# Predictive Sliding Mode Controller for Continuous Bio-Fermenter Systems

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**Abstract:** Model predictive controller predicts the system performance and accordingly improves the controller performance. So it works well for time delayed systems. But the problem is that, it is not robust under certain cases. Sliding mode controller is robust but with a very slow response rate. Its limitation is its delay handling capacity. In this paper, a predictive controller with good delay handling capability is combined with robust sliding mode controller. It is found to have a quick rise time and settling time with minimum overshoot. It is more robust and produced no offset, or oscillation. In this work, predictive sliding mode control is designed for cylindrical, conical and cylindroconical bio-fermenter systems and their performances are analysed. The controller is implemented in real time for a cylindrical system and a conical system and it is found that the controller could handle delays and is also robust to parameter variations.

*Keywords:* Sliding mode control, Predictive Control, Non minimum phase system, Lyapunov stability, robustness

## 1. INTRODUCTION

Fermentation technology involves studying, controlling and optimizing fermentation process. Fermenter are vessels designed and built to grow high concentration of cells. They are different from chemical reactors in the way that, they should handle living organisms, and so they should be designed in such a way that, they overcome the difficulties of process upsets and contaminations as these living organisms are more sensitive and less stable says Jagani et al. (2010).

Continuous fermenter, is a closed loop system, where exponential growth of microbes is maintained for long period by the addition of fresh liquid medium continuously and the product is extracted from the fermenter continuously. In this type of fermenter, the environmental conditions are kept constant, and the growth rate of the microbes is also maintained. To perform all these, the reactor requires more complex controls to be carried out, for which all possible measurements are to be made.

The physical parameters that are measured and controlled in a bioreactor are temperature, pressure, reactor weight, liquid level, foam level, agitator speed, power consumption, gas flow rate, medium flow rate, feed flow rate, viscosity of the culture, and gas hold up. The chemical parameters measured and controlled are pH, dissolved oxygen, dissolved carbon-dioxide, redox potential, conductivity and broth composition. The biological parameters such as biomass concentration, enzyme concentration, biomass composition, viability and morphology are also measured and controlled.

The major parameters that affect the growth of the organism are (i) temperature, which is maintained constant by means of a coolant jacket, (ii) pH of the liquid which varies based on the acid or base which is added to the reactor to support fermentation, (iii) concentration of nutrients, and (iv) dissolved oxygen which are continuously added to the fermenter.

The mixture of cell culture and raw materials which is the broth is allowed to settle. As broth settles, fermentation begins and cells start to grow. Fermentation is often accompanied by the formation of foam due to the presence of biomaterials like protein in the solution. Controlling the formation of foam in a bioreactor is important because it leads to the control of fermentation itself and also the equipment involved could be protected from damage leading to decrease in production cost. The formation of foam could be decreased by adding anti-foaming agents. But the main disadvantage is the proper selection of anti-foaming agent which is based on previous experience as said by Etoc et al. (2006).

When this foam formation is left unnoticed, it may pave way for the foam to enter into the gas suction pump, and damage the pump. This results to frequent replacement of pumps and shut down of the process for cleaning and replacement, which are undesirable in the case of continuous fermenters. So an alternate way of controlling foam is by varying the level of liquid broth in the fermenter well below the location were the pipe for suction pump is fixed. This ultimately prevents the foam reaching the pump and damaging it.

Fermentation is generally carried out either in cylindrical, conical or cylindroconical containers. The cylindrical fermenters are most commonly used for small scale applications where batch fermentation is carried out. The problem here is the cleaning of the container after each batch of fermentation. The sediments at the bottom are to be properly and fully removed and the container has to be sterilized, failure of which may cause contamination of the next batch of substance.

Conical fermenters overcome the difficulty in complete drainage of extract from bottom of the fermenter. Here the methods of liquid transfer are gravity feeding or vessel pressurization which avoid the usage of pump for media transfer. The layer of sediment at the bottom of the cone continuously disengages gas, which throws the fermentation substances into the fermentation medium, and so the highest proportion of fermentation substance in suspension occurs. But the volume of fermentation substance is less when compared to the space occupied by the fermenter.

So a cylindroconical fermenter is used in the industries. This shape was patented by Leopold Nathan in 1927, and he claims that, this fermenter supports faster fermentation rate. It has a cylindrical region at the top, which facilitate the increase in capacity and a conical bottom, which supports easy drain of the product. As these fermenters are covered, contamination is minimized reducing the cost of sterilization.

Model Predictive Control (MPC) is an algorithm by which a sequence of control signals are calculated in such a way to minimize a cost function defined over a prediction horizon. To implement MPC, a model of the plant is used to predict the future plant outputs based on the past and present values of input and output of the plant. It handles delays effectively.

Layerle et al. (2008) has reported MPC for non-minimum phase systems. Pawlowski et al. (2012) says it is similar to a feedback plus feed forward control and perfect control is possible if weight is set to zero. It lists the nonlinear MPC applications and discusses about modelling, control, optimization, and implementation issues like infinite prediction horizon which is much needed in control theory for nominal stability. Lee et al. (2002) discusses receding horizon concept of MPC to nonlinear systems. Bleris et al. (2005), Bolognani et al. (2009), Srinivasan et al. (2013) compares IMC and DMC for pH process and reports DMC performs better than IMC.

Model predictive control is not robust and so when the requirement is that the controller must be robust, sliding mode control is used. Utkin et al. (2002) explains the fundamentals of SMC with examples of motors and other electromechanical systems. Szell et al. (2014) deals with the mathematics behind sliding mode control. Ying-Jeh et al. (2008) have done a survey on the fundamental theory, and practical applications of SMC. The paper consolidates the characteristics of SMC as invariant, robust, providing order reduction, controlling chattering, and providing satisfactory performance. Gouaisbaut et al. (2002); Lei et al. (2013) have reported on SMC for time delay system.

Sliding mode control (SMC) involves defining a structure, selection of parameters for the structure and defining the travelling path. The switching surface represents the plant behaviour during the transient period. The control law is designed such that, any state outside the sliding surface is driven to reach the surface in a finite time and stay there forever. The design of sliding mode has two controls to be designed. One is the equivalent control and the other is switching control. The performance of the controller can be varied by varying the switching control. Abbas et al. (2012) deals with sliding mode control for liquid level systems using reaching law in switch control. It is found to be robust, but it suffers from chattering.

Houda et al. (2012) reports on a hybrid control structure called Predictive Sliding Mode Control (PSMC), which combines the advantages of MPC and SMC, which would overcome the drawbacks of both the controllers. Here, prediction of the sliding surface is introduced into the objective function. Even in the presence of disturbance and parameter variations, PSMC keeps the controlled variable at its set point without oscillation. The authors claim the controller robust. Garcia et al. (2009) has described the application of PSMC for solar air conditioning system. Marco et al. (2018) has reported work on PSMC for a wind turbine. Garcia et al. (2005) have reported this algorithm for time delayed system. Layerle et al. (2008) has reported PSMC results for MIMO non minimum phase system. Mansour et al. (2015) tested PSMC for multivariable systems and claims the controller to be robust in hard parameter variation cases and eliminating chattering. The controller's stability was discussed by Houda et al. (2015). Camacho et al. (2002) carried out simulation for time delayed system in which, smith predictor architecture is combined with SMC. Garcia et al. (2013) reports PSMC for non-minimum phase systems and claims that the controller avoids instability of MPC, and stays robust to model uncertainties and disturbance. In Rubagotti et al. (2011), integral sliding mode is designed, MPC is used for adding constraints, and the overall controller is an additive sum. Lingfei et al. (2006), also reports on PSMC in which, by introducing the sliding mode model, the trajectory of the sliding mode follows the expected track to the proposed reaching law exactly.

Here, the Predictive Sliding Mode Control (PSMC) is designed for cylindrical, conical and cylindroconical biofermenter systems and their performances are analysed.

## 2. CONTROLLER DESIGN

There is always a mismatch between the mathematical model and the actual plant. There are several reasons for the mismatches like not considering the transportation delay in the system, not considering the actuator valve dynamics etc. These mismatches will always affect the controller design. Though MPC has a very good capability of handling delays and improving the system performance, it is not able to handle mismatches in model dynamics. But controllers should always be designed such that these mismatches does not affect the system performance. Controllers designed with this quality are called robust controllers and one such robust controller is sliding mode controller.

### 2.1 Sliding Mode Controller

Sliding Mode Control is a non-linear control technique which alters the behaviour of the system by applying a discontinuous signal. The application of this control signal forces the system to slide along the cross section of the system's usual behaviour. It is designed to drive the system states to a particular surface in state space called sliding surface. Its salient features are accuracy, robustness, easy tuning and implementation. Also in SMC, no precise information about the original system dynamics is required and the controlled system is a completely uncertain black box object. The dynamic behaviour of the system may be tailored by the choice of the sliding surface and the closed loop behaviour is totally insensitive to uncertainties like model parameter uncertainty, disturbance and non-linearity.

The controller design involves 2 parts: the equivalent control design to keep the system state on the sliding surface and the switching control design to force the system to slide on the sliding surface.

At first, the sliding surface, which is a scalar function of the system states has to be defined and it can take the following forms.

$$s = \dot{e} + c_0 e \tag{1}$$

or

$$s = \ddot{e} + c_1\dot{e} + c_0 e \tag{2}$$

where, s is the sliding surface, e is the error, $c_0, c_1$ are constants, $\dot{e}$ is the first derivative of error, $\ddot{e}$ is the second derivative of error.

The values of c are to be selected such that it supports the aim of making s zero at the earliest. The equation $s = 0$, defines a surface in error space called the sliding surface. The choice of the sliding surface by the above said functions have a drawback that, the number of constants to be tuned are more and the complexity increases as the order of the system increases. So the choice of the sliding surface can be made such that,

$$s = \left(\frac{d}{dt} + c\right)^{k_i} e \tag{3}$$

where, $k_i$ defines the order of the equation and s depends on a single scalar parameter c. The choice of c must be such that it is always positive, because it defines the poles of the resulting reduced dynamics of the system when it is sliding.

The next step is the design of control law, which aims to bring the sliding surface to zero in a finite time. Based on the choice of the control law used for designing the switching, the SMC is differentiated into various types. Here, the sliding mode based on reaching law is used.

The reaching law increases the reaching speed when the state is far away from the switching manifold and reduces the rate at which the state is reached when it is near the manifold. Using this technique, the system's approaching speed decreases which leads to weakening of the chattering effect.

This law constrains the switching variable to reach the switching manifold at a constant rate. The merit of this law is its simplicity. The reaching law is defined as,

$$\dot{s} = -\epsilon \ sgn(s), \epsilon \ > 0 \tag{4}$$

The system is considered to be of the form,

$$\ddot{l}(t) = f(l,t) + b_1 u(t) \tag{5}$$

where, f(l,t) and $b_1$ are known and $b_1$ is a constant greater than 0. Here, l is the level of liquid which varies with respect to time and u is the controller output. The sliding mode function is chosen as in equation (1). Using general feedback mechanism,

$$\ddot{e} = \ddot{r} - \ddot{l} \tag{6}$$

Taking the first derivative of equation (1),

$$\dot{s} = \ddot{e} + c_0\dot{e} \tag{7}$$

$$\dot{s} = \ddot{r} - \ddot{l} + c_0\big(\dot{r} - \dot{l}\big) \tag{8}$$

Substituting equation (5) in equation (8),

$$\dot{s} = \ddot{r} - f(l,t) - b_1 u(t) + c_0\big(\dot{r} - \dot{l}\big) \tag{9}$$

Equating equation (4) and equation (9),

$$\ddot{r} - f(l,t) - b_1 u(t) + c_0\big(\dot{r} - \dot{l}\big) = -\epsilon \ sgn(s) \tag{10}$$

From this, the sliding mode controller using reaching law is obtained as,

$$u(t) = \frac{1}{b_1}\Big(\ddot{r} - f(l,t) + c_0\big(\dot{r} - \dot{l}\big) + \epsilon \ sgn(s)\Big) \tag{11}$$

### 2.2 Predictive Sliding Mode Controller

In this control technique, the sliding surface is predicted with the present values of input and future values of control signals. Here, MPC is used to force the state into the required region within a finite horizon while the state is away from the region and SMC is used while the state is within the region. Here, the prediction of the control surface is included as one factor in control objective. Using future control moves allows better prediction of future control values in the sliding surface and so, it is very useful in implementing control action for systems with dead time.

The main function of this controller is to make the controlled variable track the set point, which is supported by the objective of predicting the future sliding surface and making the state reach to zero at the earliest. This controller has two parts: a continuous part, the MPC, which is responsible for making the controlled variable track the set point, and a discontinuous part which is the non-linear predictive element that includes the switching element of the control law.

The system sampled at k intervals is considered as,

$$x_m(k+1) = A_m x_m(k) + B_m u(k) \tag{12}$$

where, $A_m$ is the state matrix, $B_m$ is the input matrix, $x_m$ is the state vector and u(k) is the input vector to the system, which is the controller output at the $k^{th}$ instant.

The sliding function is defined as,

$$s(k) = C_s x_m(k) \tag{13}$$

where, $C_s$ is the vector of sliding constant. The necessary and sufficient condition assuring sliding motion and convergence to the sliding surface is,

$$|s(k+1)| - |s(k)| < 0 \tag{14}$$

According to sliding mode function represented by equation (13), the sliding value at the $(k_i + N_p)$ instant can be obtained as,

$$s(k_i + N_p) = C_s x_m(k_i + N_p) \tag{15}$$

where, $N_p$ is the prediction horizon.

To confirm the convergence of the sliding function s(k) to zero, reaching law is introduced. According to reaching law,

$$s(k+1) = -\epsilon \; sgn(s(k)) \tag{16}$$

where, $\epsilon$ is the switch gain constant.

To enable fast convergence, the equation is modified as,

$$s(k+1) = s(k) - \epsilon \; sgn(s(k)) \tag{17}$$

where, sgn is the sigmoidal function defined as,

$$sgn(k) = \begin{cases} -1, & \text{if } x < 0, \\ 1, & \text{otherwise.} \end{cases} \tag{18}$$

For prediction of the sliding mode, a reference sliding mode trajectory is chosen based on equation (17) as,

$$s_r(k_i+N_p) = s_r(k_i+N_p-1) - \epsilon(sgn(s_r(k_i+N_p-1)) \tag{19}$$

The objective is to design a sliding mode predictive control. As this is a prediction mechanism, it involves prediction of future sliding mode trajectory based on the present values of the system states. Using equation (15), the sliding function at any future instant $(k_i + N_p)$ can be obtained in terms of present and past values of the system state and future control input as,

$$s(k_i + N_p|k_i) = C_s A_m^p x(k_i) + \sum_{j=1}^{p} C_s A_m^{j-1} B_m u(k_i + N_p - j) \tag{20}$$

Equation (20) can be represented in vector form as,

$$S_p(k+1) = F_{psmc} x(k) + \phi_{psmc} U(k) \tag{21}$$

where,

$$S_p(k+1) = [s(k_i+1|k_i) \; . \; . \; . \; s(k_i+N_p|k_i)]^T \tag{22}$$

$$U = [u(k_i) \;\; u(k_i+1) \; . \; . \; . \; u(k_i+N_c-1)]^T \tag{23}$$

$$F_{psmc} = \begin{bmatrix} (C_s A_m)^T) \\ (C_s A_m^2)^T) \\ . \\ . \\ . \\ (C_s A_m^{N_p})^T) \end{bmatrix} \tag{24}$$

and,

$$\phi_{psmc} = \begin{bmatrix} C_s B_m & . \; . & 0 \\ C_s A_m B_m & . \; . & 0 \\ . & . \; . & . \\ . & . \; . & . \\ C_s A_m^{N_p-1} B_m & . \; . & C_s A_m^{N_p-N_c} B_m \end{bmatrix} \tag{25}$$

The predicted sliding mode value can be represented in terms of practical sliding mode value as,

$$\hat{s}_p(k_i + N_p) = s(k_i + N_p) + h_p e(k_i) \tag{26}$$

where, $e(k_i)$ is the error and $h_p$ is correction coefficient.

Using equation (20),

$$\hat{s}_p(k_i + N_p|k_i) = C_s A_m^p x(k_i) + h_p e(k_i) + \sum_{j=1}^{p} C_s A_m^{j-1} B_m u(k_i + N_p - j) \tag{27}$$

In vector form, it can be represented as,

$$\hat{S}_p(k+1) = S_p(k+1) + H_p E(k)) \tag{28}$$

where,

$$\hat{S}_p(k+1) = [\hat{s}_p(k_i+1) \; . \; . \; . \; \hat{s}_p(k_i+N_p)]^T \tag{29}$$

$$H_p = diag \begin{bmatrix} h_1 \;\; h_2 \; . \; . \; . \; h_{N_p} \end{bmatrix} \tag{30}$$

Optimization is done using the cost function,

$$J = \sum_{j=1}^{N_p} [\hat{s}_p(k+j) - s_r(k+j)]^2 + \sum_{j=1}^{N_c} [g[u(k+j-1)]]^2 \tag{31}$$

where, $N_c$ is the control horizon.

To find the optimal $\Delta u$ that will minimize J, partial differentiation of J with respect to $\Delta u$ is done and equated to zero.

$$\frac{\partial J}{\partial U} = 0 \tag{32}$$

On solving,

$$U(k) = -(\phi_{psmc}^T \phi_{psmc} + G)^{-1} \phi_{psmc}^T [F_{psmc} x(k_i) + H_p E(k) - S_r(k+1)] \tag{33}$$

where,

$$S_r(k+1) = [s_r(k_i+1) \; . \; . \; . \; s_r(k_i+N_p)]^T \tag{34}$$

$$G_{psmc} = [g \;\; g \; . \; . \; .g] \tag{35}$$

and g is the tuning parameter.

Receding horizon control can be implemented by using the first value of the sequence of control signal calculated which is given by,

$$U(k) = -[1 \;\; 0 \; . \; . \; .0] \, (\phi_{psmc}^T \phi_{psmc} + G)^{-1} \phi_{psmc}^T [F_{psmc} x(k_i) + H_p E(k) - S_r(k+1)] \tag{36}$$

## 3. RESULTS AND DISCUSSIONS

### *3.1 System Model*

The transfer functions of the cylindrical and conical systems are obtained by experimentally modelling them using the set ups shown in Fig. 1 and Fig. 2. The physical dimensions of the systems are given in table 1.From the open loop responses obtained, the systems transfer functions were found and approximated to a first order system with dead time.

Table 1. Dimensions of the physical setup

| specification | cylindrical | conical | cylindroconical |
|---|---|---|---|
| Tank height | 43 cm | 60 cm | cylindrical 50 cm |
| | | | conical 50 cm |
| Maximum diameter | 25 cm | 42 cm | 40 cm |
| slide angle | | 70 degree | 66 degree |

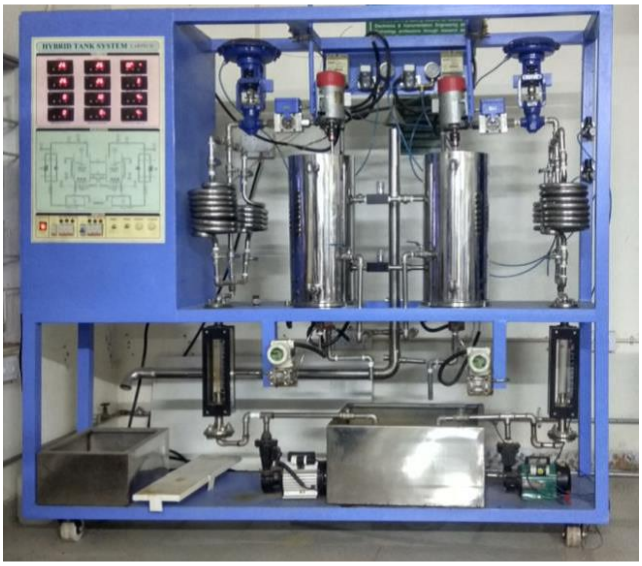

Fig. 1. Experimental setup of cylindrical system

Fig. 2. Experimental setup of conical system

Case 1: For cylindrical system
The transfer function G(s)is given by,

$$G(s) = \frac{11.165}{1185.185s + 1} e^{-40s} \tag{37}$$

It is approximated using first order Pade approximation as,

$$G(s) = \frac{-223.3s + 11.165}{23703.7s^2 + 1205.185s + 1} \tag{38}$$

It is represented in state space as,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -4.22 * 10^{-5} & -0.0508 \end{bmatrix} x + \begin{bmatrix} 0 \\ 4.22 * 10^{-5} \end{bmatrix} u \tag{39}$$

$$y = [11.165 \;\; -223.3]\, x \tag{40}$$

Case 2: For conical system

$$G(s) = \frac{0.3013}{21.11s + 1} e^{-17s} \tag{41}$$

It is approximated using first order Pade approximation as,

$$G(s) = \frac{-2.56105s + 0.3013}{179.435s^2 + 29.61s + 1} \tag{42}$$

It is represented in state space as,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -0.00557 & -0.165 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0.00557 \end{bmatrix} u \tag{43}$$

$$y = [0.3013 \;\; -2.56105]\, x \tag{44}$$

Case 3: For cylindroconical system
The model of the cylindroconical system is obtained by considering the system into two regions, based on the dynamics of the system: the top region is the cylindrical region and the bottom region is the conical region. The dimensions were chosen from Kesavan (2014).

$$G_{cy}(s) = \frac{1.27}{1597.28s + 1} e^{-42s} \tag{45}$$

$$G_c(s) = \frac{0.68}{90s + 1} e^{-42s} \tag{46}$$

It is approximated using first order Pade approximation as,

$$G_{cy}(s) = \frac{-22.67s + 1.27}{33542s^2 + 1617.28s + 1} \tag{47}$$

$$G_c(s) = \frac{-14.28s + 0.68}{1890s^2 + 111s + 1} \tag{48}$$

The cylindrical region is represented in state space as,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2.98 * 10^{-5} & -0.0482 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2.98 * 10^{-5} \end{bmatrix} u \tag{49}$$

$$y = [1.27 \;\; -22.67]\, x \tag{50}$$

The conical region is represented in state space as,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -5.29 * 10^{-4} & -0.0587 \end{bmatrix} x + \begin{bmatrix} 0 \\ 5.29 * 10^{-4} \end{bmatrix} u \tag{51}$$

$$y = [0.68 \;\; -14.28]\, x \tag{52}$$

### *3.2 Controller Response*

The level of broth in the fermenter is to be maintained constant by manipulating the raw material inflow to the fermenter and varying the speed of the agitator which is also a cause for producing foam. This prevents the foam level from rising and damaging of pumps.

The predictive sliding mode control is designed for all the three bio-fermenter systems using the parameters values shown in table 2. Same set of controller parameters except the $C_s$ were chosen for sliding mode controller for the purpose of comparison. The closed loop responses of the systems when PSMC controller is used are obtained as shown in Fig. 3, Fig. 4 and Fig. 5 respectively.

Table 2. Controller Parameters

| Parameters | cylindrical | conical | cylindroconical |
|---|---|---|---|
| Cs | [3 1] | [3 1] | [3 1] |
| Np | 12 | 12 | 12 |
| Nc | 2 | 2 | 2 |
| epsilon | 10 | 10 | 10 |

When the prediction horizon is decreased, the oscillations increase. When the value of the tuning parameter g is increased, offset decreases. As the size of control horizon is increased, the computational complexity increases. The parameter has very less impact on the offset of the system response. Offset variations are less as $\epsilon$ value increases. From all the three responses, it is evident that, there is a quick rise in the system response with minimum overshoot when PSMC is used than SMC thus preventing the foam

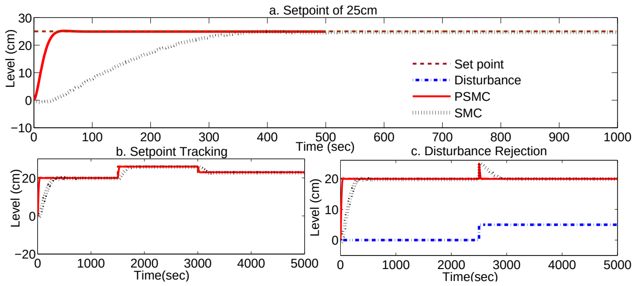


Fig. 3. Response of cylindrical bio-fermenter

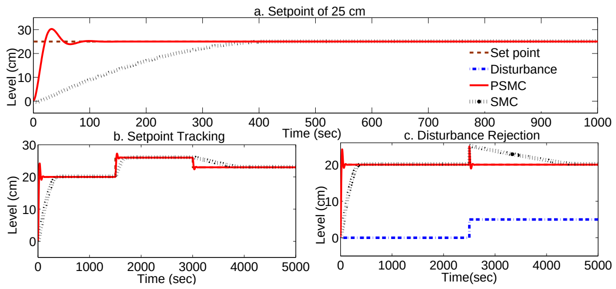


Fig. 4. Response of conical bio-fermenter

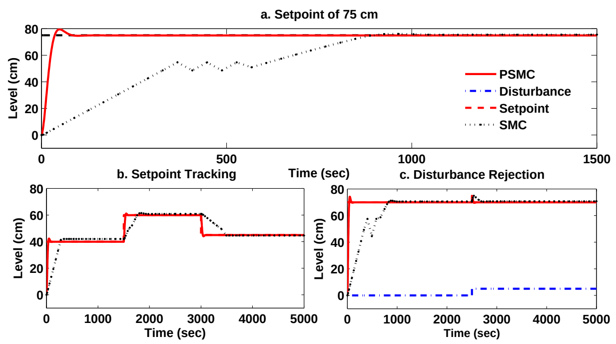


Fig. 5. Response of cylindroconical bio-fermenter

entering into the pump. The set point tracking capability and disturbance rejection capability of the system is good, which helps the system to accommodate to any variations and react efficiently.

### *3.3 Stability Analysis*

Stability of the closed loop system is analysed based on the direct method of Lyapunov. It is based on the concept of energy and the relation between stored energy and system stability.
If the system of the form,

$$\frac{dx}{dt} = f(x(t)) \tag{53}$$

has a solution $\mathrm{x}(\mathrm{x}\ (\mathrm{t}_0),\ \mathrm{t})$, then the energy function associated with it may be represented as $\mathrm{V_L(x)}$ . If $\mathrm{dV_L(x)}/dt$ is negative for all $\mathrm{f(x(t_0),\ t)}$, except the equilibrium point, then it means that the energy of the system decreases as time increases and finally, the system will reach its equilibrium point. This holds because, the energy is a non-negative function of system state which reaches a minimum, only if the system motion stops. The effectiveness of determination of stability by this method is based on the choice of $\mathrm{V_L(x)}$.

Considering that the system is linearized around its operating point, state equation of the system is given by,

$$\dot{x} = Ax \tag{54}$$

The system is asymptotically stable in the large at the origin, if and only if, given any symmetric, positive definite matrix Q, there exists a symmetric positive definite matrix P which is the unique solution of

$$A^T P + PA = -Q \tag{55}$$

Assuming Q to be an identity matrix, the P matrix is calculated for the system. Using Sylvester's theorem, the positive definiteness of P matrix is checked. If the P matrix is positive definite, then the origin of the system under consideration is asymptotically stable in the large.

If the state matrix is

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \tag{56}$$

P matrix is,

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{57}$$

and Q matrix is an identity matrix,

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{58}$$

then, P matrix is calculated by solving the equation,

$$A^T P + PA = -Q \tag{59}$$

Case 1: For cylindrical system
The stability of the systems are checked using Lyapunov stability criteria.
When SMC is implemented for a cylindrical system, the closed loop system matrix and its P matrix are obtained as,

$$A = \begin{bmatrix} -0.0258 & -0 \\ 1 & 0 \end{bmatrix} \tag{60}$$

$$P = 10^5 \begin{bmatrix} 5.9977 & 0.0550 \\ 0.0550 & 0.0042 \end{bmatrix} \tag{61}$$

Here, the determinant of P and all its submatrices are positive. Therefore, the system is stable.

When PSMC is implemented for a cylindrical system, the closed loop system matrix and its P matrix are obtained as,

$$A = \begin{bmatrix} -0.0945 & -0.0116 & -0.0005 & -0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{62}$$

$$P = 10^8 \begin{bmatrix} 1.4292 & 0.1350 & 0.0121 & 0.0002 \\ 0.135 & 0.0171 & 0.0016 & 0.0001 \\ 0.0121 & 0.0016 & 0.0002 & 0 \\ 0.0002 & 0.0001 & 0 & 0 \end{bmatrix} \tag{63}$$

Here, the determinant of P and all its submatrices are positive. Therefore, the system is stable.

Case 2: For conical system
When SMC is implemented for a cylindrical system, the closed loop system matrix and its P matrix are obtained as,

$$A = \begin{bmatrix} -0.1062 & -0.0003 \\ 1 & 0 \end{bmatrix} \tag{64}$$

$$P = 10^4 \begin{bmatrix} 1.5696 & 0.0916 \\ 0.0916 & 0.0062 \end{bmatrix} \quad (65)$$

Here, the determinant of P and all its submatrices are positive. Therefore, the system is stable.

When PSMC is implemented for a conical system, the closed loop system matrix and its P matrix are obtained as,

$$A = \begin{bmatrix} -0.5354 & -0.0875 & -0.0045 & -0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (66)$$

$$P = 10^6 \begin{bmatrix} 4.1702 & 2.2327 & 0.3649 & 0.0182 \\ 2.2327 & 1.1969 & 0.1961 & 0.0098 \\ 0.3649 & 0.1961 & 0.0324 & 0.0017 \\ 0.0182 & 0.0098 & 0.0017 & 0.0001 \end{bmatrix} \quad (67)$$

Here, the determinant of P and all its submatrices are positive. Therefore, the system is stable.

Case 3: For cylindroconical system
When SMC is implemented for a cylindrical system, the closed loop system matrix and its P matrix are obtained as,

$$A = \begin{bmatrix} -0.0349 & -0.0003 \\ 1 & 0 \end{bmatrix} \quad (68)$$

$$P = 10^4 \begin{bmatrix} 4.9254 & 0.1719 \\ 0.1719 & 0.0074 \end{bmatrix} \quad (69)$$

Here, the determinant of P and all its submatrices are positive. Therefore, the system is stable.

When PSMC is implemented for a cylindroconical system, the closed loop system matrix and its P matrix are obtained as,

$$A = \begin{bmatrix} -0.0967 & -0.0092 & -0.0004 & -0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (70)$$

$$P = 10^8 \begin{bmatrix} 2.6826 & 0.2595 & 0.0206 & 0.0006 \\ 0.2595 & 0.0291 & 0.0024 & 0.0001 \\ 0.0206 & 0.0024 & 0.0002 & 0 \\ 0.0006 & 0.0001 & 0 & 0 \end{bmatrix} \quad (71)$$

Here, the determinant of P and all its submatrices are positive. Therefore, the system is stable. So, both the controllers designed produce stable output for all the systems.

### *3.4 Robustness Analysis*

Here, the robustness of the controllers are checked by altering the values of the system matrix. For the purpose of analysis, uniformity in altering the system dynamics is maintained. The value of $A_{21}$ of state matrix and $B_{21}$ of input matrix are increased ten times in all the systems uniformly to introduce variations in dynamics of the system. Fig. 6 shows the response of the bio-fermenter systems when the system dynamics is altered. The actual system dynamics and the altered system dynamics are as mentioned from equation(72)to equation(83).
Case 1: For cylindrical system

a. Actual dynamics

$$A_m = \begin{bmatrix} 0 & 1 \\ -4.22 * 10^{-5} & -0.0508 \end{bmatrix} \quad (72)$$

$$B_m = \begin{bmatrix} 0 \\ -4.22 * 10^{-5} \end{bmatrix} \quad (73)$$

b. Altered dynamics

$$A_m = \begin{bmatrix} 0 & 1 \\ -4.22 * 10^{-4} & -0.0508 \end{bmatrix} \quad (74)$$

$$B_m = \begin{bmatrix} 0 \\ -4.22 * 10^{-4} \end{bmatrix} \quad (75)$$

Case 2: For conical system
a. Actual dynamics

$$A_m = \begin{bmatrix} 0 & 1 \\ -0.00556 & -0.1646 \end{bmatrix} \quad (76)$$

$$B_m = \begin{bmatrix} 0 \\ -0.00556 \end{bmatrix} \quad (77)$$

b. Altered dynamics

$$A_m = \begin{bmatrix} 0 & 1 \\ -0.0556 & -0.1646 \end{bmatrix} \quad (78)$$

$$B_m = \begin{bmatrix} 0 \\ -0.0556 \end{bmatrix} \quad (79)$$

Case 3: For cylindroconical system
a. Actual dynamics

$$A_m = \begin{bmatrix} 0 & 1 \\ -0.00556 & -0.1646 \end{bmatrix} \quad (80)$$

$$B_m = \begin{bmatrix} 0 \\ -0.00556 \end{bmatrix} \quad (81)$$

b. Altered dynamics

$$A_m = \begin{bmatrix} 0 & 1 \\ -0.0556 & -0.1646 \end{bmatrix} \quad (82)$$

$$B_m = \begin{bmatrix} 0 \\ -0.0556 \end{bmatrix} \quad (83)$$

The responses show that the closed loop dynamics does not vary much in the case of PSMC when compared to SMC even when the system and input parameters are varied. This shows that the controller is robust.

### *3.5 Real Time Implementation*

The experiment is carried out in real time in cylindrical and conical set ups shown in Fig. 1 and Fig. 2. The physical system was put in closed loop through control algorithm in a embedded processor. Based on the online level measurements made by the level transmitters, the corrective actions were generated by the controller which is given to a pneumatic control valve connected at the inlet pipeline, to vary the flow of liquid thereby controlling the level of liquid. Also the speed of the agitator was varied so as to limit the agitation rate, thereby varying the foam formation to some extent, which also contributes to variation in level of liquid.

Fig. 7 shows the real time responses of the cylindrical and conical system respectively, for a set point of 25 cm. Delay

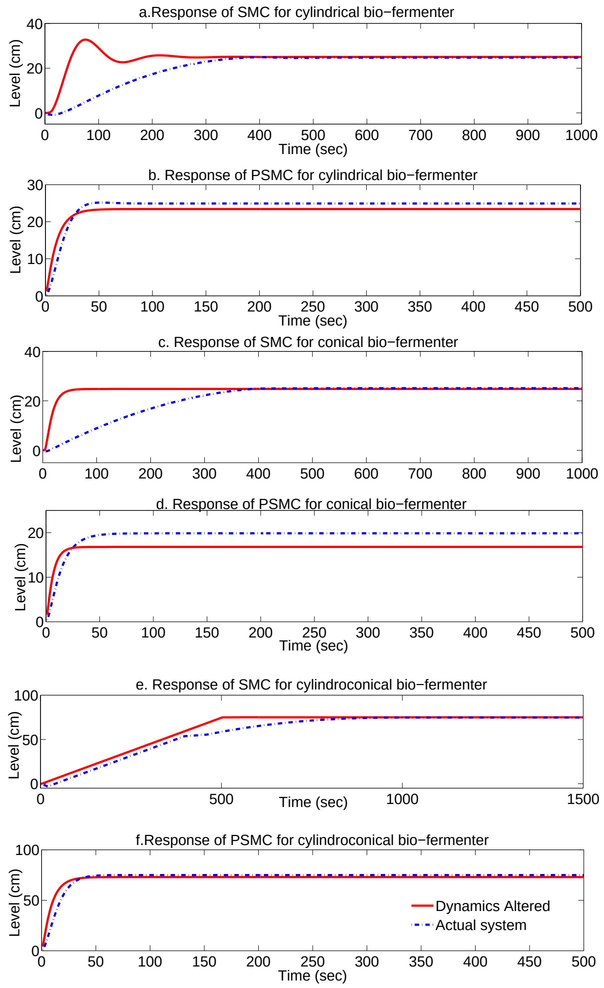


Fig. 6. Response of the bio-fermenter systems when dynamics is altered

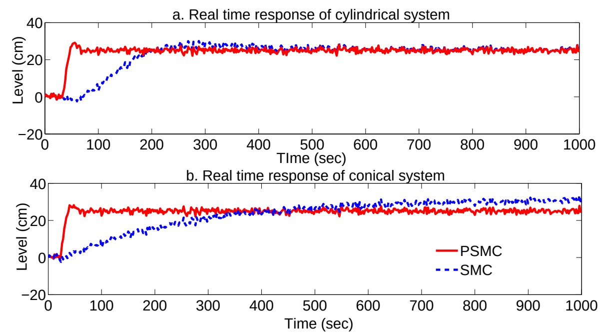


Fig. 7. Real time response of systems

is intentionally introduced into the system using delay coils. In the case of PSMC, there is no much deviation of the real time system responses from that of the responses simulated, which again proves the robustness of the system which has overcome the modelling mismatches. The offset produced is almost zero for PSMC, which means that the basic requirement of a bio-fermenter that the volume of broth has to be maintained constant is achieved. Also, both systems have a quick rise and settling time for PSMC when compared to SMC, which shows that the controller responds quickly and efficiently.

## 4. SUMMARY

The response of the bio-fermenter systems when subjected to SMC and PSMC are observed and their performance are analysed. Based on their performance, it is found that, the rise time, settling time, overshoot, offset, oscillations are very minimum and its set point tracking and disturbance rejection capabilities are high for PSMC. Real time implementation of SMC and PSMC are made in a cylindrical system and a conical system and their responses are also analysed. The disturbance rejection capability, fast convergence, with oscillation makes PSMC superior to SMC. The predictive sliding mode controller is also found to be robust.

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## Appendix A. PROGRAM FOR PSMC IN LABVIEW