ANALYSIS OF FUZZY CONTROL SOLUTIONS FOR ANTI-LOCK BRAKING SYSTEMS

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Abstract: *This paper performs a critical analysis of five fuzzy control solutions dedicated to Anti-lock Braking Systems (ABSs). The requirements in ABS control are clearly stated to help the analysis. The detailed mathematical model of controlled plant is derived and simplified for control design with focus on tire slip control. A new fuzzy control solution based on a class of Takagi-Sugeno fuzzy controllers is proposed. This class of fuzzy controllers combines separately designed PI and PID controllers corresponding to a set of simplified models of controlled plant linearized in the vicinity of important operating points. Simulation results validate the suggested fuzzy control solution in controlling the relative slip of a single wheel.*

Keywords: *mathematical modelling, ABS control, Takagi-Sugeno fuzzy controllers, modal equivalence principle, digital simulation results.*

1. INTRODUCTION

Anti-lock Braking Systems (ABSs) play a key role in the complex steering systems for modern cars [12]. The main requirement in ABS control systems (CSs) is the prevention of wheel-lock during braking to ensure high friction and maintain the steer ability of the car. The ABS control solutions can be grouped in two categories, wheel acceleration control and tire slip control, associated to several actuator types. The first category solves the tire slip control indirectly by controlling wither the wheel deceleration / acceleration or the braking torque commanded by the driver [16]. Widely used approaches in the second category include reaching the maximum friction point based on the measurement of wheel's the angular velocity and brake pressure [6], feedback linearization [18], model-based hybrid controllers [9], LQ controllers [9], robust PID controllers [8, 25], gain scheduling [25] or fuzzy controllers [14].

Modelling the controlled plant is a very important stage in designing an ABS controller. Even if a simplified model is available, it can provide valuable information about the system's main parameters, like time constants, delays, the stable / unstable character or the factors that can influence the stability. The literature generally suggests a simplified mathematical model, the so called quarter-vehicle model [4, 11, 16, 19, 20, 26, 27]. This model has the advantage that ignores the interactions between the four wheels and the vehicle's body, and also the additional phenomenon that complicates it. However, the literature ignores the effects of mass shift in longitudinal direction towards the front of the vehicle and in the lateral direction when the vehicle is cornering. The mass shift leads to an increase of normal force on the front wheels and/or on the wheels situated outside the cornering. Most of the results reported in the literature ignore these effects and the modification of the normal force is seen as a simple noise. Therefore, it is important to present the detailed controlled plant (CP) model in this paper and this presentation is one of the paper goals.

Due to the linear dependence of each rule on the input variables of the fuzzy controller (FC), the Takagi-Sugeno fuzzy controllers (TS-FCs) [28] are good interpolating supervisors of multiple linear controllers applied to different operating conditions of nonlinear CPs. The TS-FCs are extremely well-suited to play the role of bumpless interpolators between the linear controllers applied across the input space; therefore, these controllers are natural and efficient gain schedulers [23]. These are the reasons why it is suggested here a new class of TS-FCs that performs that merges a set of PI or PID local controllers. For the design of the local controllers it employed here a benchmark which involves a simplified nonlinear model that describes the slip dynamics for a wheel [26]. In the first step, by linearization of the friction curves in the vicinity of important steady-state operating points there are obtained 64 first-order lag plus time delay models of the CP representing local linear models. Next, the local controllers meant for controlling these local linear models are developed in terms of a frequency domain approach.

The paper is organized as follows. The following section deals with the presentation of the requirements in ABS control accompanied by the derivation of CP's mathematical model further simplified aiming its use in control design. Then, section 3 is dedicated to the critical analysis of five fuzzy control solutions dedicated to ABS control. Section 4 offers details on the original fuzzy control solution based on a class of TS-FCs presenting the controller structure and design. The theoretical approach is validated in section 5 by digital simulation results in case of a TS-FC controlling the relative slip of a single wheel, and the conclusions are drawn in the final section.

2. REQUIREMENTS IN ABS CONTROL AND MATHEMATICAL MODELS OF CONTROLLED PLANT

The requirements in ABS control are important because they represent targets to be followed in the control design and allow the assessment of control system performance indices. Using authors' experience and [3], these requirements can be arranged as follows:

- optimal brake torque after the tire-road friction coefficient (μ)-jump from low to high has to be reached in less than 500 ms. Slip controller must not be bottle neck. Fast μ-estimation and accurate reference speed needed to satisfy this requirement,
- the brake control for forward driving must work over the whole vehicle velocity range down to standstill,
- there should be an ABS for backward driving,
- the brake control has to guarantee stability and controllability (by steering) for any road surface,
- the ABS has to show full performance in the temperature range from –20°C to +120°C and must not fail below –20°C,
- a build-up of oscillations due to (chassis) vibrations must be avoided,
- priorities of requirements for ABS when driving at high speeds: 1. vehicle stability, 2. braking distance and 3. comfort,
- priorities of requirements for ABS to brake in straightforward direction: 1. braking distance, 2. vehicle stability and 3. comfort,
- priorities of requirements for ABS to brake on μ-split: 1. braking distance, 2. vehicle stability, 3. steer ability, 4. lateral displacements from the centre of the track have to be less than 0.5 m, 5. comfort.

To derive the mathematical model of controlled plant it is important to observe that a crucial physical parameter that influences the braking process is the adherence of the tire-road system, characterized by μ. This coefficient depends mainly on the state of the road, (dry, wet, etc.) but also on the longitudinal slip of the tire against the road referred to as the relative slip, λ. When the braking force exceeds the adherence offered by the wheel-road contact, the wheel of the car begins to slide. The relative slip is defined in terms of (1):

$$\lambda = (v - v_{\mathrm{wheel}}) / v = (v - \omega R) / v, \qquad (1)$$

where: v – longitudinal velocity of the vehicle (car), v_{wheel} – tangential velocity of the wheel, ω – angular velocity of the wheel, R – outer radius of the wheel.

There are several models that approximate the λ-μ curve. In literature, the most known are Pacejka's magic formula and Lu-Gre Model, which is a result of a study done in cooperation by two universities, Lund Institute of Technology and Grenoble Institute of Technology. In [27], some equations are provided to describe the dependence between the longitudinal slip and the friction coefficient, as follows (see Fig. 1):

- for dry asphalt:

$$\mu = 0.9[1.07(1 - e^{-0.2773\lambda}) - 0.0026\lambda],$$

- for wet asphalt:

$$\mu = 0.7[1.07(1 - e^{-0.5\lambda}) - 0.0006\lambda],$$

- for snow:

$$\mu = 0.3[1.07(1 - e^{-0.1773\lambda}) - 0.0003\lambda],$$

- for ice:

$$\mu = 0.1[1.07(1 - e^{-0.38\lambda}) - 0.0003\lambda].$$

In [10], the Pacejka's magic formula is stated as:

$$f(x) = S_y + D\sin\{C\tan^{-1}[B(x - S_x)\cdot (1 - E)] + E\tan^{-1}[B(s - S_x)]\}, \qquad (2)$$

where x is the slip angle, $f(x)$ is the output force, S_x and S_y are shift factors which locate the centre of magic formula curve relative to the origin and B, C, D and E are coefficients chosen to fit measured tire data. These four coefficients modify the shape of the curve. A few "rules of thumb" are now stated as:

- the peak of the curve occurs at $f(x) = D$,
- the initial slope of the curve is given by the product BCD,
- B is related to the location of the peak along the x-axis,
- E controls the shape of the curve in the vicinity of the peak.

In practice, the effects of the coefficients are more complicated as they all have secondary effects. A much more detailed discussion of the parameters is given in [24].

A Lu-Gre model according to [21] deals with the dependence of friction on velocity. This model is focused on the definition of a pseudo steady-state expression for the absolute value of friction force as:

$$F_x = F_n[h(v_r)/\theta]\{1 + [2\gamma h(v_r)/(\sigma_0\theta L|\eta|)]\cdot \exp[-\sigma_0\theta L|\eta|/2/h(v_r)]\} + F_n\sigma_2 v_r, \qquad (3)$$

where:

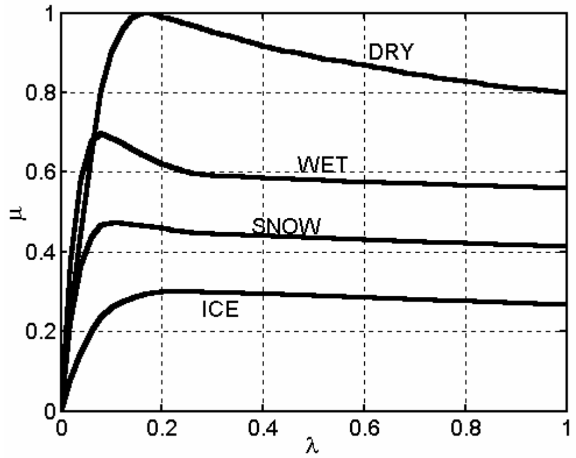

Fig. 1. Tire-road friction coefficient (μ) versus relative slip (λ).

$$\eta = -\frac{s}{1-s}, \gamma = 1 - \frac{\sigma_1\theta|v_r|}{h(v_r)},$$

$$h(v_r) = \mu_C + (\mu_s - \mu_C)\exp[-\left|\frac{v_r}{v_s}\right|^{\alpha}]. \qquad (4)$$

All the models give an approximation in case of the four so-called standard surfaces, already mentioned: dry asphalt, wet asphalt, snow and ice. The models admit that there is a maximum in the λ-μ curve, somewhere around 12 % for dry asphalt. Since there is a maximum at 12 %, it is easy to observe that the maximum possible deceleration is obtained at this relative slip, which is, in general, an important ABS requirement associated to the requirements presented here.

Besides the longitudinal λ-μ curve, there is another characteristic that describes the lateral friction coefficient depending on longitudinal slip. This second coefficient is very important for the stability of the vehicle. In Fig. 1 one can see that if the longitudinal slip increases, the lateral coefficient dramatically decreases, which eventually leads to a loss in vehicle's stability. This unwanted phenomenon is easier to notice if the braking is done during cornering. If one or both rear wheel lock, the vehicle will start spinning.

The quarter vehicle model is proposed in [26, 27] related to Fig. 2. It starts with the well-known Newton relationship:

$$\vec{F} = m\vec{a}\,, \tag{5}$$

where: F – friction force, m – vehicle's mass, a – vehicle's acceleration. The friction force F_f has the form (6):

$$F_f = \mu G\,, \tag{6}$$

where: μ – friction coefficient, G = m·g – vehicle's weight, m – vehicle's mass and g – gravitational acceleration. From (5) and (6), by equalizing the forces (in module), one obtains:

$$|\mu m g| = |ma|\,. \tag{7}$$

By dividing to the vehicle's mass, the modulus of acceleration's value is obtained:

$$|a| = |\mu g|\,. \tag{8}$$

Regarding the wheel, the forces that occur and the corresponding parameters are shown in Fig. 3. The total torque is given in (9):

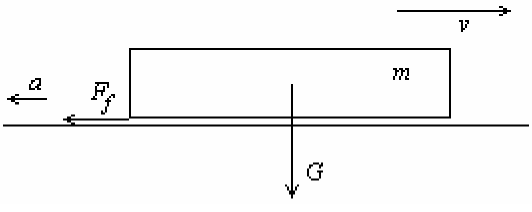

Fig. 2. An object's interaction with the ground.

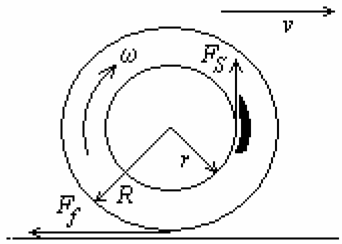

Fig. 3. The wheel's geometry and its contact with the ground..

$$\vec{M}_{\text{Res}} = R\vec{F}_f - r\vec{F}_s\,, \tag{9}$$

so the wheel's equation can be immediately found:

$$J\dot{\omega} = RF_f - rF_S\,. \tag{10}$$

The friction force of the brake pad, $\underline{F_B}$, can be written in the form (11):

$$F_B = \mu_B p_{Act} S_B\,, \tag{11}$$

where μ_B is the friction coefficient between the brake pad and the discus (not to be mistakenly taken as the friction coefficient between the tire and the ground), p_{Act} is the pressure from the hydraulic system and S_B is the contact surface between the brake pad and the brake discus. Of course, in electric brakes (like Electro Magnetic Brake also known as EMB or Electronic Wedge Brake also known as EWB), the pressure is no longer relevant (as in hydraulic brakes) since the actuators are electrically driven and thus the power conversion is done to N directly, without having the pressure as a relevant value any longer.

The vehicle's dynamics during braking can be expressed in terms of (12):

$$\begin{aligned} \dot{\omega} &= (R\mu m - r\mu_B F_B)/J, \\ \dot{v} &= \mu g, \end{aligned} \tag{12}$$

with: m – 1/4 of vehicle's total mass, μ_B – friction coefficient between the braking discus and the wheel, and some parameters in (12) are variable with respect to time.

The same mathematical model, written in different ways, is suggested also in [4, 17, 19, 20]. In [15] it is taken into account the increase of the normal force on the front wheel and the decrease in the rear wheels, but do not take into consideration the vehicle's dynamics during cornering. The total tractive force, denoted by F_{tot}, is represented as the sum in (13):

$$F_{tot} = F_{tf} + F_{tr}, F_{tf} = \mu(\lambda_f)F_{zf}, \\ F_{tr} = \mu(\lambda_r)F_{zr}. \tag{13}$$

Because of the vehicle's geometry, the static normal forces for the front tire and rear tire are F_{zf1} and F_{zr1}, respectively:

$$F_{zf1} = m_{tot} gb/(a+b), \\ F_{zr1} = m_{tot} ga/(a+b). \tag{14}$$

Concluding, the state-space mathematical model can be expressed as:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -g\frac{\mu(\lambda_f)m_1 + \mu(\lambda_r)m_2}{m_{tot} + m_3[\mu(\lambda_r) - \mu(\lambda_f)]} = c, \\ \dot{x}_3 &= [-T_B + \mu(\lambda_f)m_1 R_w g - \\ &\quad - \mu(\lambda_f)m_3 R_w c + T_e], \\ \dot{x}_4 &= [-k_{br}T_{bf} + \mu(\lambda_r)m_2 R_w g - \\ &\quad - \mu(\lambda_r)m_3 R_w c]/(2J_f), \\ y &= [x_1 \quad x_2 \quad 1 - \frac{R_w x_3}{x_2} \quad 1 - \frac{R_w x_4}{x_2}]^T, \end{aligned} \tag{15}$$

where x_1 represents the distance, x_2 is the vehicle's speed, $x_3 = w_f$, $x_4 = w_r$, and the parameters in (15) represent: a and b – distance from centre of gravity to front and rear axle, respectively, h_s, h_f and h_r – height of sprung mass, front unsprung mass and rear unsprung mass, respectively, m_{tot} – total mass of the vehicle, m_s – sprung mass of the vehicle, mf and m_r – front and rear unsprung mass, respectively, J_f and J_r – moment of inertia of the front and rear wheel, respectively, R_w – radius of tire, T_e – engine torque, k_{br} – brake displacement proportionality constant. As one can easily see, the result is a nonlinear mathematical model, much more complicated than the quarter vehicle model. However, this model still does not account for the weight shift due to cornering. In this model it is assumed that the car is braking in straight forward direction.

The actuator is generally neglected. Only few papers mention it and give a simplified model of the actuator. In [26] a continuous-time model is given with the transfer function (16):

$$A(s) = \frac{0.0091s + 3.9545}{0.0001s^2 + 0.0402s + 3.9545}. \tag{16}$$

In [19] a hydraulic system is presented:

$$C_f \dot{P}_{bi} = A_1 C_{d_1 i}\sqrt{2(P_p - P_{bi})/\rho} - \\ - A_2 C_{d_2 i}\sqrt{2(P_{bi} - P_{low})/\rho}, \tag{17}$$

where $C_{d_1 i}$ and $C_{d_2 i}$ are the control signals, which can take values between 0 and 1, depending on the corresponding valve being open or closed, P_b is the output hydraulic pressure, P_p is the pump pressure, P_{low} is the reservoir pressure and $A_{1,2}$ are the orifice areas of hydraulic valves. In [29] a first-order lag model for an actuator dedicated to aircraft braking systems is suggested.

These actuators work in a range of forces (for passenger cars) between 0 and 30 kN, all of them are non-linear mechanical elements characterized by delays and other limitations (like limited increase). A linear mathematical model is incorrect and incomplete. However, for a limited range they can provide an accurate response and can be used in the design phase.

A simplified model of the braking wheel as CP, similar to (12), under the action of brake torque and ground contact reaction force was proposed in [26] and will be used here:

$$\begin{aligned} \dot{\omega} &= \alpha\mu(\lambda(t)) - \beta T_b(t-\tau)\cdot \\ &\quad \cdot \mu_b(\omega(t)),\ T_b \ge 0, \\ \dot{v} &= -\gamma\mu(\lambda(t)), \end{aligned} \tag{18}$$

where $T_b = u$ is the brake torque (control signal), μ_b the friction coefficient in the brakes, τ the time delay, and α, β and γ are positive constants resulting from the physical parameters of the car. The following values of CP parameters will be considered in the sequel: r=0.3 m, α=1500, β=1 and γ=10. For simplicity it is assumed that μ_b=min(ω/ε,1), for small ε >0, and the signals ω, v and μ_H (the maximum road-tire friction coefficient) can be measured / estimated.

In order to simplify the TS-FC design the nonlinear model (1), (18) will be linearized in the vicinity of a steady-state operating point $A_0(v_0,\ \omega_0,\ \lambda_0,\ \mu_0)$, where the lower index 0 applied to a certain variable highlights the steady-state value of that variable. Only the nonlinear components will be linearized here except for $\mu_b(\omega(t))$ which will be approximated to 1 in the control design:

$$\mu_b(\omega(t)) = 1\,. \tag{19}$$

Next, (1) can be written in its equivalent form:

$$v(t) = \lambda(t)v(t) + r\omega(t)\,. \tag{20}$$

Linearizing the product in (20) in the vicinity of A_0 and differentiating the result (with respect to time), the linearized form of the second equation in (18) can be expressed as:

$$(1-\lambda_0)\dot{v}(t) = v_0\dot{\lambda}(t) + r\dot{\omega}(t)\,. \tag{21}$$

But, the nonlinear component $\mu(\lambda(t))$ can be also linearized around A_0:

$$\mu(\lambda(t)) = m_0(\lambda(t)-\lambda_0) + \mu_0\,, \tag{22}$$

where m_0 represents the slope of the tire friction curve at the considered operating point A_0:

$$m_0 = (\mathrm{d}\mu/\mathrm{d}\lambda)\big|_{A_0}\,. \tag{23}$$

The substitution of the expression (22) of $\mu(\lambda(t))$ into (18) in the condition (19) leads to the modified version of (18) expressed in terms of (24):

$$\begin{aligned}\dot{\omega}(t) &= \alpha m_0\lambda(t) - \beta u(t-\tau) + \\ &\quad + \alpha(\mu_0 - \lambda_0 m_0),\\ \dot{v}(t) &= -\gamma m_0\lambda(t) - \gamma(\mu_0 - \lambda_0 m_0).\end{aligned} \tag{24}$$

Then, (24) is substituted into (21) and the linearized model of wheel dynamics can be expressed as:

$$T_1\dot{\lambda}(t) + \lambda(t) = k_1 u(t-\tau) + d(t)\,, \tag{25}$$

with the parameters k_1 and T_1 expressed in (26):

$$\begin{aligned}k_1 &= r\beta/m_0/[\gamma(1-\lambda_0)+\alpha r],\\ T_1 &= v_0/m_0/[\gamma(1-\lambda_0)+\alpha r],\end{aligned} \tag{26}$$

and the disturbance input $d(t)$ expressed as:

$$d(t) = (m_0\lambda_0 - \mu_0)/(\beta r m_0)\,, \tag{27}$$

with the remark that it is necessary the choice of A_0 such as $m_0 \neq 0$.

Taking into consideration the fact that the slope m_0 can take positive or negative values, the rest of parameters in (26) take only positive values, it comes out that the parameters k_1 and T_1 will take positive or negative values. So it will be accepted in the sequel two model types for the wheel dynamics in tire slip control. The unified expression of these model types in zero initial conditions outlines the CP transfer function, $P(s)$:

$$\lambda(s) = P(s)u(s) + d(s)\,, \tag{28}$$

and the expressions of this transfer function for the two model types are:

- in case of $m_0 > 0$:

$$P(s) = k_P\exp(-s\tau)/(1+sT_P)\,, \tag{29}$$

- in case of $m_0 < 0$:
-

$$P(s) = k_P\exp(-s\tau)/(-1+sT_P)\,, \tag{30}$$

where k_P, T_P and τ are the CP gain, lag time constant and time delay, respectively. The parameters k_P and T_P can be derived calculating the absolute value in (31):

$$\begin{aligned}k_P &= |k_1| = r\beta/|m_0|/[\gamma(1-\lambda_0)+\\ &\quad +\alpha r] > 0,\\ T_P &= |T_1| = v_0/|m_0|/[\gamma(1-\lambda_0)+\\ &\quad +\alpha r] > 0.\end{aligned} \tag{31}$$

The linear models (29) and (30) represent widely used first-order lag plus time delay models. Challenging design problems, reported in [22], arise in case of (30). Since the new class of TS-FC to be presented in section 4 will merge several local PI / PID controllers, for the design of these local controllers devoted to the local models (29) / (30) of CP it is necessary to derive local models valid in the vicinity of significant operating points. For the choice of the operating points it must be highlighted that μ_0 does not appear in (31) so it is sufficient to calculate the slope m_0 for several values of μ_H (Fig. 1). The operating points are characterized by the following sets of steady-state values: $\mu_H \in \{0.1, 0.3, 0.7, 1\}$, $\lambda_0 \in \{0.05, 0.1, 0.2, 0.3\}$, $v_0 \in \{1, 10, 20, 30\}$. So, there will result 64 local models of the CP, of type (29) or (30).

3. FUZZY CONTROL SOLUTIONS

Most of the fuzzy control solutions analyzed involve just fuzzy control [4, 16, 29] or fuzzy logic combined with genetic algorithms [15, 16, 19] or fuzzy logic combined with sliding mode [17]. This section is dedicated to a short analysis of currently used fuzzy control solutions in ABS.

In [4] a method is proposed to determine the friction coefficient between the tire and the road and the solution for this was a Kalman filter. Two Mamdani fuzzy logic blocks are included to the control loop. The first block determines the optimum relative slip for braking in order to ensure the maximum friction coefficient and it uses two input linguistic variables (LVs):

- the calculated relative slip with three linguistic terms (LTs),
- the estimated friction coefficient μ_x with five LTs,

and one output LV, the road type, with five LTs. The second fuzzy block, a Mamdani fuzzy controller, employs two input LVs:

- the control error with five LTs,
- the derivative of control error with five LTs,
-

and one output LV, the pressure, with six LTs. The results reveal a good behaviour for high-μ surfaces; however, in the case of lower friction surfaces oscillations occur. However, if there is a transition from wet surface to snow, the overshot of the slip is quite small; the slip reaches a maximum value of 30 %. However, there is no measurement for a transition from dry asphalt to ice and from ice to dry asphalt. These two situations are very important when judging the overall performance of an ABS.
The mathematical model of CP in [15] is the most detailed compared to similar literature. However, the actuator is not described but it is neglected. The suggested controller involves a combination between fuzzy logic and genetic algorithms using a genetic algorithm in tuning the fuzzy controller parameters. Two controllers are implemented, one for the front wheels and one for the rear ones. The population of the genetic algorithm consists of 20 members. Each member of the population contains all parameters of the fuzzy controllers. In order to decide which chromosome is the best one, a fitness function is used:

$$fitness = \frac{1}{\int_0^{t_{sim}} [e_f^{\,2}(t) + e_r^{\,2}(t)]\mathrm{d}t}, \qquad (32)$$

where e_f and e_r stand for the control error in case of front and rear wheel, respectively. The performance of each chromosome is tested using the same environment for 2 seconds. The best two chromosomes are kept into the next population. Crossover and mutation are applied to the chromosomes in the mating pool. The algorithm takes care of keeping the coherence of the fuzzy rules into the parameters of the new population.

The main disadvantage of the solution proposed in [15] is the big number of simulations in order to achieve good results. For each chromosome, a whole simulation must be run. Since there are 20 chromosomes, for each generation, 20 simulations must be performed. Considering the fact that the authors needed 248 generations, one could calculate the total number of simulations: 20 x 248 = 4960. Besides this huge number of simulations, a considerably computational effort is needed also for the crossover and mutation of the new generation. In this particular case, the complete operation was performed 247 times. On the other hand, the results do not show the μ-jump results or the measurements done for low-μ. From practical point of view, this method can't be used. It's impossible to tune an ABS algorithm like this on a real car, because it's practically impossible to have 4960 identical tests in order to determine the best chromosome combination.

The paper [29] suggests a fuzzy controller with three input LVs, road type, current slip and predicted slip. The road type is determined by an algorithm that works as a state machine, which can provide the four types of road, i.e. the standard surfaces mentioned in the previous section. The controller takes the type of the estimated road and, depending on it; it applies some fuzzy rules for each situation. The interesting part is that not all combinations are

taken into account, only those that are considered to be relevant (the well-accepted four ones). But, if the current situation is not found in the fuzzy rules database, the output is the last control signal. The controller involves also the calculation of a compensation value that multiplies the control signal.
The results reported in [29] prove exhibit strong oscillations, beyond the accepted range in normal passenger car. However, in this case the ABS works on a military jet plane and the comfort is not the main concern. Even so, from the performance point of view, the big oscillations lead to a dramatic reduction of braking performance and thus to an increase of the braking distance. Besides, also the actuator could have a much shorter life.

In [17] the inclination of the road in modelling the CP is accounted for. Due to this factor, the normal force is modified and the mathematical model of CP becomes more complex. The so-called self-learning fuzzy sliding mode controller consists of four blocks: the fuzzy controller, the adaptive law that perform on-line tuning of fuzzy controller parameters, the robust controller that handles model uncertainties, and the bound estimation algorithm necessary in the implementation of a sliding mode-like behaviour of the fuzzy control system. The results that were presented reveal a very large overshot (around 50% slip, which means about 500 % overshot) in the beginning then the system is stable. Also, a transition from high to low μ is shown, but from the system's response one can see that the actuator is completely disregarded; a real mechanical system could not provide a decrease of approx 900 Nm to 0 practically instantly, which is totally unrealistic.

The paper [19] suggests a TS-FC using three inputs: the slip, its derivative and the vehicle’s deceleration. A genetic algorithm is employed in tuning the fuzzy controller parameters, where the chromosomes performance function is:

$$F_{obj}\left(\lambda(t)\right) = \sum_{t=o}^{t_{sim}} [\lambda(t) - \lambda_r(t)]^2 , \qquad (33)$$

with λ_r – reference input (desired value of relative slip). The genetic algorithm is run in parallel with two separate populations of 50 chromosomes with the probability to crossover p_c and mutation p_m under the supervision of a roulette wheel selection mechanism. In order to avoid losing the best solutions, the best chromosome is used into the next population.

The results presented in [19] are compared with the ones using a PI controller in identical conditions. The PI controller results reveal very high oscillations especially at low speeds, and the TS-FC results prove smaller oscillations and also a better braking distance. A very interesting detail is that the output torque seems to be limited to a certain value. However, this is not a proper way to implement an algorithm since a real vehicle can have different loads due to passengers or the luggage. Also, a transition from a downhill road to a horizontal road even at medium speeds can lead to a certain increase in the vehicle’s load so the upper limitation in the requested brake torque is not acceptable. On the other hand, [19] suggests the tuning to be done using a genetic algorithm.

As it was shown earlier, a real vehicle cannot be tuned like this because it is really impossible to have a huge number of identical tests. The majority of fuzzy control solutions reported in the literature work only in simulated environments. Therefore, it is necessary to suggest new fuzzy control solutions capable to work in real-time. One such solution will be presented in section 4.

4. A CLASS OF TAKAGI-SUGENO FUZZY CONTROLLERS

The suggested CS structure, dedicated to tire slip control has the structure presented in Fig. 4 and it is based on the measurement / estimation of the signals $\{\mu_H, v, \lambda\}$ available from the CP. The other elements in Fig. 4 are: λ_r – reference input, $e = \lambda_r - \lambda$ – control error. It can be seen that the relative slip λ plays the role of controlled output. For the sake of simplicity the measuring elements and the estimators are accepted to belong to the CP.

The design of TS-FC starts, as already mentioned, with the design of local controllers to control the local CP models of type (29) or (30). In the first phase there are designed continuous-time controllers, PI controllers for (29) and PID ones for (30) in the frequency domain using the approach in [22]. Next, the discretization of continuous-time PI and PID controllers is done resulting in the incremental version of quasi-continuous digital PI / PID controllers expressed in the unified form (34):

$$\Delta u_k = K_P (\Delta e_k + \alpha_e e_k + \alpha_f f_k) , \qquad (34)$$

where e_k is the control error, $\Delta e_k = e_k - e_{k-1}$ and $\Delta u_k = u_k - u_{k-1}$ stand for the increment of control error and control signal, respectively, and f_k is the derivative term. The parameters K_P, α_e, and α_f depend on the discretization method and sampling period T_s = 0.005 s for the accepted benchmark. In the case of the PI controller α_f=0. In the case of the PID controller, the derivative term f_k is obtained in terms of the recurrent equation (35):

$$f_k = -f_{k-1} + e_k - 2e_{k-1} + e_{k-2}. \tag{35}$$

The suggested structure for the class of TS-FCs is presented in Fig. 5 and it contains the block B-FC representing the fuzzy controller without dynamics and the blocks with dynamic needed to be inserted to the structure. In the initial phase, the fuzzification is done in terms of the membership functions presented in Fig. 6. The block B-FC employs the MAX and MIN operators in the inference engine, and the weighted sum method for defuzzification.

A complete rule base of 64 rules with the general form (36) assists the inference engine in the block B-FC:

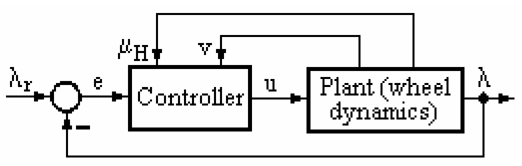

Fig. 4. Tire slip control system structure.

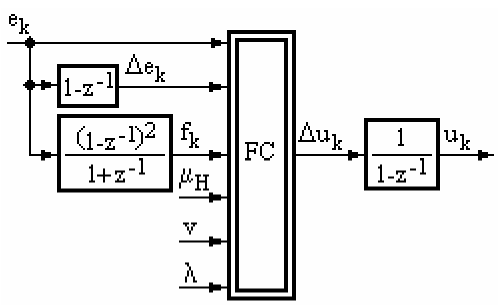

Fig. 5. Takagi-Sugeno fuzzy controller structure.

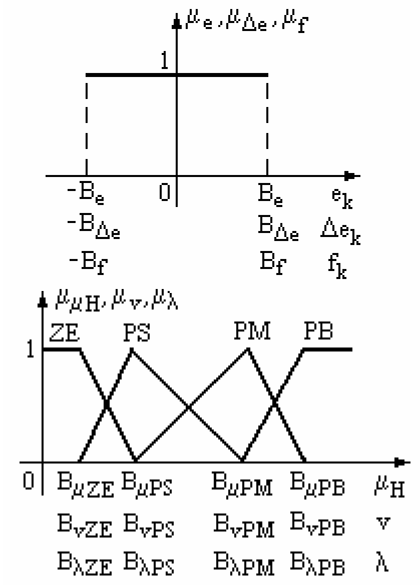

Fig. 6. Input membership functions.

$$\begin{aligned}
&\text{IF } (e_k = LT_e) \text{ AND } (\Delta e_k = LT_{\Delta e}) \\
&\text{AND } (f_k = LT_f) \text{ AND } (\mu_H = LT_\mu) \\
&\text{AND } (v = LT_v) \text{ AND } (\lambda = LT_\lambda) \\
&\text{THEN } (\Delta u_k^i = K_P(\Delta e_k + \alpha_e e_k + \\
&+ \alpha_f f_k)),
\end{aligned} \tag{36}$$

the upper index i, $i = \overline{1,64}$, corresponds to the index of the local model of CP for which the digital PI / PID controller was designed.

It should be outlined that the paper suggests a class of TS-FC because it can involve other parameters in all modules of B-FC. These modifications are of great interest in order to compensate for controlled plant nonlinearities and to further ensure control system performance enhancement.

Fig. 6 illustrates the specific parameters of the TS-FC, tuned by the modal equivalence principle, referred to also as parallel distributed compensation, connected to the coordinates of accepted operating points. In these conditions the following values of TS-FC parameters were obtained: $B_{\mu ZE}$=0.1, $B_{\mu ZE}$=0.3, $B_{\mu ZE}$=0.7, $B_{\mu ZE}$=0.9, B_{vZE}=1, B_{vZE}=10, B_{vZE}=20, B_{vZE}=29, $B_{\lambda ZE}$=0.05, $B_{\lambda ZE}$=0.1, $B_{\lambda ZE}$=0.2, $B_{\lambda ZE}$=0.25, B_e=0.2, $B_{\Delta e}$=0.2, B_f=0.2. The parameters B_e $B_{\Delta e}$ and B_f depend on the universes of the variables e_k, Δe_k and f_k, respectively, and were calculated to ensure the scaling factors which are not shown in Fig. 5.

Summarizing the results presented before, the design method for the new class of TS-FCs consists of the following steps:

- step 1: choose the significant operating points of the CP, linearize the mathematical model around these points and express the local linearized models in the forms (29) or (30),
- step 2: using a conventional design method (e.g., the frequency domain approach), tune the continuous-time local PI controllers dedicated to (29) and (30), respectively,
- step 3: choose a sufficiently small sampling period T_s accepted by quasi-continuous digital control and take into account the presence of a zero-order hold,
- step 4: discretize the continuous-time local PI / PID controllers and compute the parameters K_P, α_e and α_f in (34),
- step 5: set one TS-FC from the class of TS-FC in terms of choosing the operators involved in the inference engine, the membership function shapes, scaling factors and defuzzification method,
- step 6: apply the modal equivalence principle to calculate the TS-FC parameters.

5. CASE STUDY. DIGITAL SIMULATION RESULTS

The validation of the fuzzy control solution presented in the previous section is done considering a case study characterized by the simplified mathematical model in terms of (1) and (18) corresponding to the CP involved in tire slip control, as part of an ABS control system.

The CS structure is presented in Fig. 4. The parameter values in the case study were presented in section 2, together with τ=0.014 s and ε=0.001.

The TS-FC is designed according to the design steps presented in section 4. For the accepted case study, the simulation scenario consists of feeding a reference input λ_r=0.015 (acceptable for several surfaces), and the braking is started on a high friction surface (μ_H=0.9 for 0.9 s). Next, a low friction surface is encountered (μ_H=0.1 for the next 1.5 s), and the braking is finished on a medium friction surface (μ_H=0.5 for the final 0.6 s). The initial value of v is set to v_0=25 m/s. Part of the digital simulation results corresponding to the fuzzy control system with TS-FC is presented in Fig. 7 and Fig. 8.

6. CONCLUSIONS

The paper suggests an original fuzzy control solution dedicated to relative slip control. The solution consists of a class of Takagi-Sugeno fuzzy controllers and a design method.

The fuzzy control solution is backed up by the presentation of the detailed mathematical model of CP in ABS control accompanied by its simplification to enable the control design. A critical analysis of current approaches employed in fuzzy controlled ABSs presented in this paper emphasizes the necessity for fuzzy control solutions capable of being implemented in real-time control of modern cars.

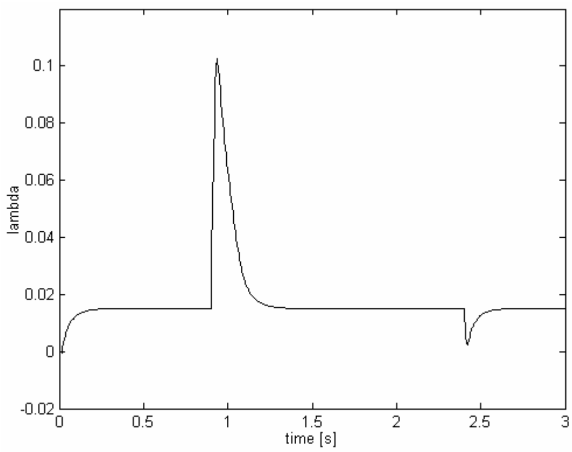

Fig. 7. Relative tire slip (λ) versus time.

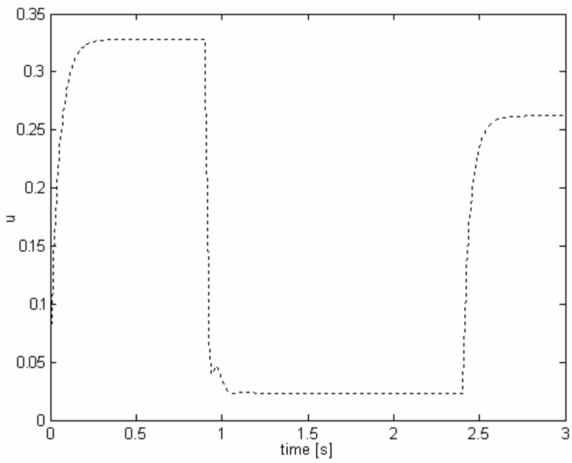

Fig. 8. Control signal versus time.

The solution presented here in a very simple form (with 64 local controllers merged in the TS-FC although there can be used more, 80 at least depending on the performance of the automotive embedded systems that equip the cars) meets this necessity and preliminary tests done with the new brake-by-wire braking system, called Electronic Wedge Brake (EWB) confirm the control system performance enhancement. The fuzzy control solution described in section 5 fulfils the requirements in ABS control highlighted in section 2.

Further research will be focused on real-time experiments and the reduction of rule base that may result in other classes of fuzzy controllers with supplementary features [1, 7]. On the other hand, since the controlled plants in audio signal processing have the same structure but with smaller time constants [2, 5, 13], the authors' intention is to design fuzzy controllers for these plants, too.

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