

ADVANCED PROCEDURES FOR BOND GRAPH MODEL ADAPTATION

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Abstract: This study proposes a methodology that elaborates the structures of linear bond graph models in order to allow them to reproduce observed behaviour. This systematic technique is used for conceptual design of a prototype velocity sensor on the basis of bond graph fundamental components.

Keywords: bond graph, reconstruction, adaptation, conceptual design, junction structure

1. INTRODUCTION

Over the last decade different approaches have been investigated in order to automate the modelling process. The decomposition of models into sub-models in a similar way as the system is composed of sub-systems is an approach widely adopted by most of the present modelling techniques.

The alternative is to provide the user with a unique model which can be manipulated in order to obtain the level of accuracy needed according to specifications or context. This approach is particularly interesting when modelling is a tool used for conceptual design. The model proposed is constantly adapted and revised until it satisfies the designer's needs and a large part of the modelling process consists in revision and adaptation.

This study provides an insight into conceptual modelling using bond graph techniques and dynamic performance specifications. It involves specifying input-output frequency requirements and techniques of translating these requirements into bond graph primitives from where we may retrieve the actual topology of the system.

Section 1 introduces the device that needs to be remodelled, an inertial velocity sensor. A thoroughly analysis is made on a concept existing in the literature. This concept resulted from an impedance synthesis procedure. In section 2 a new automated procedure for model adaptation is presented. An algorithm is detailed that takes input-output specifications and matches them with possible bond graph models ([6]). On the developed structures causality rules are applied and element placement mechanism is discussed. All different structures verifying

the previously defined rules and satisfying the specifications are obtained. Each solution may then be interpreted in terms of physical mechanisms. New concepts for an inertial velocity indicator are suggested.

2. INERTIAL VELOCITY SENSOR

This section is meant to describe the component that needs to be remodelled and to show why the need of an adaptation/reconstruction tool. There are multiple solutions for determining a velocity including: integration of an acceleration signal, differentiating a displacement signal, employing Kalman filters or using the impedance synthesis method described in ([7]). Being aware of the drawbacks of all ancient methods, the author in ([7]) provided its own approach, using the idea of one-port impedance manipulation suggested by the electrical networks synthesis. The design approach of his work begins with a known input, a proposed input-output specification and a design procedure that generates an unknown sub-system such that the specification is met if possible.

The starting point for further designs of the velocity sensor is the configuration of figure 1 which was inspired by the seismic sensors. For measuring the velocity V , an initial configuration is proposed, consisting of a mass suspended by a spring. The frequency based specification considers the ratio of the input velocity to an output we can measure (in this case a relative displacement). In figure 1(b) is the bond graph model for this initial configuration. An unknown dynamic system is to be added in parallel to the spring such that spring displacement (q) or the spring force(F) is some measure of V . This unknown sub-system will create an appropriate relationship (a proportional relation is desired) between V and q (or F). The unknown system could also have been added in series with the spring thus generating a different concept.

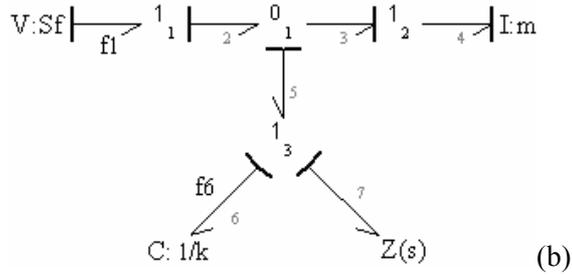
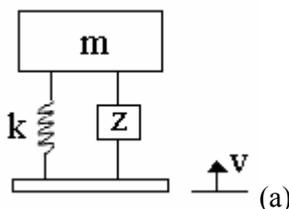


Fig.1. Initial configuration for the velocity sensor and corresponding bond graph model

The transfer function between the input velocity (f_1) and the spring displacement(q_6) is calculated as described below:

$$H(s) = \frac{q_6(s)}{f_1(s)} = \frac{ms}{ms^2 + Z(s)s + k} \tag{1}$$

$Z(s)$ being the impedance of the unknown sub-system.

A number of sensor architectures are possible, depending on performance requirements. A detailed discussion of several input-output performances is presented in ([7]) along with several concepts for the inertial velocity sensor. The conclusion of the cited author is that the performance is markedly improved with a band pass filter (equation (2)) that provides a nearly flat magnitude response and small phase shift over the frequencies of interest (a band of frequencies centred in $f_0 = 10^0 Hz$).

$$Z(s) = \frac{\beta_2 s^2 + \beta_1 s + \beta_0}{\alpha_2 s^2 + \alpha_1 s + 1} \tag{2}$$

By introducing multiple poles at each of the break frequencies, the phase is much flatter in the frequency range of interest. These frequency performances impose a transfer function as described below:

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{3}$$

The general concept behind the synthesis procedure in ([7]) is to decompose the unknown subsystem's impedance function $Z(s)$ into terms that represent the impedances of simple electrical elements: resistors(R), capacitors(C), and inductors (I). This is achieved through a process of repeated partial fraction expansion and polynomial division. Individual terms are

compared to a library of bond graph primitives to determine whether their individual forms match the impedance/admittance definition of one of the primitive bond graph elements. If a particular impedance term does not match, then a further decomposition of the term is necessary. This process is repeated until all impedance and admittance terms are of basic form. From the individual impedance terms, the actual topology of the system can then be determined.

The impedance synthesis procedure has successfully found a possible decomposition for the impedance transfer function and therefore a possible concept for the velocity meter as shown in figure 2.

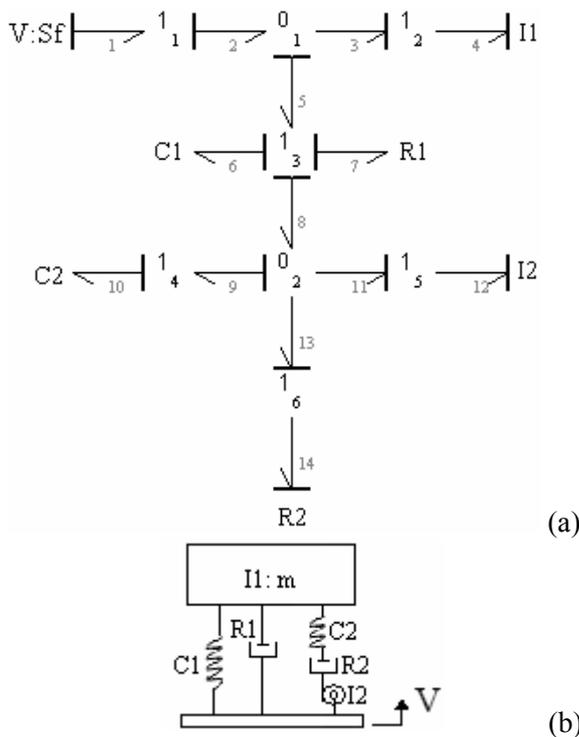


Fig. 2. Final concept for the velocity sensor derived from the bond graph model

Its physical realisation implies a mass (I_2) moving at a relative velocity ($V - V_m - V_{R2} - V_{C2}$). Since with a translational mass this would not be possible ([8]), a rotational mass has been used. This parameter is a function of the rotational inertia J and the rotational mass radius r : $I_2 = J/r^2$. But this technique may yield active elements also, as the same procedure is used for the synthesis of active elements where decomposed impedances are not always positive. Bond graph elements that represent system elements with negative impedances are

generally non-physically realizable. A solution would be to synthesize active elements by electronic means or by using controlled actuators.

Since the author did not take into consideration this aspect, further study on the identifiability of this model is needed in order to find the limits of this configuration design. When replacing $Z(s)$ from equation (1) with the band pass filter (see equation(2)), the final system becomes:

$$H: \frac{I_1 s(\alpha_2 s^2 + \alpha_1 s + 1)}{C_1 I_1 \alpha_2 s^4 + (C_1 I_1 \alpha_1 + C_1 \beta_2) s^3 + (C_1 I_1 + C_1 \beta_1 + \alpha_2) s^2 + (\alpha_1 + C_1 \beta_0) s + 1} \quad (4)$$

When choosing the specifications to be a fourth order system of gain A , with a particular Bode plot:

$$H(s) = \frac{As(T_3 s + 1)(T_4 s + 1)}{(T_1^2 s^2 + 2\zeta_1 T_1 s + 1)(T_2^2 s^2 + 2\zeta_2 T_2 s + 1)} \quad (5)$$

we can extract the parameters of the impedance depending of the measured frequencies.

$$\begin{aligned} I_1 &= A && \text{-original mass} \\ C_1 &= \frac{T_1^2 T_2^2}{AT_3 T_4} && \text{-original spring} \\ \alpha_1 &= T_3 + T_4 \\ \alpha_2 &= T_3 T_4 \\ \beta_0 &= 2(\zeta_1 T_1 + \zeta_2 T_2) - (T_3 + T_4) \frac{AT_3 T_4}{T_1^2 T_2^2} \\ \beta_1 &= \frac{4A\zeta_1 \zeta_2 T_1 T_2 T_3 T_4 + AT_3 T_4 (T_1^2 + T_2^2) - AT_1^2 T_2^2 - AT_3^2 T_4^2}{T_1^2 T_2^2} \\ \beta_2 &= \frac{2T_3 T_4 (\zeta_2 T_1 + \zeta_1 T_2) - AT_1 T_2 (T_3 + T_4)}{T_1 T_2} \end{aligned} \quad (6)$$

In order to synthesize only passive elements, the parameters of the unknown impedance $Z(s)$ must be all positive:

$$\alpha_i, \beta_i > 0 \quad (7)$$

From the inequalities above we are able to extract all conditions necessary for the frequencies measured in order for the parameters of the unknown impedance to be physically realizable.

$$\begin{cases} T_3 < T_1 < T_2 < T_4; T_1 < 1 < T_2 \\ 0 < \zeta_i < 1 \\ 2(\zeta_1 T_1 + \zeta_2 T_2) > T_3 + T_4 \\ 4A\zeta_1 \zeta_2 T_1 T_2 T_3 T_4 + T_3 T_4 (T_1^2 + T_2^2) > T_1^2 T_2^2 + T_3^2 T_4^2 \\ 2\zeta_2 T_1 T_2 T_3 T_4 + 2\zeta_1 T_1 T_2 T_3 T_4 > T_1^2 T_2^2 (T_3 + T_4) \end{cases} \quad (8)$$

The set of specifications (T_i, ζ_i) must fulfil these conditions because they are necessary (but not sufficient) for the impedance to be physically implementable.

A set of specifications that fulfil conditions is given below:

$$\begin{cases} T_1 = 0.01; T_2 = 10; T_3 = 0.9T_1; T_4 = 1.1T_2 \\ \zeta_1 = 0.6; \zeta_2 = 0.59488; \\ A = 1 \Rightarrow I_1 = 1kg \end{cases} \quad (9)$$

meaning that

$$H(s) = \frac{0.099s^3 + 11.01s^2 + s}{0.01s^4 + 1.201s^3 + 100.1s^2 + 11.91s + 1} \quad (10)$$

Until this point in our study only general conditions were generated, necessary for any impedance the design concept might contain. But the implementation capability of the solution depends also on the architecture of the generated impedance. For the solution in figure 2, by resolving the following set of non-linear equations, we can obtain specific values for the components of impedance $Z(s)$:

$$\begin{cases} C_2 I_2 = 0.099 \\ \frac{I_2}{R_1} = 11.01 \\ C_1 C_2 I_2 = 0.01 \\ R_2 C_1 C_2 I_2 + \frac{C_1 I_2}{R_1} = 1.201 \\ C_2 I_2 + C_1 I_2 + C_1 + \frac{R_2 C_1 I_2}{R_1} = 100.1 \\ R_2 C_1 + \frac{I_2}{R_1} = 11.91 \end{cases} \quad (11)$$

The previous system contains 6 equations and only 5 unknown variables. Apparently this system has no solution because the last equation can not be fulfilled with the solution from the previous equations. But the last equation may be reinterpreted as a new constraint in terms of frequencies. The definitions of R_1 , R_2 , C_1 , C_2 and I_2 may be expressed with variables from equation(8), resulting in a 6th constraint on the specifications that will be added to the others in system(8):

$$2\zeta_2 T_1^2 T_2 T_3 T_4 + 2\zeta_1 T_1 T_2^2 T_3 T_4 - T_1^2 T_2^2 (T_3 + T_4) + T_3 T_4 (T_3 + T_4) - 2T_3^2 T_4^2 (\zeta_1 T_1 + \zeta_2 T_2) = 0 \quad (12)$$

This condition is much stronger than the inequalities found before (see equation (8)) and, because of it, even smaller modifications of the

specifications given in equation (9) lead to negative parameters. Because this system has 6 equations with 5 unknown parameters, this solution proves to be very sensitive to specifications. This solution is valid as long as the last equation is redundant. A small modification of parameters will permit the last equation to be independent and the solution will no longer be valid. The conclusion so far is that the impedance synthesis technique finds only one solution for the design concept. If this solution proves to be very sensitive to specifications or has negative parameters, then we find ourselves with no choice.

This study presents another technique that may generate multiple solutions for the synthesis of the desired transfer function giving the designer the possibility to choose a certain concept that fits better with the application or cost requirements.

3. STRUCTURE GENERATION

3.1 Building of the structures

This section introduces an algorithm that matches a certain transfer function with possible bond graph models. The starting point for this algorithm consists in the bond graph model of the arbitrary design in figure 1. Each model provides its own specifications and constraints and that is why information must be provided to the generation procedure in order to establish:

- What are the specifications for the desired dynamic behaviour?
- Where to incorporate the unknown subsystem that provides the desired performances?

The initial bond graph model will be used as a base for further models with a more elaborate structure and more dynamic elements in the quest for those models that comply with specifications. In order to evolve a bond graph model several steps are needed. The first task is to develop all possible junction structures that consist only in interconnections of 0- and 1-junctions ([5,1,2]). These junction structures are the backbone of each model. For representing a skeleton, a Boolean matrix is introduced where a connection between a 0-junction and a 1-junction is signalled by adding the value "1" in the corresponding cell of the matrix as shown in figure 3.

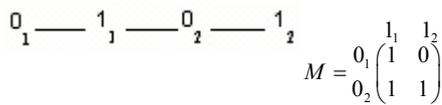
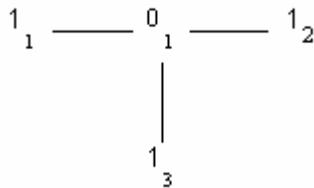


Fig.3. A bond graph skeleton and its structure matrix

Starting from a predefined skeleton, the algorithm constructs other junction structures by adding 0- and 1- junctions. In our case the starting point will be:



with the matrix $M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

The algorithm follows certain assumptions:

A₁: All external elements are added on the 1- junctions (derived from the systematic procedure of constructing bond graph models described in ([4])).

A₁ imposes the introduction of 1_1 and 1_2 in order to support Sf and I . This choice of modelling is compatible with all physical domains and does not change the physical realisation of the bond graph model.

So far, the only information imposed by the initial system is the matrix M . Each step of the procedure will add further information that will customize the evolution process.

3.2 Adaptation technique

Models evolve following two steps that repeat each other until a solution is found: adding new elements and adding new junctions.

➤ **Adding new elements**

In the process of adding new elements, each 1-junction is considered a support for new elements (other than sources and detectors). If the 1- junction already detains dynamic or resistive elements from the initial design, the algorithm will not permit the addition of elements of the same type. In the example of figure 4 only an I-element in derivative causality may be introduced.

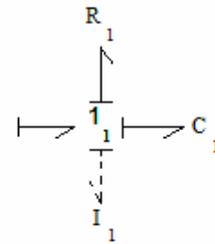


Fig.4. Adding dynamic elements

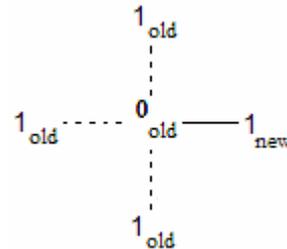
That was the case of 1-junctions already existing on the original skeleton and probably holding elements from the initial design. But if a modification in structure was triggered, there will appear in the structure new junctions free of elements. These junctions will be completed with storage elements in all possible combinations.

➤ **Adding new junctions**

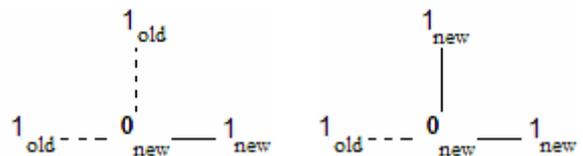
If new elements were considered and no solution was found, a modification in structure is triggered. The algorithm is reloaded from the junction structure phase, where an already existing structure is completed with new junctions.

Adding junctions is a different task depending where the new junctions are introduced.

➤ *Adding on a 0-junction already in the model*



➤ *Adding on a 1-junction already in the model*



Because of the assumption A₁, 0-junctions suffer a special treatment: every time a new 0-junction is introduced it must be connected with three other bonds. Otherwise they may be eliminated as seen in figure 5:

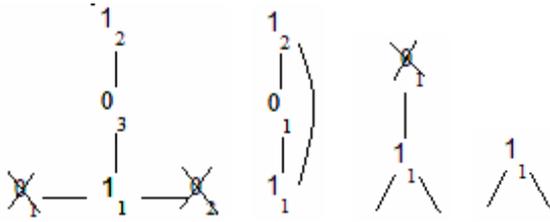
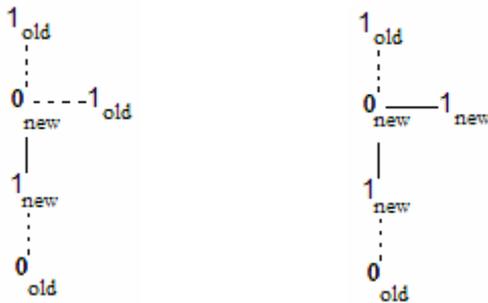


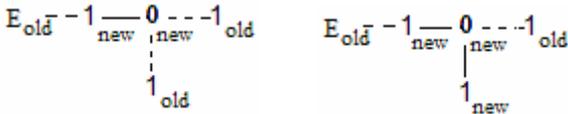
Fig.5. Reduction of 0-junctions with less than three bonds

These bonds may be: a new 1-junction and two others which already exist in the system or, two new 1-junctions and another initial 0-junction.

➤ Adding between two junctions already in the model



➤ Adding between an element and a junction already in the model



In order to avoid too many multiplications in a wrong direction, the designer should indicate initially where to start adding junctions giving a specific site or a list of possible sites to start with. This corresponds exactly with the position needed for the newly introduced impedance. Observe that multiple degrees of freedom are also permitted (see fig. 6) since there is no restriction in the number of sites considered for completion. More than one component is free to vary if one impedance led to no solution after a decent number of iterations.

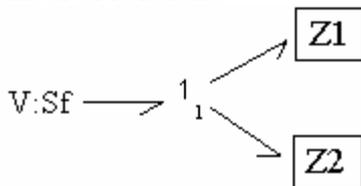


Fig.6. Two unknown impedance concepts

As it can be observed, adding junctions means adding rows and columns to already existing

matrices (rows for 0-junctions and columns for 1-junctions).

3.3 Causality and element placement

The initial bond graph model retains elements that must be retrieved in all possible solutions that follow. It is the case of the mass and the spring and this information must be supplied to the generation procedure as, the more precise is the base the more accurate will be the solution. At the junction structure level the algorithm retained information needed for developing the skeleton (the matrix M). At this level it will take into consideration dynamic elements that hold their place in the initial design. For the figure 1, this information will be represented as a matrix holding the positions of each element into the bond graph model. Every 1-junction has several internal bonds (connecting them to 0-junctions) and one or more external bonds for supporting elements. The external bonds are treated as 0-junctions and are represented by new rows in this matrix.

$$P = \begin{matrix} \text{internal bonds} \\ \text{external bonds} \end{matrix} \begin{cases} \begin{pmatrix} 1 & 1 & 1 \\ Sf & I_1 & C_1 \\ 0 & 0 & Z \end{pmatrix} \end{cases}$$

The generation procedure has provided all possible and unique configurations of 0 and 1-junctions.

At a next step each 1-junction is populated with elements always taking into consideration the matrix described above. Also at this level causality takes its place into the new developed structures. Causality is represented by introducing in the P - matrix (+) and (-) signs: a causal stroke “far” from a 1-junction is coded with -1 and a causal stroke “close” to the 1-junction is coded with 1. The rule for 1-junctions states that one causal stroke “far” will impose all the others “close”. This means that only one (-1) is permitted for every column. For rows (corresponding to the 0-junctions, not to the external elements) the logic is reversed: one (-1) means a causal stroke “close” to the 0-junction and this imposes all other causal strokes “far” i.e. coded with (+1).

$$C = \begin{pmatrix} 1 & 1 & -1 \\ -1 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Fig.7. Example of causality representation

If a modification in structure was triggered and there are 1-junctions free, the causality for these junctions will be imposed by the system. These junctions will be populated with elements that respect the causality already imposed by components from the initial design. A (-1) in the causality matrix indicates that a member of the “outgoing flow” family $\{I, R, C_{der}\}$ will be chosen as element for that 1-junction, (+1) indicating a candidate from the “outgoing effort” family $\{C, R, I_{der}\}$.

If there are multiple solutions for the causality, each one will be explored and each one will take into consideration various combinations for the storage and resistive elements.

There are two cases when a new installed element may change the causality of the system.

- Adding a resistive element that forms a causal path with a storage element in derivative causality

In this case the resistive element will change its causality so that the storage element falls into integral causality.

- Adding a storage element in derivative causality which forms a causal path with another storage element in derivative causality.

In this case both elements will fall into integral causality changing also the causality along the path. These two cases are important as both of them create new independent dynamics for the system.

3.4 Transfer function

In order to apply Mason’s rule and calculate the transfer function for the bond graphs obtained so far, we need to establish the power flow. As we discuss here about SISO models, we set the following assumption:

A₂: The power propagates from the source to every branch of the structure.

For the power flow representation, the same logic as for causality is used, by introducing a power flow dedicated matrix for which terms are equal to (+1) if the half-arrow enters the 1-junction, (-1) if the half-arrow leaves the 1-junction (see fig.8). The half-arrow for external bonds always points out of the sources and into the other elements.

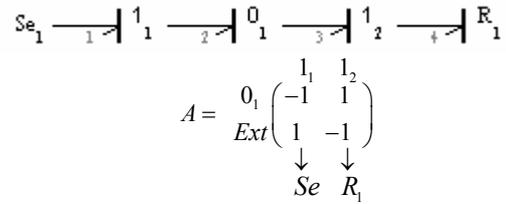


Fig. 8. Power oriented BG and its corresponding matrix

Using causal loops and path gains, the Mason’s rule ([3]) may be implemented. For each generated solution we calculate the transfer function and compare it with the one given as specification.

4. NEW DEVELOPED CONCEPTS

Using the bond graph adaptation technique, 42 different design concepts were obtained and between them we find also the solution pointed by the engineers in fig. 2. Some of them are significantly more complex. Three classes of solutions were developed depending on their number of equations and unknown variables as seen in system (11). For each class one example of concept is presented in fig. 9.

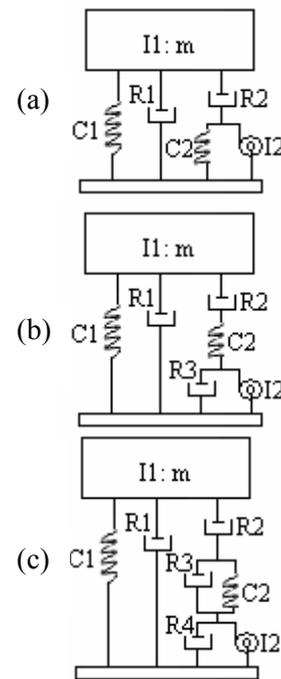


Fig. 9. (a) system with 5 unknown parameters;
 (b) system with 6 unknown parameters;
 (c) system with ≥ 7 unknown parameters.

The next step would be to introduce numerical specifications for the dynamical behaviour that

would permit to quantify the sensor's performance and to select its parameters.

As seen in the discussion in section 1, between specification and the appropriate architecture of impedance $Z(s)$, there is a tight relation. Some numerical specifications may lead to a configuration with passive elements while for other numerical specifications this configuration might not be useful any more. A new structure with new parameters would have to be retrieved from the set of solutions.

All this adaptive process is a difficult task to resolve by hand. Every time a new system of non-linear equations has to be resolved in order to tune the sensor's parameters. For this task an automated procedure based on a Levenberg-Marquardt algorithm has been developed, that permits us to choose the appropriate configuration and its parameters that satisfy best introduced specifications.

This application offers an interface where the user can introduce specifications in terms of frequencies and retrieve the exact concept that complies with demands. Parameter identification is done via Matlab functions. For the specifications in equation (9), 2 solutions were pointed out by the application. One of them is the original concept found using techniques of conceptual design, seen in fig. 2. The second solution is a similar concept that provides almost the same behaviour but with a supplementary dissipative element, R_3 that assures better damping (fig. 10).

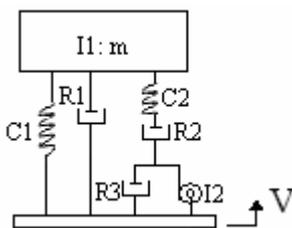


Fig. 10. New concept with supplementary dissipative element

This solution belongs to a non-linear system with 6 equations and 6 unknown parameters that is much more robust than the solution presented in figure 2. The presence of this 6th variable of a non-negligible value allows a better identification of parameters (the residue of the objective function $F(x)$ at the solution x is minimized in absolute value) and assures better performances in terms of sensibility to specifications.

5. CONCLUSIONS

The bond graph tool presented in this study may suggest to relatively inexperienced modellers appropriate elaborations of models that comply with a set of specifications given in terms of frequency response. In the case of a velocity sensor concept, the system was capable of proposing multiple architectures in order to help the designer choose the best fit for the given specifications. The solution found by the designers in ([7]) proved to be extremely sensible to specifications and our method was able to propose a numerically improved concept.

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