Robust Finite Time Extended State Observer Design for SISO Systems with additive Time-varying Disturbances

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Abstract: Extended state observer (ESO), which works with the augmented state variable of the nonlinear system, is proposed and employed to straightforwardly reject disturbances by the online disturbance reconstruction (estimation) and attenuation technique under the active disturbance rejection control (ADRC) scheme. Focused on more precise disturbance estimation on unknown high order perturbation signals or fast-varying disturbances, which are difficult to be dealt with by most of traditional ESO, yet widely emerging in practical application control systems, a novel global robust finite-time ESO is designed and expounded for rapidly and accurately tracking such internal and external disturbances. The proposed approach employs finite-time control theory and compensation control method by means of the sliding mode switching term to ensure that the state variable estimation error can converge in finite time to zero for various forms of system disturbances. The corresponding stability analysis is presented in terms of the Lyapunov method. In addition, the validity of the robust finite-time ESO is verified via simulations and revealed to be more effective than the traditional linear and nonlinear extended state observer.

Keywords: Extended state observer, total disturbance, sliding mode control, finite-time stability.

1. INTRODUCTION

In physical systems, various forms of disturbances and uncertainties, or their superposition, are ubiquitous, however, it may bring about unpredictable adverse effects on control performance, let alone instability of the system (Gao, 2014; Chen, 2016). Robust and adaptive control methods have been the main focus of many recent studies on controller design for such systems (Xie, 2000; Jiang, 2015). Although, the common prerequisite for these approaches is to acquire a fairly accurate mathematical model of the control system, or to limit the uncertainty to a sufficiently small range. In fact, the accuracy of physical system models and the magnitude of uncertainties often exceed the assumptions. An effective solution is to estimate the impact of uncertainty or interference from measurable variables, and then take measures to compensate this impact based on these estimates. Compared with uncompensated feedback control systems, a smaller gain is allowed. There are a number of observers used for online estimation, such as disturbance observers (Ohishi, 1987), extended state observers (Han, 2009; Gao, 2003), unknown input observers (Guan, 1991), equivalent input observers (She, 2011), neural network observers (Abdollahi, 2006) and so on. All above disturbance observers are different in modeling information of the plant and assumptions made for the stability, but they have been proved to be effective practical solutions. Particularly expounding is that conventional observers need to obtain the disturbance contained in the system based on a pre-established accurate mathematical model excluding the ESO method (Zheng, 2009). Therefore, the leaved question of dynamic uncertainty has not been dealt with completely (Miklosovic, 2006; Tian, 2009).

Extended state observer (ESO), which plays an important role in the active disturbance rejection control theory and technical method first proposed by Han in (Han, 1995), has been well applied in the design of output feedback controller (Huang, 2000; Gao, 2001; Guo, 2016) because of the ability to estimate the undetermined system nonlinearities, dynamic uncertainties and the unknown external disturbances (Tian, 2007; Li, 2016; Guo, (2017a,b); Huang, 2017). For practical applications (Wang, 2013; Qi, 2016), the linearized and parameterized LESO, as a special case of the NESO, was proposed in (Gao, 2003). The ESO is much more effective by reasonably chosen nonlinear functions and related parameters (Wang, 2003; Han, 2009; Li, 2012; Wu, 2019; Zhao, 2015b), nevertheless, the observer requires rarely information about the plant model, only the input and output signals and the relative order of the control system. In ADRC framework, due to the estimated disturbance totality and the ingenious feedback, the remarkable performance of the nonlinear system control can be achieved, which also depends on the efficiency of ESO.

With the increasing significance of the ESO-based control methods, a large number of analysis results of the convergence of the observer are discussed (Yang, 2009; Yoo, 2006; Zheng, 2007). Furthermore, the convergence and stability of multiple forms of ESO or ESO-based control methods are also discussed in (Guo, 2011a, 2012, 2013, 2015; Pu, 2015; Zhao, 2015a, 2016, 2017, 2018; Wu, 2016, 2019). Although the traditional ESO has made great progress in theory and practice, it has not achieved enough high precision performance (Madoński, 2015; Xiong, 2015), especially when the disturbance is complex with a higher order and fast-varying disturbance quantity. And another unsatisfying feature is either infinite time convergence or else convergent to a region of the error equation for tracking disturbances, which are also usually limited to constant and slow changing. The reason lies in that the derivative of the lumped disturbance in the observed system is not zero. In this paper, our solution is to find a method to eliminate the disturbance and ensure the finite time convergence, so that the observation error can be reduced to zero, and then improve the estimation capability, instead of adjusting the parameters on the basis of the original ESO design. As we all know, convergence performance is an important indicator of the observer, and the rapidity and tracking ability under various forms of disturbances are the eternal criteria to evaluate the design of the ESO (also applicable to other disturbance observers). In practical applications, the equivalent system disturbance mostly contains the composite time-varying disturbance of various components. Hence, robustness and convergence are fundamental problem in the design and analysis of extended state observer, with which this paper concerned.

From the perspective of robust control, variable structure control (VSC) (Utkin, 1992; Zinober, 1994) can provide an effective estimation of the unknown part by means of equivalent control method. Inspired by the desirable properties of this approach, a new robust finite-time convergence extended state observer reconstructing with an integral-like chain structure by the discontinuous function is proposed in this paper. The basic principle of the ESO is derived from the following three steps. Firstly, the first time derivative of total disturbance is treated as an expanded state. In this way, the new type of integral chained observer is established for the new extended system, and the obtained observation error equation contains uncertainties. Secondly, a non-smooth control law is designed to make the initial observation error value reach the selected sliding surface in a limited time. Once the equivalent control is achieved, the uncertainty in the first step is estimated and compensated appropriately. Finally, combining the homogeneity theory, a finite-time control law is chosen for the error equation convergent to zero in the absent of uncertainty.

The main contribution of this paper is the novel ESO proposed with excellent accuracy in terms of robustness to external and internal disturbances (namely total or lumped disturbances) and finite-time convergence to zero.

i. Benefiting from the advantages of variable structure control and finite-time control theory, the proposed observer shows higher precision of online disturbance reconstruction compared with traditional ESO, which is mainly manifested in the convergence of error to zero.

ii. For fast-varying disturbances, and even high order polynomial-type disturbance, the approach in this paper can also be suitable and effective instead of the ESO bandwidth increasing sufficiently larger. The linear or nonlinear ESO can only estimate the extended state variable with zero steady-state error in case of invariant disturbances, while the method discussed can still track accurately more complex perturbations such as ramp-like or sinusoidal-like signals.

iii. Another distinctive feature is that it is easy to design and tune for practical applications, which provides an idea for designing efficient observers in the future.

The rest of the paper is organized as follows. The considered problem of ESO is described and some preliminaries are presented in Section 2. The design and analysis of proposed finite-time extended state observer are expounded in Section 3. Numerical simulations are performed to verify the performance of the designed extended state observer in Section 4. At last, In Section 5, this paper is concluded.

2. PROBLEM FORMULATION AND PRELIMINARIES

Considering the following n-dimensional SISO (possibly nonlinear) system

$$\begin{cases} x^{(n)}(t) = f(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) + w(t) + u(t) \\ y(t) = x(t) \end{cases} \tag{1}$$

The state equation of system (1) can be concisely described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_i = x_{i+1}, i = 2, \cdots, n-1 \\ \dot{x}_n = f(x_1, x_2, \cdots, x_n) + w(t) + u(t) \\ y = x_1 \end{cases} \tag{2}$$

where y is the measured output, u the control input , w the external disturbance, and f is possibly an unknown system function. Here, $f + w$ is called total disturbance which can be denoted by .

Assuming that the total disturbance function is differentiable, we can chose a new state as

$$\begin{cases} x_{n+1} = f_1(x_1, x_2, \cdots, x_n, w) \\ \dot{x}_{n+1} = d(t) \end{cases} \tag{3}$$

with

$$d(t) = \dot{f}_1(x_1, x_2, \cdots, x_n, w) \tag{4}$$

then an expanded system is established as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_i = x_{i+1}, i = 2, \cdots, n-1 \\ \dot{x}_n = x_{n+1} + u \\ \dot{x}_{n+1} = d(t) \\ y = x_1 \end{cases} \tag{5}$$

Referring to (Guo, 2011b), the ESO is designed as a Luenberger state observer in the form of

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \beta_1 g_1(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_i = \hat{x}_{i+1} - \beta_i g_i(\hat{x}_1 - x_1), i = 2, \cdots, n-1 \\ \dot{\hat{x}}_n = \hat{x}_{n+1} - \beta_n g_n(\hat{x}_1 - x_1) + u \\ \dot{\hat{x}}_{n+1} = -\beta_{n+1} g_{n+1}(\hat{x}_1 - x_1) \end{cases} \tag{6}$$

where $\beta_i(i = 1,2,\cdots,n+1)$ denote the observer gains to be designed, and $g_i(i = 1,2,\cdots,n+1)$ denote the linear or nonlinear functions which are appropriately chosen to estimate the states of $x_i(i = 1,2,\cdots,n)$ and the total disturbance x_{n+1}. It is the main idea of the ESO that the states $\hat{x}_i(i = 1,2,\cdots,n)$ and $\hat{x}_{n+1}$ of (6) can approach the states $x_i(i = 1,2,\cdots,n+1)$ as accurately as possible by regulating constants β_i.

Subtracting (5) from (6), the error equation of ESO can be written by

$$\begin{cases}\dot{e}_1 = e_2 - \beta_1 g_1(e_1) \\ \dot{e}_i = e_{i+1} - \beta_i g_i(e_1), i = 2,\cdots,n-1 \\ \dot{e}_n = e_{n+1} - \beta_n g_n(e_1) \\ \dot{e}_{n+1} = -\beta_{n+1} g_{n+1}(e_1) - d(t)\end{cases} \quad (7)$$

where $e_i = \hat{x}_i - x_i$ denote the state estimation error. Eq. (6) is designed to estimate all states of the expanded system, including the extended state of lumped disturbance x_{n+1}, which is composed of unmodelled dynamics (or uncertainties) and exogenous disturbances, as the result of equation (7) convergent through regulating β_i. The structure diagram of the extended state observer is shown in Fig. 1.

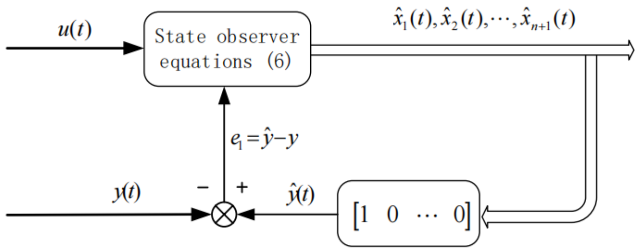


Fig. 1. Schematic diagram of the extended observer structure.

Letting $g_i(e_1) = k_i e_1, k_i \in \mathbb{R}$, the ESO is reduced to the linear extended state observer. The stable convergence of the ESO is guaranteed by properly choosing the value of β_i supposing that $d(t)$ is bounded (Yang, 2009; Talole, 2010). The advantage of employing the LESO is that the observer gains can be chosen through pole placement (e.g. via Ackerman's formula (Franklin, 1998)), while the gains of nonlinear ESO sometimes need to be determined by the trial-and-error method. Numerical simulation and theoretical analysis have proved the effectiveness of the nonlinear function, nevertheless, it is worth exploring other types of extended state observer with stronger robustness and efficient convergence performance.

Reviewing systems (6) and (7), it's easy to choose the control law to ensure the nominal system of observation error converge to the origin if there are no uncertainties, i.e. $d(t) = 0$. In other words, if the structure and stabilization law of the observer are appropriately redesigned in the presence of disturbances, the system (7) can converge globally in finite time as defined below, thus the performance of the observer can be greatly improved. This is exactly the motivation underlying this paper; that is, robust stability design based on finite-time control theory and variable structure equivalent control. The specific design and corresponding proofs are presented in the following chapters.

The basic elements of finite-time theory are given below.

Definition 1. (Hong, 2002) Consider a system given by

$$\dot{x} = h(x), h(0) = 0, x \in \mathbb{R}^n, x(0) = x_0 \quad (8)$$

where $h(x): \Omega \to \mathbb{R}^n$ is a continuous function on an open neighborhood $\Omega\backslash\{0\}$ of the origin $x = 0$. If the system is asymptotically stable in the open neighborhood $\Omega_0 \subseteq \Omega$ of the origin $x = 0$, the equilibrium of the system is finite-time stable. That is, there is a settling time $T > 0$such that the solution $x(t, x_0)$ of (8) for any initial condition $x_0 \in \Omega_0\backslash\{0\}$ satisfies $x(t, x_0) \in \Omega_0\backslash\{0\}$ for $t \in [0, T)$, $\lim_{t\to T} x(t, x_0) = 0$ and $x(t, x_0) = 0$ when $t > T$. When $\Omega = \Omega_0 = \mathbb{R}^n$, $x = 0$is globally finite-time stable.

Definition 2. (Qian, 2012) A vector field $h(x) = (h_1(x), \cdots, h_n(x))^T$ is called homogeneous of degree $m \geq -max\{\lambda_i, i = 1,\cdots n\}$ with respect to $(\lambda_1, \cdots, \lambda_n)$, if

$$h_i(\varepsilon^{\lambda_1} x_1, \cdots, \varepsilon^{\lambda_n} x_n) = \varepsilon^{m+\lambda_i} h_i(x), \varepsilon > 0, \lambda_i > 0, i = 1,\cdots,n \quad (9)$$

Lemma 1. (Bhat, 2005) Suppose the system (8) is homogeneous of degree $k < 0$. Then the system equilibrium $x = 0$ of (8) is globally finite-time stable if $x = 0$ is globally asymptotically stable.

Lemma 2. (Hong, 2002) Suppose that $V(x)$ is a continuous differentiable function and it is positive definite, there exists an open neighborhood $\Omega_0 \subseteq \Omega$ of the origin $x = 0$ such that

$$\dot{V}(x) + cV^{\alpha}(x) \leq 0, c > 0, \alpha \in (0,1) \quad (10)$$

Then the equilibrium $x = 0$ of the system (8) is finite-time stable. Moreover, when $\Omega = \Omega_0 = \mathbb{R}^n$, $x = 0$ is globally finite-time stable.

3. EXTENDED STATE OBSERVER DESIGN WITH GLOBALLY FINITE-TIME STABILITY

This section will propose the approach for the ESO with global finite-time convergence for systems being confronted with completely unknown disturbances. Here, we use the finite-time control law to make the ESO converge in a finite time when the system disturbance $d(t) = 0$, and the equivalent control method of variable structure control theory is used to compensate and eliminate the disturbance$d(t) \neq 0$, so that the system error can be globally finite-time stable at the origin.

To this end, we design a new ESO which is similar to the high-order integral chain structure as follows:

$$\begin{cases}\dot{\hat{x}}_1 = \hat{x}_2 \\ \dot{\hat{x}}_i = \hat{x}_{i+1}, i = 2,\cdots,n-1 \\ \dot{\hat{x}}_n = \hat{x}_{n+1} + u \\ \dot{\hat{x}}_{n+1} = v\end{cases} \quad (11)$$

where $v = G(e,t)$ is a finite-time convergent control law with respect to the time t and the vector e that denote the errors between the state estimation and the actual value. The control law is to be designed such that $\hat{x}_i(t)(i \in n+1)$ will approach the states $x_i(t)(i \in n+1)$ of system (5). From the structural form, it is completely different from equations (6), but the design objective is consistent, that is to say, the dynamic process of the observer error equation can converge stably.

Then, subtracting (5) from (11), the error equation can be written as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_i = e_{i+1}, i = 2, \cdots, n-1 \\ \dot{e}_n = e_{n+1} \\ \dot{e}_{n+1} = v - d(t) \end{cases} \tag{12}$$

It can be observed that if equations (12) is convergent to the equilibrium $e(t) = 0$ from any initial condition $e_0 \in \mathbb{R}^n$, when $t < T$, then (11) is designed as a global finite-time convergent observer.

For the objective of that the ESO can work in practice as a global finite-time convergent observer under a wide range of time-varying disturbances, the function $v = G(e,t)$ is made for system (11) to render the equation (12) convergent to the origin within finite time, so that the total disturbance is estimated accurately. To this end, our approach will be first presented to ensure that the origin of system (12) is a finite-time convergent equilibrium based on the control law (14) in the following, and the stability of which has been extensively studied in (Bhat, 1997; Bhat, 1998) and references therein. The following lemma is adapted from Proposition 8.1 of (Bhat, 2005).

Lemma 3. Consider the system error (12) with $d(t) = 0$

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_i = e_{i+1}, i = 2, \cdots, n-1 \\ \dot{e}_n = e_{n+1} \\ \dot{e}_{n+1} = v_1 \end{cases} \tag{13}$$

The origin of (13) is a globally finite-time stable equilibrium in the system, if

$$v_1 = -\sum_{i=1}^{n+1} k_i \, sign(e_i) \, |e_i|^{\alpha_i} \tag{14}$$

where $k_1, \cdots, k_{n+1} > 0$ are selected such that the polynomial $s^{n+1} + k_{n+1}s^n + \cdots + k_2 s + k_1$ is Hurwitz, and $\alpha_{i-1} = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, i = 2, \cdots, n$, with $\alpha_{n+1} = 1$, $\alpha_n = \alpha$, $0 < \alpha < 1$.

Remark 1. The proof of lemma 3 is provided in (Bhat, 2005). Noting that it is easy to verify that the vector field $p(e)$ of (13) is homogeneous of degree $k < 0$by Definition 2, since the conditions of system (13) guarantee the asymptotic stability, then the system is a globally finite-time stable according to lemma 1. By Theorem 6.1 of **Error! Reference source not found.**(Bhat, 2005), the origin of (13) is globally asymptotically stable, with respect to which there exists a strictly positively invariant nonempty compact set. Obviously, the proof has not involved a Lyapunov function.

The following assumption is adopted:

Assumption 1. The derivative of the lumped disturbances $f_1(x_1, x_2, \cdots, x_n, w)$ in (3) and (4) is bounded, satisfying $|d(t)| \le L$ for $\forall t > 0$, where $L > 0$ is a given constant.

In order to design the proposed ESO which can accurately estimate the total disturbance of the system (2) in finite time, an auxiliary variable and a sliding mode variable are introduced to design the control law v of the equations (11) and (12), given by

$$v = v_1 + v_2 \tag{15}$$

where $v_1$is given by (14), and the sliding control is designed as $v_2 = -\rho sign(s)$ with $\rho > L$. The sliding variable s is chosen as

$$\begin{cases} s = \sigma + e_{n+1} \\ \dot{\sigma} = -v_1, \sigma(0) = \sigma_0 \end{cases} \tag{16}$$

where $\sigma$is an auxiliary variable.

Theorem 1. Suppose that Assumption 1 holds. Considering the nonlinear extended state observer (11), given the sliding mode surface (16) and the control law (15), then the error equations (12) is convergent within finite time to the origin.

Proof The derivative of the proposed sliding variable (16), along system dynamics (12) can be obtained

$$\dot{s} = \dot{\sigma} + \dot{e}_{n+1} = v - d(t) - v_1 = -d(t) + v_2 \tag{17}$$

A candidate Lyapunov function is selected with the form

$$V = \frac{1}{2} s^2 \tag{18}$$

Calculating the derivative of (18) under (17), it can be obtained

$$\dot{V} = s\dot{s} = s(-d(t) + v_2) = -sd(t) - s\rho sign(s) \tag{19}$$

According to Assumption 1, it follows that

$$\begin{aligned} \dot{V} &\le |s|L - s\rho sign(s) \\ &= |s|L - |s|\rho \\ &= -|s|(\rho - L) \\ &\le -\eta|s| \end{aligned} \tag{20}$$

where $\eta = \rho - L > 0$. Noting that $V^{1/2} = |s|/\sqrt{2}$, substituting it into (20), it can be obtained

$$\dot{V} \le -\sqrt{2}\eta V^{1/2} \tag{21}$$

Thus, by Lemma 2, the equilibrium $s = 0$is guaranteed and will achieve finite-time convergence. In fact, by separating variables and integrating both sides of the inequation (21) over the time from 0 to t, we obtain

$$V^{1/2}(t) \le -(\sqrt{2}/2)\eta t + V^{1/2}(0) \tag{22}$$

Consequently, $V(t)$ reaches zero in a finite time t_r that is bounded by $t_r \le \sqrt{2}V^{1/2}(0)/\eta$.

Once the state trajectory is moving towards the sliding surface $s = 0$, namely

$$\dot{s} = d(t) + v_2 = 0 \tag{23}$$

then a control function satisfying (23) can be easily calculated as

$$v_2^{eq} = -d(t) \tag{24}$$

Substituting (24) into the system (12), it is obviously that the system will reduce to (13) of Lemma 3. The proof is completed.

Remark 2. Considering the structure of the proposed ESO, the disturbances in the state error equation are first compensated equivalently, and then a finite-time control law is designed to guarantee the finite-time convergence for the system (see Lemma 3). Compared with the classical ESO, the validity of the designed ESO is verified based on the simulation results. The design flow chart of the proposed ESO is shown in the Fig. 2.

Remark 3. The aforementioned analysis indicates that the estimation error of (11) can converge to zero in finite time with properly selected parameters. Moreover, it is important that the observer's assumption becomes one and only one, that is, the lumped disturbances of the system are differentiable and the derivative is bounded, which enlarges the method’s application scope. Therefore, a new type of ESO, which obtains the estimation of system states and disturbances and feeds back to the controller, is proposed in this paper.

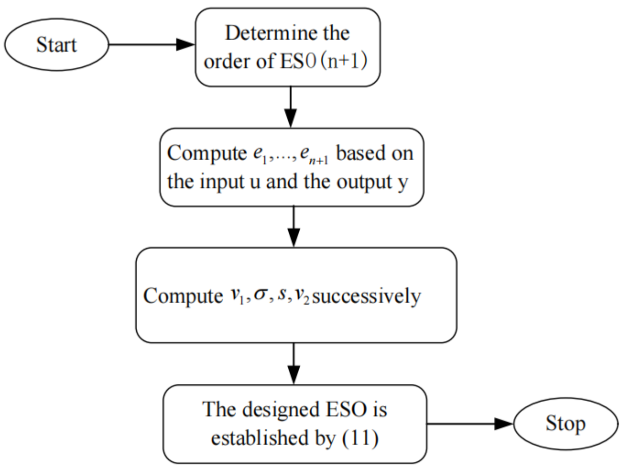


Fig. 2. Flowchart of the designed extended state observer.

Remark 4. The designed observer (11) adopts the high order integral chain structure, coupled with the switching function $-\rho sign(s)$, which is simple but effective to estimate states and uncertain part of the system, also the integration of both. Its effectiveness mainly manifests that the observer can be designed without perturbation model knowledge, and can achieve unbiased estimation in a certain time, nevertheless, most of the current extended state observers converge to a sufficiently small region in finite time (Yang, 2009; Xue, 2014; Shao, 2017). The structure diagram of the designed extended state observer is shown in Fig. 3.

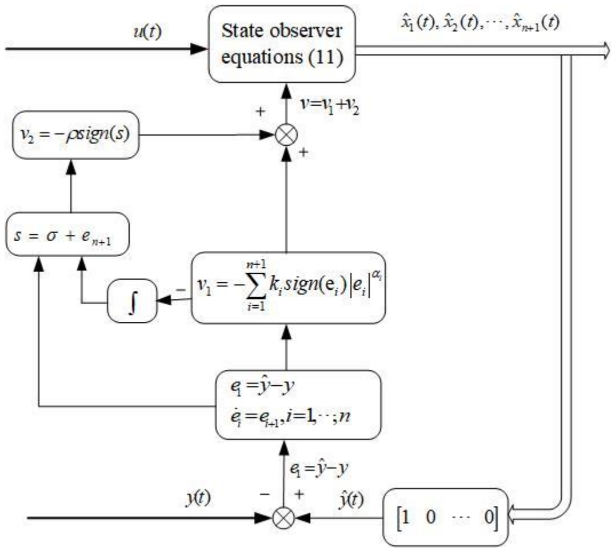


Fig. 3. Schematic diagram of the designed extended observer structure

4. ILLUSTRATIVE EXAMPLES

In this section, the estimation efficiency of the proposed finite-time convergent extended state observer is verified by numerical simulation, especially the convergence of error to zero in a finite time.

For simplicity, the proposed novel ESO in (11) and the ESO in (11) with $v = v_1$ are denoted as RFTESO and FTESO, respectively. To demonstrate the effectiveness of the designed RFTESO, the LESO and FTESO are employed for performance comparison.

Example 1. Consider the nonlinear system with lumped disturbances as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(t, x_1(t), x_2(t), w(t)) + u(t) \\ y(t) = x_1(t) \end{cases} \tag{25}$$

where $f(t, x(t), x_2(t), w(t))$ denotes the lumped disturbance, which is regarded as the total disturbance including the unknown dynamics of the system, also the unknown external disturbance.

The LESO for system (25) can be constructed as follows:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \beta_1 e_1 \\ \dot{\hat{x}}_2 = \hat{x}_3 - \beta_2 e_1 + u \\ \dot{\hat{x}}_3 = -\beta_3 e_1 \end{cases} \tag{26}$$

where $\hat{x}_1, \hat{x}_2, \hat{x}_3$ denote the estimation of state x_1, x_2, x_3 of (25) respectively, and $x_3 \triangleq f(t, x_1, x_2, w)$ is the extended state variable for system (25).

In simulation, $w(t)$ and the unknown dynamic function $f(\cdot)$ in (25) are considered as

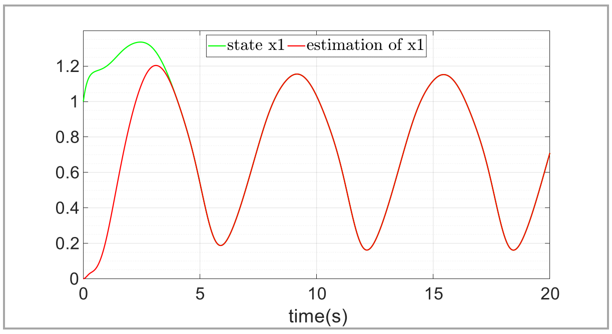
$$w(t) = sin(2t + 1)$$
$$f(t, x, w) = sin\, t - 2x_1 - 4x_2 + w + cos(x_1 + x_2 + w)$$
$$u(t) = 1 + sin(t) \tag{27}$$

The parameters in the proposed ESO are chosen as $\rho = -10, \alpha_1 = 0.5, \alpha_2 = 0.6, \alpha_3 = 0.75$, $\beta_1 = 4.5$, , $\beta_3 = 3.375$. It can be verified that the roots of $s^3 + \beta_1 s^2 + \beta_2 s + \beta_3 = 0$ are in the open left-half plane. The parameters of FTESO and LESO are the same as those of the RFTESO for the validity of contrast verification.

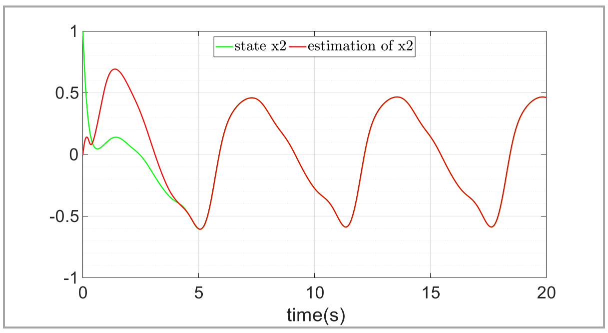
This example uses the data $\dot{x}_1(0) = \dot{x}_2(0) = \dot{x}_3(0) = 0$, $x_1(0) = x_2(0) = 1$. The results of state estimation are shown in Fig. 4 and Fig. 5. Fig. 4 shows the performance of the RFTESO estimating states of nonlinear uncertain system (25). It is observed that not only can the system states x_1, x_2, and the extended state x_3, which denotes the completely unknown uncertainties, be accurately estimated, but also that the error converges to zero in a finite time.

Fig. 5 shows the state estimation error of RFTESO, LESO and FTESO for comparison. It is easily observed that the estimation errors of both FTESO and LESO converge to a neighborhood of the origin. In addition, the size of the convergence region for the FTESO is smaller than that for the LESO. By using RFTESO, the estimation error can converge to zero in finite time, which is also the most precious feature for observers, especially in case of lumped disturbances.

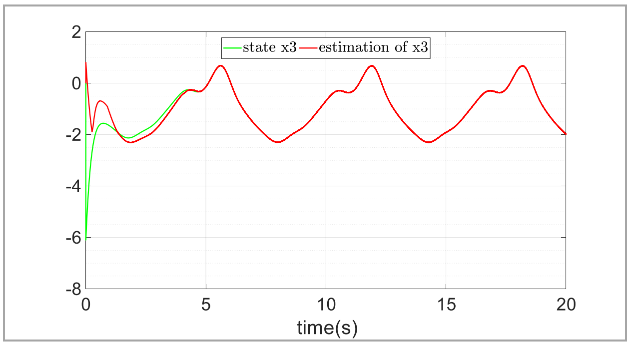
Remark 5. In the simulation, for obvious comparison, three forms of ESO have the same gain parameters, and from (27), the uncertain information of the system includes unknown states, external disturbances and their mixture. It can be seen in Fig. 5 that after a certain time, the total disturbance will be estimated by the unbiased value. With the help of input of feedback control signal, both linear time-varying system and nonlinear time-varying system can be transformed into a canonical form system (cascade integrators), i.e. so-called dynamic linearization (Huang, 2014).



(a) $x_1(t)$ and its estimation

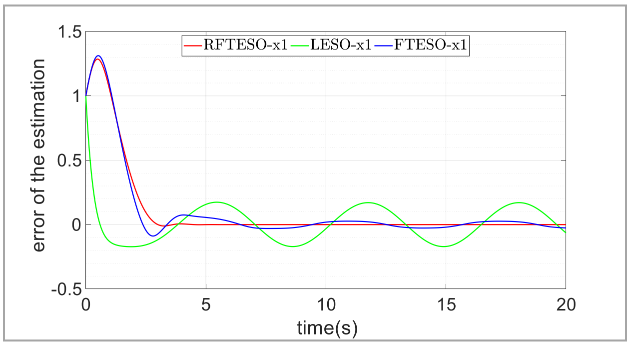


(b) $x_2(t)$ and its estimation

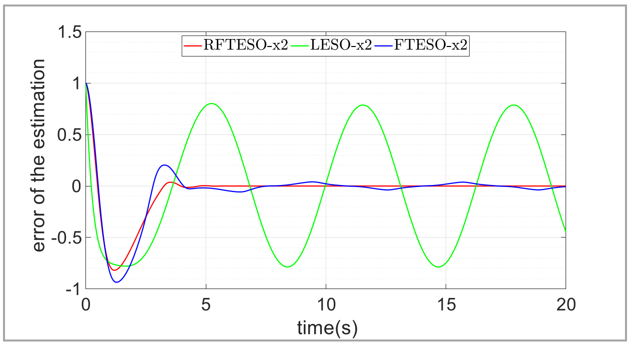


(c) $x_3(t)$ and its estimation

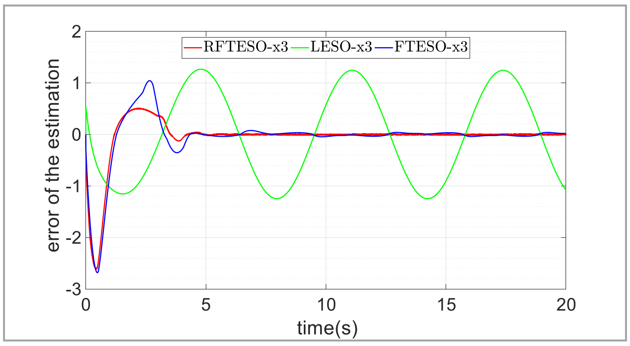
Fig. 4. Estimation of the state $x_1(t), x_2(t), x_3(t)$ of system (31) by RFTESO.



(a) Estimation error of $x_1(t)$



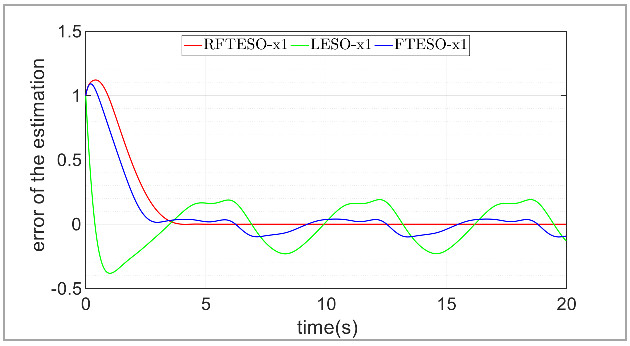
(b) Estimation error of $x_2(t)$



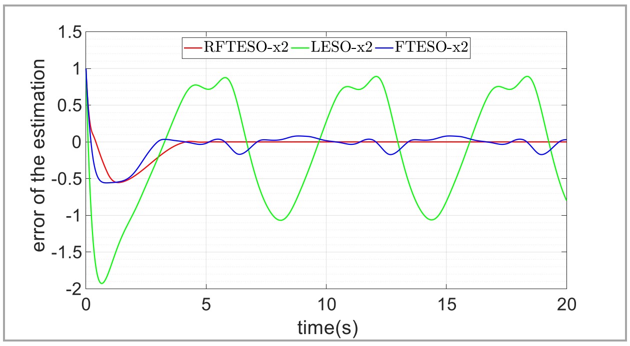
(c) Estimatin error of $x_3(t)$

Fig. 5. Estimation error of the state $x_1(t), x_2(t), x_3(t)$ of system (31) by RFTESO, FTESO, and LESO.

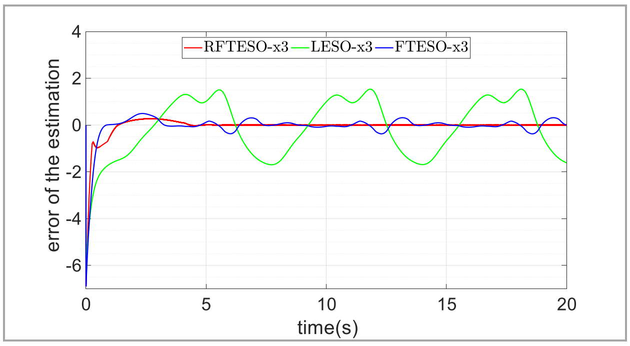
Example 2. Next, the performance of RFTESO under a total disturbance that contains only external disturbances will be discussed, i.e. $f(t,x,w)=w(t)$. Using the same parameters as the previous example, the numerical results are presented in Fig. 6 with $w(t)=cos(t+1)$ and Fig. 7 with $w(t)=cos(5t+1)$. Comparing Fig. 6 and Fig. 7, it can be seen that RFTESO still achieve accurate estimation when the disturbance change accelerates, while estimation errors of LESO and FTESO inevitably exist. Especially using FTESO, as displayed in Fig. 6 (c), the tracking error of disturbance restrains to a neighborhood of $x=0$ which is about 0.5, while the corresponding value in Fig. 7 (c) for FTESO is roughly 2.4.



(a) Estimation error of $x_1(t)$

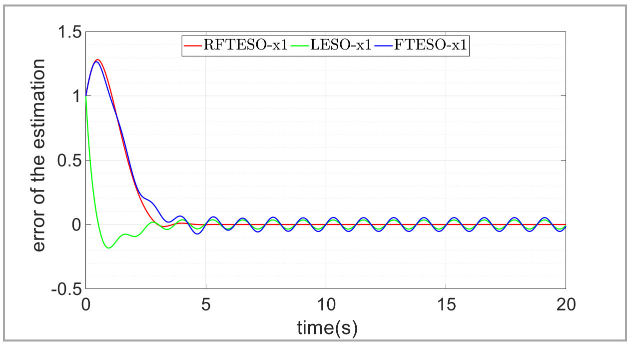


(b) Estimation error of $x_2(t)$

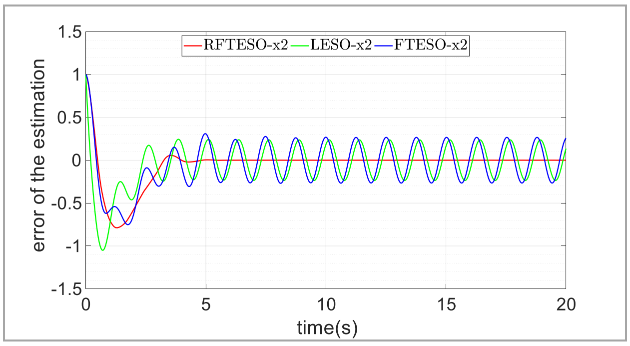


(c) Estimation error of $x_3(t)$

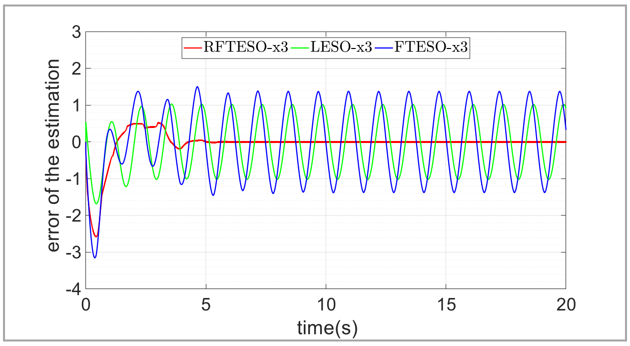
Fig. 6. Estimation error of the state $x_1(t), x_2(t), x_3(t)$ of system (31) with $w(t)=cos(t+1)$.



(a) Estimation error of $x_1(t)$



(b) Estimation error of $x_2(t)$



(c) Estimation error of $x_3(t)$

Fig. 7. Estimation error of the state $x_1(t), x_2(t), x_3(t)$ of system (31) with $w(t)=cos(5t+1)$

5. CONCLUSIONS

In this paper, the novel robust finite-time extended state observer is proposed for accurate estimation (i.e. achieving estimation error convergence to zero in a finite time) in the presence of total disturbance, which is lumped by system states, dynamic uncertainties, also external time-varying disturbances, and even high order disturbances. Compared with the conventional extended state observers, only the inaccurate estimation with a acceptable error convergent region can be achieved in a infinite time (for LESO) or a finite time (for NESO) under identical conditions. Finally, numerical simulation examples demonstrate the feasibility and efficiency of proposed method.

ACKNOWLEDGEMENT

This work was supported by the National Key Research and Development Program of China under Grant 2016YFC0302800.

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