Industrial Wastewater Treatment Control by a Minimax Principle Over Weakly Measured Pollution

Patrycja Trojczak*, Piotr Lizakowski, Hanna Tkachuk*****

**Faculty of Navigation and Naval Weapons, Polish Naval Academy,*
69 Śmidowicza Street, Gdynia, 81-127, Poland (e-mail: p.trojczak@amw.gdynia.pl)
***[3]Faculty of Command and Naval Operations, Polish Naval Academy,*
69 Śmidowicza Street, Gdynia, 81-127, Poland (e-mail: p.lizakowski@amw.gdynia.pl)
****Department of Chemistry and Chemical Engineering, Khmelnitskiy National University,*
11 Instytutska Street, Khmelnitskiy, 29016, Ukraine (e-mail: 190670anna363@ukr.net)

Abstract: In this paper, a problem of rationally treating industrial wastewater is considered. The treatment is conditioned with that a one or more plants should execute this job along a river but a schedule of treating the polluted water is not correlated with the schedule of manufacturing. So, river water is treated separately, without considering manufacturing processes. Another condition is that they deal with scattered and dependent-on-location measurements of water pollution. Thus, it is hard to determine exactly for each plant a volume of water that should be necessarily treated for ensuring the water renewal for the whole river. Because of multiple, weakly correlated measurements, industrial water treatment should be controlled both for dealing with water pollution and prevention of detrimental effects. A robust approach is proposed to industrial wastewater treatment control with using a minimax principle, which fits long, deep, and wide rivers. Pollution at a shallow river can be analyzed as well only if its contaminations are registered in wide ranges. The control realized by a minimax model and algorithm is meant by effectively assigning treatment jobs for wastewater treatment plants. In three special cases, the model gives hints at plausible inaccurate measurements or overestimations.

Keywords: uncertainty, inaccurate measurements, water pollution, interval estimates, wastewater treatment control, minimax principle.

1. INTRODUCTION

1.1 Industrial wastewater treatment control

It is impossible to overestimate the importance of controlling non-polluted waters for both aquatic life and humans. Obviously, it is hard to be numerically sure of how much of Europe is polluted and what kind of threat that poses. However, the tempo of pollution has not been decreasing. Researchers from Poland, Romania, Germany, Latvia, France, Spain and other countries of European Union report about strong evidence that chemicals threaten the ecological integrity (Skuras and Tyllianakis, 2018; Gitis and Hankins, 2018) and consequently the biodiversity of almost half of the water bodies on a continental scale. After sorting through government data from 4,001 monitoring sites, they found pollution levels high enough to kill aquatic species at 14 % of the sites (see an article "Almost half of Europe's water is threatened by pollution" at riverlink.server299.com). Meanwhile, chemicals swirling in the water were prevalent enough to cause chronic health effects at 42 % of the sites. At that, the risks of the true pollution are underestimated due to limitations in measuring the pollution.

The main cause of pollution of surface waters is the uncontrolled discharge of untreated municipal and industrial wastewater directly into water bodies and through urban sewerage systems. Industrial wastewater treatment is fulfilled with a series of strategies to remove the water contamination and thus decrease the pollution. Mainly, they consist in removing dissolved salt ions from waste stream (Kim and Logan, 2013; Krstić et al., 2018), removing solids using sedimentation techniques (Cancino-Madariaga and Aguirre, 2011), oils and grease removal by separators (Alther, 2008; Jafarinejad, 2017), removal of biodegradable organics by activated sludge and trickling filter processes (Kornaros and Lyberatos, 2006; Zieliński et al., 2013; Ogidan, 2017). The wastewater treatment with trickling filters is one of the most well characterized treatment technologies. However, problems may arise if the wastewater is excessively diluted with washing water (Gitis and Hankins, 2018; Jafarinejad, 2017). Eventually, the presence of cleaning agents, disinfectants, pesticides, or antibiotics impacts detrimentally on the wastewater treatment process. Therefore, the industrial wastewater treatment has its own restrictions. On the one hand, its costs are not small. Not all industrial enterprises (factories, plants, mills) can fulfil the treatment without government subsidies (Gikas, 2017; Abraham, 2017; Fung and Wibowo, 2013). On the other hand, despite the treatment produces effluents reused to a sanitary sewer or to surface water in the environment, the wastewater treatment process is not innocuous itself. Continuous wastewater treatment may inversely affect the environment of a terrain where an industrial wastewater treatment plant is mounted. This concerns especially those plants that are located near rivers and natural reservoirs and pour out the treated water back into them.

1.2 Motivation

An industrial wastewater treatment plant is a complex technological object, wherein the technique of treating polluted water depends on manufacturing processes and production of the respective industry (Abraham, 2017; Jafarinejad, 2017). When river water is directly recycled, locations of water drawoff and discharge may be located close to each other (Jafarinejad, 2017; Alther, 2008). Then a schedule of treating the polluted water is easily correlated with the schedule of manufacturing (Kornaros and Lyberatos, 2006). Otherwise, if river water is treated separately, without considering manufacturing processes, it is hard to determine exactly a volume of water that should be necessarily treated for ensuring the water renewal for the whole river (Fung and Wibowo, 2013; Leonzio, 2017; Abraham, 2017). Moreover, when a few plants work along the same river (for instance, along Danube, Tisza, Rhine, Elbe), it is impossible to accurately distribute treatment assignments for those plants because the sites where they work are usually polluted non-uniformly (Krstić et al., 2018). Besides, accurate measurements of water pollution, if any, are always tied to localities and depths where concentration of contaminants in water is registered (Abraham, 2017; Vlad et al., 2012). This fact is a main cause of why a problem of distributing treatment assignments is an open question. Because of multiple, weakly correlated measurements, industrial water treatment should be controlled both for dealing with water pollution and prevention of detrimental effects.

1.3 Goal of the paper

The goal is to derive formulae and compose an algorithm for industrial water treatment control, while dealing with scattered and dependent-on-location measurements of water pollution. The control is meant by effectively assigning treatment jobs for wastewater treatment plants. The goal will be achieved by plotting a model of treatment control with considering any number of the plants. The algorithm will be based on this model. Additionally, it will be shown how to apply the algorithm and list issues of its application.

2. MODEL OF TREATMENT CONTROL

2.1. Variables and Constraints

Denote a maximum allowable concentration of contaminants in water of a river by c_0 (in mg/m^3). Then a volume of allowable contamination is

$$v_0 = \lambda\left(c_0\right) \text{ (in m}^3\text{)},$$

where λ is a function that maps the concentration into cubic meters of untreated water. If c is a current average concentration (in mg/m^3) of contaminants in water of the given river, then

$$v = \lambda\left(c\right) \text{ (in m}^3\text{)} \tag{1}$$

is a current average volume of untreated water. Hence, an average volume of contamination to be treated is $v - v_0$. A period of time (in hours) during which a plant should work for drawing the water pollution back to rate c_0 (or, it can be expressed via v_0) is

$$h_0 = g\left(v_0, v\right) \text{ (in hours)}, \tag{2}$$

where g is a function that maps the average volume of contamination to be treated into a grand total of hours. Note that period (2) is not constrained to 24 hours. This is just a period of the pending treatment work. For example, if $h_0 = 48$ then it implies that the treatment work can be done with at least two sources (at a plant, considering a single plant) of water treatment. Besides, these treatment hours can be transferred into water volumes (in cubic meters), if such units are convenient.

However, for the k -th plant, such a period is

$$h_k = g\left(v_0, v_k, S_k\right) \text{ (in hours)}, \tag{3}$$

where S_k is a set of geographic peculiarities of the plant site (e. g., atmospheric pressure, a rate of atmospheric precipitates, land erosion, subsurface erosion, liquid impact erosion, proximity of highway areas, etc.), which additionally affect the treatment process and values v_0 and v_k are corrected thereof (in m^3). Let us call time period (3) the treatment load.

Nevertheless, the average concentration of contaminants (1) may differ from site to site due to non-uniform pollution and specific geographic characteristics. So, at the site where the k -th plant works, an average volume to be treated is $v_k - v_0$ by an average concentration of contaminants c_k (in mg/m^3). A real concentration registered at different seasons, weather, periods of a day, is a varying value. So, let the concentration at the k -th plant site be registered from $c_k^{<\min>}$ to $c_k^{<\max>}$, i. e.

$$c_k^{<\min>} \leqslant c_k \leqslant c_k^{<\max>}, \quad k = \overline{1, P}, \tag{4}$$

where P is a total number of plants along the given river.

According to this and general mapping (1), the volume of the to-be-treated contamination changes from value $v_k^{<\min>} - v_0$ to value $v_k^{<\max>} - v_0$:

$$v_k^{<\min>} - v_0 \leqslant v_k - v_0 \leqslant v_k^{<\max>} - v_0 . \tag{5}$$

Moreover, according to mapping (3), a grand total of the water treatment hours for plants is constrained:

$$\sum_{k=1}^{P} h_k \leqslant h_0 . \tag{6}$$

So, a case $\sum_{k=1}^{P} h_k < h_0$ is possible when, except for the considered industrial water pollution, additional pollution exists. From the other side, inequality (6) is introduced due to the surplus water treatment causes detrimental effects.

Mapping (3) along with inequality (5) imply that there are lower and upper values of those water treatment hours for the k -th plant, respectively:

$$a_k = g\left(v_0, v_k^{<\min>}, S_k\right), \quad b_k = g\left(v_0, v_k^{<\max>}, S_k\right). \tag{7}$$

Thus, duration of water treatment can be reliably presented only as interval estimates

$$h_k \in [a_k; b_k] \quad \text{for} \quad k = \overline{1, P}. \tag{8}$$

Here, let us remember that overestimating the treatment loads may cause undesirable effects, and underestimating the loads will not draw the water pollution back to rate c_0. In both cases, an impact of the water treatment weakens. For controlling the wastewater treatment in order to ensure its best impact, treatment loads $\{h_k\}_{k=1}^{P}$ should be handled as interval estimates (8) without supplementary statistical data. This is possible by using the minimax principle, which guarantees the best result under worst conditions (Romanuke, 2011).

2.2 Load for One Aggregate (Plant) of Water Treatment

Suppose that water of a river is served with just a single plant (or an aggregate of water treatment). Note that, even in this case, there may be a few sources of the treatment. Instead of (8), there is an interval

$$h_1 \in [a_1; b_1] \tag{9}$$

for that plant. It is obvious that $a_1 < h_0$ here, but both cases of equalities $b_1 \leqslant h_0$ and $h_0 < b_1$ are possible. It does not violate constraint (6) because here this constraint is

$$h_1 \leqslant h_0, \tag{10}$$

and constraint (10) holds on average.

For case $b_1 \leqslant h_0$, there is just interval uncertainty (9). First of all, the load is standardized by dividing by h_0. Denote the standardized load formally assigned to the plant by u. The standardized load that is factually required is denoted by w. For removing uncertainty between the assigned load and the fairly effective load, an antagonistic game model is applied with a resulting (payoff) function

$$\theta(w, u) = \max\left\{w/u, (1-w)/(1-u)\right\}. \tag{11}$$

By the definition, maximum of function (11) should be minimized across

$$u \in [a_1/h_0; b_1/h_0] \subset (0; 1] \tag{12}$$

by

$$w \in [a_1/h_0; b_1/h_0] \subset (0; 1].$$

The solution of the game model with payoff function (11) is an optimal pure strategy (Romanuke, 2011)

$$u^* = b_1/(h_0 + b_1 - a_1), \tag{13}$$

which shows a part of total load h_0 that should be executed by the plant (here and further, the strategies are dimensionless quantities). The rest part, which is

$$1 - u^* = (h_0 - a_1)/(h_0 + b_1 - a_1),$$

should be executed additionally as a treatment load

$$h_0 \cdot (h_0 - a_1)/(h_0 + b_1 - a_1)$$

does not concern the plant (there is another origin of water contamination). Note that strategy (13) always satisfies condition (12), without the left and right endpoints (neither the minimal registered pollution rate is included, nor the maximal one is):

$$u^* \in (a_1/h_0; b_1/h_0).$$

This means that, in the case $b_1 = h_0$, it is $u^* < 1$, i. e. the plant will not be entirely responsible for the pollution.

For the case

$$a_1 < h_0 < b_1 \tag{14}$$

the load is standardized by dividing by b_1. Then maximum of function (11) is minimized across

$$u \in [a_1/b_1; 1] \subset (0; 1]$$

by

$$w \in [a_1/b_1; 1] \subset (0; 1].$$

The optimal strategy in this case is deduced based on (Romanuke, 2017):

$$u^* = b_1/(2b_1 - a_1). \tag{15}$$

It is obvious that strategy (15) satisfies the following condition:

$$u^* \in (1/2; 1). \tag{16}$$

Condition (16) implies that, in the case of (14), the plant will be always responsible for no less than 50 % of the pollution. Generally, strategy (15) shows a part of load b_1 (according with minimal registered pollution rate) that should be executed by the plant. In fact, this load is $u^* b_1$. But what happens if $u^* b_1 > h_0$ is obtained? Strictly speaking, if

$$b_1/(2b_1 - a_1) > h_0/b_1$$

then the plant should treat the pollution wholly because it is entirely responsible for the pollutants in water of the given river. Otherwise, if

$$b_1/(2b_1 - a_1) \leqslant h_0/b_1 \tag{17}$$

then the plant should treat only for

$$u^* b_1 = b_1^2/2b_1 - a_1 \tag{18}$$

hours. With condition (17), however, it may happen occasionally that $u^{*}b_1 = h_0$, and then the plant will be entirely responsible again.

2.3 Regular Load Strategies for Multiple Aggregates (Plants)

When water of a river is served with two or more plants (or aggregates of water treatment), similar standardizations are done depending on whether

$$\sum_{k=1}^{P} b_k \leqslant h_0 \tag{19}$$

or

$$\sum_{k=1}^{P} a_k < h_0 < \sum_{k=1}^{P} b_k . \tag{20}$$

Like it was previously, inequality

$$\sum_{k=1}^{P} b_k < h_0$$

as a partial case of inequality (19) means that water in the given river is polluted additionally, apart from the pollution related to those P plants.

Denote the standardized load formally assigned to the k-th plant by u_k. The standardized load that is factually required for this plant is denoted by w_k. For case (19) the load is standardized by dividing by h_0. The uncertainties, generated by interval estimates (8), between the assigned loads and the fairly effective loads are removed via minimaxing a payoff function

$$\theta_P\left(\left\{w_k\right\}_{k=1}^{P}, \left\{u_k\right\}_{k=1}^{P}\right) =$$

$$= \max\left\{\left\{w_j / u_j\right\}_{j=1}^{P}, \left(1-\sum_{i=1}^{P} w_i\right) \Big/ \left(1-\sum_{i=1}^{P} u_i\right)\right\} \tag{21}$$

across every

$$u_k \in \left[a_k / h_0 ; b_k / h_0\right] \subset (0; 1)$$

by every

$$w_k \in \left[a_k / h_0 ; b_k / h_0\right] \subset (0; 1) .$$

Note that here

$$b_k / h_0 < 1 \quad \forall k = \overline{1, P}$$

and that is why function (21) is defined on an open unit $2P$-dimensional hyperparallelepiped (Romanuke, 2011; Romanuke, 2017).

The solution of the game model with payoff function (21) are optimal pure strategies for those P plants (Romanuke, 2011). For the k-th plant, the optimal strategy is

$$u_k^{*} = b_k \Big/ \left(h_0 + \sum_{i=1}^{P} b_i - \sum_{i=1}^{P} a_i\right), \quad k = \overline{1, P}, \tag{22}$$

although strategies $\left\{u_k^{*}\right\}_{k=1}^{P}$ by (22) are valid if only (Romanuke, 2011)

$$\left(b_k - a_k\right) \Big/ \left(\sum_{i=1}^{P} b_i - \sum_{i=1}^{P} a_i\right) \geqslant \frac{a_k}{h_0} \quad \forall k = \overline{1, P} . \tag{23}$$

So if inequalities (23) all are true, then strategy (22) shows a part $u_k^{*} h_0$ of total load h_0 that should be executed by the k-th plant. The part remained beyond the parts by strategies $\left\{u_k^{*}\right\}_{k=1}^{P}$, which is

$$1-\sum_{k=1}^{P} u_k^{*} = 1-\sum_{k=1}^{P} b_k \Big/ \left(h_0 + \sum_{i=1}^{P} b_i - \sum_{i=1}^{P} a_i\right),$$

should be executed additionally as a treatment load

$$h_0 - h_0 \sum_{k=1}^{P} b_k \Big/ \left(h_0 + \sum_{i=1}^{P} b_i - \sum_{i=1}^{P} a_i\right)$$

does not concern those P plants (another origin or origins of water contamination).

Nonetheless, some of inequalities (23) can be false. Strictly speaking, if a subset of inequalities (23) turns to be false, then the optimal strategies $\left\{u_k^{*}\right\}_{k=1}^{P}$ are different. According to this, strategies (22), for which all inequalities (23) are true, are called regular, otherwise they are called them irregular (Romanuke, 2011).

2.4 Irregular Load Strategies for Multiple Aggregates (Plants)

Continuing with case (19), let $J \subset \left\{\overline{1, P}\right\}$ be a subset of indices such that inequalities

$$\left(b_j - a_j\right) \Big/ \left(\sum_{i=1}^{P} b_i - \sum_{i=1}^{P} a_i\right) < \frac{a_j}{h_0}$$

$$\forall j \in J \subset \left\{\overline{1, P}\right\} \tag{24}$$

hold along with inequalities (Romanuke, 2011)

$$\left(b_q - a_q\right) \Big/ \left(\sum_{i=1}^{P} b_i - \sum_{i=1}^{P} a_i\right) \geqslant \frac{a_q}{h_0}$$

$$\forall q \in \left\{\overline{1, P}\right\} \setminus J . \tag{25}$$

Then the irregular optimal strategies are found by the following formulae:

$$u_j^{*} = a_j / h_0 \quad \forall j \in J \subset \left\{\overline{1, P}\right\}, \tag{26}$$

$$u_q^{*} = \frac{b_q\left(1-\sum_{j \in J} a_j / h_0\right)}{h_0 + \sum_{k=1}^{P} b_k - \sum_{j \in J} b_j - \sum_{k=1}^{P} a_k}$$

$$\forall q \in \left\{\overline{1, P}\right\} \setminus J . \quad (27)$$

Strategy (26) implies that the j-th plant should execute its minimal load a_j. However, strategies $\left\{u_q^*\right\}_{q\in\left\{\overline{1, P}\right\}\setminus J}$ by (27) are valid if only (Romanuke, 2011)

$$\frac{b_q\left(1-\sum_{j\in J} a_j / h_0\right)}{h_0+\sum_{k=1}^{P} b_k-\sum_{j\in J} b_j-\sum_{k=1}^{P} a_k} \geqslant \frac{a_q}{h_0}$$

$$\forall q \in \left\{\overline{1, P}\right\} \setminus J . \quad (28)$$

So if inequalities (28) all are true, then strategy (27) shows a part $u_q^* h_0$ of total load h_0 that should be executed by the q-th plant. The part remained beyond the parts by strategies $\left\{u_q^*\right\}_{q\in\left\{\overline{1, P}\right\}\setminus J}$ by (27) and $\left\{u_j^*\right\}_{j\in J\subset\left\{\overline{1, P}\right\}}$ by (26), which is

$$1-\sum_{q\in\left\{\overline{1, P}\right\}\setminus J} u_q^*-\sum_{j\in J} a_j / h_0 ,$$

should be executed additionally.

2.5 Total Load in the Middle

For case (20) the load is standardized by dividing by $\sum_{k=1}^{P} b_k$. By doing so, the right endpoints of the standardized intervals are increased by the factor

$$\sum_{k=1}^{P} b_k \Big/ h_0 . \quad (29)$$

Subsequently, if inequalities

$$\left(b_k-a_k\right) \Big/ \left(\sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right) \geqslant a_k \Big/ \sum_{i=1}^{P} b_i \quad (30)$$

hold $\forall k=\overline{1, P}$, then the optimal strategy for the k-th plant is

$$u_k^*=b_k \Big/ \left(2 \sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right), \quad k=\overline{1, P} . \quad (31)$$

However, the right endpoints should be divided by factor (29). Thus, strategy (31) is valid if inequality (Romanuke, 2017)

$$b_k \Big/ \left(2 \sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right) \leqslant \frac{b_k}{\sum_{i=1}^{P} b_i} \cdot \frac{h_0}{\sum_{i=1}^{P} b_i} ,$$

re-written simpler as

$$\left(\sum_{i=1}^{P} b_i\right)^2 \leqslant h_0 \cdot\left(2 \sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right), \quad (32)$$

holds. Note that inequality (32) does not contain an index of the plant. So, if (32) holds and inequalities (30) are true $\forall k=\overline{1, P}$, then strategies $\left\{u_k^*\right\}_{k=1}^{P}$ are determined by (31), and the k-th plant is assigned its water treatment load

$$b_k \sum_{i=1}^{P} b_i \Big/ \left(2 \sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right), \quad k=\overline{1, P} . \quad (33)$$

Otherwise, if (32) does not hold, the optimal strategy for the k-th plant is

$$u_k^*=b_k h_0 \Big/ \left(\sum_{i=1}^{P} b_i\right)^2, \quad k=\overline{1, P}, \quad (34)$$

if only

$$b_k h_0 \Big/ \sum_{i=1}^{P} b_i \geqslant a_k \quad \forall k=\overline{1, P}, \quad (35)$$

which is deduced from

$$b_k h_0 \Big/ \left(\sum_{i=1}^{P} b_i\right)^2 \geqslant a_k \Big/ \sum_{i=1}^{P} b_i \quad \forall k=\overline{1, P} .$$

Strategy (34) implies that the k-th plant is assigned its water treatment load

$$b_k h_0 \Big/ \sum_{i=1}^{P} b_i, \quad k=\overline{1, P}, \quad (36)$$

i. e. the k-th plant should execute a part equal to $b_k \Big/ \sum_{i=1}^{P} b_i$ of the total load. Thus, strategies (34) could be called proportional strategies.

Continuing with case (20), let $J \subset\left\{\overline{1, P}\right\}$ be a subset of indices such that inequalities

$$\left(b_j-a_j\right) \Big/ \left(\sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right)<a_j \Big/ \sum_{i=1}^{P} b_i$$

$$\forall j \in J \subset\left\{\overline{1, P}\right\} \quad (37)$$

hold along with inequalities (Romanuke, 2011)

$$\left(b_q-a_q\right) \Big/ \left(\sum_{i=1}^{P} b_i-\sum_{i=1}^{P} a_i\right) \geqslant a_q \Big/ \sum_{i=1}^{P} b_i \quad \forall q \in\left\{\overline{1, P}\right\} \setminus J . \quad (38)$$

Then the irregular optimal strategies are found by the following formulae:

$$u_j^* = a_j \Big/ \sum_{i=1}^{P} b_i \quad \forall j \in J \subset \left\{\overline{1, P}\right\}, \tag{39}$$

$$u_q^* = \frac{b_q \left(1 - \sum_{j \in J} a_j \Big/ \sum_{i=1}^{P} b_i \right)}{2\sum_{k=1}^{P} b_k - \sum_{j \in J} b_j - \sum_{k=1}^{P} a_k} \quad \forall q \in \left\{\overline{1, P}\right\} \setminus J . \tag{40}$$

Strategy (39) implies that the j-th plant should execute its minimal load a_j. However, strategies $\left\{u_q^*\right\}_{q \in \left\{\overline{1, P}\right\} \setminus J}$ by (40) are valid if only (Romanuke, 2011)

$$\frac{b_q \left(1 - \sum_{j \in J} a_j \Big/ \sum_{i=1}^{P} b_i \right)}{2\sum_{k=1}^{P} b_k - \sum_{j \in J} b_j - \sum_{k=1}^{P} a_k} \geqslant a_q \Big/ \sum_{i=1}^{P} b_i$$

$$\forall q \in \left\{\overline{1, P}\right\} \setminus J \tag{41}$$

and inequality (32) holds. Then strategy (40) shows a part $u_q^* \sum_{i=1}^{P} b_i$ of the summed maximal load estimations $\sum_{i=1}^{P} b_i$ that should be executed by the q-th plant. The part remained beyond the parts by strategies $\left\{u_q^*\right\}_{q \in \left\{\overline{1, P}\right\} \setminus J}$ by (40) and $\left\{u_j^*\right\}_{j \in J \subset \left\{\overline{1, P}\right\}}$ by (39), which is

$$1 - \sum_{q \in \left\{\overline{1, P}\right\} \setminus J} u_q^* - \sum_{j \in J} a_j \Big/ \sum_{i=1}^{P} b_i ,$$

should be executed additionally.

3. ALGORITHM OF DISTRIBUTING WATER VOLUMES TO BE TREATED

In fact, whichever strategies $\left\{u_k^*\right\}_{k=1}^{P}$ are, only load hours (or water volumes in cubic meters) matter. Owing to the model stated above, there are eight possibilities (cases), in which treatment loads are determined. There also are three possibilities where determining the treatment hours or volumes is unfeasible. Two of them are issued from cases when at least either an inequality of (28) or an inequality of (41) does not hold. Such cases hint that the measurements of concentrations (4) were conducted inaccurately or improperly (or both). The third possibility of unfeasible load determination is when at least an inequality of (35) does not hold. This is an evidence of a plausible overestimation of either the maximal or minimal load (Gitis and Hankins, 2018).

An algorithm of distributing water treatment loads based on the model is completely presented in Figure 1. The branch for a one plant is much shorter than that for a few plants. The one plant branch has three versions of the plant's load determination and none of unfeasible load determination.

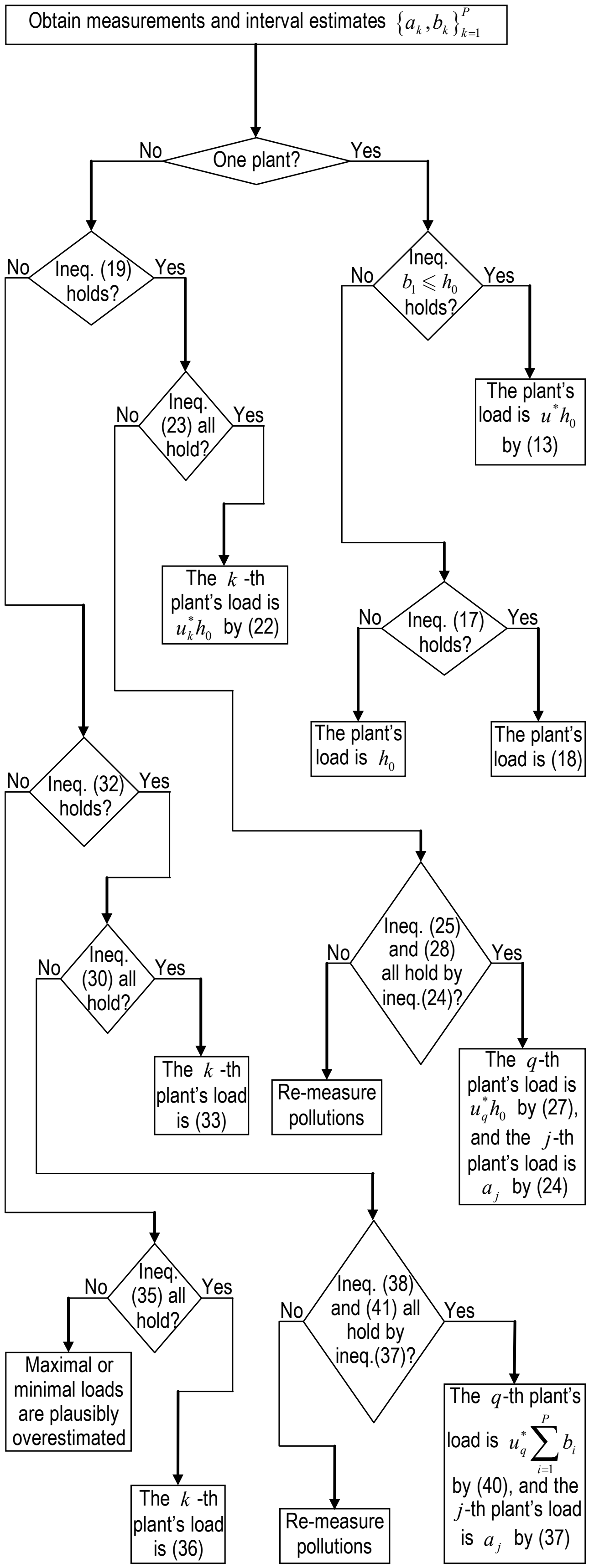


Fig. 1. Algorithm of distributing treatment loads.

4. APPLICATION AND ISSUES

This section is inserted for better understanding how to apply the algorithm. The application consists of four main steps:

1. Measuring concentrations of contaminants and associate them with plants.

2. Registering minimal and maximal concentrations for each plant, i. e. determining concentrations' range (4).

3. Transferring the range into treatment hours (volumes) by using mappings (1) and (3) with corrections due to influence of geographic peculiarities of the plant site.

4. Running the algorithm in accordance with Figure 1.

A generalized example for one of three plants is shown in Figure 2. Interval estimates for this plant are such that $b_k / a_k = 2$, and relative widths of two other estimates are 1.5 and 1.7333. It is seen that the pending treatment work increases quasi-linearly with a little curvilinearity. At greater values of maximal load estimations, strategies by (31) are more likely, where the total load does not influence. Additionally to that, overestimations of either maximal or minimal loads may occur when the total load decreases.

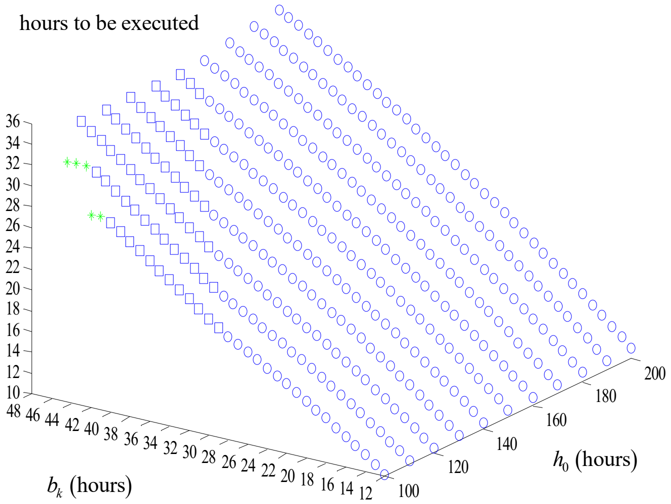
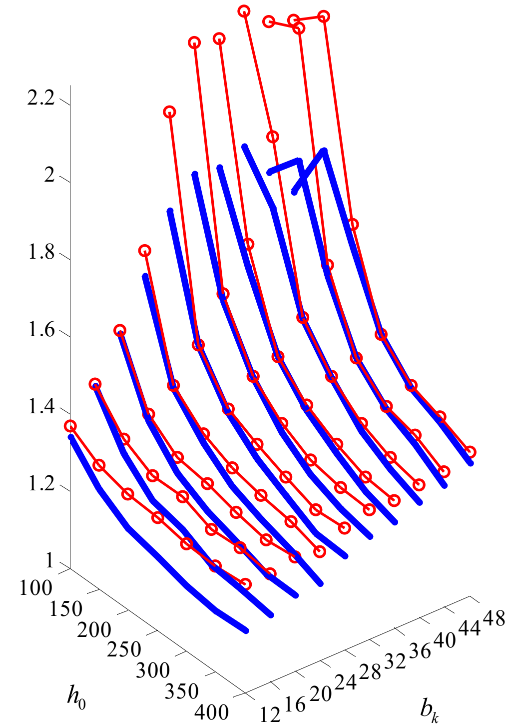


Fig. 2. The plant's load as a function of total load and maximal load estimation: circled dots stand for regular strategies by (22), squared dots stand for regular strategies by (31), and starred dots correspond to proportional strategies (34).

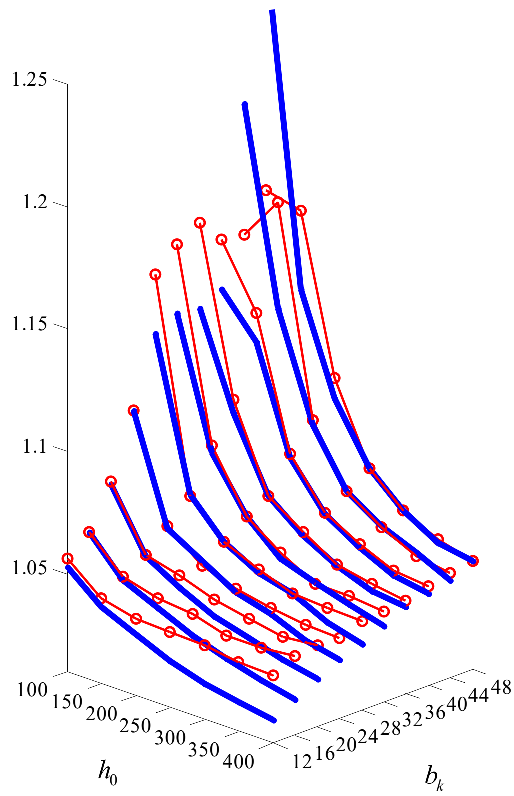
An issue is, in the case of overestimation, it is hard to say how bad either maximal or minimal loads go beyond. Another issue is independence of strategies by (31) from the total load. However, effects of applying the algorithm are positive if to compare the optimal strategies to 0.5-strategies (0.5's) when factually required loads are pretty close to minimal or maximal load estimations (Figure 3a). The effects are weaker when factual required loads are random (Figure 3b).

For the first real-world example, let us consider how a single plant can be loaded. Suppose that function g in (3) is quasilinear:

$$h_1 = 2.4 \cdot 10^{-4} \cdot \left(v_1 - v_0\right)^{1.05}, \tag{42}$$



a)



b)

Fig. 3. Effects of applying the algorithm with values (21) best to be close to 1: bold lines stand for the optimal strategies, circled dots stand for 0.5's

where coefficient $2.4 \cdot 10^{-4}$ relates to a speed of treating the polluted water. If the volumes are given in cubic meters, this coefficient would be hours per m^3 by a condition that the power was set at 1. However, as the volume of water to be treated increases, the capacity of the treatment plant may decrease. For instance, according to formula (2), 10^5 m^3 are treated in 42.68 hours, whereas $2 \cdot 10^5$ m^3 will be treated in 88.37 hours.

Suppose that we have to treat between $1.2 \cdot 10^5$ m^3 and $1.4 \cdot 10^5$ m^3 of the polluted water by $h_0 = 96$ hours. Then, according to formulae (7), the water treatment should be executed for $a_1 = 51.6835$ (minimal estimation) to $b_1 = 60.7639$ hours (maximal estimation). As here $b_1 < h_0$, then we use optimal strategy (13). According to this strategy, the plant should execute $u^* = 0.5783$ of those 96 hours (total load), i. e.,

$$u^* h_0 = 0.5783 \cdot 96 = 55.5168 \text{ hours}$$

is a treatment job for the plant. If the total load is between minimal and maximal estimations, e. g., $h_0 = 54$, then we have case (14). In this case, the optimal strategy is found by (15): $u^* = 0.87$, and the plant should execute

$$u^* b_1 = 0.87 \cdot 60.7639 = 52.8646 \text{ hours,}$$

which constitute 97.9 % of the total load. Here, due to bad uncertainty in the estimation of water treatment, the minimax principle allows to reveal another possible source (or sources) of pollution, whose part is that 2.1 %. Besides, it is obvious that if the total load drops lower than those 52.8646 hours, then the plant will be entirely responsible for the pollution.

Cases with two plants are not much that difficult. For a slight simplification, nominal capacities of the plants will be considered roughly equal. Suppose, one plant has to treat between $1.2 \cdot 10^5$ m^3 and $1.4 \cdot 10^5$ m^3, and the other one – between $1.3 \cdot 10^5$ m^3 and $1.5 \cdot 10^5$ m^3 of the polluted water by $h_0 = 192$ hours. Although the absolute uncertainty in the estimation of water treatment is the same, the pollution is worse in the neighborhood of the second plant. Here, according to formulae (7), minimal and maximal estimations remain the same for the first plant, and

$$a_2 = 56.2149, \quad b_2 = 65.3292 .$$

So, condition $b_1 + b_2 < h_0$ holds and, subsequently,

$$u_1^* = 0.2891, \quad u_2^* = 0.3108 .$$

These strategies are valid owing to both inequalities in (23) hold. According to these strategies, the plants' will be obliged to execute

$$u_1^* h_0 = 0.2891 \cdot 192 = 55.5072$$

and

$$u_2^* h_0 = 0.3108 \cdot 192 = 59.6736 \text{ hours}$$

of water treatment, respectively. Note that despite the second plant's maximal estimation is 7.14 % greater than the maximal estimation for the first plant, the second plant's job is 7.51 % longer than the job for the first plant. This is so due to that power of 1.05 in function (42): the second plant should execute that 0.37 % more by the reason of the worse pollution.

When an interval estimate for the first plant comes too narrow, e. g., with just $1.32 \cdot 10^5$ m^3 to $1.4 \cdot 10^5$ m^3 of the polluted water to be treated, the algorithm catches the irregularity of the corresponding decision for this plant: inequality (24) becomes true for $j = 1$, whereas inequality (25) holds for $q = 2$. Eventually, the first plant is obliged to execute its minimal estimation job, i. e. $a_1 = 57.1234$ hours. At the same time, despite the interval estimate for the second plant is the same as in the previous example, its job becomes longer:

$$u_2^* h_0 = 0.3187 \cdot 192 = 61.1904 \text{ hours.}$$

There is an interesting case with three plants, where the second one has to treat between $1.27 \cdot 10^5$ m^3 and $1.5 \cdot 10^5$ m^3, whereas the first and third ones have to treat between $1.32 \cdot 10^5$ m^3 and $1.4 \cdot 10^5$ m^3. Here, we obtain two irregular strategies – obviously, for those identical interval estimates. So,

$$u_1^* = u_3^* = 0.2975, \quad u_2^* = 0.2999,$$

and the treatment jobs for the plants with the identical interval estimates are equal to the minimal estimation job, i. e. $a_1 = a_3 = 57.1234$ hours again. Meanwhile, the second plant is obliged to execute

$$u_2^* h_0 = 0.2999 \cdot 192 = 57.5808 \text{ hours}$$

of the treatment job. However, the fact of that we obtain two irregular strategies is a manifestation of overestimation of minimal loads ($1.32 \cdot 10^5$ m^3). If they are decreased to $1.25 \cdot 10^5$ m^3, then all the three inequalities in (23) hold, and the treatment jobs for the plants appear to be just between their minimal and maximal estimations.

The case with the total load in the middle is solved much the same. The plant's load depends on the maximal estimation due to (33) if only inequalities (30) and inequality (32) hold. If inequality (32) does not hold, we may reveal overestimations. For instance, having the same estimations for the second plant, and $1.22 \cdot 10^5$ m^3 and $1.4 \cdot 10^5$ m^3 for the first and third plants, by $h_0 = 160$ we get:

$$a_1 = a_3 = 52.5883, \quad a_2 = 54.8536,$$

$$b_1 = b_3 = 60.7639, \quad b_2 = 65.3292,$$

and loads (36) would come into force by proportional strategies (34). Then, however, the jobs for the first and second plants would be 52.0303 hours, which is less than the minimal estimation (52.5883 hours). Therefore, there is an overestimation either in minimal or maximal loads. In this case, the pollution measurements are recommended to be repeated.

5. DISCUSSION AND CONCLUSIONS

Obviously, the minimax principle is applicable only when statistical data are just partially available. This is a case of long rivers, at which measuring pollution is a very hard task, especially if the riverbed is large and the bottom is deep (Abraham, 2017). Therefore, the model and algorithm in Figure 1 is not applicable for shallow waters, unless their pollution is scattered (non-uniform). It is a plain demerit of this too pessimistic model not working with narrow intervals. However, a merit is its robustness. Besides, it gives a hint at plausible inaccurate measurements or overestimations when two or more irregular strategies are obtained. A special attention to the place where a single irregular strategy is obtained should be paid also.

So, a novel approach has been proposed to industrial wastewater treatment control with using a minimax principle for cases when measurements of water pollution are scattered and dependent on locations. This approach fits long, deep, and wide rivers. Pollution at a shallow river can be analyzed as well only if its contaminations are registered in a wide range, i. e. $c_k^{<\max>} \big/ c_k^{<\min>} > 1.5$ or about that for most indices. The proposed approach allows as calculating water treatment jobs under weakly measured pollution, as well as spotting non-listed sources of pollution, controlling thus the pollution origination. Moreover, it allows to estimate reliability of the interval estimates themselves, wherein the pollution measurements should be repeated more accurately or intensively. A promising advance of the approach is connected with including Bayesian decisions.

ACKNOWLEDGEMENTS

This work is supported by the Faculties of Navigation and Naval Weapons and Command and Naval Operations at the Polish Naval Academy, Gdynia, Poland.

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