Non-linear Model-based Stochastic Fault Diagnosis of 2 DoF Helicopter

M Raghappriya * **S Kanthalakshmi** **

* *Government College of Technology, Coimbatore,India (e-mail: ragha_mms@gct.ac.in*
** *PSG College of Technology, Coimbatore,India (e-mail: skl.eee@psgtech.ac.in)*

Abstract: Fault diagnosis of non-linear helicopter systems are affected by inherent characteristics such as non-linear behaviour, high cross coupling effects, external disturbances such as atmospheric turbulence and wind effects. Fault diagnosis in non-linear systems gains importance due to its high complexity and this work focuses on fault detection of helicopter system with the consideration of the inherent non-linearity effects. This paper deals with the detection, identification and classification of sensor, actuator and component faults in non-linear helicopter systems using model-based state estimation approaches. Approaches include Interacting Multiple Model based Extended Kalman Filter and Interacting Multiple Model based Unscented Kalman Filter. To address problem of fault detection, statistical measures of residual analysis, stochastic likelihood ratio and model probability is proposed. A Comparison of these approaches is presented based on the ability to detect, identify and classify faults in spite of system non-linearity. Algorithm is applied to 2 degrees of freedom helicopter and the results for various fault cases are presented. The results yield better fault detection performance using Interacting Multiple Model based Unscented Kalman Filter.

Keywords: Fault detection and diagnosis, Non-linear, Aerospace, FDI for non-linear systems, Sensor and actuator faults, Model-based estimation and filtering.

1. INTRODUCTION

Safety, reliability and improved performance of modern day applications depends on sophisticated control algorithms. Any fault or failure in engineering systems can disrupt the normal functionality leading to decreased or unacceptable performance, less reliability and safety. Thus Fault Detection and Diagnosis (FDD) has a paramount importance to ensure reliable Fault Tolerant Control (FTC). See Zhang et al. (2008). FDD schemes plays a major role in safety related applications such as aerospace, Unmanned Aerial Vehicles (UAV), spacecraft and robotic applications. Most of the practical systems seen today are non-linear systems. Non-linear systems are affected by non-linear dynamics, cross couplings and uncertainty. Thus non-linear fault detection and diagnosis gains importance because of its ability to identify faults in spite of the effects of non-linear dynamics. See Du et al. (2014), Tan et al. (2015) and Yang et al. (2015). The control of small scale helicopters draws attention due to its popularity for short distance transportations because of its ability to take off and land without runways in small areas. The applications include transportation, air-sea rescue, firefighting, military and surveillance applications. These helicopter systems exhibits highly non-linear behavior and high cross coupling between different axis. It is also affected by external disturbances such as atmospheric turbulence, greater wind speed, icing, parametric uncertainties and unmodelled dynamics. See Marzat et al. (2015) and Van Eykeren et al. (2014). Fault diagnosis and classification of these highly non-linear cross coupled helicopter systems poses a major challenge. Various faults that occurs in helicopter systems include sensor faults, faults in aerodynamic propellers, and any other system components.

Many FDDs schemes have been developed mostly for monitoring purposes. Model based FDD schemes are predominantly used because of its fast speed of response. Model based methods are based on state estimation, parameter estimation or parity based methods which uses dynamic process or system models. See Isermann. (2005). Zolghadri. (2012) and Zolghadri et al. (2016) emphasizes the advantages of model-based fault detection methods to aerospace systems.

One of the effective methods is the Multiple Model (MM) scheme. Since MM approach provides sophisticated solutions to control, estimation and modelling problems, research on the multiple-model approach has attracted considerable interest in the last decades. See Blom, et al. (1988), Li, et al. (1996) and Li, et al. (2000). Some well-known examples to MM approach is the target tracking problem and Fault Detection and Diagnosis. The main advantage of this multiple model is a larger class of faults can be modelled. MM approach allows modelling of sensor, actuator as well as component faults since each local fault model can represent different dynamics. The first step of MM scheme is to design a normal system model and a set of models representing different fault conditions. MM scheme consist of a set of filters that runs in parallel where the overall state estimate is the probabilistically weighed sum of all estimates. The above MM filter is based on non-interaction between the set of models considered. MM estimation effectively handles problem with an unknown structure or parameter without structural or parametric changes. System undergoes structural or parametric changes hence this method will not produce effective results.

Another variant of MM filter is the Interacting Multiple Model (IMM) filter see Zhang et al. (1999), Mazor, et al. (2000),Menke, et al. (1995), Ding, et al. (1997) and Zhang et al. (2001) where the set of models considered interacts with each other by switching from one model to another providing better ability to identify the faulty condition or model occurring in that instant. Unlike MM estimation where there is no interaction between models, IMM interacts with set of models and switches from one model to another as a Markov chain system according to the probabilistic manner thereby resulting in improved performance. The mixture of all estimates from single model based filters acts as an initial estimate at the beginning of each cycle. The mixing of estimates enables a more accurate estimate of the system. The other characteristic of IMM is that the probability of each mode is calculated which clearly indicates the current mode in effect and mode transition at each time. Identification of single and multiple failures is another advantage of multiple model based approaches. While other FDD schemes uses residual based fault classification, see Zhang et al. (2008) and Wang, et al. (2015), IMM uses a stochastic based approach to detect and classify faults in the system. The set of model probabilities summarizes all the past and current information about the mode in effect and is reliable for fault decision making. Another stochastic approach is the set of likelihoods based on the innovations is white noise with zero mean and known covariance provides information about the abrupt and continuous sensor and actuator faults occurring in the system for classification of faults. See Deshpande et al. (2009), Kiasi et al. (2015), Dhler et al. (2016) and Jayaprasanth et al. (2018).

State estimation approach is one way to design a set of filters for the multiple models. Traditional IMM scheme as in Zhang et al. (1999) uses Kalman Filter (KF) to propagate the mean and covariance of the system states over time. This holds good for the linear varying systems. Helicopter systems posses higher degree of non-linearity. Since fault detection and diagnosis of non-linear system is proposed, variant of Kalman filter, see Chowdhary et al. (2010), known as Extended Kalman filter (EKF) is applied to non-linear system. EKFs propagate the state mean and covariance through linear approximation around the operating point by calculating the Jacobian matrices for non-linear systems. For higher order systems, another state estimation filter named as Unscented Kalman Filter (UKF) is developed for non-linear systems. The Unscented Kalman filter propagates the system through a set of points in Gaussian probability distribution function. See Kandepu et al. (2008).

Kargar et al. (2015) discussed an interactive bank of Extended Kalman Filter is used in IMM framework for the estimation of actuator faults. Residual based isolation technique is used to quantify and isolate the actuator faults in the system. Another IMM based fault detection approach is employed in high redundancy actuation system. A set of actuator fault models representing overheating, loose faults and lockup faults are considered and the fault detection is carried out using IMM framework in Davies et al. (2009). Another multiple model approach for FDI schemes is hierarchical multiple model approach which detects the sensor and actuator faults in the system is discussed in Hsiao et al. (2012). It covers a variety of faults such as incipient faults, abrupt faults and simultaneous faults. A likelihood based evaluation test reflects the missed detection probabilities.

Lu, et al. (2015) proposed sensor fault detection and estimation using augmented UKF for a kinematic model of a quadrotor UAV. The faults in sensors are modeled using random walk process and it is augmented with system states. UKF detects the faults and the system states simultaneously. Zhong, et al. (2019) proposed a fault diagnosis scheme for quadrotor helicopter covering sensor faults via adaptive two stage extended Kalman filter. A set of forgetting factors is introduced into the adaptive scheme to estimate the system with faults occurring separately and simultaneously. The main advantage here is FDD scheme is designed for a non-linear model of a quadrotor while it suffers from shortcomings such as consideration of only bias and drift sensor faults.

Freddi et al. (2010) developed a Thau observer which addresses the actuator fault detection of a mini-quadrotor. FDD strategy uses a residual generation and residual evaluation module which generates residuals and detects the change in it. It uses upper and lower threshold bound values to detect faults. The main disadvantage here is it detects and isolate actuator faults and does not estimate the faults. Simultaneous state and parameter estimation techniques for actuator faults in unmanned helicopter is dealt in Wu, et al. (2015). An Actuator Health Coefficients (AHC) defining the effect of actuator faults with the augmented flight states transforms the general structure into a non-linear state and parameter estimation problem. Actuator FDD performance is analyzed using three adaptive schemes; KF-UKF, MIT-UKF and MIT-ESMF. This method deals only with actuator faults affecting the system. Fault detection of 3 DOF helicopter based on Nonlinear Unknown Input Observer (NUIO) is developed in the paper, see Lan et al. (2017). Here the actuation faults, oscillation faults and saturation faults are considered. This achieve asymptotic estimation of faults without the need of system output derivatives.

Heredia et al. (2008) presented a sensor and actuator fault detection systems for small autonomous vehicles. An observer is designed for detection of fault using residual generation. However it can only detect the faults and not isolate and estimate the faults. Caliskan et al. (2016) proposed an adaptive two state Kalman Filter based FDD scheme detects the sensor and actuator faults in UAV. To estimate the loss of control effectiveness, degree of struck magnitude and fault isolation, control effectiveness factor and struck magnitude parameters are given to active FTC scheme. Zhong, et al. (2018) proposed a methodology to detect actuator and sensor faults for unmanned quadrotor helicopters. They proposed IMM approach with state augmentation strategy to reduce the computational load on large model set design. The loss of control effectiveness in actuators and bias in sensor faults are handled. The main shortcoming is the model is linearized and the FDD strategy is designed for a linear model excluding the inherent non-linearity.

As seen from literature, most of the FDD strategies deals with either actuator faults or sensor faults. Few works are available for both actuator and sensor fault detection. However many of the works mentioned above uses a linear model of helicopter rather than a non-linear model. Since the helicopter model considered has severe non-linearity effects and high degree of interaction between its variables, occurrence of faults tends to magnify this non-linearity and interaction. Hence only a non-linear model can clearly portray these effects. Hence an FDD scheme is proposed for this non-linear helicopter model with IMM scheme as the backbone. IMM scheme with its significant advantages, tends to represent the fault models effectively. It also gives a good statistical measure of fault decision based on the set of likelihood and mode probabilities. The non-linearity and interaction is handled using two model-based state estimation approaches EKF and UKF which estimates the states and covariance based on system model. Apart from sensor and actuator faults, component faults occurring in the system is also considered in this paper. The main objective is to design a model-based non-linear FDD scheme combining the Interacting Multiple Model (IMM) with Extended Kalman filter (EKF) and Unscented Kalman Filter (UKF). Two strategies, IMM based EKF and IMM based UKF were developed for handling sensor, actuator and component faults. The helicopter model chosen is Quanser 2 Degrees of Freedom (DOF) helicopter.

The paper is organized as follows. In section 2, mathematical modeling of 2 DoF helicopter is provided. IMM-EKF and IMM-UKF based fault detection strategy is discussed in section 3. Experimental results from simulations are provided in section 4 and conclusions in section 5.

2. MATHEMATICAL MODELING OF 2 DOF HELICOPTER

2.1 Non-linear equations of motion

Two degrees of freedom helicopter consist of two propellers driven by DC motors which is mounted on a fixed base. The motion of front propeller elevates the pitch axis and the motion of back propeller elevates the yaw axis. The pitch and yaw axis angles are measured using high resolution encoders. The two degrees of freedom are pitch angle represented by θ and yaw angle represented by ψ. The mathematical modeling of 2 DoF helicopter is derived using the following conventions.

(1) When pitch angle is zero, the helicopter is horizontal and pitch angle increases positively, when nose is moved upwards and body rotates in counter clockwise direction.
(2) Yaw angle increases positively when the body rotates in clockwise direction.
(3) Both pitch and yaw angle increases when their respective thrust forces are positive.

Fig. 1 Kumar et al. (2015) describes the kinematic diagram of 2 DoF helicopter. The thrust forces F_p and F_y are applied across the pitch and yaw axis respectively. The torques acts at a distance r_p and r_y from the respective axis. F_g is the gravitational force which acts on helicopter body. The centre of mass acts at a distance l_{cm} from the helicopter body.The transformation matrix for translation and rotation are defined as

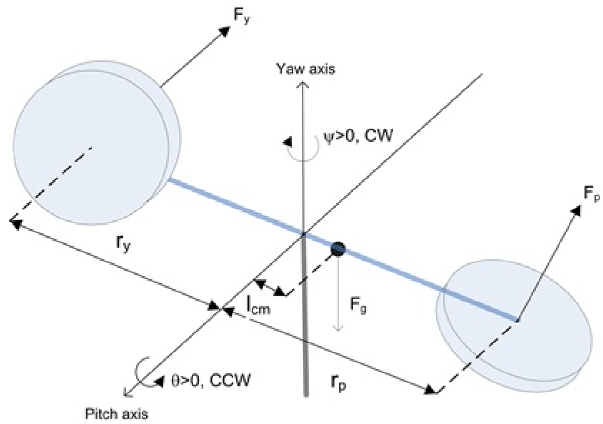

Fig. 1. Kinematics of 2 DOF Helicopter

T_θ - Rotation about pitch axis

$$T_\theta = \begin{bmatrix} cos(-\theta) & 0 & sin(-\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(-\theta) & 0 & cos(-\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

T_ψ - Rotation about yaw axis

$$T_\psi = \begin{bmatrix} cos(-\psi) & -sin(-\psi) & 0 & 0 \\ sin(-\psi) & cos(-\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

T_{cm} - Translation about centre of mass

$$T_{cm} = \begin{bmatrix} 1 & 0 & 0 & l_{cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Thus the resultant transformation matrix is

$$T_o = T_\psi T_\theta T_{cm} \tag{4}$$

$$T_o = \begin{bmatrix} cos\psi cos\theta & sin\psi & -cos\psi sin\theta & l_{cm} cos\psi cos\theta \\ -sin\psi cos\theta & cos\psi & sin\psi sin\theta & -l_{cm} sin\psi cos\theta \\ sin\theta & 0 & cos\theta & l_{cm} sin\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The potential energy V due to gravity is

$$V = m_{heli} g sin\theta l_{cm} \tag{6}$$

The total kinetic energy T of the system is the sum of kinetic energy due to rotation of pitch $T_{r,p}$ and kinetic energy due to rotation of yaw $T_{r,y}$ and kinetic energy due to translational movement of centre of mass T_t.

$$T = T_{r,p} + T_{r,y} + T_t \tag{7}$$

The rotational kinetic energy is related to angular movement and total moment of inertia of pitch $J_{eq,p}$ and yaw axis $J_{eq,y}$ respectively. The equivalent moment of inertia of pitch and yaw about the centre of mass are

$$J_{eq,p} = J_{m,p} + J_{body,p} + J_p + J_y \tag{8}$$

$$J_{eq,y} = J_{m,y} + J_{body,y} + J_p + J_y + T_{shaft} \tag{9}$$

Where $J_{m,p}$ and $J_{m,y}$ are rotor moment of inertia of pitch and yaw motor respectively; $J_{body,p}$ and $J_{body,y}$ are moment of inertia of helicopter body about pitch and yaw axis respectively; J_p and J_y are moment of inertia of shield assembly about pitch and yaw pivot respectively; J_{shaft} is moment of inertia of metal shaft about yaw axis end pivot.

$$J_{body,p} = \frac{m_{body,p} L_{body}^2}{12} \quad (10)$$

$$J_{body,y} = \frac{m_{body,y} L_{body}^2}{12} \quad (11)$$

$$J_{shaft} = \frac{m_{shaft}(R_1^2 + R_2^2)}{2} \quad (12)$$

$$J_p = (m_{m_p} + m_{support,p}) r_p^2 \quad (13)$$

$$J_y = (m_{m_y} + m_{support,y}) r_y^2 \quad (14)$$

The rotational kinetic energy of the pitch and yaw axis is

$$T_{r,p} = \frac{1}{2} J_{eq,p} \dot{\theta}^2 \quad (15)$$

$$T_{r,y} = \frac{1}{2} J_{eq,y} \dot{\psi}^2 \quad (16)$$

The kinetic energy due to translational movement is

$$\begin{aligned} T_t = \frac{1}{2} m_{heli} [(-sin\psi\dot{\psi}cos\theta l_{cm} - cos\psi sin\theta\dot{\theta} l_{cm})^2 + \\ (-cos\psi\dot{\psi}cos\theta l_{cm} + sin\psi sin\theta\dot{\psi} l_{cm})^2 \\ + (cos\theta\dot{\theta} l_{cm})^2] \end{aligned} \quad (17)$$

Simplifying,

$$T_t = \frac{1}{2} m_{heli} l_{cm}^2 [\dot{\theta}^2 + \dot{\psi}^2 cos^2\theta] \quad (18)$$

Thus the total kinetic energy is

$$T = \frac{1}{2} J_{eq,p} \dot{\theta}^2 + \frac{1}{2} J_{eq,y} \dot{\psi}^2 + \frac{1}{2} m_{heli} l_{cm}^2 [\dot{\theta}^2 + \dot{\psi}^2 cos^2\theta] \quad (19)$$

The non-linear motion model of helicopter system is derived using Euler's-Lagrange equation. The Lagrange variable L is defined as difference between kinetic energy and potential energy.

$$\begin{aligned} L = \frac{1}{2} J_{eq,p} \dot{\theta}^2 + \frac{1}{2} J_{eq,y} \dot{\psi}^2 + \frac{1}{2} m_{heli} l_{cm}^2 [\dot{\theta}^2 + \dot{\psi}^2 cos^2\theta] \\ - m_{heli} g sin\theta l_{cm} \end{aligned} \quad (20)$$

The generalized co-ordinates of the helicopter system that describes the non-linear dynamics are pitch angle, yaw angle, pitch velocity, yaw velocity represented as $[\theta, \psi, \dot{\theta}, \dot{\psi}]$ The Euler's–Lagrange equation of motion is

$$[\frac{\partial^2}{\partial t \partial\theta} L] - [\frac{\partial}{\partial\theta} L] = Q_1 \quad (21)$$

$$[\frac{\partial^2}{\partial t \partial\psi} L] - [\frac{\partial}{\partial\psi} L] = Q_2 \quad (22)$$

where Q_1 and Q_2 are generalized forces corresponding to generalized co-ordinates. The generalized forces of the system are

$$Q_1 = \tau_p(V_{m,p}, V_{m,y}) - B_p \dot{\theta} \quad (23)$$

$$Q_2 = \tau_y(V_{m,p}, V_{m,y}) - B_y \dot{\psi} \quad (24)$$

where B_p and B_y are the viscous rotary friction acting about the pitch and yaw axis respectively. The torque about pitch and yaw axis is

$$\tau_p(V_{m,p}, V_{m,y}) = K_{pp} V_{m,p} + K_{py} V_{m,y} \quad (25)$$

$$\tau_y(V_{m,p}, V_{m,y}) = K_{yp} V_{m,p} + K_{yy} V_{m,y} \quad (26)$$

where $V_{m,p}$ and $V_{m,y}$ are input pitch and yaw motor voltage. The torque constants that acts at a distance r_p and r_y from the centre of mass is

$$K_{pp} = K_{f,p} r_p \quad (27)$$

$$K_{yy} = K_{f,y} r_y \quad (28)$$

$$K_{py} = \frac{K_{t,y}}{R_{m,y}} \quad (29)$$

$$K_{yp} = \frac{K_{t,p}}{R_{m,p}} \quad (30)$$

where $K_{f,p}$ and $K_{f,y}$ are thrust force constants of pitch and yaw respectively and $K_{t,y}$ and $K_{t,p}$ are current torque constants of pitch and yaw respectively. The electrical resistance of pitch and yaw motor are $R_{m,p}$ and $R_{m,y}$. Thus Q_1 and Q_2 become

$$Q_1 = K_{pp} V_{m,p} + K_{py} V_{m,y} - B_p \dot{\theta} \quad (31)$$

$$Q_2 = K_{yp} V_{m,p} + K_{yy} V_{m,y} - B_y \dot{\psi} \quad (32)$$

The Lagrangian is determined as

$$\frac{\partial^2}{\partial t \partial\theta} L = J_{eq,p} \ddot{\theta} + m_{heli} l_{cm}^2 \ddot{\theta} \quad (33)$$

$$\frac{\partial}{\partial\theta} L = -m_{heli} g l_{cm} cos\theta - m_{heli} l_{cm}^2 sin\theta cos\theta \dot{\psi}^2 \quad (34)$$

$$\frac{\partial^2}{\partial t \partial\psi} L = J_{eq,y} \ddot{\psi} + m_{heli} l_{cm} cos^2\theta \ddot{\psi} \quad (35)$$

$$\frac{\partial}{\partial\psi} L = -2 m_{heli} l_{cm}^2 sin\theta cos\theta \dot{\theta} \dot{\psi} \quad (36)$$

Substituting equations (31) to (36) in Lagrangian described in equations (21) and (22), the non-linear equations of motion of helicopter model is

$$\begin{aligned} (J_{eq,p} + m_{heli} l_{cm}^2) \ddot{\theta} = K_{pp} V_{m,p} + K_{py} V_{m,y} - B_p \dot{\theta} \\ - m_{heli} g l_{cm} cos\theta - m_{heli} l_{cm}^2 sin\theta cos\theta \dot{\psi}^2 \end{aligned} \quad (37)$$

$$\begin{aligned} (J_{eq,y} + m_{heli} l_{cm}^2 cos^2\theta) \ddot{\psi} = K_{yp} V_{m,p} + K_{yy} V_{m,y} - B_y \dot{\psi} \\ + 2 m_{heli} l_{cm}^2 sin\theta cos\theta \dot{\theta} \dot{\psi} \end{aligned} \quad (38)$$

2.2 Helicopter State Space model

The state-space model of helicopter system is given by

$$\dot{X} = f(x) + g(X, u) + w \quad (39)$$

$$y = h(x) + v \quad (40)$$

where $X = [x_1, x_2, x_3, x_4]$ and $u = [u_1, u_2]$, considering state vectors as pitch angle θ, yaw angle ψ, pitch velocity $\dot{\theta}$ and yaw velocity $\dot{\psi}$ and the inputs u_1 as pitch motor voltage $V_{m,p}$ and u_2 as yaw motor voltage $V_{m,y}$. w and v are parameters which addresses unknown disturbances affecting the pitch and yaw axis respectively. These unknown disturbances account for non-modeled dynamics, parametric uncertainties and other external disturbances like wind effects, icing and atmospheric turbulence affecting the system. Non-modeled dynamics are due to gyroscopic effects and couplings caused by the pitch and yaw propeller. Thus the non-linear state space model is

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{1}{J_{eq,p}+m_{heli}l_{cm}^2}[-(B_p x_3 + \\ m_{heli}l_{cm}^2 x_4^2 sinx_1 cosx_1 + m_{heli} g l_{cm} cosx_1) + w] \\ \frac{1}{J_{eq,y}+m_{heli}l_{cm}^2 cos^2 x_1}[-(B_y x_4 + \\ 2m_{heli}l_{cm}^2 x_3 x_4 sinx_1 cosx_1) + w] \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{eq,p}+m_{heli}l_{cm}^2} & \frac{K_{py}}{J_{eq,p}+m_{heli}l_{cm}^2} \\ \frac{K_{yp}}{J_{eq,y}+m_{heli}l_{cm}^2 cos^2 x_1} & \frac{K_{yy}}{J_{eq,y}+m_{heli}l_{cm}^2 cos^2 x_1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (41)$$

The output vector y is the measurement vector which contains the pitch and yaw co-ordinates.

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v \quad (42)$$

Thus w and v are assumed as Gaussian process and measurement noise vectors with covariances Q and R respectively. They are assumed to be white, independent, zero mean with normal probability distributions.

$$p(w) = N(0, Q) \; and \; p(v) = N(0, R) \quad (43)$$

2.3 Helicopter Normal and Fault Models

For FDD using multiple model approach, it is assumed that the actual system at any given time is modeled by the following stochastic hybrid non-linear system.

$$x(k+1) = f(k, x(k), u(k), m(k+1)) + w_k \quad (44)$$
$$y(k+1) = h(k, x(k+1), u(k), m(k)) + v_k \quad (45)$$

where f and h are functions of state and measurement output respectively. The mode sequence (normal and fault modes) of the system at time k is selected by transition probabilities $\pi_{ij}(k) = P\{m_j(k+1)|m_i(k)\}, \forall m_i, m_j \in M$ and $\sum_j \pi_{ij} = 1, \; i = 1, ...M$. The multiple models normal and faulty modes are a set of models $M = \{m_1....m_N\}$, where m_1 denotes normal mode and $\{m_2....m_N\}$ denotes faulty modes. The mode j in effect at time k is denoted by $m_j(k)$.

The predominant faults that affect the helicopter system includes sensor faults, actuator faults and component faults. Component faults represents the changes in the physical parameters of the helicopter like change in mass, change in aerodynamic co-efficient, change in centre of gravity etc., Partial faults such as stuck actuator due to lack of lubrication, a hydraulic leakage or decrease in supply voltage results in actuator faults. Sensor faults comprises reduction in gain, constant offset or bias, sensor drift etc., A total of five faulty modes are considered comprising two sensor faults, two actuator faults and a component fault. The various fault modes are applied as additive and multiplicative faults to the system which are considered partial system failures.

The normal and faulty models chosen are

- $m_1(j = 1)$ Normal mode
- $m_2(j = 2)$ Component fault mode with 20 % change in mass of helicopter
- $m_3(j = 3)$ Actuator 1 pitch motor fault mode with 20 % decrease in supply to actuator
- $m_4(j = 4)$ - Actuator 2 yaw motor fault mode with 20 % decrease in supply to actuator
- $m_5(j = 5)$ Sensor 1 pitch encoder fault mode with 20 % sensor bias
- $m_6(j = 6)$ Sensor 2 yaw encoder fault mode with 20 % sensor bias

Considering various faults occurring in the system, the general non-linear model structure becomes

$$x(k+1) = f(k, x(k), u(k), m(k+1), \gamma(k)) + w_k \quad (46)$$
$$y(k+1) = h(k, x(k+1), u(k), m(k)) + v_k \quad (47)$$

where γ denotes the fault bias vector or the fault scaling factor of effectiveness representing the faults.

3. INTERACTING MULTIPLE MODEL BASED FDD SCHEME

3.1 Basic IMM Algorithm

IMM based filter forms the basis for fault detection. IMM algorithm is a recursive estimator which carries out switching among various models. The set of models are chosen as models $m_1....m_N$, where N denotes the total number of models. The various steps of IMM algorithm is detailed as follows.

Step 1: Interaction or mixing of the model-conditioned estimates: The input to the non-linear filter is acquired by combining the estimates of all non-linear filters with the probabilities from previous time instant. The combined estimate and covariance is given by
Mixing estimate:

$$\hat{x}_j^0(k|k) = \sum_i \hat{x}_i(k|k)\mu_{i|j}(k), i, j = 1....N \quad (48)$$

Mixing covariance:

$$\hat{P}_j^0(k|k) = \sum_i \{\hat{P}_i(k|k) + [\hat{x}_j^0(k|k) - \hat{x}_i(k|k)] \\ [\hat{x}_j^0(k|k) - \hat{x}_i(k|k)]' \mu_{i|j}(k)\} i, j = 1....N \quad (49)$$

where

$$\mu_{i|j}(k) = \frac{\pi_{ij}\mu_i(k)}{\mu_j(k+1|k)} \quad (50)$$

The calculation of mixing probability $\mu_{(i|j)}(k)$ is done with mode switching probability matrix π_{ij} which denotes transition from mode i to mode j and the predicted mode probability $\mu_j(k+1|k)$. Predicted mode probability $\mu_j(k+1|k)$ is given by $\mu_j(k+1|k) = \sum_i \pi_{ij}\mu_i(k)$

Step 2:Model Conditioned Filtering: A set of non-linear filters like EKF, UKF in parallel corresponding to different models is chosen and the individual updated state $\hat{x}_j(k+1|k+1)$ and covariance $P_j(k+1|k+1)$ of models are obtained.

Step 3: Mode Probability Update: Based on innovation and the likelihood ratio, mode probability is updated. The likelihood ratio and the mode probability denotes the target mode in effect at time k. For fault detection and diagnosis, both likelihood ratio and mode probability provides indication of faults occurring in the system. Likelihood ratio is calculated based on measurement residual and residual covariance.

Likelihood function:

$$L_j(k+1) = \frac{1}{\sqrt{|2\pi S_j(k+1)|}} exp\{-\frac{1}{2}v_j(k+1)' S_j^{-1}(k+1) \\ v_j(k+1)\} \quad (51)$$

Model probability:

$$\mu_j(k+1) = \frac{\mu_j(k+1|k)L_j(k+1)}{\sum_i \mu_j(k+1|k)L_j(l+1)} \quad (52)$$

Step 4: Estimate Combination: The combined state estimate is calculated as a probability-weighted sum of the updated state estimates from all the filters.

Combined state estimate:

$$\hat{x}(k+1|k+1) = \sum_j \hat{x}_j(k+1|k)\mu_j(k+1) \quad (53)$$

Overall covariance:

$$P(k+1|k+1) = \sum_j \{P_j(k+1|k+1)+ \\ [\hat{x}(k+1|k+1) - \hat{x_j}(k+1|k+1)] \\ [\hat{x}(k+1|k+1) - \hat{x_j}(k+1|k+1)]'\}\mu_j(k+1) \quad (54)$$

3.2 Model Conditioned EKF Filter

Extended Kalman filter is a recursive predictive filter which operates by propagating the mean and covariance of state through time. The EKF acts as model conditioned filter in the IMM-EKF algorithm. The non-linear model is approximated at mean estimate at time k by using first order Taylor series expansion. The approximated Jacobian model $F(k)$ and $H(k)$ are given by

$$F(k) = \bigtriangledown f_k|_{\hat{x}(k|k)} \quad (55)$$

$$H(k) = \bigtriangledown h_k|_{\hat{x}(k+1|k)} \quad (56)$$

Where f and h are non-linear functions and $\bigtriangledown$ is Jacobian. The time update equations for state and covariance are as follows

$$\hat{x_j}(k+1|k) = f_k(\hat{x}(k|k)) \quad (57)$$

$$P_j(k+1|k) = F_(k)P(k|k)F^T(k) + Q(k) \quad (58)$$

In the measurement update, Kalman gain $K(k+1)$ is computed and the state and covariance are updated according to Kalman gain and current measurement at instant k.

$$v_j(k+1) = y_{k+1} - h_{k+1}(\hat{x}(k+1|k)) \quad (59)$$

$$\hat{x_j}(k+1|k+1) = \hat{x}(k+1||k)+K(k+1)[y_{k+1} - h_{k+1}(\hat{x}(k+1|k))] \quad (60)$$

$$S_j(k+1) = H(k+1)P(k+1|k)H^T(k+1) + R(k+1) \quad (61)$$

$$K_j(k+1) = P(k+1)H^T(k+1)[H(k+1)P(k+1|k) \\ H^T(k+1) + R(k+1)]^{-1} \quad (62)$$

$$P_j(k+1|k+1) = [I - K(k+1)H(k+1)P(k+1|k)] \quad (63)$$

3.3 Model Conditioned UKF Filter

The Unscented Kalman Filter (UKF) is based on Unscented Transformations which transforms the probability density function of non-linear systems through a set of individual points called sigma points in state space. The steps of UKF algorithm are detailed further. The $(2L+1)$ sigma points are computed as

$$X_{k|k} = [\hat{x}(k|k) \quad \hat{x}(k|k) + \sqrt{(L+\lambda)P(k|k)} \\ \hat{x}(k|k) - \sqrt{(L+\lambda)P(k|k)}] \quad (64)$$

where $\lambda = \alpha^2(L+\kappa) - L$. L denotes the dimension of state space model. α and κ are tuning parameters. α represents the scaling parameter which determines the spread of sigma points around mean state where α varies as $0.0001 \leq \alpha \leq 1$. $\kappa = 0$ ensures the semi-positive definiteness of covariance matrix.

The computed sigma points are transformed via state update functions as in (65) and the predicted state and covariance matrix from time instant k to k+1 are calculated through (66) and (67) respectively.

$$X_{k+1|k} = F(k, X_{k|k}, u_k) \quad (65)$$

$$\hat{x}(k+1|k) = \sum_{i=0}^{2L} W_i^m X_{i,k+1|k} \quad (66)$$

$$P(k+1|k) = \sum_0^{2L} W_i^c[(X_{i,k+1|k} - \hat{x}(k+1|k)) \\ (X_{i,k+1|k} - \hat{x}(k+1|k))^T] + Q \quad (67)$$

where the weights in the function are updated as follows

$$W_0^m = \frac{\lambda}{L+\lambda} \quad (68)$$

$$W_0^c = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta) \quad (69)$$

$$W_i^m = W_i^c = \frac{1}{2(L+\lambda)}, i = 1.....2L \quad (70)$$

β is the parameter which incorporates the prior knowledge about the distribution of state. For Gaussian distributions, β is set to 2. For time instant $k+1$, the output sigma point matrix is computed as in (71) and the predicted output is obtained through (72).

$$Y_{k+1|k} = H(k, X_{k+1|k}, u_k) \quad (71)$$

$$\hat{y}(k+1|k) = \sum_{i=0}^{2L} W_i^m Y_{i,k+1|k} \quad (72)$$

The measurement residual for each mode j is computed as

$$v_j(k+1) = z(k+1) - \hat{y}_j(k+1|k), j = 1, 2, ...N \quad (73)$$

where $z(k+1)$ signifies the new measurement of active mode and $\hat{y}_j(k+1|k)$ denotes the predicted output for mode j.

The output covariance or residual covariance and the cross covariance matrix are

$$P_{yy}(k+1|k) = \sum_0^{2L} W_i^c[(Y_{i,k+1|k} - \hat{y}(k+1|k)) \\ (Y_{i,k+1|k} - \hat{y}(k+1|k))^T] + R_v \quad (74)$$

$$P_{xy}(k+1|k) = \sum_0^{2L} W_i^c[(X_{i,k+1|k} - \hat{x}(k+1|k)) \\ (Y_{i,k+1|k} - \hat{y}(k+1|k))^T] \quad (75)$$

The Kalman gain $K(k+1)$ is

$$K(k+1) = P_{xy}(k+1|k)(P_{yy}(k+1|k))^{-1} \quad (76)$$

The updated state and covariance matrix at instant k+1 are

$$\hat{x_j}(k+1|k+1) = \hat{x_j}(k+1|k) + K(k+1)v_j(k+1), j = 1, 2, ...N \quad (77)$$

$$P_j(k+1|k+1) = P_j(k+1|k) - K(k+1)P_{yy}(k+1|k)^{-1} K(k+1)^T \quad (78)$$

3.4 FDD Strategy

In this paper, sensor, actuator and components faults are detected based on the generation of residuals. Continuous evaluation of residuals, v_j between model conditioned filter and the measurements is computed.

$$v_j(k+1) = y_{k+1} - h_{k+1}(\hat{x}(k+1|k)) \quad (79)$$

Under normal conditions, the mean value of residuals will be closer to zero. When fault occurs, residual value varies beyond zero based on fault magnitude. In addition, computation of likelihood ratio, L_j is another parameter considered for fault detection. Likelihood is generally computed using residual and residual covariance.

$$L_j(k+1) = \frac{1}{\sqrt{|2\pi S_j(k+1)|}} exp\{-\frac{1}{2} v_j(k+1)' S_j^{-1}(k+1) v_j(k+1)\} \quad (80)$$

The likelihood of normal mode is computed and when there is fault, the likelihood of normal mode in effect tends to zero. Likelihood value approaching zero indicates the occurence of faults. Though these strategies, FDD scheme computes the type of fault occurred, magnitude of fault and the time of fault occurrence.

FDD ensures fault occurrence using another statistical parameter of mode probability at any given instant. Mode probability vector is computed as

$$\bar{\mu}(k+1) = [\mu_1(k+1) \; \mu_2(k+1) ......\mu_N(k+1)] \quad (81)$$

The maximum value of mode probability vector is

$$\mu_{max} = max_j\{\mu_j(k+1)\} \quad (82)$$

and the index j is computed as

$$j = find(\bar{\mu} == max(\bar{\mu})) \quad (83)$$

If $\mu_{max} > \mu_{Threshold}$, then fault has occurred in index mode j. Otherwise there is no fault occurrence in the system.

4. RESULTS AND DISCUSSIONS

Simulation studies were carried out for non-linear fault detection and diagnosis using the model-based state estimation techniques, IMM-EKF and IMM-UKF for the helicopter model. The initial state is assumed at the origin and system is excited with a step input. Model conditioned filter designed actively tracks the helicopter states for any abnormalities in the system response due to faults. Thus the filter designed gives only the information and decision of fault occurrence in helicopter. In case fault occurs, decision on fault occurence is made based on the measurement residuals generated and on computation of likelihood ratio. Also the decision to classify the faults into sensor, actuator and component fault is accomplished using mode probability. Sensor, actuator and component faults are simulated and the results are discussed below.

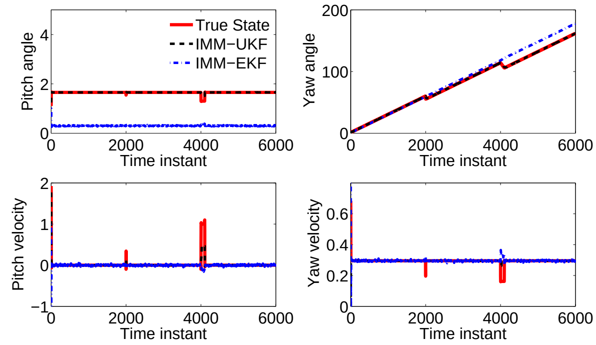

Fig. 2. Helicopter states with additive component fault

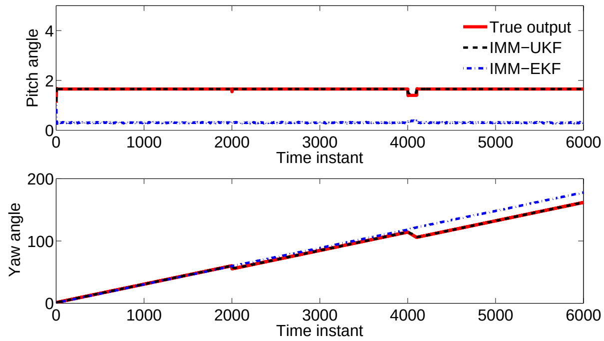

Fig. 3. Pitch and yaw angle with additive component fault

4.1 Design Parameters

The initial covariance for all possible cases is assumed to be $P = diag\{10 \;\; 10 \;\; 10 \;\; 10\}$. The process and measurement noise covariance are assumed as $Q = diag\{0.002 \;\; 0.002 \;\; 0.002 \;\; 0.002\}$ and $R = diag\{0.9 \;\; 0.9\}$. It is assumed that the initial mode probability for all models is $\frac{1}{N}$ where $N = 6$ represents the number of modes. The mode transitions are restricted to *normal to normal* mode and *normal to fault* modes. Since there is non-availability of prior knowledge of partial faults causing simultaneous faults, *fault to fault* mode transitions are not considered. Thus the transition mode probability matrix is set as

$$\pi_{ij} = \begin{bmatrix} 0.958 & 0.008 & 0.008 & 0.008 & 0.008 & 0.008 \\ 0.042 & 0.958 & 0 & 0 & 0 & 0 \\ 0.042 & 0 & 0.958 & 0 & 0 & 0 \\ 0.042 & 0 & 0 & 0.958 & 0 & 0 \\ 0.042 & 0 & 0 & 0 & 0.958 & 0 \\ 0.042 & 0 & 0 & 0 & 0 & 0.958 \end{bmatrix} \quad (84)$$

4.2 FDD Results

Simulation is carried out for 6000 iterations. Various fault cases are tested with both IMM-EKF and IMM-UKF filters and the results are plotted together and compared. The fault cases occurs as abrupt and continuous fault at time instant $t = 2000$ and $t = 4000$ *to* 4100 respectively.

Component faults. The state, output tracking and fault diagnosis of non-linear helicopter model with additive and multiplicative component faults using IMM-EKF and IMM-UKF algorithm is given in Fig. 2 to Fig. 6. As

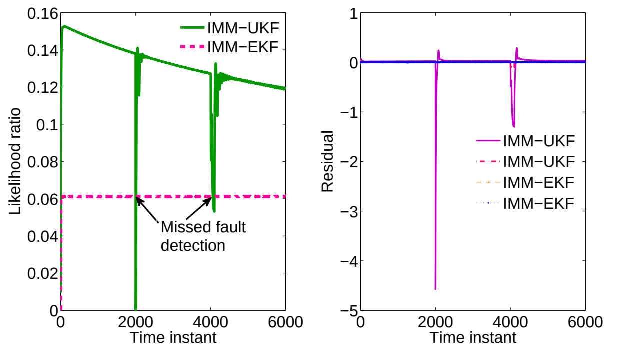

Fig. 4. Likelihood ratio and residual of additive component fault

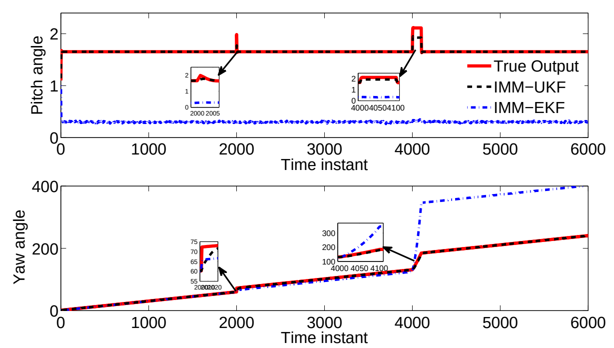

Fig. 5. Pitch and yaw angle with multiplicative component fault

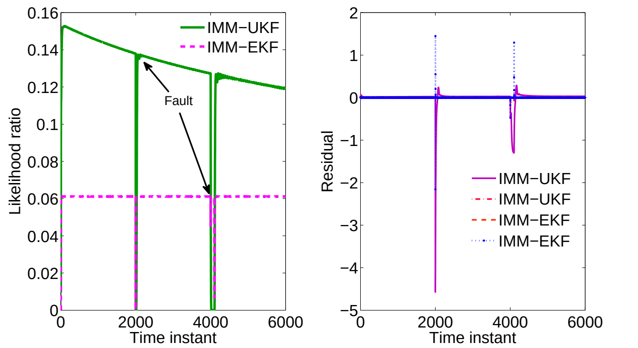

Fig. 6. Likelihood ratio and residual of multiplicative component fault

seen from Fig. 3 and Fig. 5 both IMM-EKF and IMM-UKF continuously tracks the pitch angle and yaw angle of helicopter. When fault occurs, the change in parametric values of helicopter mass affects system output instantly and the increase in pitch and yaw angle is seen as there is a variation in their corresponding velocities. Since the model being highly non-linear with cross coupling effects as seen in Fig. 2, fault in change in mass of helicopter affects all other system states. Also IMM-UKF tracks faults with minimum error than IMM-EKF.

The fault detection and diagnosis is carried over with error residual analysis and the likelihood information. Fig. 4 and 6 illustrates the residual error and model likelihood ratio of component fault. The results shows the model likelihood ratio provides the reliable fault detection as compared with residual error since likelihood ration is based on both measurement residual and residual covariance. Also IMM-UKF yields better fault detection, as IMM-EKF fails to detect the presence of additive fault because of its inability to overcome the non-linearity behavior.

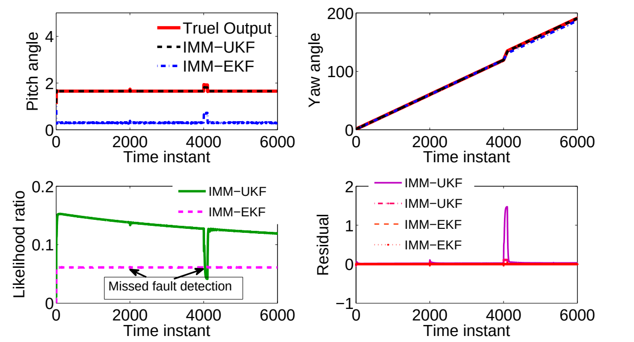

Fig. 7. Additive actuator fault

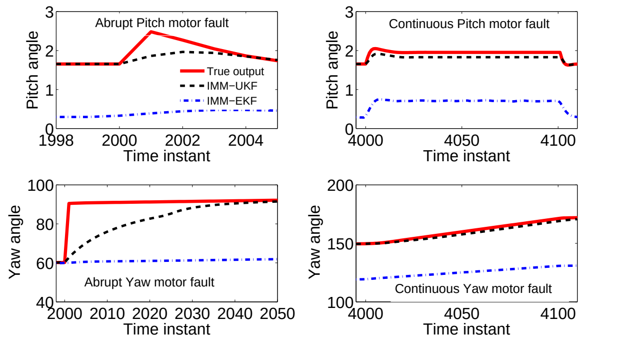

Fig. 8. Pitch and yaw angle with multiplicative actuator fault

Actuator faults. Fig. 7 depicts the helicopter output and fault detection with the occurrence of additive actuator fault. Effects of incidence of multiplicative pitch and yaw motor fault is shown in Fig. 8. The front and back propellers which are actuated by their respective motors produces thrust forces which makes the helicopter pitch and yaw angle positive.

The pitch angle and yaw angle output are produced by pitch and yaw propellers. Actuator faults affect the propellers resulting in malfunction. A sudden rise in pitch angle of 1 deg results in deviation from the true path and the IMM-UKF algorithm estimates the deviation with an offset of 0.5 degree compared with 1 degree offset from IMM-EKF. The likelihood ratio and error residual for actuator fault is depicted in Fig. 9. The likelihood ratio responds better to the system faults than the residual error analysis. IMM-EKF fails to detect the presence of faults resulting in missed fault detection.

Sensor faults. Output of pitch and yaw angle due to variation in voltage to pitch and yaw encoder is shown as sensor faults in Fig. 10. Sensor faults produces an offset bias in pitch and yaw angle, making the motors to exert a higher velocity on the propellers. Again additive and multiplicative faults cases are displayed, which shows

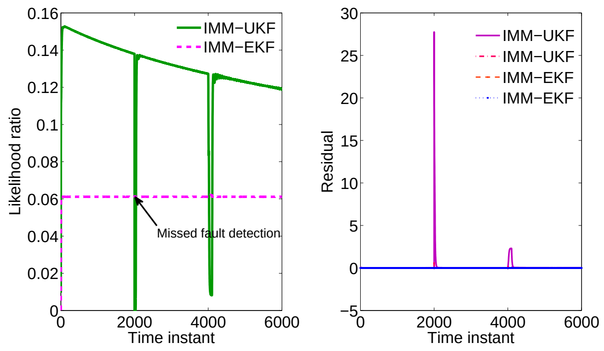

Fig. 9. Likelihood ratio and residual of multiplicative actuator fault

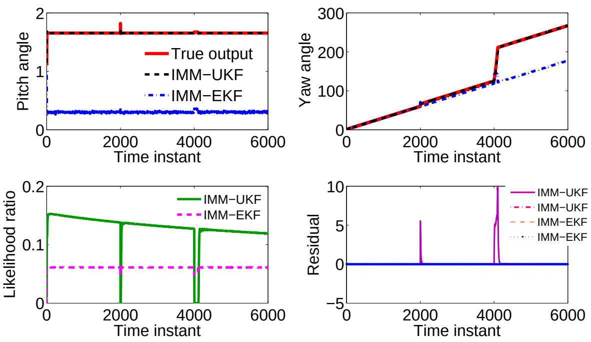

Fig. 12. Multiplicative sensor fault

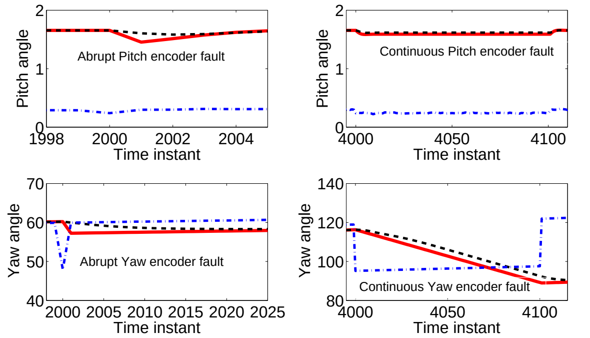

Fig. 10. Additive sensor fault

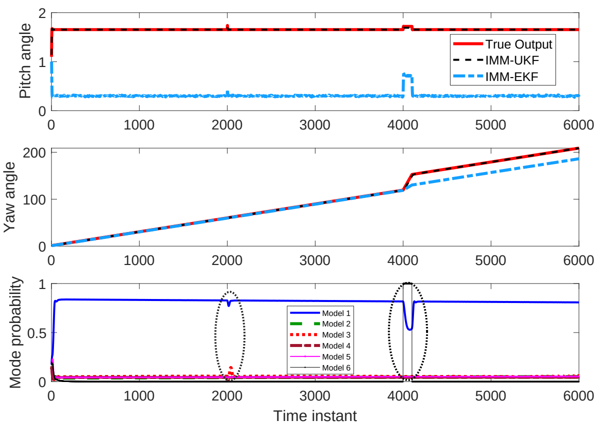

Fig. 13. Simultaneous faults

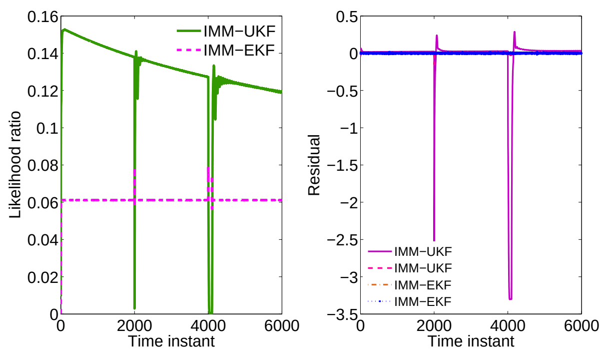

Fig. 11. Likelihood ratio and residual of additive sensor fault

the IMM-UKF tracks the faults occurring in the system more accurately. Fig. 11 and Fig. 12 shows the results of likelihood ratio and error residual of additive and multiplicative sensor fault cases.

Fig. 13 shows the occurrence of additive actuator fault and continuous sensor fault. It indicates the tracking of pitch and yaw angle with IMM-UKF providing better estimation of faults. Mode probability shows the actuator fault mode in effect at $t = 2000$ and sensor fault mode in effect from $t = 4000$ to 4100.

Fault Detection. The traditional residual analysis for the detection of faults uses a threshold bound. A lower threshold significantly reduces missed fault detection but increases the rate of false indication of fault occurrence. Thus fault decision based on residual analysis alone is not sufficient. To reduce false fault indications, another statistical decision using likelihood ratio is considered for fault detection. Likelihood ratio calculates the likelihood of each event with its corresponding residual as well as covariance. As seen from figures, a more reliable fault detection is made using likelihood ratio, significantly reducing the false indication of faults.

Another major advantage that IMM provides is more reliable FDD information using mode probability update. As mentioned in sec 3.4, FDD strategy uses detection of mode probabilities of all modes considered. The model probabilities by itself provides a meaningful insight of how likely each fault mode is in effect at a particular instant of time. The choice of detection threshold is universal in nature and performance of FDD varies little with respect to the choice of detection threshold chosen. However, it is convenient to detect a fault using a detection threshold. Thus the detection threshold is set to 0.9. The set of model probability for the different fault scenarios is displayed in Fig. 14 to Fig. 16. Model probabilities are calculated for set of models considered and for normal mode of operation, normal model has a probability of 0.9 and all other faulty model mounts to a value of 0.1. For the faulty cases, the faults occurrence at different time instants, switches the corresponding fault model to a value of 0.9, indicating the mode in effect at that time. Zhang et al. (1999) has shown the effect of model probability for a linearized aircraft model. The model probability results shown here are taken with the inherent non-linearity of the helicopter. The model probability factor indicates a type of faulty mode in effect at any instant resulting in the timely identification of the type of fault occurring in the helicopter. This provides a more reliable fault decision along with the residual analysis and model likelihood ratio.

Table 1. Comparative analysis of IMM-EKF and IMM-UKF based on fault tracking

Fault analysis criteria	IMM-EKF	IMM-UKF
Component fault	Less suitable	Suitable
Actuator fault	Less suitable	Suitable
Sensor fault	Suitable	Suitable
Fault detection	Missed Fault Detections	Detects faults
Fault classification	Not accurate	Accurate
Speed of detection	Slower	Faster
Multiple models	Applicable	Applicable
Non-linearity	Less suitable	Suitable

Table 1 presents the comparative analysis of quantitative model-based approaches, IMM-EKF and IMM-UKF. The analysis is based on diagnosis of faults with features associated with the model chosen. The features are different types of faults associated , ability to detect the presence of faults, capability to classify faults, timely detection of faults, representation of multiple model and the applicability to nonlinear system for fault identification. In general, both IMM-EKF and IMM-UKF as presented, uses residual analysis, stochastic based likelihood ratio and model probability approaches for fault diagnosis. The main difference in performance arises due to challenges posed by inherent non-linearity of helicopter model. EKF uses linearization around the current mean estimate, the original inherent nonlinearity is not propagated as such. When faults occurs in the system, it tends to disrupt the system with an underlying increase in non-linearity that occurs due to complex system dynamics. As evident from the results shown above, this increase in the non-linearity is neither completely captured nor propagated by IMM-EKF which leads to the significant performance reduction in identification of faults. Unlike EKF, Unscented Kalman Filter propagates the non-linear systems through a set of sigma points approximating a Gaussian distribution. Thus the ability to handle fault identification with IMM-UKF has a significant improvement in terms of accurate fault classification, speedy detection of faults and nonlinearity.

The results shows the importance of IMM over standard single model-based filters. IMM clearly has the ability to handle multiple faults of sensors, actuator and components. IMM clearly provides the accurate estimate of states both in normal and faulty condition. Also switching between normal and faulty models provides accurate indication of fault detection. Thus the multiple model combined with UKF allows significant tracking of abrupt and continuous faults. IMM-UKF enhances the performance of the system by providing the information pertaining to faults in the system.

5. CONCLUSION

In this paper, non-linear fault detection and diagnosis for 2 DOF helicopter model is proposed using model based stochastic state estimation approaches. The approaches include IMM-EKF and IMM UKF for several fault conditions. The fault diagnosis and classification is based upon residual analysis, likelihood ratio and model probability. The results presented shows that stochastic fault classification approach yields better fault detection compared with the residual analysis. Also IMM-UKF is able to deal with the inherent nonlinearity of helicopter in a promising manner. The validation of results shows IMM-UKF has comparatively higher performance for fault detection, diagnosis and fault classification of nonlinear systems.

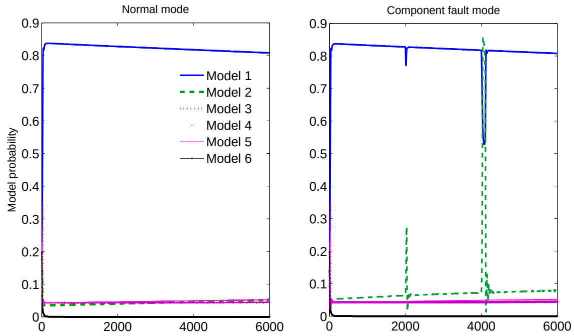

Fig. 14. IMM-UKF model probability Model 1 and Model 2

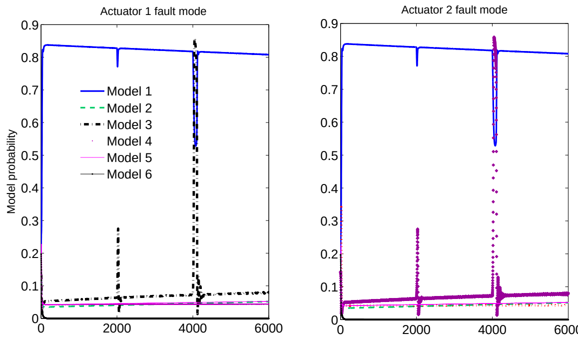

Fig. 15. IMM-UKF model probability Model 3 and Model 4

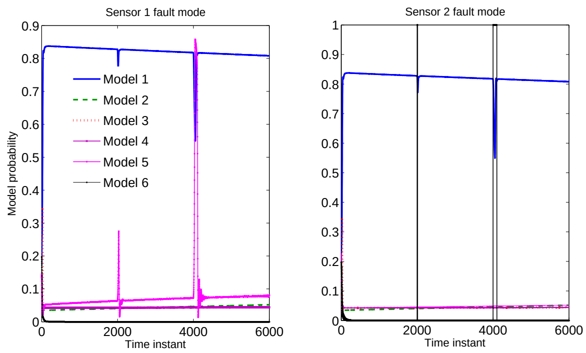

Fig. 16. IMM-UKF model probability Model 5 and Model 6

REFERENCES

Blom, H. A., and Bar-Shalom, Y. (1988). The interacting multiple model algorithm for systems with Markovian switching coefficients. *IEEE Transactions on Automatic Control*,33, 780–783.

Caliskan, F., and Hajiyev, C. (2016). Active fault-tolerant control of UAV dynamics against sensor-actuator failures. *Journal of Aerospace Engineering*, 29, 01-17.

Chowdhary, G., and Jategaonkar, R. (2010). Aerodynamic parameter estimation from flight data applying extended and unscented Kalman filter. *Aerospace Science and Technology*, 14, 106-117.

Davies, J., Tsunashima, H., Goodall, R. M., Dixon, R., and Steffen, T. (2009). Fault Detection in High Redundancy Actuation using an Interacting Multiple-Model Approach. *Proceedings of the 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, 1228–1233.

Deshpande, A. P., Patwardhan, S. C., and Narasimhan, S. S. (2009). Intelligent state estimation for fault tolerant nonlinear predictive control. *Journal of Process Control*, 19, 187–204.

Dhler, M., Mevel, L., and Zhang, Q. (2016). Fault detection isolation and quantification from Gaussian residuals with application to structural damage diagnosis. *Annual Reviews in Control*, 42, 244–256.

Ding, Z., and Hong, L. (1997). An interacting multiple model algorithm with a switching Markov chain. *Mathematical and Computer Modelling*,25, 1–9.

Du, M., and Mhaskar, P. (2014). Isolation and handling of sensor faults in nonlinear systems. *Automatica*, 50, 1066-1074.

Freddi, A., Longhi, S., and Monteriu, A. (2010). Actuator fault detection system for a mini-quadrotor. *IEEE International Symposium on Industrial Electronics (ISIE)*, 2055-2060.

Heredia, G., Ollero, A., Bejar, M., and Mahtani, R. (2008). Sensor and actuator fault detection in small autonomous helicopters. *Mechatronics*, 18, 90-99.

Hsiao, T., and and Weng, M.C. (2012). A hierarchical multiple-model approach for detection and isolation of robotic actuator faults. *Robotics and Autonomous Systems*,60, 154–166.

Isermann, R. (2005). Model-based fault-detection and diagnosis status and applications. *Annual Reviews in Control*, 29, 71-85.

Jayaprasanth, D., and Kanthalakshmi, S. (2018). Fault Detection and Isolation in Stochastic Nonlinear Systems using Unscented Particle Filter based Likelihood Ratio Approach. *Journal of Control Engineering and Applied Informatics*, 20, 75–85.

Kandepu, R., Foss, B. and Imsland, L. (2008). Applying the unscented Kalman filter for nonlinear state estimation. *Journal of Process Control*, 18, 753–768.

Kargar, S. M., Salahshoor, K., and Yazdanpanah, M. J. (2015). Integration of multiple model based fault detection and nonlinear model predictive fault-tolerant control. *IEEJ Transactions on Electrical and Electronic Engineering*,10, 547–533.

Kiasi, F., Prakash, J., and Shah, S. L. (2015). Detection and diagnosis of incipient faults in sensors of an LTI system using a modified GLR-based approach. *Journal of Process Control*, 33, 77–89.

Kumar, E. V., Raaja, G. S., and Jerome, J. (2016). Adaptive PSO for optimal LQR tracking control of 2 DoF laboratory helicopter. *Applied Soft Computing*, 41, 77–90.

Lan, J., Patton, R. J., and Zhu, X. (2017). Integrated fault-tolerant control for a 3-DOF helicopter with actuator faults and saturation. *IET Control Theory and Applications*, 11,2232–2241.

Li, X.R., and Bar-Shalom, Y. (1996). An interacting multiple model algorithm with a switching Markov chain. *Mathematical and Computer Modelling*,25, 1–9.

Li, X.R., Bar-Shalom, Y., and Blair, W. D. (2000). Engineer's guide to variable-structure multiple-model estimation for tracking. *Multitarget-multisensor tracking: Applications and advances*,3, 499–567.

Lu, P., Van Eykeren, L., van Kampen, E.J., and Chu,Q.P. (2015). Sensor fault detection and estimation for quadrotors using kinematic equations. *Advances in Aerospace Guidance, Navigation and Control*,363–379.

Marzat.J.H, Piet-Lahanier., Damongeot, F., and Walter.E. (2011). Model-based fault diagnosis for aerospace systems:a survey. *Aerospace Engineering*, 226,1329–1360.

Menke, T. E., and Maybeck, P. S. (1995). Sensor/actuator failure detection in the Vista F-16 by multiple model adaptive estimation. *IEEE Transactions on Aerospace and Electronic Systems*,31, 1218–1229.

Mazor, E., Averbuch, A., Bar-Shalom, Y., and Dayan, J. (1998). Interacting multiple model methods in target tracking: a survey. *IEEE Transactions on Aerospace and Electronic Systems*,34, 103–123.

Tan, D., and Patton, R. J. (2015). Integrated Fault Estimation and Fault Tolerant Control: A Joint Design. *IFAC-PapersOnLine*,48, 517–522.

Van Eykeren, L., and Chu, Q. P. (2014). Sensor fault detection and isolation for aircraft control systems by kinematic relations. *Control Engineering Practice*, 31, 200–210.

Wang, Z., and Shang, H. (2015). Kalman filter based fault detection for two-dimensional systems. *Journal of Process Control*,28, 83–94.

Wu, C., Qi, J., Song, D., Qi, X., and Han, J. (2015). Simultaneous state and parameter estimation based actuator fault detection and diagnosis for an unmanned helicopter. *International Journal of Applied Mathematics and Computer Science*,25, 175–187.

Yang, H., Jiang, B., Staroswiecki, M., and Zhang, Y. (2015). Fault recoverability and fault tolerant control for a class of interconnected nonlinear systems. *Automatica*, 54, 49-55.

Zhang, P., and Ding, S. X. (2008). An integrated trade-off design of observer based fault detection systems. *Automatica*, 44, 1886-1894.

Zhang, Y., and Jiang, J. (1999). An Interacting Multiple-Model Based Fault Detection, Diagnosis Fault-Tolerant Control Approach. *Proceedings of the 38" Conference on Decision and Control*, 1999 Arizona USA.

Zhang, Y., and Jiang, J. (2001). Integrated Active Fault-Tolerant Control Using IMM Approach. *IEEE Transactions on Aerospace and Electronic Systems*, 37,1221–1235.

Zhang, Y., and Jiang, J. (2008). Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control*, 32, 229–252.

Zhong, Y., Zhang, Y., Zhang, W., and Zhan, H. (2018). Actuator and Sensor Fault Detection and Diagnosis for an Unmanned Quadrotor Helicopters. *IFAC PapersOnLine*, 51, 998–1003.

Zhong, Y., Zhang, W., Zhang, Y., Zuo, J., and Zhan, H. (2019). Sensor Fault Detection and Diagnosis for an Unmanned Quadrotor Helicopter. *Journal of Intelligent and Robotic Systems*, 1–18.

Zolghadri, A. (2012). Advanced model-based FDIR techniques for aerospace systems: Today challenges and opportunities. *Progress in Aerospace Sciences*, 53, 18–29.

Zolghadri, A., Cieslak, J., Efimov, D., Goupil, P., and Dayre, R. (2016). Practical design considerations for successful industrial application of model-based fault detection techniques to aircraft systems. *Annual Reviews in Control*, 42, 224-231.

Appendix A. LIST OF ACRONYMS

FDD - Fault Detection and Diagnosis

FDI - Fault Detection and Identification

FTC - Fault Tolerant Control

DOF - Degrees of Freedom

UAV - Unmanned Aerial Vehicles

MM - Multiple Model

IMM - Interacting Multiple Model

KF - Kalman Filter

EKF - Extended Kalman Filter

UKF - Unscented Kalman Filter

ESMF - Extended Set Membership Function

NUIO - Non-linear Unknown Input Observer

AHC - Actuator Health Co-efficent