tau i}+\Delta A_{\tau i}\right)x\left(t-\tau_{1,r(t)}\right)\\+H_i\int_{t-\tau_{2,r(t)}}^{t}f\left(x\left(s\right)\right)ds+W_i\omega\left(t\right)\\y\left(t\right)=C_ix\left(t\right)+G_if_s\left(t\right)\end{cases} \qquad (48)$$

Theorem2 Defines $\overline{\tau}_i=\max\left\{\tau_{ij},j\in S\right\}$, $\underline{\tau}_i=\min\left\{\tau_{ij},j\in S\right\}$, $\overline{\delta}=\max\left\{\delta_{ii},i\in S\right\}$. If exist the constants $\alpha>0$, $\gamma>0$ and positive definite symmetric matrices P_i, X_i, Q, O and R. Let make the following linear matrix inequality

$$\Psi=\begin{bmatrix}\Psi_{11} & \Psi_{12} & P_iH & \breve{Z}_2 & P_iW_i & & & \\ * & \Psi_{22} & 0 & 0 & 0 & & & \\ * & * & -\frac{1}{\tau_{2,i}}R & 0 & 0 & \Psi_{\dot{x}} & \vec{U}_2 & \vec{V}_2^T \\ * & * & * & \Psi_{44} & 0 & & & \\ * & * & * & * & -\gamma^2 I & & & \\ & & * & & & -O & & \\ & & & & & & -\alpha^{-1}I & 0 \\ & & & * & & & & \\ & & & & & & * & -\alpha I\end{bmatrix}<0 \qquad (49)$$

holds. Then the designed fault-tolerant controller is $K_{ci}=B_i^{-1}P_i^{-1}X_i$. The system (48) is robust asymptotically stable and meets the H_∞ performance index γ.

where $\Psi_{\dot{x}}=\left[\tau_{1,i}A_i^TO\quad \tau_{1,i}A_{\tau i}^TO\quad \tau_{1,i}H_i^TO\quad 0\quad \tau_{1,i}W_i^TO\right]^T$

$$\Psi_{11}=P_iA_i+X_iC_i+A_i^TP_i+C_i^TX_i^T+\sum_{j=1}^{S}\delta_{ij}E^TP_jE$$
$$+\left(1+\overline{\delta}\left(\overline{\tau}_1-\underline{\tau}_1\right)\right)Q-E^TOE-\breve{Z}_1+I$$
$$\Psi_{12}=P_iA_{\tau i}+E^TOE,\quad \Psi_{22}=-Q-E^TOE,$$
$$\Psi_{44}=\left[\tau_{2,i}+\frac{\overline{\delta}}{2}\left(\overline{\tau}_2^{\,2}-\underline{\tau}_2^{\,2}\right)\right]R-I$$

Proof: Choose the Lyapunov-Krasovskii function as follows

$$\begin{aligned}&V\left(x\left(t\right),r\left(t\right)\right)=x^T\left(t\right)P_ix\left(t\right)+\int_{t-\tau_{1,i}}^{t}x^T\left(\theta\right)Qx\left(\theta\right)d\theta\\&+\overline{\delta}\int_{\underline{\tau}_1}^{\overline{\tau}_1}\int_{t+s}^{t}x^T\left(\theta\right)Qx\left(\theta\right)d\theta ds+\\&\tau_{1,i}\int_{-\tau_{1,i}}^{0}\int_{t+s}^{t}\dot{x}^T\left(\theta\right)E^TOE\dot{x}\left(\theta\right)d\theta ds\\&+\int_0^{\tau_{2,i}}\int_{t-s}^{t}f^T\left(x\left(\theta\right)\right)Rf\left(x\left(\theta\right)\right)d\theta ds+\\&\overline{\delta}\int_{\underline{\tau}_2}^{\overline{\tau}_2}\int_0^{a}\int_{t+s}^{t}f^T\left(x\left(\theta\right)\right)Rf\left(x\left(\theta\right)\right)d\theta dsda\end{aligned} \qquad (50)$$

For $r\left(t\right)=i,i\in S$, a weak infinitesimal operator is ℓ with a Markov process. get

$$\begin{aligned}
&\ell V(x,i) = 2x^T P_i E\dot{x} + x^T \sum_{j=1}^{S}\delta_{ij}E^T P_j E x + x^T Q x - x_{\tau_{1,i}}^T Q x_{\tau_{1,i}} \\
&+\sum_{j=1}^{S} \delta_{ij}\int_{t-\tau_{1,j}}^{t} x^T(s)Qx(s)ds + \bar\delta(\bar\tau_1 - \underline{\tau}_1)x^TQx \\
&-\bar\delta\int_{\underline{\tau}_1}^{\bar\tau_1} x^T(t-s)Qx(t-s)ds + \tau_{1,i}{}^2\dot x^T E^T O E\dot x \\
&-\tau_{1,i}\int_{t-\tau_{1,i}}^{t} \dot x^T(s)E^TOE\dot x(s)ds \\
&+\tau_{2,i}f^T(x(t))Rf(x(t)) - \int_0^{\tau_{2,i}} f^T(x(t-s))Rf(x(t-s))ds \\
&+\sum_{j=1}^{S}\delta_{ij}\int_0^{\tau_{2,j}}\int_{t-s}^{t} f^T(x(\theta))Rf(x(\theta))d\theta ds \\
&+\bar\delta\int_{\underline\tau_2}^{\bar\tau_2}\int_0^a f^T(x(t))Rf(x(t))dsda \\
&-\bar\delta\int_{\underline\tau_2}^{\bar\tau_2}\int_0^a f^T(x(t-s))Rf(x(t-s))dsda
\end{aligned} \tag{51}$$

Then

$$\begin{aligned}
&\ell V(x,i) \le 2x^T P_i E\dot x + x^T\left[\sum_{j=1}^{S}\delta_{ij}E^TP_jE + \left(1+\bar\delta(\bar\tau_1-\underline\tau_1)\right)Q - E^TOE\right]x \\
&-x_{\tau_{1,i}}^T\left(Q+E^TOE\right)x_{\tau_{1,i}} + \tau_{1,i}{}^2\dot x^TE^TOE\dot x \\
&+x_{\tau_{1,i}}^TE^TOEx + x^TE^TOEx_{\tau_{1,i}} + \left[\tau_{2,i}+\frac{\bar\delta}{2}\left(\bar\tau_2{}^2-\underline\tau_2{}^2\right)\right]f^T(x(t))Rf(x(t)) \\
&-\frac{1}{\tau_{2,i}}\left(\int_{t-\tau_{2,i}}^{t} f(x(s))ds\right)^T R\int_{t-\tau_{2,i}}^{t} f(x(s))ds
\end{aligned} \tag{52}$$

Let $E\dot x = \varphi\mu(t)$, where

$$\mu(t) = \begin{bmatrix} x(t) & x(t-\tau_{1,i}) & \int_{t-\tau_{2,i}}^{t} f(x(s))ds & f(x(t)) & \omega \end{bmatrix}^T,$$

$$\varphi = \begin{bmatrix} A_i+\Delta A_i+K_{ci}C_i & A_{\tau i}+\Delta A_{\tau i} & H_i & 0 & W_i \end{bmatrix}$$

then

$$\begin{aligned}
&\ell V(x,i) + x^T(t)x(t) - \gamma^2\omega^T(t)\omega(t) \\
&\le \mu^T(t)\left[\tilde\Psi + \tau_{1,i}{}^2\varphi^TO\varphi\right]\mu(t)
\end{aligned} \tag{53}$$

where

$$\tilde\Psi = \begin{bmatrix} \tilde\Psi_{11} & \tilde\Psi_{12} & P_iH & \breve Z_2 & P_iW_i \\ * & \Psi_{22} & 0 & 0 & 0 \\ * & * & -\frac{1}{\tau_{2,i}}R & 0 & 0 \\ * & * & * & \Psi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{54}$$

$$\begin{aligned}
&\tilde\Psi_{11} = P_i\left(A_i+\Delta A_i+B_iK_{ci}C_i\right) + \left(A_i+\Delta A_i+B_iK_{ci}C_i\right)^T P_i \\
&+\sum_{j=1}^{S}\delta_{ij}E^TP_jE + \left(1+\bar\delta(\bar\tau_1-\underline\tau_1)\right)Q - E^TOE - \breve Z_1 + I \\
&\tilde\Psi_{12} = P_i\left(A_{\tau i}+\Delta A_{\tau i}\right) + E^TOE
\end{aligned}$$

Let the $\Psi = \tilde\Psi + \tau_{1,i}{}^2\varphi^TO\varphi$, according to Lemma3, the uncertainties are eliminated and $\Psi = \hat\Psi + \alpha\vec U_2\vec U_2{}^T + \alpha^{-1}\vec V_2{}^T\vec V_2$ can be obtained, where the $\vec V_2 = \begin{bmatrix} V_1 & V_2 & 0 & 0 & 0 & 0\end{bmatrix}$, $\vec U_2{}^T = \begin{bmatrix} U^TP_i & 0 & 0 & 0 & 0 & \tau_{1,i}U^TO\end{bmatrix}$,

$$\hat\Psi = \begin{bmatrix} \hat\Psi_{11} & \hat\Psi_{12} & P_iH & \breve Z_2 & P_iW_i & \Psi_{16} \\ * & \Psi_{22} & & & & \Psi_{26} \\ * & * & -\frac{1}{\tau_{2,i}}R & & & \Psi_{36} \\ * & * & * & \Psi_{44} & & 0 \\ * & * & * & * & -\gamma^2 I & \Psi_{56} \\ * & * & * & * & * & -O \end{bmatrix} < 0 \tag{55}$$

$$\begin{aligned}
&\hat\Psi_{11} = P_i\left(A_i+B_iK_{ci}C_i\right) + \left(A_i+B_iK_{ci}C_i\right)^T P_i + \sum_{j=1}^{S}\delta_{ij}E^TP_jE \\
&+\left(1+\bar\delta(\bar\tau_1-\underline\tau_1)\right)Q - E^TOE - \breve Z_1 + I \\
&\hat\Psi_{12} = P_iA_{\tau i} + E^TOE
\end{aligned}$$

and let $P_iB_iK_c = X_i$ .Then

$$\ell V(x,i) + x^T(t)x(t) - \gamma^2\omega^T(t)\omega(t) \le \mu^T(t)\Psi\mu(t) < 0 \quad ,$$

According to the Dynkin formula, there is

$$\begin{aligned}
&\{V(x,i)\} - \{V(x(0),r(0))\} \\
&< -\left\{\int_0^\infty x^T(s)x(s)ds\right\} + \gamma^2\left\{\int_0^\infty \omega^T(s)\omega(s)ds\right\} < 0
\end{aligned}$$

We can get

$$\left\{\int_0^\infty x^T(s)x(s)ds\right\} - \gamma^2\left\{\int_0^\infty \omega^T(s)\omega(s)ds\right\} < \{V(x(0),r(0))\}$$

By the definition 1

$$\lim_{t\to\infty}\left\{\int_0^\infty x^T(s)x(s)ds\right\}^{\frac{1}{2}} \le \left[\gamma^2\|\omega(s)\|_2^2 + V(x(0),r(0))\right]^{\frac{1}{2}} \tag{56}$$

The proof is completed.

In summary, the design of the system structure figure is shown in Fig. 2.

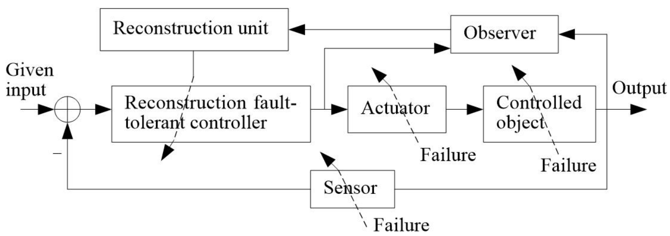

Fig. 2. System structure figure.

5. SIMULATION APPLICATION

5.1 Numerical Simulation

In order to verify the effectiveness of the observer designed in this paper, a numerical simulation is done.

Consider a Markov switching system (16) with two modes $S = \{1,2\}$, according to some references (Lili Zhang et al., 2019; Feifei Chen et al., 2019; Rathinasamy Sakthivel et al., 2015), the following parameters are selected.

The state probability transition matrix $\delta=\begin{bmatrix}-0.4 & 0.4\\ 0.3 & -0.3\end{bmatrix}$.

Mixed time-delay $\tau=\begin{bmatrix}0.4 & 0.3\\ 0.55 & 0.5\end{bmatrix}$.Let $E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$

$$A_1=\begin{bmatrix}-5 & 0 & 1\\ 0 & -7.5 & 0\\ 2 & 0 & -5\end{bmatrix}, A_2=\begin{bmatrix}-4.7 & 0 & -2\\ 0 & -8 & -3\\ 0 & 4 & -3\end{bmatrix}$$

$$D_1=\begin{bmatrix}0.2\\ 0.1\\ 0.1\end{bmatrix},\ D_2=\begin{bmatrix}0.1\\ 0\\ 0.2\end{bmatrix}, A_{\tau_{1,1}}=\begin{bmatrix}-0.1 & 0 & 0.2\\ 0 & 0.2 & 0\\ 0.1 & 0.2 & 0.1\end{bmatrix}$$

$$A_{\tau_{1,2}}=\begin{bmatrix}-1 & 0 & 0.05\\ 0.05 & 0 & 0\\ 0 & 0.05 & 0\end{bmatrix}, B_1=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},$$

$$B_2=\begin{bmatrix}1 & 0 & 0\\ 0 & 0.8 & 0\\ 0 & 0 & 1\end{bmatrix}, W_1=W_2=\begin{bmatrix}0.2\\ 0.1\\ 0.3\end{bmatrix}, G_1=G_2=\begin{bmatrix}-0.1\\ 0.3\\ 0.2\end{bmatrix}$$

$$H_1=H_2=\begin{bmatrix}0.01 & 0 & 0\\ 0 & 0.01 & 0\\ 0 & 0 & 0.02\end{bmatrix}, C_1=C_2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},$$

The uncertain matrices ΔA and $\Delta A_{\tau i}$ satisfy

$$\begin{bmatrix}\Delta A & \Delta A_{\tau i}\end{bmatrix}=UF(t)\begin{bmatrix}V_1 & V_2\end{bmatrix}$$

where choosing

$$U=\begin{bmatrix}0.2\\ 0.1\\ 0.1\end{bmatrix},\ V_1=\begin{bmatrix}0.1 & 0.1 & 0.3\end{bmatrix},\ V_2=\begin{bmatrix}0.2 & 0.1 & 0.2\end{bmatrix},$$

$F(t)=\sin t$, $\omega(t)=0.02\cos t$.

Non-linear function parameters are

$$Z_1=\begin{bmatrix}-1.6 & 0.2\\ 0.2 & -1.7\end{bmatrix},\ Z_2=\begin{bmatrix}0.4 & 0.2\\ 0.2 & 0.3\end{bmatrix}$$

According to references (Xiaohang Li et al., 2018; Li H Y et al.,2014), the actuator failure is assumed to be $f_a=\sin(5t)+e^{-2t}+2\cos(t)$, and sensor failure is assumed to be $f_s=\sin(t)+2\cos(5t)$. In the simulation, the initial states are $x_0=\begin{bmatrix}3 & 0 & 1\end{bmatrix}^T$ and $z_0=\begin{bmatrix}0 & 2 & 0\end{bmatrix}^T$, the mixed time-delay $\tau_i\in[0.1,0.4]$, and α=0.2 and γ=1.4 .

From equation (25), Theorem 1 and equation (26), we can get

$$T_1=T_2=\begin{bmatrix}0.5357 & -0.1071 & -0.0714\\ -0.1071 & 0.8214 & 0.2143\\ -0.0714 & 0.2143 & 0.6429\\ 0 & 0 & 0\\ 0.7143 & -2.1429 & -1.4286\end{bmatrix}$$

$$N_1=N_2=\begin{bmatrix}0.4643 & 0.1071 & 0.0714\\ 0.1071 & 0.1786 & -0.2143\\ 0.0714 & -0.2143 & 0.3571\\ 0 & 0 & 0\\ -0.7143 & 2.1429 & 1.4286\end{bmatrix}$$

$$M_1=\begin{bmatrix}-317.4898 & -32.0052 & 451.4227 & 0.0893 & 111.7303\\ -45.484 & 7.4505 & 98.5425 & 0.0822 & 28.6723\\ -74.5284 & -28.7417 & 70.3273 & 0.0714 & 14.1994\\ -10.483 & -3.7743 & 4.223 & 0 & 0.7606\\ -4.7499 & 9.619 & 3.6892 & -0.2143 & -2.9373\end{bmatrix}$$

$$M_2=\begin{bmatrix}77.7933 & -331.6777 & -172.7396 & 0.0393 & -142.1465\\ 9.5766 & -64.042 & -36.8562 & 0.0321 & -25.1982\\ 23.2395 & -66.4593 & -16.6112 & 0.1214 & -25.3218\\ -5.2023 & 6.5623 & 13.0979 & 0 & 5.1085\\ -9.4725 & 16.8842 & 6.5527 & -0.2143 & 1.7015\end{bmatrix}$$

The observer matrixes are as follows

$$L_1=\begin{bmatrix}116.2528 & 135.7762 & -145.5191\\ 12.6831 & 23.1726 & -28.4145\\ 33.3682 & 29.376 & -27.376\\ 4.9698 & 2.7024 & -1.568\\ -0.4924 & 0.577 & -1.1112\end{bmatrix},$$

$$L_2=\begin{bmatrix}9.4873 & 13.7552 & -15.9192\\ 3.8818 & 1.8156 & -0.7873\\ -2.3303 & 7.2335 & -12.0227\\ 0.7759 & 2.1927 & -2.9004\\ 2.778 & -1.2125 & 3.2093\end{bmatrix}$$

According to Theorem 2, the robust fault-tolerant controller can be obtained.

$$K_{c1}=\begin{bmatrix}-7.9832 & -4.8072 & 3.7404\\ 2.9263 & 18.0089 & 3.7485\\ -7.389 & -49.8477 & 8.1019\end{bmatrix},$$

$$K_{c2}=\begin{bmatrix}0.6245 & 6.9672 & -15.0092\\ -5.191 & 12.1203 & -7.9273\\ -5.0437 & 4.4937 & -10.2476\end{bmatrix}$$

If the system is normal, that is, no fault is considered in the system. The states estimation of the designed observer is shown in Fig.3 to Fig.5. The simulation results show that the estimated states of the observer can better track the actual states.

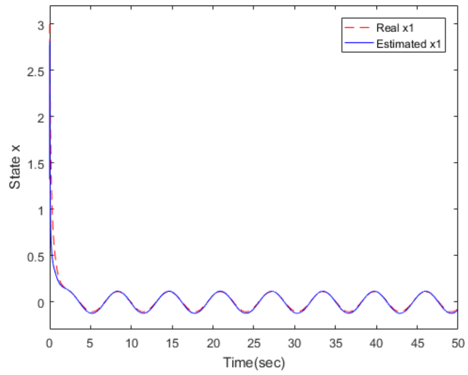

Fig. 3. State x_1 curves without failure.

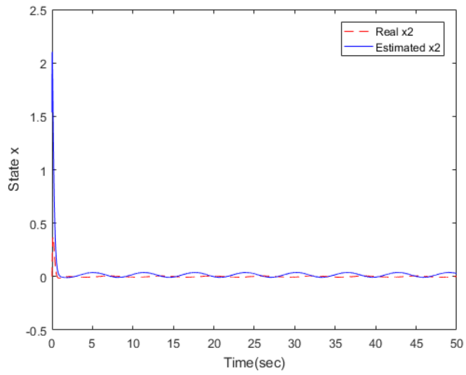

Fig. 4. State x_2 curves without failure.

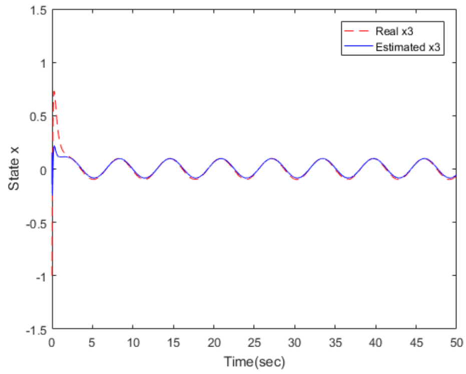

Fig. 5. State x_3 curves without failure.

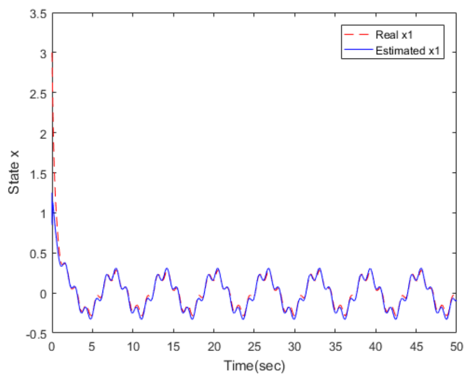

Fig. 6. State x_1 curves with failure.

If there are actuator failure and sensor failure in the system, the states of the observer are shown in Fig. 6 to Fig. 8. The simulation results show that the observer can still estimate the system states, and the robust fault tolerance is relatively strong, which is effective.

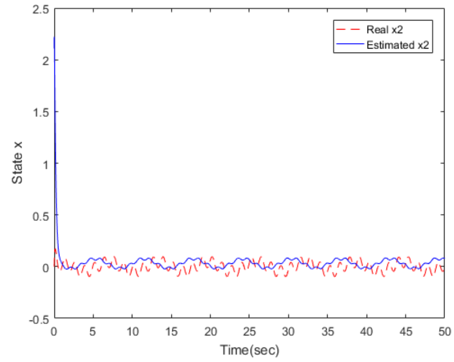

Fig. 7. State x_2 curves with failure.

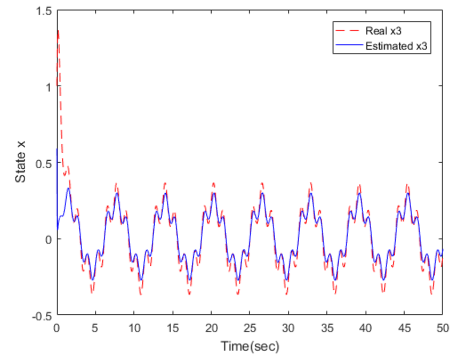

Fig. 8. State x_3 curves with failure.

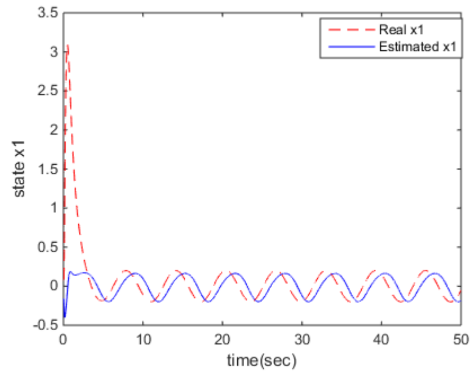

Fig. 9. State x1curves with failure (other reference).

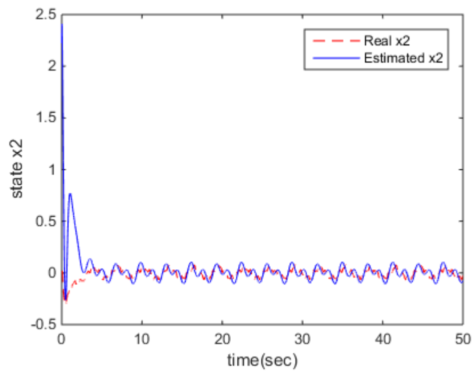

Fig. 10. State x2 curves with failure (other reference).

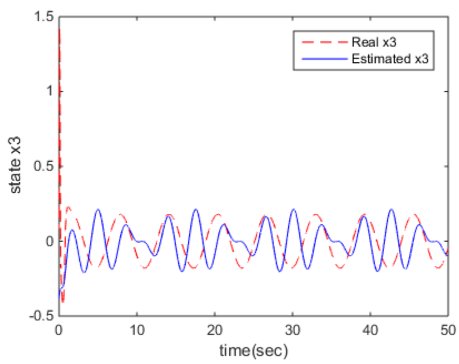

Fig. 11. State x3 curves with failure (other reference).

As a comparison, under the same condition of actuator failure and sensor failure, according to the method in the reference (Xiaohang Li et al., 2017), the state of the observer is shown in Fig.9 to Fig.11. It can be seen that the method in this paper is more accurate for states estimation.

5.2 UAV Application Simulation

Through the Linear parameter varying (LPV) method(GAO Zhenxing, 2018), the dynamic model of the four-rotor UAV can be transformed into a nominal linear space equation, as shown in the following equation (57).

$$\begin{cases} \dot{x}=Ax+Bu \\ y=Cx+Du \end{cases} \tag{57}$$

Now, consider a UAV motion system. The system state is $x(t)=\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T=\begin{bmatrix} \Delta\phi & \Delta\psi & \Delta\theta \end{bmatrix}^T$. Where $\Delta\phi$ is the roll rate, $\Delta\psi$ is the yaw rate, and $\Delta\theta$ is the pitch angle rate. The main parameters are referenced in (Zhao Xingcheng, 2019).

$$A=\begin{bmatrix} -8.4 & 2.19 & 0 \\ -0.35 & -0.761 & 0 \\ 1 & 0 & 0 \end{bmatrix}, B=\begin{bmatrix} 23.09 \\ -4.16 \\ 0 \end{bmatrix}, C=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Refer to other methods in (Xiaohang Li et al., 2017; Zhao Xingcheng, 2019; Lu Dong et al., 2018), other parameters are selected as follows

$$A_{\tau 1}=\begin{bmatrix} 0 & 0 & 0 \\ 0.5 & -0.02 & 0 \\ 0.06 & 0 & 0 \end{bmatrix}, H=\begin{bmatrix} 0.003 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.002 \end{bmatrix},$$

$$W=\begin{bmatrix} 0.001 \\ 0.002 \\ 0.004 \end{bmatrix}, D=\begin{bmatrix} 0 \\ -0.004 \\ 0.001 \end{bmatrix}, G=\begin{bmatrix} 0.008 \\ 0.002 \\ 0.006 \end{bmatrix}$$

The state probability transition matrix is $\delta=\begin{bmatrix} -0.4 & 0.4 \\ 0.3 & -0.3 \end{bmatrix}$, and other parameters value and the failure mode (f_a and f_s) are the same as section 5.1.

The robust fault-tolerant controller is

$$K_c=\begin{bmatrix} 0.1309 & 1.1515 & 0.5285 \end{bmatrix}$$

From Fig. 12 to Fig. 15, it can be seen that even if the UAV system has the faults, the UAV attitude can still maintain stability.

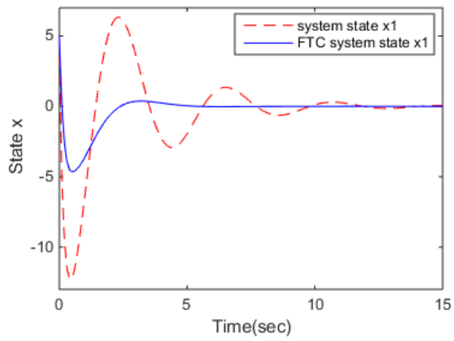

Fig. 12. The roll angle curve $\Delta\phi$ (x_1).

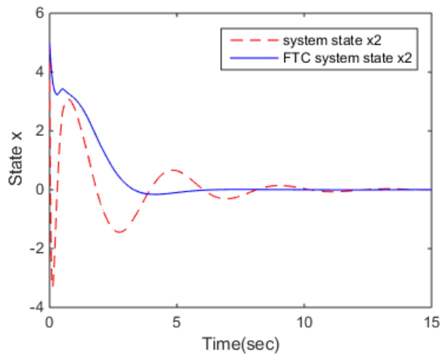

Fig. 13. The yaw angle curve $\Delta\psi$ (x_2).

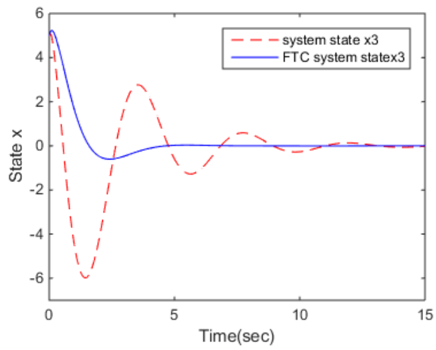

Fig. 14. The pitch angle curve $\Delta\theta$ (x_3).

In order to compare the results with similar control strategy, the simulation is studied according to the control method in the reference (Zhao Xingcheng, 2019). When there is the same actuator and sensor failure as this paper, the UAV attitude angles change as shown in Fig.15-Fig.17. It can be seen that the attitude angle curves can reach stability, but there are fluctuations, and the fluctuations will last a long time. This is because when the UAV system fails, the control system lacks fault tolerance to the external environment or its own failure. Using the control method in the reference (Zhao Xingcheng, 2019), the system can correct this uncertainty to a certain extent, but cannot continuously adjust and correct the failure effect, so the final control result will fluctuate. By comparison, it shows that the fault-tolerant control method in this paper is effective.

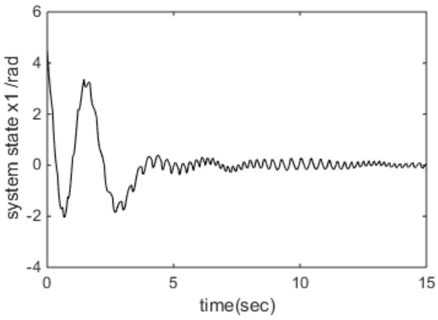

Fig. 15. The roll angle curve $\Delta\phi$ (x_1) (other reference).

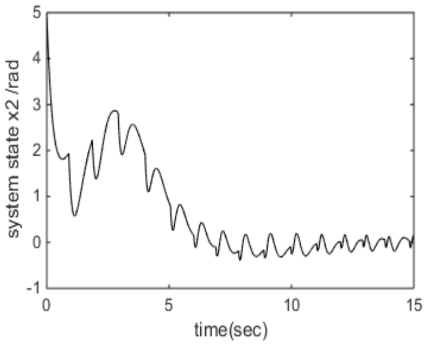

Fig. 16. The yaw angle curve $\Delta\psi$ (x_2)(other reference).

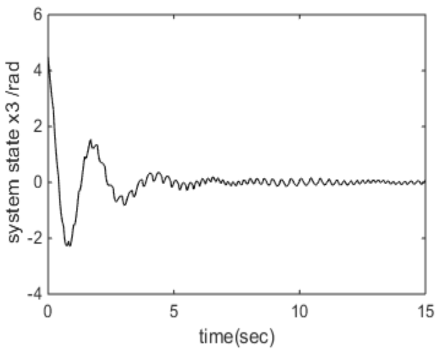

Fig. 17. The pitch angle curve $\Delta\theta$ (x_3)(other reference).

6. CONCLUSIONS

This paper focuses on a class of singular Markov switching complex systems with the mixed time-delays, and discusses the states estimation and fault-tolerant control of the system in the event of sensor failure, actuator failure, and external disturbances. Sensor and actuator faults are combined with system states to form a singular system. By designing adaptive observers, under sensor and actuator faults, the estimation of system states are achieved. In addition, for sensor and actuator failures, a fault-tolerant controller is designed to eliminate the effects of failures and disturbances to ensure the stability of the singular Markov switching system. The fault-tolerant control scheme based on control law compensation and fault signal reconstruction is adopted in this paper. The feasibility of the method is verified by numerical simulation. The conclusion is applied to the UAV's lateral motion system to prove the effectiveness of the method.

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