Controllers Design for Stabilization of Non-Minimum Phase of Switched Nonlinear Systems

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Abstract: This article deals with the global stabilization of a class of switched nonlinear systems where each mode represents non-minimum phase. We design, in this research work, nonlinear feedback controllers and switching law for this class of systems by considering both common and multiple Lyapunov functions. Sufficient conditions under which the globally asymptotically stabilization problem is solvable are also given. The global stability of the resulting switched system can be guaranteed by using the designed approach. In order to solve the global robust stabilization problem, the proposed method is extended to the uncertain switched nonlinear systems where each mode represents non-minimum phase. Two examples are given to show the effectiveness of the developed techniques.

Keywords: Switched nonlinear system, Non-minimum phase, Stabilization, Lyapunov method.

1. INTRODUCTION

A switched system is an important class of hybrid systems that comprises a finite number of continuous or discrete sub-systems by applying a switching law rule between these sub-systems (Decarlo et al., 2000; Jouili el al., 2016; Long et al., 2017; Xiang et al., 2008; Liu et al., 2017; Yao et al., 2017; Zheng et al., 2017; Liu et al., 2016). In recent years, switched systems, such as networked control systems (Zhao et al., 2009), near space vehicle control systems (Wang et al., 2013), circuit and power systems (Hamee et al., 2014), have increasingly attracted the attention of the scientific community since they can be used to describe a large number of physical and engineering applications. In fact, stability analysis and control synthesis, the most important issues dealt with when studying the switched nonlinear systems, are discussed extensively by many researchers, and excellent results were obtained for various types of switched systems (Liu et al., 2017; Long et al., 2017; Liang et al., 2013; Long et al., 2013; Sakly et al. 2015).

Despite these promising good results, few attempts were made to stabilize non-minimum phase nonlinear systems where each mode can be a non-minimum phase. In general, stabilization of non-minimum phase nonlinear systems is a challenging problem in the field of control since a nonlinear control system is non-minimum phase if it's internal or zero dynamics is unstable (Isidori, 1995). Indeed, zero dynamics play an important role in the area of control analysis and synthesis of nonlinear systems. Some contributions investigated non-minimum phase switched nonlinear systems where each mode represents non-minimum phase. For example, in (Wang et al., 2008), control purpose was realized for a class of non-minimum phase cascade switched nonlinear systems where the internal dynamics of each mode was assumed to be asymptotically stabilizable. A control approach for the stabilization of a class of non-minimum phase switched nonlinear systems based on the concept of multi-diffeomorphismwas contemplated in (Jouili et al., 2015). Output tracking of non-minimum phase switched nonlinear systems was considered in (Oishi et al., 2000) where an approximated minimum phase model was utilized. In (Yang et al., 2012), the stabilization of non-minimum phase switched nonlinear systems applied to multi-agent systems was proposed. In these systems, the states of linearized dynamics of all modes, which compose the whole state space and state dependent stabilization switching laws, were provided by considering both common and multiple Lyapunov functions. The same problem was also investigated, in (Benosman et al., 2007), using an inversion-based control strategy.

The main idea of the above-stated results is to design a controller for each mode in order to recompense for its own unstable internal dynamics such that all modes become independently stable. Then, we used the common and multiple Lyapunov functions methods to achieve the stability of the whole switched system.

In this paper, we discuss the stabilization problem of non-minimum phase switched nonlinear systems with Lyapunov function method. By extending the result presented in (Jouili et al., 2015), the studied switching system consists of two parts: The first part represents the linearized dynamics (input-output behavior), while the second part shows the unstable internal dynamics (zero dynamics). Lyapunov functions for the whole switched system for each mode are constructed by using the multiple Lyapunov functions of the input-output behavior part with single control input and common Lyapunov function of the internal dynamics part.

By assuming that the unstable internal dynamics part is uniformly global and quadratically stable, sufficient conditions, under which the global asymptotical stabilization problem is solved, are provided. In fact, the global stability of the resulting switched system can be guaranteed by using the proposed approach.

A nonlinear feedback controllers and switching law stabilization are explicitly designed by considering both common and multiple Lyapunov functions.

The introduced approach can be also extended to uncertain a switched nonlinear system where each mode may be non-minimum phase, which was not studied in the literature so far. The main contributions of this article include: (i) introducing an effective design method for the construction of nonlinear feedback controllers and switching law based on multiple Lyapunov functions and (ii) providing robust stabilization results for a more general class of switched nonlinear systems with uncertainty where each mode may be non-minimum phase.

The remainder of the manuscript is organized as follows: In section 2, we describe the considered class of switched systems. In section 3, we address the problem formulation to provide the necessary background for applying the nonlinear feedback controllers of switched nonlinear systems addressed in section 4. The extension of the proposed approach to the switched nonlinear systems with uncertainty is given in section 5. In section 6, two numerical examples are presented to illustrate the effectiveness of the proposed approach. In the final part of this paper, we present a brief conclusion and future works.

2. SYSTEM DESCRIPTION

We consider a class of switched nonlinear systems of the following form:

$$\dot{x} = f_i(x) + g_i(x) u_i \tag{1}$$

with i is a set of indices specifying the active subsystem.

where $x \in \Re^n$ are available states. Define $M = \{1, 2, \ldots, m\}$, where m is the number of modes. $\forall i \in M$, $u_i \in \Re$ is the input. $f_i(.)$ and $g_i(.)$ are smooth functions such that $f_i(0) = g_i(0) = 0$.

Lemma 1 (Decarlo et al., 2000):

Considering system (1), if there exists continuous differentiable positive function V_i , $i \in M$, $\dot{V}_i \langle 0$ and $V_i(x(\tau_{i,k})) \leq V_i(x(\tau_{i,k-1}))$, where $\tau_{i,k}$ represents the time when the i^{th} sub-system is activated at the k^{th} time. In this case, the switched system is stable.

Based on the approach presented in (Jouili et al., 2015), it is assumed that every mode $i \in M$ can be rewritten as the normal form. We can find a function y_i and a partition $x = [\xi \quad \eta]^T$, where $\xi \in \Re^{r_i}$, $\eta \in \Re^{(n-r_i)}$, to rewrite the system (1) into the normal form [15] specified by the following equation system:

$$\begin{cases} \dot{\xi} = A_i\, \xi + b_i(\xi, \eta) + a_i(\xi, \eta) u_i \\ \dot{\eta} = Q_i(\xi, \eta) \end{cases} \tag{2}$$

where

$$A_i \in \Re^{(r_i \times r_i)} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix},$$

$b_i(\xi, \eta) \in \Re^{(r_i \times 1)} = \begin{bmatrix} 0 & \cdots & 0 & \bar{b}_i(\xi, \eta) \end{bmatrix}^T$ and $a_i(\xi, \eta) \in \Re^{(r_i \times 1)} = \begin{bmatrix} 0 & \cdots & 0 & \bar{a}_i(\xi, \eta) \end{bmatrix}^T$, with $\bar{b}_i(.)$ and $\bar{a}_i(.)$ are scalar functions, $\bar{a}_i(.) \neq 0$.

Mode i is a non-minimum phase if its zero dynamics $\dot{\eta} = Q_i(0, \eta)$ is unstable. Otherwise, it is a minimum phase. In this paper, the problem to be resolved consists in stabilizing the non-minimum phase switched nonlinear systems where the internal dynamics of each mode are unstable and uncontrollable.

3. PROBLEM FORMULATION AND BASIC ASSUMPTIONS

In order to design a switching law and nonlinear feedback controllers to stabilize switched nonlinear control system (1), we first rewrite the switched nonlinear system (2) in the form:

$$\begin{cases} \dot{\xi} = \bar{f}_i(\xi, \eta) + \underbrace{\bar{g}_i(\xi, 0) + \tilde{g}_i(\xi, \eta)\eta}_{\bar{g}_i(\xi, \eta)} u_i \\ \dot{\eta} = Q_i(\xi, \eta) \end{cases} \tag{3}$$

where $\bar{g}_i(\xi, \eta) = a_i(\xi, \eta)$ and $\bar{f}_i(\xi, \eta) = A_i\, \xi + b_i(\xi, \eta)$

For each mode i of system (3), it is assumed that there exists a continuous non-negative function of the following form:

$$\mathrm{V}_i(\xi, \eta) = \bar{\mathrm{V}}_i(\xi) + k_i\, \tilde{\mathrm{V}}(\eta) \tag{4}$$

where $\bar{\mathrm{V}}_i(\xi)$ are the Lyapunov functions for the each sub-system i of the first part with single control input of switched system (3), $\tilde{\mathrm{V}}(\eta)$ is the common Lyapunov function of the second part of the switched system (3) without control input, and k_i, $i = 1, \ldots, m$ are positive constants.

Then, we need to impose the following assumptions on the slow sub-systems (input-output behavior) of equation (3):

Assumption 1

For any compact set $\bar{\Omega}$, there exists positive definite, radially unbounded C^1 function $\bar{\mathrm{V}}_i(\xi)$ for each mode $i = 1, \ldots, m$ under which there are a continuous non-negative C^0 function $\vartheta_{il}(\xi) \geq 0$ $l = 1, \ldots, m$ and some positive constants ρ_i, λ_i and θ_i, such that:

$$\begin{cases} \dfrac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{f}_i(\xi,0) + \dfrac{1}{(2\lambda_i)^2}\left[\dfrac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{g}_i(\xi,0)\right]^2 \\ +\rho_i \|\xi\|^2 + \left(\displaystyle\sum_{l=1}^{m} \vartheta_{il}(\xi)\left(\bar{V}_l(\xi) - \bar{V}_i(\xi)\right)\right) \le 0 \\ \left|\dfrac{\partial \bar{V}_i(\xi)}{\partial \xi}\right| \le \theta_i \|\xi\|^2 \qquad \forall i \in M \end{cases} \tag{5}$$

Assumption 2

For every $\alpha_i > 0$, there exist a positive and radially-unbounded function $\tilde{V}(\eta)$ in such a way that:

$$\frac{\partial \tilde{V}(\eta)}{\partial \eta} Q_i(\xi,\eta) \le \alpha_i \left(\|\xi\|^2 + \|\eta\|^2\right) \quad \forall i \in M \tag{6}$$

Assumption 3

For system (2), if there exist $\gamma_i > 0$, the vector fields $\bar{f}_i(\xi,\eta)$ will be globally Lipschitz continuous for each $i \in M$, in such a way that:

$$\left|\bar{f}_i(\xi,\eta) - \bar{f}_i(\xi,0)\right| < \gamma_i \|\eta\| \tag{7}$$

After imposing the above assumptions on the sub-systems of equation (3), we describe, in the next section, the proposed approach.

4. NONLINEAR FEEDBACK CONTROLLERS DESIGN AND SWITCHING LAW

In this section, we present the main obtained results, an integrated design of switching laws and nonlinear feedback controllers used to stabilize non-minimum phase switched nonlinear system where the internal dynamics of each mode are unstable. We also construct Lyapunov functions for each mode *i*. Simultaneous, a nonlinear feedback controller for system (3) and a switching law are formulated explicitly based on multiple Lyapunov functions. The studied non-minimum phase switched nonlinear system (3) consists of two parts:
- The first part represents the linearized dynamics (input-output behavior).
- The second part represents the unstable internal dynamics (zero dynamics).

The switched nonlinear system (3) can be stabilized by using Lyapunov function method. Under the assumption that the unstable internal dynamics is uniformly and globally quadratic stable, sufficient conditions are given, which guarantees the global asymptotical stabilizbility of the non-minimum phase switched nonlinear system (3). A nonlinear switched state feedback and a switching law are constructed based on the structure characteristics of the switched system. The switching law is constructed based on partial state of the switched system.

Let i^{th} be the sub-system of the switched nonlinear system (3). Along the trajectory of the system (3), the time derivative of $V_i(\xi,\eta)$ is as follows:

$$\begin{aligned} \dot{V}_i(\xi,\eta) &= \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{f}_i(\xi,\eta) + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{g}_i(\xi,\eta)u_i \\ &\quad + k_i \frac{\partial \tilde{V}(\eta)}{\partial \eta} Q_i(\xi,\eta) \\ &= \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\left(\bar{f}_i(\xi,\eta) - \bar{f}_i(\xi,0)\right) + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{f}_i(\xi,0) \\ &\quad + k_i \frac{\partial \tilde{V}(\eta)}{\partial \eta} Q_i(\xi,\eta) + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\tilde{g}_i(\xi,\eta)\eta u_i \\ &\quad + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{g}_i(\xi,0)u_i \end{aligned} \tag{8}$$

For every *i* and from assumption 1, we deduce that:

$$\begin{aligned} &\frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{f}_i(\xi,0) + \frac{1}{(2\lambda_i)^2}\left[\frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{g}_i(\xi,0)\right]^2 \\ &\quad + \rho_i \|\xi\|^2 \le 0 \end{aligned} \tag{9}$$

Thus,

$$\begin{aligned} \dot{V}_i(\xi,\eta) \le & \left|\frac{\partial \bar{V}_i(\xi)}{\partial \xi}\right| \left|\left(\bar{f}_i(\xi,\eta) - \bar{f}_i(\xi,0)\right)\right| \\ & + \frac{1}{(2\lambda_i)^2}\left[\frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{g}_i(\xi,0)\right]^2 \\ & + (\lambda_i u_i)^2 + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\tilde{g}_i(\xi,\eta)\eta u_i \\ & + k_i \frac{\partial \tilde{V}(\eta)}{\partial \eta} Q_i(\xi,\eta) + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\left(\bar{f}_i(\xi,0)\right) \end{aligned} \tag{10}$$

Based on assumption2, we obtain the following equation:

$$\begin{aligned} \dot{V}_i(\xi,\eta) \le & + \left|\frac{\partial \bar{V}_i(\xi)}{\partial \xi}\right| \left|\left(\bar{f}_i(\xi,\eta) - \bar{f}_i(\xi,0)\right)\right| \\ & + \frac{1}{(2\lambda_i)^2}\left[\frac{\partial \bar{V}_i(\xi)}{\partial \xi}\bar{g}_i(\xi,0)\right]^2 \\ & + (\lambda_i u_i)^2 + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\tilde{g}_i(\xi,\eta)\eta u_i \\ & + k_i \alpha_i \left(\|\xi\|^2 + \|\eta\|^2\right) + \frac{\partial \bar{V}_i(\xi)}{\partial \xi}\left(\bar{f}_i(\xi,0)\right) \end{aligned} \tag{11}$$

Moreover, from assumption 3 and condition 9, we get the following inequality:

$$\dot{V}_i(\xi,\eta) \le k_i\,\alpha_i\left(\|\xi\|^2+\|\eta\|^2\right)-\rho_i\,\|\xi\|^2$$
$$+\theta_i\,\gamma_i\,\|\xi\|\|\eta\|+(\lambda_i\,u_i)^2+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\left(\tilde{g}_i(\xi,\eta)\eta\right)u_i$$
$$\le k_i\,\alpha_i\left(\|\xi\|^2+\|\eta\|^2\right)-\rho_i\,\|\xi\|^2+\theta_i\,\gamma_i\,\|\xi\|\|\eta\| \tag{12}$$
$$+\left(\lambda_i\,u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2$$
$$-\frac{1}{(2\lambda_i)^2}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)^2$$

We also obtain:

$$\dot{V}_i(\xi,\eta) \le k_i\,\alpha_i\,\|\xi\|^2+k_i\,\alpha_i\,\|\eta\|^2$$
$$-\rho_i\,\|\xi\|^2+\frac{\rho_i}{2}\|\xi\|^2+\frac{\theta_i\,\gamma_i}{2\rho_i}\|\eta\|^2$$
$$+\left(\lambda_i\,u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2$$
$$\le\left(k_i\,\alpha_i-\frac{\rho_i}{2}\right)\|\xi\|^2+\left(\frac{\theta_i\,\gamma_i}{2\rho_i}+k_i\,\alpha_i\right)\|\eta\|^2 \tag{13}$$
$$+\left(\lambda_i\,u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2$$
$$\le min\left(k_i\,\alpha_i-\frac{\rho_i}{2},\ \frac{\theta_i\,\gamma_i}{2\rho_i}+k_i\,\alpha_i\right)\left(\|\xi\|^2+\|\eta\|^2\right)$$
$$+\left(\lambda_i\,u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2$$

For each $i=1,\ldots,m$, if we choose $\frac{(\theta_i\,\gamma_i\,\rho_i)}{2}>k_i\,\alpha_i$, then the nonlinear feedback controller $u_i(\xi,\eta)$ satisfying $\dot{V}_i(\xi,\eta)\le 0$ is as shown below:

$$u_i(\xi,\eta)=-\frac{\bar{g}_i(\xi,\eta)\eta}{2\lambda_i^2}\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi} \tag{14}$$

(13) can be rewritten as:

$$\dot{V}_i(\xi,\eta)\le$$
$$min\left(k_i\,\alpha_i-\frac{\rho_i}{2},\ \frac{\theta_i\,\gamma_i}{2\rho_i}+k_i\,\alpha_i\right)\left(\|\xi\|^2+\|\eta\|^2\right) \tag{15}$$

If the i^{th} sub-system is switched to the j^{th} sub-system at switching time τ_n, $n=1,2,\ldots$, the following inequalities will be satisfied:

$$V_i\left((\xi,\eta)(\tau_n)\right)\ge V_j\left((\xi,\eta)(\tau_n)\right) \tag{16}$$

Then, $V_i\left((\xi,\eta)(\tau_{i,k})\right)\le V_i\left((\xi,\eta)(\tau_{i,k-1})\right)$ can be obtained.

According to lemma 1, when the switched law and nonlinear feedback controllers u_i (14) satisfy equation (16), system (3) will be stable.

If the switching law from the i^{th} sub-system to the j^{th} sub-system is a sub-set of the following set

$$\Omega_k=\left\{\xi,\eta/V_{ik}\ge V_{jk},i,j\in M\ and\ \ i\ne j\right\} \tag{17}$$

then the feedback control $u_i(\xi,\eta)$ (14) can stabilize system (3) under the corresponding switching law.

We obtain the following results

Theorem 1

Considering the switched nonlinear system (3) satisfying assumptions 1-3, for which a family of Lyapunov functions $V_i(\xi,\eta)$ $i=1,\ldots,m$ satisfies (16), then

$$\dot{V}_i(\xi,\eta)\le min\left(k_i\,\alpha_i-\frac{\rho_i}{2},\ \frac{\theta_i\,\gamma_i}{2\rho_i}+k_i\,\alpha_i\right)\left(\|\xi\|^2+\|\eta\|^2\right)$$

under the following family of nonlinear feedback controllers:

$$u_i(\xi,\eta)=-\frac{\bar{g}_i(\xi,\eta)\eta}{2\lambda_i^2}\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}$$

Let Ω_k satisfy (17). If the switched law guarantees that, when the i^{th} subsystem is switched to the j^{th} sub-system $S_{ij}\subseteq\Omega_k$, then the nonlinear switched feedback controller (14) can guarantee the asymptotically-stable system (2).

5. EXTENSION OF THE UNCERTAIN SWITCHED NONLINEAR SYSTEMS

In this section, we consider the switched system (3) with uncertainties described by the following equations:

$$\begin{cases}\dot{\xi}=A_i\,\xi+b_i(\xi,\eta)+a_i(\xi,\eta)u_i+\Delta_{i\,\xi}(\xi,\eta)\\ \dot{\eta}=Q_i(\xi,\eta)+\Delta_{i\,\eta}(\xi,\eta)\end{cases} \tag{18}$$

where $\Delta_i(.)$ are uncertain functions. Furthermore, functions $\Delta_{i\,\xi}(\xi,\eta)$ and $\Delta_{i\,\eta}(\xi,\eta)$ are bounded and satisfy the inequalities below:

$$\left|\Delta_{i\,\xi}(\xi,\eta)\right|\le\bar{\gamma}_i\,\|(\xi,\eta)\|_2 \tag{19}$$

and

$$\left|\Delta_{i\,\eta}(\xi,\eta)\right|\le\bar{\ell}_i\,\|(\xi,\eta)\|_2 \tag{20}$$

with $\bar{\gamma}_i$ and $\bar{\ell}_i$ are positive real values and the other symbols are the same as those provided by system (2).Subsequently, system (18) can be changed into the following switched form:

$$\begin{cases} \dot{\xi}=\bar{f}_i(\xi,\eta)+\underbrace{\bar{g}_i(\xi,0)+\tilde{g}_i(\xi,\eta)}_{\bar{g}_i(\xi,\eta)}\eta u_i+\Delta_{i\xi}(\xi,\eta) \\ \dot{\eta}=Q_i(\xi,\eta)+\Delta_{i\eta}(\xi,\eta) \end{cases} \tag{21}$$

Our objective is to extend the result obtained in section 4 to solve the global stabilization problem for switched nonlinear system (21). We consider explicitly the Lyapunov function $\mathrm{V}_i(\xi,\eta)=\bar{\mathrm{V}}_i(\xi)+k_i\tilde{\mathrm{V}}(\eta)$ and its time derivative along the trajectories of (18).

$$\begin{aligned} \dot{V}_i(\xi,\eta)&=\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\left(\bar{f}_i(\xi,\eta)+\bar{g}_i(\xi,\eta)u_i+\Delta_{i\xi}(\xi,\eta)\right) \\ &+k_i\frac{\partial\tilde{\mathrm{V}}(\eta)}{\partial\eta}\left(Q_i(\xi,\eta)+\Delta_{i\eta}(\xi,\eta)\right) \\ &=\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\left(\bar{f}_i(\xi,\eta)-\bar{f}_i(\xi,0)\right)+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\bar{f}_i(\xi,0) \\ &+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\Delta_{i\xi}(\xi,\eta)+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\bar{g}_i(\xi,0)u_i \\ &+k_i\frac{\partial\tilde{\mathrm{V}}(\eta)}{\partial\eta}Q_i(\xi,\eta)+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta u_i \\ &+k_i\frac{\partial\tilde{\mathrm{V}}(\eta)}{\partial\eta}\Delta_{i\eta}(\xi,\eta) \end{aligned} \tag{22}$$

Based on the assumptions 2 and 3, we get an inequality of the following form:

$$\begin{aligned} \dot{V}_i(\xi,\eta)&\le\frac{1}{(2\lambda_i)^2}\left[\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\bar{g}_i(\xi,0)\right]^2 \\ &+\left|\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\right|\left|\left(\bar{f}_i(\xi,\eta)-\bar{f}_i(\xi,0)\right)\right|+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\left(\bar{f}_i(\xi,0)\right) \\ &+(\lambda_i u_i)^2+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta u_i+k_i\alpha_i\left(\|\xi\|^2+\|\eta\|^2\right) \\ &+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\Delta_{i\xi}(\xi,\eta)+k_i\frac{\partial\tilde{\mathrm{V}}(\eta)}{\partial\eta}\Delta_{i\eta}(\xi,\eta) \end{aligned} \tag{23}$$

Let consider the assumptions below:

Assumption 4

For each mode $i\in M$ of system (21), there exist a continuous positive Lyapunov function $\bar{V}_i(\xi)$ and positive numbers $\lambda\!\!\!^{-}_i>0$ such that:

$$\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\Delta_{i\xi}(\xi,\eta)\le\lambda\!\!\!^{-}_i\left(\|\xi\|^2+\|\eta\|^2\right) \tag{24}$$

Assumption 5

There exist a continuous differentiable positive function $\tilde{V}(\eta)$ and constants $\hbar_i>0$ such as, for every $i\in M$,

$$\frac{\partial\tilde{\mathrm{V}}(\eta)}{\partial\eta}\Delta_{i\eta}(\xi,\eta)\le\hbar_i\left(\|\xi\|^2+\|\eta\|^2\right) \tag{25}$$

From assumptions 2 and 5, we have:

$$\begin{aligned} \dot{V}_i(\xi,\eta)&\le\lambda\!\!\!^{-}_i\left(\|\xi\|^2+\|\eta\|^2\right)-\rho_i\|\xi\|^2 \\ &+\theta_i\gamma_i\|\xi\|\|\eta\|+(\lambda_i u_i)^2+\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\left(\tilde{g}_i(\xi,\eta)\eta\right)u_i \\ &+k_i\hbar_i\left(\|\xi\|^2+\|\eta\|^2\right)+k_i\alpha_i\left(\|\xi\|^2+\|\eta\|^2\right) \end{aligned} \tag{26}$$

$$\begin{aligned} \dot{V}_i(\xi,\eta)&\le-\frac{1}{(2\lambda_i)^2}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)^2 \\ &+\left(\lambda_i u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2 \\ &+\left(k_i\hbar_i+k_i\alpha_i+\lambda\!\!\!^{-}_i\right)\left(\|\xi\|^2+\|\eta\|^2\right) \\ &-\rho_i\|\xi\|^2+\theta_i\gamma_i\|\xi\|\|\eta\| \end{aligned}$$

Then, we can obtain, from (26), that

$$\begin{aligned} \dot{V}_i(\xi,\eta)&\le\left(k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i\right)\|\xi\|^2 \\ &+\left(k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i\right)\|\eta\|^2-\rho_i\|\xi\|^2+\frac{\rho_i}{2}\|\xi\|^2 \\ &+\left(\lambda_i u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2+\frac{\theta_i\gamma_i}{2\rho_i}\|\eta\|^2 \\ &\le\left(\frac{\theta_i\gamma_i}{2\rho_i}+k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i\right)\|\eta\|^2 \\ &+\left(\lambda_i u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2 \\ &+\left(k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i-\frac{\rho_i}{2}\right)\|\xi\|^2 \\ &\le min\begin{pmatrix} k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i-\frac{\rho_i}{2}, \\ \frac{\theta_i\gamma_i}{2\rho_i}+k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i \end{pmatrix}\left(\|\xi\|^2+\|\eta\|^2\right) \\ &+\left(\lambda_i u_i+\frac{1}{2\lambda_i}\left(\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}\tilde{g}_i(\xi,\eta)\eta\right)\right)^2 \end{aligned} \tag{27}$$

Therefore, in the case of $\frac{\rho_i}{2}>\left(k_i\alpha_i+\lambda\!\!\!^{-}_i+k_i\hbar_i\right),i\in M$, we can derive $\dot{V}_i(\xi,\eta)\le 0$ .The nonlinear controller is chosen as $u_i=-\frac{\bar{g}_i(\xi,\eta)\eta}{2\lambda_i^2}\frac{\partial\bar{\mathrm{V}}_i(\xi)}{\partial\xi}$ such that the positive function $V_i(\xi,\eta)$ satisfies:

$$\dot{V}_i(\xi,\eta) \le min \begin{pmatrix} k_i\,\alpha_i + \bar{\lambda}_i + k_i\,\hbar_i - \dfrac{\rho_i}{2}, \\ \dfrac{\theta_i\,\gamma_i}{2\rho_i} + k_i\,\alpha_i + \bar{\lambda}_i + k_i\,\hbar_i \end{pmatrix} \left(\|\xi\|^2 + \|\eta\|^2\right) \tag{28}$$

According to theorem 1 presented in the previous section, if the switching law from the i^{th} sub-system to the j^{th} sub-system satisfies (16), and $\frac{\rho_i}{2} > \left(k_i\,\alpha_i + \bar{\lambda}_i + k_i\,\hbar_i\right), i \in M$, system (17) is stable, we will obtain the following results:

Theorem 2

For $i \in M$, $V_i(\xi,\eta)$ are continuous positive Lyapunov functions for each sub-system of the switched nonlinear system (18) satisfying

$$\dot{V}_i(\xi,\eta) \le min \begin{pmatrix} k_i\left(\alpha_i + \hbar_i\right) + \bar{\lambda}_i - \dfrac{\rho_i}{2}, \\ \dfrac{\theta_i\,\gamma_i}{2\rho_i} + k_i\left(\alpha_i + \hbar_i\right) + \bar{\lambda}_i \end{pmatrix} \left(\|\xi\|^2 + \|\eta\|^2\right)$$

Let Ω_k satisfy (17). If the switched law guarantees that, when the i^{th} sub-system is switched to the j^{th} sub-system $S_{ij} \subseteq \Omega_k$, then the robust nonlinear feedback controllers $u_i(\xi,\eta) = -\dfrac{\bar{g}_i(\xi,\eta)\eta}{2\bar{\lambda}_i^2}\dfrac{\partial \bar{V}_i(\xi)}{\partial \xi}$ can guarantee the asymptotically-stable system (18).

6. SIMULATIONS EXAMPLES

In this section, we show the applicability and effectiveness of our approach by presenting two examples illustrating the main results obtained in this research work.

6.1 Example 1

In this section, a nonlinear non-minimum phase inverted cart-pendulum system is presented to illustrate the effectiveness of the proposed control structure compared to the stabilization of non-minimum phase switched nonlinear systems method presented in (Jouili et al., 2015).

Description of the inverted cart-pendulum system

We consider the familiar inverted cart-pendulum system (Jouili et al., 2015) shown in figure 1.

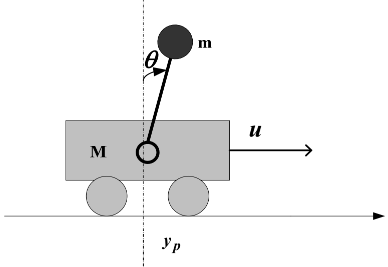

Fig. 1. Schematic of the inverted cart-pendulum.

The cart must be moved using the force u so that the pendulum will be in the upright position. Let the mass of the cart be M, the mass of the pendulum be m, the length of the stick be $L$and the acceleration of the gravity be g. The mass of the stick is smaller compared to the mass m and it will be neglected together with the effect of friction if the pendulum angle θ and the cart position y_p are chosen as the generalized position coordinates for the considered system.

The inverted cart-pendulum equations are:

$$\begin{cases} \ddot{y}_p = \dfrac{m\left[L\,\dot{\theta}^2 - gcos(\theta)\right]sin(\theta) + u}{M + m\left(sin(\theta)\right)^2} \\ \ddot{\theta} = \dfrac{1}{L}\left[gsin(\theta) - \ddot{y}_p cos(\theta)\right] \end{cases} \tag{29}$$

Let $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} y_p & \dot{y}_p & \theta & \dot{\theta} \end{bmatrix}^T$ and $y = x_3$.

Then, we obtain the following state space equation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{m\left[L\,x_4^2 - g\,cos(x_3)\right]sin(x_3)}{M + m\left(sin(x_3)\right)^2} \\ \qquad + \dfrac{1}{M + m\left(sin(x_3)\right)^2}u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{\left[(M+m)g - mLx_4^2 cos(x_3)\right]sin(x_3)}{ML + m\,L\left(sin(x_3)\right)^2} \\ \qquad - \dfrac{cos(x_3)}{ML + m\,L\left(sin(x_3)\right)^2}u \end{cases} \tag{30}$$

Stabilization control simulations

To stabilize the angle $\theta \to \cong 0$ (upright position), when the cart position is not limited, we apply the proposed approach of system (30). The synthesized control approach, in this case, has to maintain the load in a perpendicular position during the cart motion. Thus, the balancing angle θ remains bounded so that we can consider that: $\theta \to \cong 0$ .Based on the latter hypothesis, we can write the following approximations: $sin(x_3) = x_3$ and $cos(x_3) = 1$. If we apply the approach presented in section 4, the system given by (30) satisfying the lemma 1 will be transformed into the two following sub-systems:

- Sub-system 1:

$$\begin{cases} \dot{\xi} = \bar{f}_1(\xi,\eta) + \underbrace{\bar{g}_1(\xi,0) + \tilde{g}_1(\xi,\eta)}_{\bar{g}_1(\xi,\eta)}\eta u_1 \\ \dot{\eta} = Q_1(\xi,\eta) \end{cases} \tag{31}$$

with $\bar{f}_1(\xi,\eta) = \begin{bmatrix} \xi_2 \\ \dfrac{\left((M+m)g - m\,L\xi_2^2\right)\xi_1}{ML + m\,L\,\xi_1^2} \end{bmatrix}$,

$$\tilde{g}_1(\xi,\eta)=\begin{bmatrix}0\\ -\dfrac{1}{\eta_2\left(M\,L+m\,L\,\xi_1^2\right)}\end{bmatrix},$$

$$Q_1(\xi,\eta)=\begin{bmatrix}L(\eta_2-\xi_2)\\ \dfrac{M\,g\,\xi_1-2m\,L\,\xi_1\,\xi_2^2}{M\,L+m\,L\,\xi_1^2}\end{bmatrix}\text{ and }\bar{g}_1(\xi,0)=0.$$

- Sub-system 2:

$$\begin{cases}\dot{\xi}=\bar{f}_2(\xi,\eta)+\underbrace{\bar{g}_2(\xi,0)+\tilde{g}_2(\xi,\eta)}_{g_2(\xi,\eta)}\eta u_2\\ \dot{\eta}=Q_2(\xi,\eta)\end{cases}\tag{32}$$

with $\bar{f}_2(\xi,\eta)=\begin{bmatrix}\xi_2\\ \dfrac{\left((M+m)g-m\,L\,\xi_2^2\right)\xi_1}{ML+m\,L\,\xi_1^2}\end{bmatrix}$,

$$\tilde{g}_2(\xi,\eta)=\begin{bmatrix}0\\ -\dfrac{1}{\eta_2\left(M\,L+m\,L\,\xi_1^2\right)}\end{bmatrix},\bar{g}_2(\xi,0)=0\text{ and}$$

$$Q_2(\xi,\eta)=\begin{bmatrix}L(1+\eta_2-\xi_2)\xi_1\\ \dfrac{M\,g\,\xi_1-2m\,L\,\xi_1\,\xi_2^2}{M\,L+m\,L\,\xi_1^2}\end{bmatrix}$$

Choosing $\bar{V}_1(\xi)=\dfrac{1}{2}\left(\xi_1^2+\xi_2^2\right)$,

$$\vartheta_{12}(\xi)=\frac{\xi_1\,\xi_2\left[\left(ML+m\,L\,\xi_1^2\right)+\left((M+m)g-m\,L\,\xi_2^2\right)\right]}{\left(ML+m\,L\,\xi_1^2\right)\left(\dfrac{1}{2}\xi_1^2+\dfrac{1}{2}\xi_2^2\right)},$$

$$\vartheta_{21}(\xi)=-\frac{2\,\xi_1\,\xi_2\left[\left(ML+m\,L\,\xi_1^2\right)+\left((M+m)g-m\,L\,\xi_2^2\right)\right]}{\left(ML+m\,L\,\xi_1^2\right)\left(\dfrac{1}{2}\xi_1^2+\dfrac{1}{2}\xi_2^2\right)},$$

$\tilde{V}(\eta)=\dfrac{1}{\eta_1^2}+\dfrac{1}{\eta_2^2}$, $\bar{V}_2(\xi)=\xi_1^2+\xi_2^2$, $\rho_1=-\dfrac{1}{2}$ and $\rho_2=-1$.

It is easy to confirm that assumptions 1 and 3 are satisfied:

- For sub-system 1, we can compute:

$$\begin{cases}\dfrac{\partial\bar{V}_1(\xi)}{\partial\xi}\bar{f}_1(\xi,0)+\dfrac{1}{(2\lambda_1)^2}\left[\dfrac{\partial\bar{V}_1(\xi)}{\partial\xi}\bar{g}_1(\xi,0)\right]^2\\ +-\|\xi\|^2+\vartheta_{12}(\xi)\left(\bar{V}_2(\xi)-\bar{V}_1(\xi)\right)\le-\dfrac{1}{2}\left(\xi_1^2+\xi_2^2\right)\le 0\\ \left|\bar{f}_1(\xi,\eta)-\bar{f}_1(\xi,0)\right|\le\|\eta\|^2\\ \dfrac{\partial\tilde{V}(\eta)}{\partial\eta}Q_1(\xi,\eta)\le\left(\xi_1^2+\xi_2^2+\eta_1^2+\eta_2^2\right)\end{cases}$$

- For sub-system 2, we can compute:

$$\begin{cases}\dfrac{\partial\bar{V}_2(\xi)}{\partial\xi}\bar{f}_2(\xi,0)+\dfrac{1}{(2\lambda_2)^2}\left[\dfrac{\partial\bar{V}_2(\xi)}{\partial\xi}\bar{g}_2(\xi,0)\right]^2\\ +-\|\xi\|^2+\vartheta_{21}(\xi)\left(\bar{V}_1(\xi)-\bar{V}_2(\xi)\right)\le-\left(\xi_1^2+\xi_2^2\right)\le 0\\ \left|\bar{f}_2(\xi,\eta)-\bar{f}_2(\xi,0)\right|\le\|\eta\|^2\\ \dfrac{\partial\tilde{V}(\eta)}{\partial\eta}Q_2(\xi,\eta)\le\left(\xi_1^2+\xi_2^2+\eta_1^2+\eta_2^2\right)\end{cases}$$

We construct the controllers u_1 and u_2, for each sub-system i *(i=1,2)*, under the same form of equation (14). By choosing $k_1=6$ and $k_2=10$, we construct the following Lyapunov functions:

- $V_1(\xi,\eta)$ for sub-system 1:

$$V_1(\xi,\eta)=\frac{1}{2}\xi_1^2+\frac{1}{2}\xi_2^2+\frac{6}{\eta_1^2}+\frac{6}{\eta_2^2}\tag{34}$$

- $V_2(\xi,\eta)$ for sub-system 2:

$$V_2(\xi,\eta)=\frac{10}{\eta_1^2}+\frac{10}{\eta_2^2}+\xi_1^2+\xi_2^2\tag{35}$$

The stabilizing feedback controller u_1 is:

$$u_1=\frac{\xi_2}{2}\left(M\,L+m\,L\,\xi_1^2\right)\tag{36}$$

and the stabilizing feedback controller u_2 is:

$$u_2=\xi_2\left(M\,L+m\,L\,\xi_1^2\right)\tag{37}$$

According to equation (16), the switching law can be designed as:

$$i=i(\xi,\eta)=arg\ min\left\{V_i(\xi,\eta)\right\}\tag{38}$$

With the help of the designed controllers (36), (37) and switching law (38), the simulation was carried out using the design parameters $\theta_1=2$, $\theta_2=4$, $\gamma_1=\gamma_2=2$, $\alpha_1=\alpha_2=1$, $\lambda_1=1$ and $\lambda_2=1$.

The simulation results are presented in figures 2 and 3.

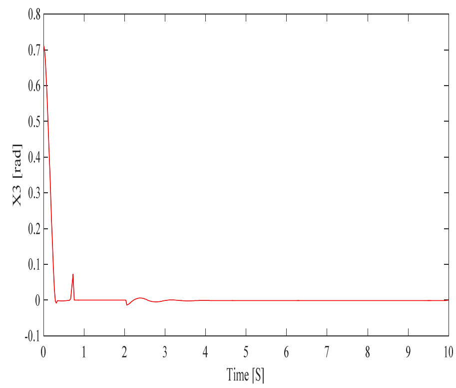

Fig. 2. Evolution of the pendulum angle.

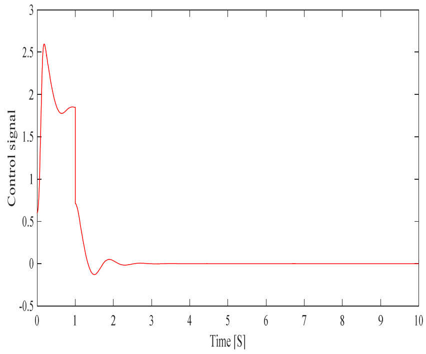

Fig. 3. Evolution of the control signal.

We observe, from these figures, that the stabilization of the cart angle is satisfactory. Figure 3 presents the evolution of the switching signal. Figure 2 shows the state response of the inverted cart-pendulum system (30) under the designed switching law (38).This state indicates that the closed-loop system is asymptotically stable. Indeed the pendulum angle stabilizes quickly to zero, which shows that the controllers (36) and (37) can globally and asymptotically stabilize the inverted cart-pendulum system (30). Thus, the simulation results illustrate well the effectiveness of the introduced method.

Comparison of the proposed approach with the concept of multi-diffeomorphism method

In order to evaluate the performance of the proposed approach on the inverted pendulum model, a comparison between the proposed approach and the stabilization of non-minimum phase switched nonlinear systems method presented (Jouili et al., 2015) is given in this subsection.

The results of this study are provided in figures (4) and (5).

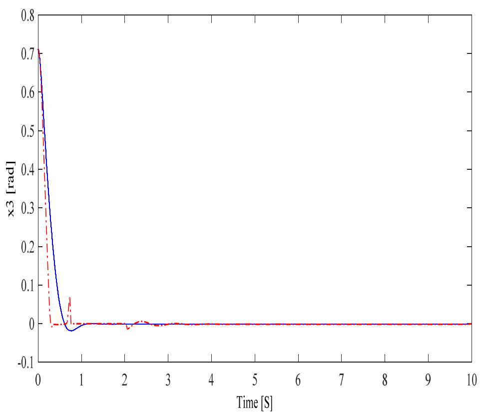

Fig. 4. Evolution of the pendulum angle (dashed line: the proposed approach; continous line: multi-diffeomorphism method).

Figure 4 shows the state response of the inverted cart-pendulum system (30). The simulation results prove that good convergence performances were achieved and the output signal of the inverted cart-pendulum system (30) were bounded. It can also be seen that the output state trajectory asymptotically converged to the origin. As a result, applying the proposed approach, we obtained asymptotic stabilization of the system (30), which led to a convergence rate faster than that provided by applying the method introduced in (Jouili et al., 2015).

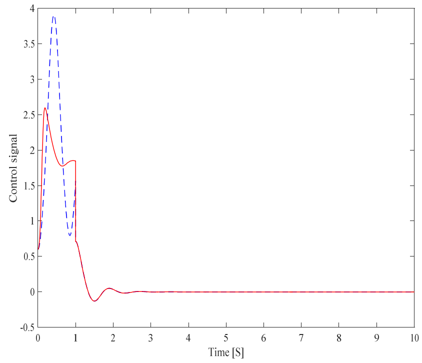

Fig. 5. Evolution of the control signal (dashed line: the proposed approach; continous line: multi-diffeomorphism method).

In figure 5, the control signal obtained by using the proposed approach is inferior to that provided by employing the method presented in (Jouili et al., 2015), which highlights a more stable evolution of the dynamic variable θ shown in figure 4. These results confirm again the remarkable performance of both methods while focusing on the advantage of the proposed method.

6.2 Example 2

In this sub-section, a switched nonlinear uncertain system with non-minimum phase modes is presented to illustrate the effectiveness of the proposed control structure. We consider the uncertain switched nonlinear system (Wang et al., 2016):

$$\dot{x} = f_i(x) + g_i(x)u_i + \Delta_i(x), i \in \{1,2\} \tag{39}$$

with

$$f_1(x) = \begin{bmatrix} x_1 - x_2 \\ 4x_1 - x_2^3 \end{bmatrix}, f_2(x) = \begin{bmatrix} x_2^2 - x_1^2 \\ -6x_1 x_2 \end{bmatrix}, g_1(x) = \begin{bmatrix} 1 \\ -x_2^2 \end{bmatrix},$$

$$g_2(x) = \begin{bmatrix} x_2 \\ -2 \end{bmatrix}, \Delta_1(x) = \begin{bmatrix} 0 \\ x_1 x_2 \end{bmatrix} \text{ and } \Delta_2(x) = \begin{bmatrix} x_2^2 \\ 0 \end{bmatrix}$$

We also apply the approach presented in previous section. The two switched nonlinear systems given by (39) can be written in the form presented by (21):

- Sub-system 1:

$$\begin{cases} \dot{\xi} = \bar{f}_1(\xi,\eta) + \underbrace{\bar{g}_1(\xi,0) + \tilde{g}_1(\xi,\eta)\eta}_{\bar{g}_1(\xi,\eta)} u_1 + \Delta_{1\xi}(\xi,\eta) \\ \dot{\eta} = Q_1(\xi,\eta) + \Delta_{1\eta}(\xi,\eta) \end{cases} \tag{40}$$

With $\bar{f}_1(\xi,\eta) = \dfrac{4}{\xi} - 4\xi - \xi^2, \bar{g}_1(\xi,0) = -\xi^2, \tilde{g}_1(\xi,\eta) = 0$,

$Q_1(\xi,\eta) = -\eta - \dfrac{4\eta}{\xi^2} - \dfrac{1}{\xi} - \dfrac{4}{\xi^3}$, $\Delta_{1\xi}(\xi,\eta) = 1 - \eta\xi$ and

$\Delta_{1\eta}(\xi,\eta) = \dfrac{\eta}{\xi} - \dfrac{1}{\xi^2}$

- Sub-system 2:

$$\begin{cases}\dot{\xi}=\bar{f}_2(\xi,\eta)+\underbrace{\bar{g}_2(\xi,0)+\tilde{g}_2(\xi,\eta)}_{\bar{g}_2(\xi,\eta)}\eta u_2+\Delta_{2\xi}(\xi,\eta)\\ \dot{\eta}=Q_2(\xi,\eta)+\Delta_{2\eta}(\xi,\eta)\end{cases} \tag{41}$$

with $\bar{f}_2(\xi,\eta)=-\frac{3}{2}\xi^3-\frac{3}{2}\xi\eta,\ \bar{g}_2(\xi,0)=-2, \tilde{g}_2(\xi,\eta)=0$

$Q_2(\xi,\eta)=4\xi-\eta^2-4\xi^2-2\xi\eta+4\xi^3+\xi^4, \Delta_{2\xi}(\xi,\eta)=0$

and $\Delta_{2\eta}(\xi,\eta)=\xi^2$

Choosing

$\bar{V}_1(\xi)=\xi^2, \bar{V}_2(\xi)=2\xi^2, \tilde{V}(\eta)=\frac{\eta^2}{4}$,

$\vartheta_{12}(\xi)=4\xi-\frac{1}{2}\xi^3,\ \vartheta_{21}(\xi)=-6\xi-\frac{4}{\xi}+2, \vartheta_{21}(\xi)=-6\xi-\frac{4}{\xi}+2$,

$\theta_1=1,\ \theta_2=3, \gamma_1=\gamma_2=\bar{\ell}_1=\bar{\ell}_2=1, \alpha_1=\alpha_2=\bar{\gamma}_1=\bar{\gamma}_2=1$

$, \lambda_1=\lambda_2=\rho_1=\rho_2=1,\ \hbar_1=\hbar_2=1$ and $\bar{\lambda}_1=\bar{\lambda}_2=1$.

It is easy to confirm that assumptions 1 and 5 are satisfied:

- For sub-system 1, we can compute:

$$\begin{cases}\frac{\partial \bar{V}_1(\xi)}{\partial\xi}\bar{f}_1(\xi,0)+\frac{1}{(2\lambda_1)^2}\left[\frac{\partial \bar{V}_1(\xi)}{\partial\xi}\bar{g}_1(\xi,0)\right]^2\\ +\rho_1|\xi|^2+\vartheta_{12}(\xi)\left(\bar{V}_2(\xi)-\bar{V}_1(\xi)\right)\le -8-7\xi^2\le 0\\ \left|\bar{f}_1(\xi,\eta)-\bar{f}_1(\xi,0)\right|\le 0\\ \frac{\partial\tilde{V}(\eta)}{\partial\eta}Q_1(\xi,\eta)\le|\xi|^2+|\eta|^2\\ \frac{\partial \bar{V}_1(\xi)}{\partial\xi}\Delta_{1\xi}(\xi,\eta)\le\left|\xi^2\right|+|\eta|^2\\ \frac{\partial\tilde{V}(\eta)}{\partial\eta}\Delta_{1\eta}(\xi,\eta)\le\left|\xi^2\right|+|\eta|^2\end{cases}$$

- For sub-system 2, we can calculate:

$$\begin{cases}\frac{\partial \bar{V}_2(\xi)}{\partial\xi}\bar{f}_2(\xi,0)+\frac{1}{(2\lambda_2)^2}\left[\frac{\partial \bar{V}_2(\xi)}{\partial\xi}\bar{g}_2(\xi,0)\right]^2\\ +\rho_2|\xi|^2+\vartheta_{21}(\xi)\left(\bar{V}_1(\xi)-\bar{V}_2(\xi)\right)\le -\xi^2\le 0\\ \left|\bar{f}_2(\xi,\eta)-\bar{f}_2(\xi,0)\right|\le|\eta|\\ \frac{\partial\tilde{V}(\eta)}{\partial\eta}Q_2(\xi,\eta)\le|\xi|^2+|\eta|^2\\ \frac{\partial \bar{V}_2(\xi)}{\partial\xi}\Delta_{2\xi}(\xi,\eta)\le\left|\xi^2\right|+|\eta|^2\\ \frac{\partial\tilde{V}(\eta)}{\partial\eta}\Delta_{2\eta}(\xi,\eta)\le\left|\xi^2\right|+|\eta|^2\end{cases}$$

By choosing $k_1=9$ and $k_2=4$, we construct the Lyapunov functions $V_i(\xi,\eta)$ as follows:

$$V_i(\xi,\eta)=\begin{cases}\xi^2+\frac{9}{4}\eta^2, & if\ \ i=1\\ 2\xi^2+\eta^2, & if\ \ i=2\end{cases} \tag{42}$$

Then, we apply theorem 2 to design a set of state feedback controllers for the switched system (39) such that the closed-loop system is globally and asymptotically stable.

Following the design procedure presented in the previous section, we can find the switching law $i=arg\ min\{V_i(\xi,\eta), i=1,2\}$ and the feedback controllers:

$$\begin{cases}u_1(\xi,\eta)=\xi^3\\ u_2(\xi,\eta)=4\xi\end{cases} \tag{43}$$

From theorem 2, the controllers (43) guarantee global robust asymptotic stability of the switched system (39). The simulation results are shown in Figures 6 and 7. Figure 6 demonstrates the output response of the closed-loop switched system. Figure 7 presents the feedback controllers (43). It can be seen that the closed-loop system is robustly asymptotically stable. These results verify that, by appropriately choosing the design parameters, we obtained a good transient performance and a moderate control effort. Thus, the simulation results validate the developed approach.

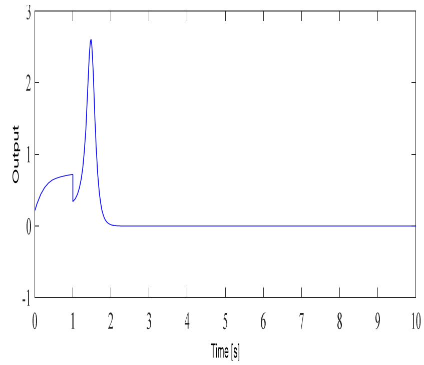

Fig. 6. Output response of the switched nonlinear system (40).

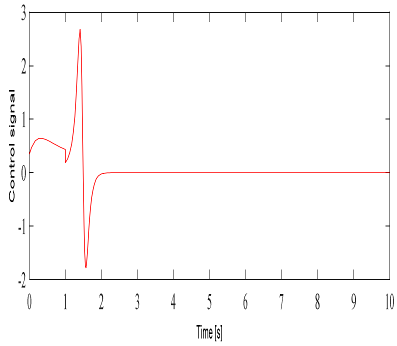

Fig. 7. Nonlinear switched feedback controller.

7. CONCLUSION

In this paper, we discussed the global stabilization problem for a class of switched nonlinear systems where each mode represents non-minimum phase. Using both common and multiple Lyapunov functions, we showed how to explicitly design nonlinear feedback controllers that guarantee the asymptotic stability of the resulting switched system. The sufficient conditions for the stabilization of the resulting switched system were obtained. As extension of the proposed design scheme, the global robust stabilization of uncertain switched nonlinear systems, where each mode represents non-minimum phase, was also studied. It was shown that the design idea can be successfully used in global stabilization of switched nonlinear systems. The obtained results can be easily extended to the multi-input multi-output case. However, the tracking control problem of the proposed approach was not studied in this paper. This issue would be investigated in the future work.

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