FAULT ISOLATION BASED ON WAVELET TRANSFORM

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Abstract: *This paper evaluates how wavelet transform can be used to detect and isolate particular faults. The diagnostic method that is proposed is based on the stationary wavelet transform. The wavelet coefficients allow analysing the signal changes over different scales. Therefore, fault detection can be performed. Each scale is related to a particular frequency band. Thus if various faults are known to affect different frequency bands, the wavelet coefficients can be used to isolate the faults. Fuzzyfication of the wavelet coefficients is first applied, followed by the fuzzy aggregation of the fuzzyfied coefficients to make the isolation decision easy to compute and gradual. Academic examples are discussed to show the efficiency of the isolation method presented here.*

Keywords: *Fault detection, Fault isolation, Stationary wavelet transform Fuzzy aggregation.*

1. INTRODUCTION

Fast detection of faults in dynamic systems is very important for safe operation. Moreover, fault isolation allows determining which physical component is failing. Fault Detection and Isolation (FDI) is necessary not only for maintenance, but also for making online correct protection decision. Signal processing is a well known tool to deal with fault diagnosis. It is used to analyse directly the signals measured online, avoiding system modelling.

Its main drawback is that a change in some signal feature, *e.g.* the signal mean or the frequency contents, must be distinctive for each fault that may occur on the system. Despite this difficulty, signal processing is widely used in industry, for instance when vibration monitoring is considered.

Detection tests that aim at detecting a change in the mean or the standard deviation of a signal are now very common [1, 2]. Frequency representations are particularly useful for studying rotating machines. Indeed, extra frequency contents may appear under the influence of a particular fault. For instance, [3] deeply studies faults in a three-phase induction machine. The spectral analysis of electric and electromagnetic signals shows that mechanical abnormalities such as broken rotor bars generate characteristic frequency contents in the signals.

Unfortunately, the Fourier Transform is unable to accurately analyse and represent a signal with non periodic features, for instance a transient signal. To study non stationary signals, time-frequency methods replace traditional spectral analysis [4, 5]. The Short Time Fourier Transform (STFT) interpretation is close to a local Fast Fourier Transform analysis. The signal to analyse is multiplied by a sliding window (for instance rectangular, Hamming, Blackman, etc.) with finite duration. Thus the spectrum is computed in real time and its variation contents are used to detect faults. This method has been applied for instance in the metallurgical industry [6]. Actually, rise in productivity in modern rolling mill plants induces an increase of the rolling speed. This also increases the potential vibrations of the system. Different vibrations appear that correspond to particular faults [7]. Thus monitoring the frequency contents can help to localise the faults.

The main drawback of STFT method is due to the constant time-frequency resolution, according to the Heisenberg-Gabor uncertainty principle. Indeed, there is a trade-off between time and frequency resolutions because an accurate time resolution requires a “short” analysis window while an accurate frequency resolution involves a “long” analysis window, which introduces an extra detection delay.

In order to obtain a variable time and frequency resolution (their product been constant), the Wavelet Transform (*WT*) has been introduced. Moreover, Wavelet analysis does not require stationarity hypothesis and it is well adapted to the analyses of signals with temporary changes. It has been investigated for monitoring and diagnosis in various industrial areas. The case of arc tracking is typical of its use [8]: no model of the physical phenomenon is available and arcing appears as random discontinuities in the current signal. Article [9] reports the use of wavelet analysis to detect faults in a high voltage direct current line (HVDC). Line faults, commutation failures in the converter and single phase short circuits at the AC side are studied and shown to produce time varying transients. For this application, isolating faults is very important because the safety procedures are very different depending on the type of fault. In [10], the surface faults of a compact disc like scratches and fingerprints are detected and handled with dedicated filters.

Most of the reported applications deal with fault detection. How to process wavelet coefficients to cope with fault isolation is investigated in this paper. The paper is organised as follows. In section 2, the Stationary Wavelet Transform (*SWT*) is introduced and the wavelet coefficients thresholding method is explained. Section 3 is devoted to the description of the isolation method proposed. It is based on the fuzzyfication of the wavelet coefficients, interpreted as partial criteria to be aggregated in order to make a decision. Isolation results achieved with the proposed method are discussed in section 4.

2. STATIONARY WAVELET TRANSFORM AND THRESHOLDING

A detection method based upon multiresolution analysis (*MRA*) has been previously proposed [8]. It has been applied to vibration monitoring in [11]. In this paper, the Stationary Wavelet Transform (*SWT*) is used instead of the *MRA* for the detection of “low” frequency vibrations. The main advantage of the *SWT* [7, 12] is its time-invariance property: the *SWT* coefficients of a delayed signal are just a time-shifted version of the original ones. A quick overview of the *SWT* is now given and the thresholding technique of the wavelet coefficients is summarised.

2.1. Wavelets Transform and Stationary Wavelets Transform

The Continuous Wavelet Transform (*CWT*) projects a signal $x(t)$ on a family of zero-mean functions $\psi_{a,b}$ (the wavelets) deduced from an elementary function ψ (the mother wavelet) by means of translations and dilatations:

$$CWT_{a,b}(x) = \int_{-\infty}^{+\infty} x(t)\psi^*_{a,b}(t)\,dt$$

where * stands for ”conjugate”, a is the scaling parameter (taking $|a| > 1$ dilates the function ψ), b is the translation parameter and $\psi_{a,b}(t) = \left(1/\sqrt{a}\right)\psi\left(\left(t-b\right)/a\right)$ [13]. The redundancy introduced by the *CWT* can be reduced by the discretization of parameters a and b, leading to the Discrete Wavelet Transform (*DWT*):

$$DWT_{j,k}(x) = \int_{-\infty}^{+\infty} x(t)\psi_{j,k}(t)\,dt \qquad (1)$$

where $\psi_{j,k}(t) = 1\Big/\sqrt{a_0^j}\;\psi\left(a_0^{-j}t - kb_0\right)$.

The choice $a_0 = 2$, $b_0 = 1$ corresponds to the dyadic sampling of the time-frequency plane (*i.e.* one set of coefficients per octave). Thanks to this particular sampling, it is possible to obtain for the set $\psi_{j,k}$ an orthonormal basis with a wavelet ψ well localized both in time and frequency (see Fig. 1 for the "shape" of a few wavelets).

At level $j = 1 : J$, the approximations a_k^j and details d_k^j (Fig. 2) are recursively computed using digital filters, respectively a lowpass filter $\tilde{h}$ and a highpass one $\tilde{g}$. Note that the computation of the coefficients lead to the evaluation of dot products which is very attractive for the implementation on Digital Signal Processors (fused add-multiply operation).

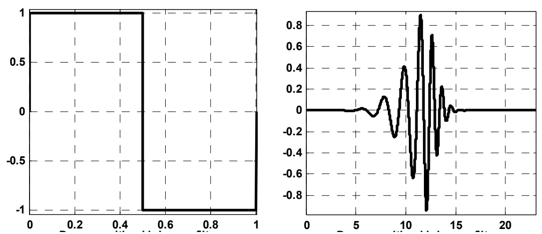

Fig. 1: Wavelets (Haar, Daubechies12 db12)

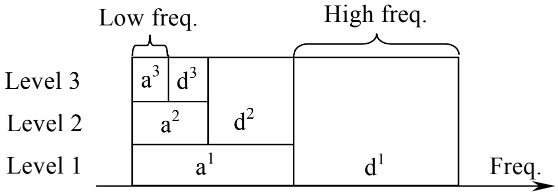

Fig. 2: Frequency domain split

This recursive algorithm is initialized by $x(k)$, i.e. $a_k^0 = x(k)$:

$$a_k^j = \sum_l \tilde{h}_{2k-l} a_l^{j-1}\,; \quad d_k^j = \sum_l \tilde{g}_{2k-l} a_l^{j-1} \qquad (2)$$

As stated above, the main advantage of the *SWT* (Fig. 3) is its time invariance. This property is fundamental for diagnosis purpose to provide a symptom that is time-invariant, *i.e.* its value does not depend on the time the fault occurs.

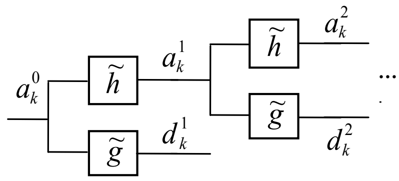

Fig. 3. *SWT* decomposition algorithm

2.2. Wavelet coefficient thresholding

The singularity occurrence in a signal is revealed by the size of the wavelet coefficients [11, 16]. Thus, the objective is to find when the noisy data:

$$x(t_i) = g(t_i) + \varepsilon(t_i), \quad i = 1:n \qquad (3)$$

changes its behaviour, without any hypothesis about the parametric form of g which contains this change. In (3), ε is usually supposed to be a zero mean and σ^2 variance independent normally distributed noise, which allows interesting theoretical results about the "optimality" of thresholding [14]. The aim is to distinguish the wavelet coefficients containing information about the singularity from the other ones which belong to the "normal" signal behaviour. In [15] Donoho proposes to extract these significant coefficients by soft thresholding:

$$\delta_k^j = \begin{cases} d_k^j + \lambda^j & d_k^j < -\lambda^j \\ 0 & 0 \le \left|d_k^j\right| \le \lambda^j \\ d_k^j - \lambda^j & d_k^j > \lambda^j \end{cases} \qquad (4)$$

where δ_k^j is the thresholded coefficient, d_k^j is given in (2) and λ^j is the threshold value.

The coefficients below their threshold are set to zero (they are assumed to represent the normal behaviour), while exceeding coefficients indicate the occurrence of a signal abnormal behaviour (Fig. 4). Note that the occurrence of a singularity may affect only a few levels. Therefore, λ^j is level-dependant.

The threshold choice is tricky. Several methods may be used, and a bibliographical study reveals many possibilities. The optimal choice requires knowledge (or at least hypotheses) about the analysed signal [11, 16]. In this paper, the *minimax threshold* has been chosen. Its main property is that the Risk function:

$$R(g,\hat{g}) = \frac{1}{n} E\left(\sum_{i=1}^{n} \left(g(t_i) - \hat{g}(t_i) \right)^2 \right) \qquad (5)$$

is minimum, $\hat{g}$ being the reconstructed signal. For each level j, the threshold λ^j is given by:

$$\begin{aligned} \lambda^j &= \sigma^j \lambda^*_{n_j} \quad if\ n_j \geq 64 \\ \lambda^j &= \sigma^j \sqrt{2\log(n_j)} \quad if\ n_j < 64 \end{aligned} \qquad (6)$$

where σ^j is the standard deviation of d_k^j, n_j is the number of coefficients d_k^j that are used for the threshold computation and $\lambda^*_{n_j}$ has been tabulated and can be found in [14].

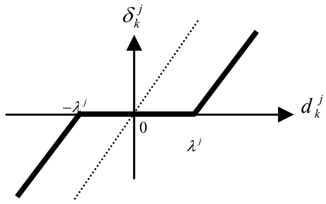

Fig. 4. *SWT* coefficient soft thresholding

2.3. Choice of the Wavelet

When time-scale methods are used, the relationship between scale and frequency is expressed through the pseudo-frequency f_a (in Hz) corresponding to a given scale a. f_a is computed thanks to the normalised "centre frequency" f_c of the wavelet [17]:

$$f_a = \frac{f_c}{a \times T_e} \quad \text{where } T_e = 1/f_e \qquad (7)$$

The underlying idea is to associate with a given wavelet a purely periodic signal of frequency f_c that maximizes the Fast Fourier Transform (FFT) of the wavelet modulus. In Fig. 5, the *db12* wavelet [18] and its associated purely periodic signal are drawn. The pseudo-frequency f_a depends on the wavelet and the decomposition level j. Thus, it can suggest the choice of the analysing function, and the number of decomposition levels J, depending on the frequency contents that reveal the appearance of a particular fault.
Another criterion for the choice of the wavelet is related to the kind of singularity that must be detected, *i.e.* the singularity that appears in the signal when a fault occurs. This choice is directly connected to the regularity of the wavelet [18]. The reader can refer to [13] for a few examples on this subject.

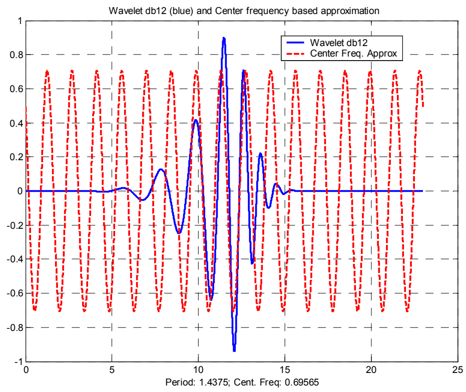

Fig. 5. Centre frequency approximation of *db12*

3. FAULT DETECTION AND ISOLATION

3.1. Principle

Wavelet decomposition can be implemented for diagnostic purpose when a fault occurrence is revealed by a signal singularity. The proposed detection method analyses the changes that appear over the different decomposition levels to detect the singularity. The hypothesis for fault isolation is that different faults induce different effects on the wavelet coefficients over the decomposition levels. The isolation method proposed in this paper analyses the modification of the wavelet coefficients over the different levels of decomposition to deduce which fault is present.

3.2. Detection

The detection procedure works in three steps.
The first step transforms the signal into wavelets coefficients. It decomposes the signal on J scales. This step also allows characterizing the frequency contents that define the "normal" behaviour of the system.

The second step corresponds to the wavelet coefficient thresholding, where the thresholds are computed as explained in section 2. Nevertheless, the classical thresholding method is a crisp one, while a gradual indicator is generally more interesting than a Boolean one. Indeed, it allows focusing attention on a component before a fault is completely installed. Therefore, a fuzzyfication of the thresholded coefficients is implemented:

$$\mu^j(\delta_k^j) = \begin{cases} 1 & \left|\delta_k^j\right| \geq 2\alpha^j\lambda^j \\ \dfrac{\left|\delta_k^j\right|}{2\alpha^j\lambda^j} & 0 < \left|\delta_k^j\right| < 2\alpha^j\lambda^j \\ 0 & \delta_k^j = 0 \end{cases} \quad (8)$$

where α^j is a parameter that defines the membership function of the coefficients (Fig. 6).

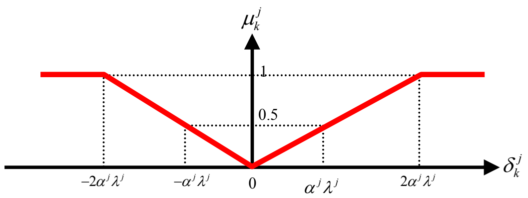

Fig. 6. Coefficient fuzzyfication

The third step corresponds to the detection decision. In order to give a unique indicator, the various fuzzy coefficients are considered as partial criteria and the detection problem is regarded as a fuzzy decision making one with partial criteria. Fuzzy decision making allows formal modelling of decision-making for imprecise and uncertain conditions. The decision (here the detection decision) is considered as a fuzzy set described by its membership function μ_d that is computed using the membership functions of the various partial points of view on the final decision $c_i(d)$:

$$\mu_d = h\left(c_1(d), c_2(d), \ldots, c_p(d)\right) \quad (9)$$

where h is a fuzzy set operator to be determined in function of the properties that are required for the decision.

Consider the most common operators, conjunctive or disjunctive ones and a small example. Suppose that the "faulty" state of a heat exchanger has to be diagnosed using two criteria, a "high" temperature and a "small" flow. A conjunctive operator h states that all the criteria must be met simultaneously (the state is faulty if the temperature is "high" **and** the flow is "small" at the same time). It can be expressed mathematically by a *min* function, for instance. A disjunctive operator h states that a single criterion is sufficient for the decision to be made (the state is faulty if **either** the temperature is "high" **or** the flow is "small") which is expressed for instance by a *max* function. A *mean* function expresses a compromise operator.

When a singularity occurs in the signal, at least one level of decomposition must reveal its appearance, and the singularity may not be present over all scales Thus, (10) is proposed for the detection decision:

$$D1_k = \max_j(\mu_k^j);\ j = 1:J \quad (10)$$

From a practical point of view, it can be observed that the wavelet coefficients may be very small, during a very short time, even when there is a singularity in the signal. Thus (11) may be preferred to (10), to favour a clearer decision:

$$D2_k = \max_j\{\max(\mu_{k-l}^j); l = 0:N-1\};\ j = 1:J \quad (11)$$

where N is a small time window. [19] proposes a comparison of different aggregation operators to detect extra vibrations (considered as faults) in a rolling mill.

3.3. Isolation

For fault isolation, the singularity appearance must modify differently the various levels of decomposition, depending on the considered fault. A learning phase shows which levels are modified by a specific fault. For example, consider a signal that is decomposed over 5 levels. Moreover, suppose that the wavelet coefficients on levels i and j are modified by the fault, while the coefficients on levels k, l, m are not modified. This situation can occur for instance when the fault gives rise to oscillations in a specific frequency range as reported by [3] for electrical drives or [7] for rolling mills. The isolation decision for this specific fault can be given by:

$$D3_k = \min\begin{pmatrix} \mu_k^i, \mu_k^j, (1-\mu_k^k), \\ (1-\mu_k^l), (1-\mu_k^m) \end{pmatrix} \quad (12)$$

(12) expresses that the coefficients on levels i and j must be "high" at the same time, and the other coefficients must be "small", to decide that this fault is present.

4. EXPERIMENTAL RESULTS

In this section, different scenarios are discussed in order to evaluate the *SWT* capabilities to detect and isolate particular faults. For a wide range of applications, particular additive frequency contents are related to the occurrence of a particular fault (e.g. faults in rolling mill process or abnormalities such a broken rotor bars in induction motor). In other applications, the signals recorded on the process exhibits impulses in amplitude or a pseudo frequency occurrence when a fault occurs. All these situations can be handled with *SWT*. Table 1 gives three academic examples that mimic these situations. These simulated signals will show the powerful of the isolation method proposed in this paper.

Table 1. Scenario number and its simulated signal

0	$x_{ref}(t) = \sin(2\pi f t) + \varepsilon(t)$
1	$x_1(t) = x_{ref}(t) + 0.7(\sin(2\pi f_1 t) + \sin(2\pi f_2 t))(u(t-\tau_1) - u(t-\tau_2)) + 0.7\sin(2\pi f_3 t)(u(t-\tau_3) - u(t-\tau_4))$
2	$x_2(t) = x_{ref}(t) + 40\sum_{k=1}^{n}(-1)^{k+1}\delta(t - k\tau_1)$
3	$x_3(t) = x_{ref}(t) + +2e^{-0.25(2\pi f_1 t)}\sin(2\pi f_1 t)(u(t-\tau_1) - u(t-\tau_2))$

The parameters are f = 50Hz, f_1 = 20Hz, f_2 = 350Hz, f_3 = 175Hz. Scenario 0 corresponds to the reference signal (i.e. "normal" behaviour): it corresponds to a noisy sinusoidal signal. ε is a Gaussian white noise with zero mean and variance σ^2 chosen such that the Signal-to-Noise Ratio is $SNR \approx 10dB$.

In scenario 1, extra frequency contents f_1 and f_2 occur at time $t = \tau_1$ during a time interval $\tau_2 - \tau_1$ and another additive frequency f_3 occurs at instant $t = \tau_3$ during a time interval $\tau_4 - \tau_3$ ($\tau_1 < \tau_2 < \tau_3 < \tau_4$). Scenario 2 corresponds to a fault characterised by the appearance of periodic impulses while scenario 3 deals with the appearance at time $t = \tau_1$ of a pseudo frequency of duration $\tau_2 - \tau_1 > 0$.

In order to detect and isolate the faults described in scenarios 1 to 3, some parameters of the *SWT* must be discussed. The sampling frequency f_e of the signal and the number of decomposition level of the wavelet transform are related to the frequency that must be detected through equation (7). The sensitivity of the FDI method proposed here is increased when f_i, i = 1:3, satisfies (7). This remark should guide the choice of the mother wavelet. Actually, the *SWT* can be performed with different wavelets based on Matlab function *swt*. For instance, the Mallat wavelet is used in [9] for detection and identification of faults in HVDC systems. The Morlet wavelet has been used in the literature for the analysis of vibration signals recorded on rotating machineries [20]. This is due to the fact that the Morlet wavelet is able to pick up impulses generated by the rotating elements. Other wavelets are used in the literature but the Daubechies' wavelets [18] are used in a wide range of applications [21, 22]. This is certainly due to their "nice" properties (compact support, number of vanishing moments, orthogonality, etc.).

For the examples in Table 1, a wavelet decomposition over 5 levels (J = 5) is sufficient to ensure a good detection. The sampling frequency is equal to 1 kHz. The Daubechies 12 "*db*12" wavelet has been used because it is able to highlight the "faulty" extra frequency contents. The thresholds λ^j have been computed with the reference signal x_{ref} thanks to equation (4). x_1, its *SWT* decomposition and the thresholds λ^j are given in Fig. 7.

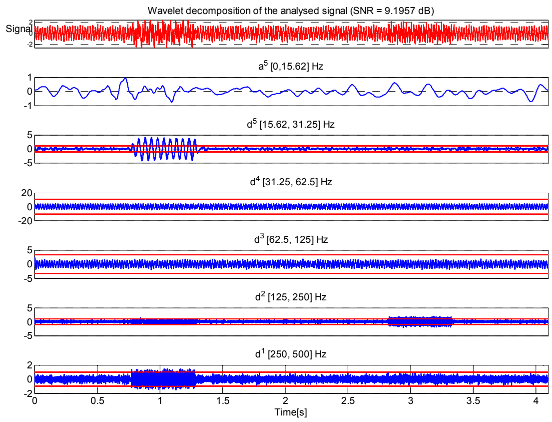

Fig. 7. x_1 and its SWT decomposition

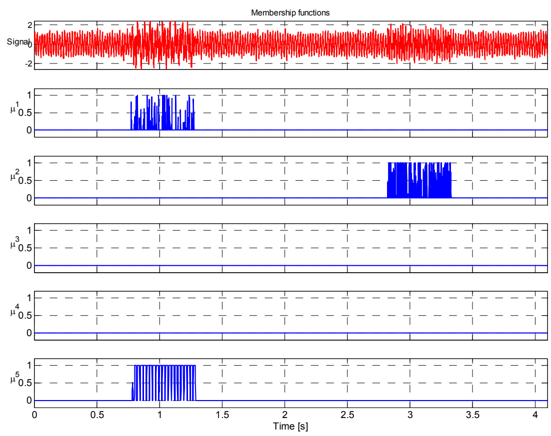

Fig. 8. Fuzzyfication of the thresholded coefficients

The *SWT* coefficients d_5 and d_1 clearly exhibit the extra frequency contents f_1 = 20Hz and f_2 =

350Hz. This can be explained by the dyadic split of the frequency domain (see Fig. 2). The other extra frequency content is characterized by $f_3 = 175\text{Hz}$. It is exhibited in the coefficients d_2 on the second level of decomposition. The thresholded coefficients are fuzzyfied with the membership functions μ^j, $j = 1:5$ calculated with (8). The result is shown in Fig. 8. The abnormality in each frequency band is clearly exhibited.

The fault detection indicator *FD* is computed with (11). It measures the appearance of an abnormal behaviour over all the levels of decomposition. When fault isolation is considered, specific aggregation operators must be defined. These new operators take into account some knowledge on the kind of singularity that appears when a particular fault occurs. Thus, the fault isolation decisions that are defined are given by:

$$FI_{F1} = \min\left\{\begin{matrix}\mu_1, (1-\mu_2), (1-\mu_3), \\ (1-\mu_4), \mu_5\end{matrix}\right\} \tag{13}$$

$$FI_{F2} = \min\left\{\begin{matrix}(1-\mu_1), \mu_2, (1-\mu_3), \\ (1-\mu_4), (1-\mu_5)\end{matrix}\right\} \tag{14}$$

Results are shown in Fig. 9.

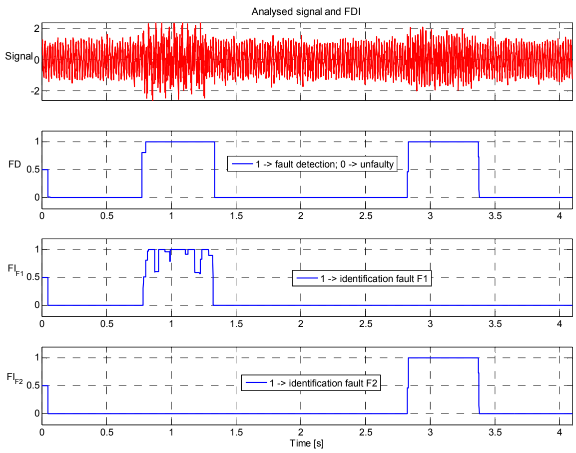

Fig. 9. Signal x_1 and *FD*, FI_{F1}, FI_{F2}

It can be observed that the isolation decision FI_{F1} that is devoted to the detection of frequencies f_1 and f_2 clearly identifies this fault. Identically FI_{F2} is able to detect the fault characterized by f_3.

The results achieved for the second scenario (signal x_2 in Table 1) are given in Fig. 10. The fault detection and isolation procedure is similar to the one presented for scenario 1 but the decision rule that takes into account all the decomposition levels is:

$$FI_{F3} = \min\{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\} \tag{15}$$

This can be explained by the fact that impulses induce wavelet coefficients that exceed their thresholds over several levels of decomposition. As expected, the fault (occurrence of impulses) is detected and localised. The results obtained for the 3rd scenario (signal x_3 in Table 1) are given in Fig. 11. Note that the pseudo frequency is equal to f_1. Thus, the wavelet coefficients on level 5 are sensitive to the occurrence of this extra frequency content. To isolate this fault, the decision rule must mainly focus on μ_5:

$$FI_{F4} = \min\left\{\begin{matrix}(1-\mu_1), (1-\mu_2), \\ (1-\mu_3), (1-\mu_4), \mu_5\end{matrix}\right\} \tag{16}$$

As expected, the fault is detected and isolated.

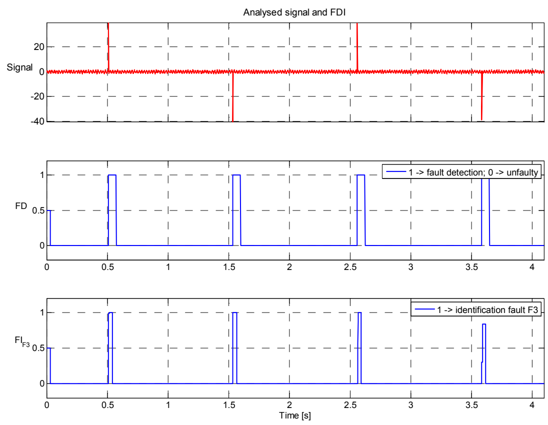

Fig. 10. Signal x_2 and *FD*, FI_{F3}

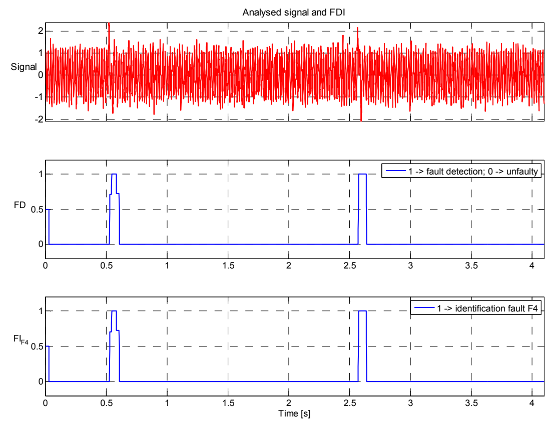

Fig. 11. Signal x_3 and *FD*, FI_{F4}

5. CONCLUSIONS

In this paper, the capability for the stationary wavelet transform to deal with different faults for fault detection and isolation has been investigated. A detection procedure based upon the thresholds of the wavelet coefficients has been considered. These coefficients are fuzzyfied and aggregated in order to provide a symptom. The tuning parameters of this procedure are the wavelet itself, the number of decomposition levels, the thresholds and the decision method. The wavelet choice depends on the features that must be detected in the signal under analysis. This selection is sometimes not unique. For detection purpose, the final choice is made in order to maximize the symptom sensitivity.

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