Event-triggered based H_∞ Consensus Control for Multi-agent Systems under Time-varying Delay

Irfan Qaisar[a], **Muhammad Shamrooz Aslam** ⋆[b], **Chuan Zhou**[c]

[b] *School of Electrical and Information Engineering, Guangxi University of Science and Technology, Liuzhou 545006, China. (e-mail: shamroz_aslam@yahoo.com)*
[a] *Technical Department, Wuxi Johnnywell Railway Equipment Technology Co.,Ltd, (e-mail: irfanqaisar92@outlook.com)*
[c] *School of Automation, Nanjing University of Science and Technology, 210094, Jiangsu, P.R. China. (e-mail: njust_zc@126.com)*

Abstract: This article addresses the event triggered based H_∞ consensus control problem for Multi-Agent Systems (MASs) with a directed graph under time-varying delay. The event-triggered communication scheme is utilized to overcome the unnecessary utilization of communication bandwidth. Based on this event-trigger scheme a consensus control protocol is presented. The consensus control problem of MAS is transferred into the stability problem of a closed-loop system under time-varying delay. Furthermore, a new version of *Lyapunov function (LF)* is presented to establish H_∞ performance index in the form of *linear matrix inequalities* (LMIs), which is different from the tradition (Liu et al., 2014) and attained less conservative results. Finally, a simulation illustration is provided to exhibit the design method.

Keywords: H_∞, Multi-agent Systems, Event-trigger scheme, Consensus, Time-varying delay.

1. INTRODUCTION

In the past two decades, there is a significant development in control techniques as well as signal processing and communication techniques, it became simpler for various dynamical systems to interface with each other. Such a system is called a multi-agent system (MAS), in which every agent has its independent dynamical system. In the network MAS, the agent and the other neighboring agents transmit their information through the network and accomplish the large and complex multi-objective task in the form of group collaboration (Zhang et al., 2014). With the rapid development of modern technology and distributed ideas, there is a rising recognition of the importance of MAS cooperation in accomplishing complex tasks, distinguishing that MASs can play a richer sense of perception, stronger stability, high efficiency, higher reliability and low cost rather than individual agents (Fink et al., 2014).

In cooperative control of MASs, the consensus is a vital and fundamental problem. The term consensus originates from Latin word *consentire* which implies a general agreement made by all or majority. Consensus means is an agreement protocol which required to be designed appropriately, during which all agents converge to some common variables of interest (such as voltage, position, and velocity) by communicating with their neighboring agents (Ren et al., 2007). In recent decades, the consensus control problem in MASs has gained considerable concern due to a wide range of applications. For example, the formation of UAV (unmanned air vehicles), spacecraft formation flying, cooperative surveillance and autonomous configuration of the mobile sensor network (Stamatescu et al., 2017), (Qin et al., 2011), (Beard et al., 2002), Peng et al., (2015), and (Liu et al., 2014). The concept of consensus was first proposed in (Olfati-Saber and Murray, 2004). Subsequently, the topic of consensus of MASs has rapidly grown and greatly motivated the researcher's interest (Hu and Feng, 2010), (Ren and Beard, 2005), (Cao and Ren, 2014),and (Zhang et al., 2011). For example in (Ren and Beard, 2005), the consensus problem solved for MASs under dynamically changing interaction topologies while unreliable and limited information exchanged among agents. The average consensus control problem of MASs with various communication delays is presented in (Zhang et al., 2011).

In efforts for accomplishing the consensus for MASs, the agents essential to exchange shared information with their neighboring agents through communication networks. In conventional control consensus problems for MASs, it is supposed to get continuous access to measurements and control signals. Such presumption support by an ideal communication network and appropriate computation resources for MAS. Unlikely in practical application, it should be noted that every agent has restricted communication and computing resources, excessive calculation and information exchange could also be unnecessary utilized the network resources (Xing and Deng, 2018). Moreover, packet dropout, network-induced delay, packet disordering, and sensor/actuator fault are primarily caused by

⋆ Corresponding Author

(Song et al., 2013), (Tan et al., 2015), (Liu et al., 2015),and (Liu et al., 2019). To cope with these issues, the distributed event-triggered scheme is presented, during which information can solely transfer to neighboring agents once it reaches to a threshold value. It is an effective way to reduce unnecessary utilization of bandwidth (Wang and Tian, 2016). During this effort, many results have been reported by using the event-triggered mechanism, for example, the consensus control problem in (You et al., 2017), and (Liu et al., 2017), and control problem of output tracking for T–S fuzzy systems have been observed in (Zhang et al., 2015). The concept of the event-based control scheme in MASs is presented in (Tabuada, 2007). The problem of average consensus control for MASs with a decentralized event-triggered mechanism is mentioned in (Dimarogonas et al., 2012) and substantiated that the presented event-triggered mechanism is successful. In (Xu et al., 2017), to attain leader following consensus a centralized clustered event triggering condition is presented. It is the primary motivation for our research. In (Aslam et al., 2020), authors have been investigated the properties of Extended dissipative filter with novel Lyapunov-Krasovskii function, while in (Aslam et al., 2020), researchers explore the impact of Multiagent system for a Markov jump system. In recent studies, the performance of MASs is influenced by numerous disturbances, such as model uncertainties, external disturbance and time delay (Lin et al., 2008). They are typically inevitable in particle networked system. The presence of disturbances may extinguish the convergence performance of MASs. Therefore, it is significant to analyze their impacts on the performance of MASs. Some associated problems regarding external disturbance have been conducted by several researchers (Li et al., 2012), (Zhang et al., 2016), (Li et al., 2011), and (Lin and Jia, 2010). As described in (Li et al., 2012), for the class of nonlinear MASs global H_∞ consensus problem is presented while considering the Lipschitz non-linear condition and directed communication graph. Robust H_∞ based consensus analysis for the class of second-order dynamical MASs with uncertainty is proposed in (Lin and Jia, 2010). furthermore, time delay problem also investigate by some researchers (Chen et al., 2011), (Mu et al., 2015), (Xiao et al., 2016), and (Chen et al., 2017). For instance, In (Chen et al., 2011) for a class of time-varying delayed MASs under noisy environment, a robust control law is presented.

The consensus control problem of MASs under time-varying delays while communication topology is fixed and undirected investigated in (Chen et al., 2017). In the above-mentioned, there is no work which considers the external disturbance and the time-varying delay in the same problem of MASs, so in this aspect, researchers do not investigate fully. It is the secondary motivation for this research work.

A real-time system is a system whose correctness depends not only on the logical result of the computation. In this regard, many researchers has been investigated for the different applications e.g. Communication systems (Peng et al., 2016), inverted pendulum (Shi et al., 2016), (Liu et al., 2016) and smart robots (Zhang et al., 2015). Because of the above discussion, it can be stated that the problem of MASs for an event-triggered scheme will not fully be investigated with the appearance of time-varying delays. The main contribution to this article is concise as:

(i) A general kind of model for MASs is established with the network-induced delay, event-triggering communication scheme, in practical application it is more applications such as for embedded system design.
(ii) Given the proposed method, a new sampled-data based consensus control protocol will be designed, by which the consensus control problem of MAS can be transferred into the stability problem of the closed-loop system under time-varying delay.
(iii) On the account of LMIs, a Lyapunov functional is presented to achieve appropriate conditions and makes the system asymptotically stable, which gives the delay-dependent conditions. Our proposed methodology provides effective utilization of bandwidth along with the event-triggered mechanism among the others. It is verified in the simulation example.

1.1 Notations

The notation will be used all through this article are following.

$A > B$ / $A \geq B$	positive definite/semi-definite
I/1_n	identity matrix with proper dimension/vector of ones
$\otimes$	Operator of Kronecker product
$\Vert \cdot \Vert$	Euclidean norm
A^{-1}/A^T	inverse/transpose of the matrix A
$diag(A_1, \cdots, A_n)$	block-diagonal matrix with diagonal values A_i, $i = 1, \cdots, n$
$L_2[0, \infty)$	vector functions over $[0, \infty)$
$*$	donates symmetry

1.2 Graph Theory

A communication topology among the agents is represented by a graph $\mathcal{G}$. The directed graph $\mathcal{G}$ of order $\mathbb{N}$ is a set of $(\mathcal{V}, \mathcal{E})$, with set of vertices $\mathcal{V} = \{v_1, \cdots, v_N\}$ and subset of edges $\mathcal{E} \subseteq \mathcal{V}, \mathcal{V}$ in which an edge is defined by the ordered pair of different vertices. For an edge (v_i, v_j),v_i and v_j are so-called parent and child vertices, respectively, and v_i is a neighboring of v_j. A graph is said to be undirected if the property $(v_i, v_j) \in \mathcal{E}$ implies $(v_j, v_i) \in \mathcal{E}$ hold. A directed route from vertex v_{i1} to v_{il} is ordered edges?sequence of the form $(v_{ik}, v_{i(k+1)})$, $k = 1, \cdots, l-1$. A directed graph comprises a spanning tree if it has a vertex known as the root, which has no parent vertex and there exists a directed path from it to every other vertex in the graph. A directed graph is strongly connected if every vertex is connected with all other vertices. If a directed graph is strongly connected its mean it has a directed spanning tree.
Let there are $\mathbb{N}$ vertices in the communication graph $\mathcal{G}$. The adjacency matrix represented by $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}}$ related to the directed communication graph is state as $a_{ii} = 0$, $a_{ij} = 1$ if $(v_j, v_i) \in \mathcal{E}$ otherwise $a_{ij} = 0$. The Laplacian matrix is represented by $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}$ is state as $\mathcal{L}_{ii} = \sum_{j\neq i} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}$,$i \neq j$. For undirected communication graph, the matrices $\mathcal{A}$ and $\mathcal{L}$ are considered as symmetric.

The rest of the article is structured as the next section is about problem formulation. Section 3 presents the consensus analysis and synthesis of the MAS. In section 4 simulation illustration is provided which validates the usefulness of the suggested design. Section 5 concludes and summarized the article.

2. PROBLEM FORMULATION

The event-triggering system in the continuous-time domain for the conventional MAS comprises a plant, its configuration includes sensors, samplers, an event generator, and a network environment as illustrated in Fig. 1. A class of wireless sensor systems can be illustrated by the aforementioned configuration, where the sampler receives the measurements from the sensor.

$$\begin{cases} \dot{x}_i(t) = g_1(x_i(t)) + g_{d1}(x_i(t-d(t))) + g_u(u_i(t)) \\ \qquad +f_1(\omega_i(t)), \quad i = 1,\cdots,\mathbb{N}, \\ y_i(t) = g_2(x_i(t)), \end{cases}$$

Assume a MAS of $\mathbb{N}$ identical linear agents which are subject to external disturbances, defined as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + A_d x_i(t-d(t)) + Bu_i(t) \\ \qquad +D_\omega \omega_i(t), \quad i = 1,\cdots,\mathbb{N}, \\ y_i(t) = Cx_i(t), \end{cases} \tag{1}$$

where $x_i(t) = [x_{i1},\cdots,x_{in}]^T \in \mathbb{R}^n$ is state of the i^{th} agent, $u_i(t) \in \mathbb{R}^p$ is the control input, A, A_d, B, C, D_ω are constant matrices with compatible dimensions, $\omega_i(t) \in L_2^{m_2}[0,\infty)$ is the external disturbance. It is supposed that the communication topology graph $\mathcal{G}$ is strongly connected and also balanced, the every agent has approaches to the relative states with concerning its neighbors. Due to

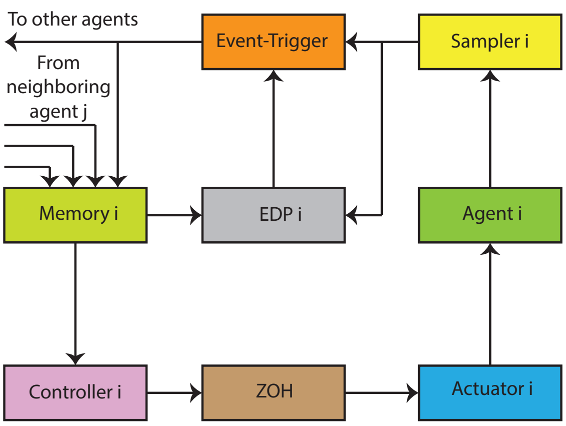


Fig. 1. A distributed event-triggered scheme for MASs

the limitation of communication and computing resources, excessive calculation and information exchange may be unnecessary utilized the network resources. Therefore, we adopted the distributed event-triggered scheme for the MASs, whose structure is explained in Fig. 1. In this system, sample information of agent i will only transmit at the sampling instant $kh(k \in \mathbb{N})$ when it reaches the threshold value. The sampler sampled the agent i state at a constant period of sample $h > 0$. The agent i sampled data $x_i(kh)(k \in \mathbb{N})$ can be well transferred to its *event-distributed processor* (EDP) when it fulfil the event-triggered condition. Here, the *EDP's* perform three responsibilities for each agent i:

(a) Form agent i getting sampled data and store it at every single sampling instant.
(b) Handling the received sampled data according to event-triggered scheme condition.
(c) When threshold value reached, producing a trigger signal and sending it to the event-trigger generator.

Once the trigger signal received by event trigger, the sampled data information of agent i is allowed to be transferred to the store of itself and its neighboring agents. Here, the store is hired to collect the sample data of agent i as well as its neighboring agents and to instantly update its data information at the moment assessing newly sampled data from agent i and also its neighboring agents. At that point, the controller of agent i brings up to date its input value by utilizing the received sampled data information from the store, which infers that the controller update of agent i is driven at its own and also neighboring agents' event instants. Then, the agent i controller output is transferred to the actuator by ZOH. Here, ZOH is utilized to hold the agent i control input unless most current sampled data from the agent i controller reaches.

It notices that the agent i sampled data should only be transferred among neighboring agents depends on an event-triggered condition instead of elapse of fixed time. The *sth* transmitted instant of the agent i sampled data indicates by $t_s^i h$. Thus, the next transmitting instant $t_{s+1}^i h$ of agent i is consequently defined by.

$$\begin{aligned} t_{s+1}^i h &= t_s^i h + min\{\Omega_i h | X_i^T(t_s^i h + \Omega_i h)\Phi X_i(t_s^i h + \Omega_i h) \\ &\geq \sqrt{\sigma_i} Y_i^T(t_s^i h + \Omega_i h)\Phi Y_i(t_s^i h + \Omega_i h)\} \end{aligned} \tag{2}$$

where $\Omega_i \in \mathbb{N}$, $\sigma_i > 0$ is represent threshold factor, $\Phi > 0$ is represent weighting matrix and h is a sampling period, $X_i(t_s^i h+\Omega_i h) = x_i(t_s^i h+\Omega_i h) - x_i(t_s^i h)$ and $Y_i(t_s^i h+\Omega_i h) = \sum_{j=1}^N w[x_i(t_s^i h) - x_j(t_{\acute{s}_j}^i h)]$ with $\acute{s}_j = arg \quad min_q\{t_s^i + \Omega_i - t_q^j | t_s^i + \Omega_i > t_q^j, q \in \mathbb{N}\}$. Obviously, from (2) one can perceive that, for agent i event triggered condition at the *kth* sampling instant is closely related to sample data error $X_i(t_s^i h)$ and the sampled data $Y_i(t_s^i h)$ as well as the latest transmitted sampled data $x_i(t_s^i h)$ of agent i and the most recent transmitted sampled data $(x_j t_{\acute{s}}^i h)$ of its neighboring agents. At that point, we assume the following sample data consensus protocol.

$$u_i = -K \sum_{j=i}^N w_{ij}[x_i(t_s^i h) - x_j(t_{\acute{s}}^i h)] \tag{3}$$

where $t \in [t_s^i h, t_{s+1}^i h)$, $\acute{s}(t) arg \quad min_q\{t - t_q^j | t_s^i + l + i > t_q^j, q \in \mathbb{N}\}$ and K indicates the controller gain consensus protocol which will be determined later.

3. CONSENSUS ANALYSIS AND SYNTHESIS

At the *kth* sampling time the sample data measurement error can be characterized as

$$e_i(kh) = x_i(kh) - x_i(t_s^i h), \quad t_s^i \leq k < t_{s+1}^i \tag{4}$$

It can be notice that the release time interval $(t_s^i h, t_{s+1}^i h) = \bigcup_{k=t^i s}^{t_{s+1}^i - 1}[kh,(k+1)h)$, hence, by dividing $(t_s^i h, t_{s+1}^i h)$ into $t_{s+1}^i - t_s^i$ sampling interval and from equations (1),(3) and (4), the closed loop MAS dynamics for $t \in [kh,(k+1)h)$ can be attained as

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + A_d x_i(t-d(t)) - kB\textstyle\sum_{j=i}^{N} w_{ij}[x_i(t_s^i h) \\ &\quad - x_j(t_s^i h)] + D_\omega \omega_i(t) \\ &= Ax_i(t) + A_d x_i(t-d(t)) - kB\textstyle\sum_{j=i}^{N} w_{ij}[x_i(kh) \\ &\quad - x_j(kh) - e_i(kh) + e_j(kh)] + D_\omega \omega_i(t) \\ y_i(t) &= Cx_i(t)\end{aligned} \tag{5}$$

Let $x(t) = [x_1^T(t), \cdots, x_N^T(t)]^T$, $\check{z}(t) = [\check{z}_2^T, \cdots, \check{z}_N^T]^T$, $e(kh) = [e_1^T(kh), \cdots, e_N^T(kh)]^T$, with $\check{z}_i(t) = x_1(t) - x_i(t)$, and $\mathcal{E}(kh) = \mathcal{E}(kh)_2^T(kh), \cdots, \mathcal{E}(kh)_N^T(kh)]^T$ with $\mathcal{E}(kh) = e_1(kh) - e_i(kh)$. In relation to (Sun and Wang, 2009) it can be write as:

$$\begin{bmatrix} \check{z}(t) \\ x(t) \\ \mathcal{E}(kh) \\ e(kh) \end{bmatrix} = \begin{bmatrix} (E_1 \otimes I_n)x(t) \\ (E_2 \otimes I_n)\check{z}(t) + (\mathbf{1} \otimes I_n)x_1 \\ (E_1 \otimes I_n)e(kh) \\ (E_2 \otimes I_n)\mathcal{E}(kh) + (\mathbf{1} \otimes I_n)e_1(kh) \end{bmatrix}$$

Then we have

$$\begin{aligned}\dot{\check{z}} &= (I_{N-1} \otimes A)\check{z}(t) + (I_{N-1} \otimes A_d)\check{z}(t-d(t)) \\ &- (\mathcal{L} \otimes BK)\check{z}(kh) + (\mathcal{L} \otimes BK)\mathcal{E}(kh) \\ &+ (I_{M-1} \otimes D_\omega)\omega_i(t), \quad kh \le t < (k+1)h\end{aligned} \tag{6}$$

where $\mathcal{L} = E_1 L E_2 \in \mathbb{R}^{(N-1)\times(N-1)}$. Consider an "communication delay" $\tau = t - kh$, $kh \le t < (k+1)h$. Obviously, τ is piecewise linear with the derivative $\dot{\tau} = 1$ at $t \neq kh$ and is discontinuous at $t = kh$. It is straightforward notable that $0 \le \tau(t) < h$. therefore, the system (6) can be composed as

$$\begin{aligned}\dot{\check{z}} &= (I_{N-1} \otimes A)\check{z}(t) + (I_{N-1} \otimes A_d)\check{z}(t-d(t)) \\ &- (\mathcal{L} \otimes BK)\check{z}(t-\tau(t)) + (\mathcal{L} \otimes BK)\mathcal{E}(t-\tau(t)) \\ &+ (I_{M-1} \otimes D_\omega)\omega_i(t), \quad kh \le t < (k+1)h\end{aligned} \tag{7}$$

The initial state of the $\check{z}$ is enhanced as $\check{z}(\theta) = \phi(\theta), \theta \in [-h, 0]$ with $\phi(0) = \check{z}(0) = [\check{z}_2^T, \cdots, \check{z}_N^T]^T$ and $\phi \in W$, where W indicates the banch space of categorically continuous functions $[-h, 0] \to \mathbb{R}^{(N-1)n}$ with square-integrable derivatives and also the norm

$$\|\phi\|_w = \underbrace{max}_{\theta \in [-h,0]} \|\phi(\theta)\| + [\int_{-h}^{0} \|\dot{\phi}(s)\|^2 ds]^{1/2}$$

Remark 1. Noticed that as compare to traditional time varying delay, τ not only depends on release times, but also on the event triggered and as well as on sampling period h.

Definition 1. Given a positive scalar η, the control protocol (3) is supposed to reach global consensus with an assured H_∞ performance η for the agents in (1), if the subsequent two conditions hold:
(1) The system (3) with $\omega_i = 0$ can achieve global consensus, if $Lim_{t\to\infty}\|x_i - x_j\| = 0, \forall i, j = 1, \cdots, N$.
(2) Under the zero-initial state, the performance variable z fulfills

$$J = \int_0^\infty [y_i^T(t)y_i(t) - \eta^2 \omega^T(t)\omega(t)]dt < 0, \tag{8}$$

where $z = [z_1^T, \cdots, z_N^T]$, $\omega = [\omega_1^T, \cdots, \omega_N^T]$

Assumption 1. Time varying delays $d(t)$, satisfy:

$$0 \le d(t) \le \bar{d}, \quad \dot{d}(t) \le \bar{d}$$

where $\bar{d} > 0$ and $\dot{d}$ are prescribed constant scalars.

Theorem 1. Given $\bar{d}$, $\bar{\tau}$, and γ, for any time-varying delays $d(t)$ fulfilling Assumption (1) under the event-triggered scheme (2), the system (7) is globally asymptotically stable, if the communication topology graph $\mathcal{G}$ has a directed spanning tree as well as there exist real matrices $\mathcal{P} > 0$, $\mathcal{Q}_\ell > 0$, $\mathcal{R}_1 > 0$, $\mathcal{M}_1$, $\ell = 1, 2, 3$ with suitable dimensions such that:

$$\Upsilon = \begin{bmatrix} \Delta_{11} & \Gamma^T \mathcal{P} \\ * & -\mathcal{R}_1^{-1} \end{bmatrix} < 0 \tag{9}$$

where

$$\begin{aligned}\Delta_{11} = (1,1) &= (I_{N-1} \otimes A\mathcal{P}) + (I_{N-1} \otimes A\mathcal{P})^T + k_1\mathcal{Q}_1 \\ &\quad + k_2\mathcal{Q}_2 - \mathcal{R}_1 + C^T C, \\ (1,2) &= \mathcal{P}(I_{N-1} \otimes A_d), \\ (1,3) &= \mathcal{M}_1, \\ (1,4) &= \mathcal{P}(\mathcal{L} \otimes BK), \\ (1,5) &= \mathcal{P}(\mathcal{L} \otimes BK) + \mathcal{R}_1 - \mathcal{M}_1, \\ (1,7) &= \mathcal{P}(I_{M-1} \otimes D_\omega), \\ (2,2) &= (1-\dot{d})k_1\mathcal{Q}_1, \\ (3,3) &= -k_2\mathcal{Q}_2 - \mathcal{R}_1, \\ (3,5) &= \mathcal{R}_1^T - \mathcal{M}_1^T, \\ (4,4) &= -\mathcal{Q}_3 + \kappa C^T \Phi C, \\ (4,6) &= -\sqrt{\sigma_i} C^T \Phi, \\ (5,5) &= -2\mathcal{R}_1 + \mathcal{M}_1 + \mathcal{M}_1^T, \\ (6,6) &= (\sigma_i - 1)\Phi \\ (7,7) &= -\eta^2 I, \\ \Gamma(h) &= \begin{bmatrix} I_{N-1} \otimes A \\ I_{N-1} \otimes A_d \\ 0 \\ -\mathcal{L} \otimes BK \\ \mathcal{L} \otimes BK \\ I_{M-1} \otimes D_\omega \end{bmatrix}^T\end{aligned}$$

Proof. Let consider a Lyapunov function for (2):

$$V(t) = \check{z}^T(t)\mathcal{P}\check{z}(t) + \textstyle\sum_{i=1}^{2} V_i(t) \tag{10}$$

where

$$\begin{aligned}V_1 &= k_1 \textstyle\int_{t-d(t)}^{t} \check{z}^T(s)\mathcal{Q}_1\check{z}^T(s)ds \\ &+ k_2 \textstyle\int_{t-\bar{d}}^{t} \check{z}^T(s)\mathcal{Q}_2\check{z}^T ds \\ V_2 &= \textstyle\int_{t-\tau(t)}^{t} \check{z}^T(s)\mathcal{Q}_3\check{z}^T(s)ds \\ &+ \bar{d}\textstyle\int_{t-\bar{d}}^{t}\int_{s}^{t} \dot{\check{z}}^T(s)\mathcal{R}_1\dot{\check{z}}(v)dvds\end{aligned} \tag{11}$$

By taking derivative of $V(t)$

$$\begin{aligned}\dot{V}(\check{z}(t), t) &= 2\check{z}^T(t)\mathcal{P}\mathbf{A} + \check{z}^T(k_1\mathcal{Q}_1 + k_2\mathcal{Q}_2)\check{z}(t) \\ &- \check{z}(t-d(t))^T(1-\dot{d}_1)k_1\mathcal{Q}_1\check{z}(t-d(t)) \\ &- \check{z}^T(t-\bar{d}_1)k_2\mathcal{Q}_2\check{z}(t-\bar{d}_1) \\ &+ \bar{d}_1\dot{\check{z}}^T(t)R_1\dot{\check{z}}(t) \\ &- \textstyle\int_{t-\bar{d}_1}^{t} \dot{\check{z}}^T(t)\mathcal{R}_1\dot{\check{z}}(t)ds\end{aligned} \tag{12}$$

where

$$\begin{aligned}\mathbf{A} &= (I_{N-1} \otimes A)\check{z}(t) + (I_{N-1} \otimes A_d)\check{z}(t-d(t)) \\ &- (\mathcal{L} \otimes BK)\check{z}(t-\tau(t)) + (\mathcal{L} \otimes BK)\mathcal{E}(t-\tau(t)) \\ &+ (I_{M-1} \otimes D_\omega)\omega_i(t)\end{aligned}$$

Recalling *Lemma 3* of (Aslam et al., 2020)

$$-\bar{d}\int_{t-\bar{d}}^{t} \dot{\check{z}}(\alpha)^T \mathcal{R}_1 \dot{\check{z}}(\alpha)d\alpha \le \varpi_l(t)^T \mathcal{M} \varpi_l(t) \tag{13}$$

where

$$\varpi_l(t) = \left[\ \check{z}^T\ \mathcal{E}(t-\tau(t))^T\ \check{z}(t-\bar{d})^T\ \right]^T$$

$$\mathcal{M} = \begin{bmatrix} -\mathcal{R}_1 & \mathcal{R}_1 - \mathcal{M}_1 & \mathcal{M}_1 \\ * & -2\mathcal{R}_1 + \mathcal{M}_1 + \mathcal{M}_1^T & \mathcal{R}_1 - \mathcal{M}_1 \\ * & * & -\mathcal{R}_1 \end{bmatrix}$$

In view of event-triggered condition (2), $t\epsilon[i_k h+\tau_{i_k}, i_{k+1}h+\tau_{i_{k+1}})$ then authors have:

$$e_k^T(t)\alpha e_k(t) \leq \sqrt{\sigma_i} y(i_k h)^T \Phi y(i_k h)$$

above equation is equivalent to

$$[\check{z}^T(t-\tau(t)) \quad e_k^T(t)] \begin{bmatrix} \kappa C^T \Phi C & -\sqrt{\sigma_i} C^T \Phi \\ * & \sqrt{\sigma_i}\Phi \end{bmatrix} \begin{bmatrix} \check{z}(t-\tau(t)) \\ e_k(t) \end{bmatrix} \leq 0 \quad (14)$$

Define an augmented vector as

$$\eta(t) = \{\check{z}(t), \check{z}(t-d(t), \check{z}(t-\bar{d}), \check{z}(t-\tau(t), \mathcal{E}(t-\bar{\tau}(t), e_k(t), w(t)\}$$

Then, it is obtained that

$$\dot{V}(\check{z}_t, t) - [y_i^T(t)y_i(t) - \eta^2\omega^T(t)\omega(t] \leq \eta(t)^T \Upsilon \eta(t) \quad (15)$$

Consequently, it can seen that the MAS (7) is asymptotically stable. Hence, the proof of this theorem can be promptly completed. □

Corollary 1. Given $\bar{d}$, $\bar{\tau}$, and γ, the system (7), for any time-varying delays $d(t)$ satisfying Assumption (1) under the event-triggered transmission strategy (2) is globally asymptotically stable if the graph G has a directed spanning tree and there exist real matrices $\tilde{\mathcal{P}} > 0$, $\mathcal{X} > 0$, $\tilde{\mathcal{Q}}_\ell > 0$, $\tilde{\mathcal{R}}_1 > 0$, $\tilde{\mathcal{M}}_1$, $\ell = 1, 2, 3$ with suitable dimensions such that:

$$\begin{bmatrix} -\tilde{\mathcal{R}}_1 & \tilde{\mathcal{M}}_1 \\ * & -\tilde{\mathcal{R}}_1 \end{bmatrix} < 0 \ (16)$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & 0 & \Psi_{17} & \Psi_{18} & \Psi_{19} \\ * & \Psi_{22} & 0 & 0 & 0 & 0 & 0 & \Psi_{28} & 0 \\ * & * & \Psi_{33} & 0 & \Psi_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & \Psi_{46} & 0 & \Psi_{48} & 0 \\ * & * & * & * & \Psi_{55} & 0 & 0 & \Psi_{58} & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} \end{bmatrix} < 0 \ (17)$$

where

$\Psi_{11} = \mathcal{X}(I_{N-1} \otimes A) + (I_{N-1} \otimes A)^T \mathcal{X} + k_1 \tilde{\mathcal{Q}}_1 + k_2 \tilde{\mathcal{Q}}_2 - \tilde{\mathcal{R}}_1$,
$\Psi_{12} = (I_{N-1} \otimes A_d)\mathcal{X}$,
$\Psi_{13} = \tilde{\mathcal{M}}_1$,
$\Psi_{14} = -(\mathcal{L} \otimes B\tilde{K})$,
$\Psi_{15} = (\mathcal{L} \otimes B\tilde{K}) + \tilde{\mathcal{R}}_1 - \tilde{\mathcal{M}}_1$,
$\Psi_{17} = (I_{M-1} \otimes D_\omega)\mathcal{X}$,
$\Psi_{18} = (I_{N-1} \otimes A)^T \mathcal{X}$,
$\Psi_{19} = \mathcal{X}C^T$,
$\Psi_{22} = (1 - \dot{d})\tilde{\mathcal{Q}}_1$,
$\Psi_{28} = (I_{N-1} \otimes A_d)^T \mathcal{X}$,
$\Psi_{33} = -\tilde{\mathcal{Q}}_2 - \tilde{\mathcal{R}}_1$,
$\Psi_{35} = \tilde{\mathcal{R}}_1^T - \tilde{\mathcal{M}}_1^T$,
$\Psi_{44} = -\tilde{\mathcal{Q}}_3 + \kappa C^T \tilde{\Phi} C$,
$\Psi_{46} = -\sqrt{\sigma_i} C^T \Phi$,
$\Psi_{48} = -(\mathcal{L} \otimes B\tilde{K})$,
$\Psi_{55} = -2\tilde{\mathcal{R}}_1 + \tilde{\mathcal{M}}_1 + \tilde{\mathcal{M}}_1^T$,
$\Psi_{58} = (\mathcal{L} \otimes B\tilde{K})$,
$\Psi_{66} = (\sqrt{\sigma_i} - 1)\Phi$,
$\Psi_{77} = -\eta^2 I$,
$\Psi_{78} = (I_{M-1} \otimes D_\omega)\mathcal{X}$,
$\Psi_{88} = \mu^2 \mathcal{R}_1 - 2\mu\mathcal{X}$,
$\Psi_{99} = -I$

Moreover, the consensus controller gain is given by

$$\tilde{K} = K\mathcal{P}^{-1} \quad (18)$$

Proof. Define $\mathcal{X} = \mathcal{P}^{-1}$, $\tilde{\mathcal{Q}}_\ell = \mathcal{X}\mathcal{Q}_\ell\mathcal{X}$, $\tilde{\mathcal{R}}_1 = \mathcal{X}\mathcal{R}_1\mathcal{X}$, $\tilde{\mathcal{M}}_1 = \mathcal{X}\mathcal{M}_1\mathcal{X}$.
Pre and post multiplying $\{\mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, I, \mathcal{X}\}$ both the sides of (9), which yields to (17). The term $\mathcal{X}\mathcal{R}_1^{-1}\mathcal{X}$ is resolved by the inequality $\mathcal{X}\mathcal{R}_1^{-1}\mathcal{X} \leq \mu^2\mathcal{R}_1 - 2\mu\mathcal{X}$.
Detail proof of inequalities define in Corollary 1 from (17) are given below:

$$\begin{aligned} &\mathcal{P}(\mathcal{L} \otimes BK) < 0 \\ &\qquad = \mathcal{X}\mathcal{P}(\mathcal{L} \otimes BK)\mathcal{X}, \qquad \mathcal{X} = \mathcal{P}^{-1} \\ &\qquad = (\mathcal{L} \otimes B\tilde{K}), \end{aligned}$$

This implies that all the conditions in Corollary 1 is fulfilled. Accordingly, by Corollary 1, the MAS (7) is extended dissipative for any time-varying delays $d_1(t)$ fulfilling Assumption 1. The proof is done. □

Remark 2. It can be noticed that the network communication topology of the MAS is reached out to an increasingly general directed graph with a spanning tree instead of an undirected connected graph which is different from (Dimarogonas et al., 2012).

4. SIMULATION EXAMPLE

In this section, to represent the viability of the presented results a numerical illustration is given. It is assumed that the MAS consist of four linear agents defined by:

$$\dot{x_i}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.4 \end{bmatrix} x_i(t) + \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix} u_i(t) \qquad i = (1, 2, 3, 4)$$

The relationship amongst the agents is represented by a directed graph with a spanning tree as presented in Fig. 2. and its Laplacian matrix is

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Let $(\dot{d}, \bar{\tau}) = (0.5, 0.35)$, $(k_1, k_2) = (0.58, 0.0001)$ and $\gamma = 2.5$. Then, it is found that the LMIs (16)–(17) are feasible with corresponding event-triggered parameter $\tilde{\Phi} = 0.6190$, and the feasible results to those LMIs are acquired as $K = [3.0765 \quad -4.4663]$.

The initial state of MAS are set as $x_1(0) = [0; 5]$, $x_2(0) = [0; 10]$, $x_3(0) = [0; 15]$, and $x_4(0) = [0; 18]$. Apply the Algorithm (see Table 1), then simulation results are

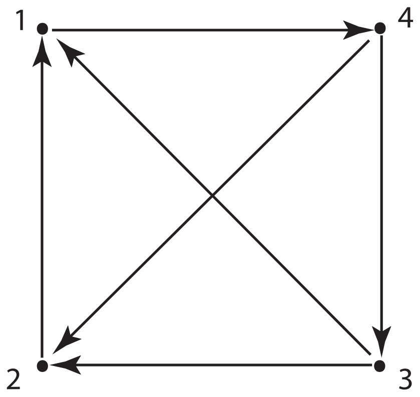
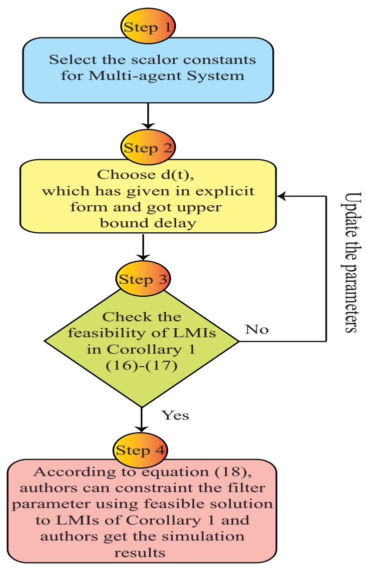


Fig. 2. A directed network topology with spanning tree.

Table 1.

Algorithm for design procedure.



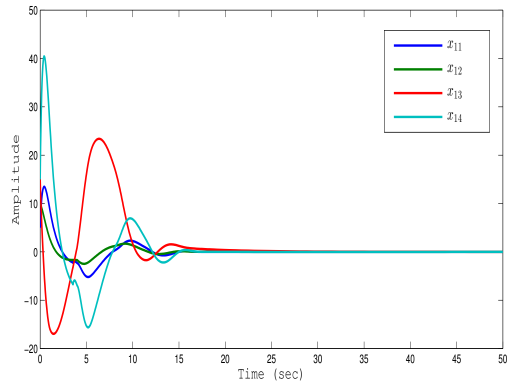


Fig. 3. State response of MASs with time varying delay.

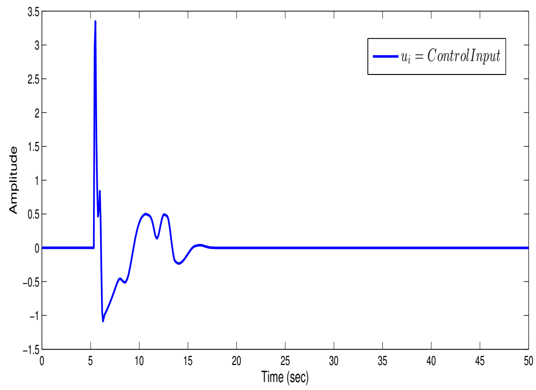


Fig. 4. Response of the control input.

Table 2.

d	0.05	0.10	0.20	0.30
(Liu et al., 2014)	0.72	0.75	infeasible	infeasible
Corollary 1	0.17	0.19	0.25	0.29

demonstrated in Fig. 3-5. The state trajectories of MASs depict in Fig. 3. From which one can observe that all systems' states can definitely achieve consensus, which approves the viability of the proposed structure. Fig. 4 exhibited the response of the control protocol law, which shows that our close loop system becomes stable from the calculated controller gain of Corollary 1. It is noted that the H_∞ Consensus of MASs with Random Time-delay has been studied in (Liu et al., 2014). Now, to compare the minimum permissible value of γ acquired by utilizing the techniques proposed in (Liu et al., 2014) and our paper. The comparison is given in Table 2.

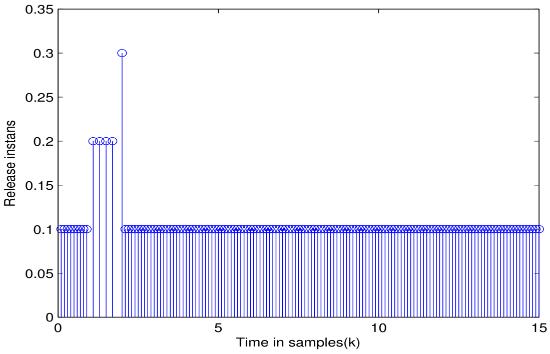
One can see from Table 2 that, for various values of the delay upper bounds, the minimum permissible values of γ gotten by utilizing our method are less than those acquired by utilizing the method in (Liu et al., 2014). This implies, for this illustration, the presented method in this article is less conservative than (Liu et al., 2014).

Fig. 5 show the transmitting and release interval of MASs. Which demonstrate that the number of transmitting sampled-data is considerably minimized by using the event-triggered scheme. The finding from Fig. 3-5 shows that the proposed delayed multi-agent system is effective, which is especially important for wireless communication and industrial control.

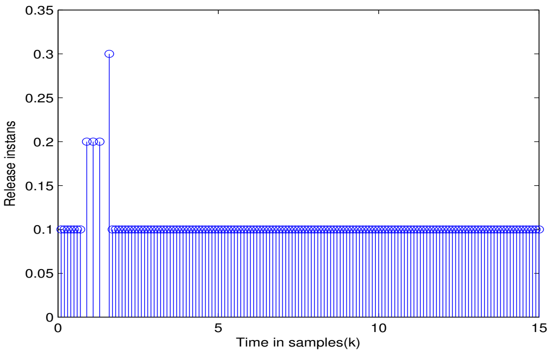
Remark 3. : It is noticed that work is done in (Liu et al., 2014) deals with the random delays. To make the system dynamics the same we only considered event 1 for traditional time-varying delay in (Liu et al., 2014).

5. CONCLUSION

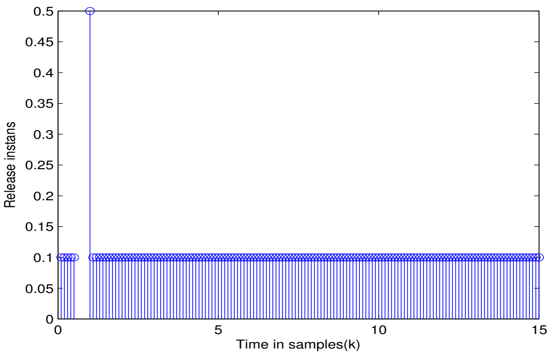
In this article, the problem of event-triggered based H_∞ consensus control for MASs with the directed graph under time-varying delay is considered. The introduced event-triggered scheme has enhanced system performance and also reduced the unnecessary utilization of network resources. A new Lyapunov function has been utilized to derive sufficient conditions to promise the system stability and to attain a defined performance with LMI techniques,



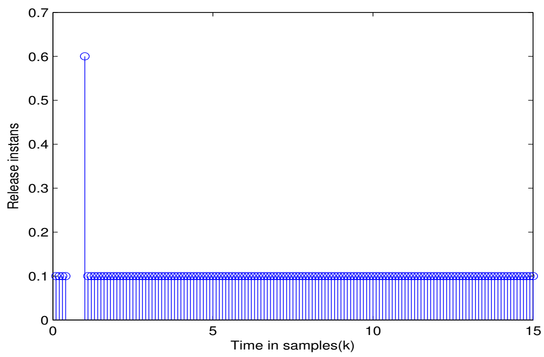
(a) Release instants and release interval by ETM for agent 1



(b) Release instants and release interval by ETM for agent 2



(c) Release instants and release interval by ETM for agent 3



(d) Release instants and release interval by ETM for agent 4

Fig. 5. Release instants and release interval by ETM.

under which all the systems' states asymptotically achieve consensus. The proposed method has shown promising results that have been verified by a simulation example. Therefore, it has been recommended to increase the efficiency of the system.

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