Observer-based Single-Phase Robustness Variable Structure Controller for Uncertain Interconnected Systems without Reaching Phase

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Abstract: This paper proposes a decentralized single phase robustness variable structure controller (SPRVSC) for complex uncertain interconnected systems satisfying mismatching condition. A key achievement of this study is to eliminate reaching phase in variable structure control theory. A switching surface is specifically designed such that a robust performance against extraneous perturbations is accurately assured from the initial time of the motion. A reduced order estimator (ROE) with lower-dimensional systems is designed to estimate the plant's state variables which are not measured. Based on the ROE and Moore-Penrose inverse technique, a decentralized SPRVSC is explored for uncertain systems with mismatched interconnections and unknown disturbances. Furthermore, a sufficient condition is derived by Lyapunov approach and linear matrix inequalities (LMI) technique for guaranteeing the robust stability of motion dynamics in sliding mode. Finally, a numerical example is also confirmed by MATLAB software for showing the effectiveness of the proposed method.

Keywords: variable structure control, reduced-order estimator, without reaching phase, interconnected systems.

1. INTRODUCTION

The variable structure control (VSC) (Edwards and Spurgeon, 1998; Hung et al., 1993; Utkin et al., 1999; Zak and Hui, 2003) is one of the main methods which have been successfully applied to a wide variety of practical world applications such as hydraulic/pneumatic, data transmission, satellite systems, robotic manipulator, automotive engines, communication and network system, etc (Boulaabi et al., 2015; Chen and Peng, 2006; Garcia et al., 2015; Rezoug et al., 2016). Based on the distinguished features of the VSC including the brilliant robustness, rapid dynamics response, exogenous disturbances rejection ability, and its insensitivity to system parameter variations (Boufounas et al., 2015; Dib et al., 2015; Din et al., 2017; Zheng et al., 2014. Recently, the stability of the mismatched uncertain systems has been attracting the interest of a significant number of quality work published in the most internationally well-known journals (Hu and Zhao, 2014; Huynh et al., 2018; Ma et al., 2015; Spurgeon and Yan, 2014; Sun et al., 2014; Wu, 2016; Xue et al., 2015; Yan et al., 2012). However, in the existing work of the VSC, the system is sensible to the uncertainties and external disturbances during the reaching phase and all the robust properties are valid during the sliding mode. In addition, a motion dynamics is determined after the state trajectories of system drive into the switching surface and the system's performance is unknown in the reaching phase. As a result, the overall stability of plant may not be assured or dangerously corrupted (Bartoszewicz and Nowacka-Leverton, 2007; Mantz et al., 2001). Consequently, it is crucial for control systems to develop a novel VSC eliminating the reaching phase that means reaching time is equal to zero.

For the purpose of the new VSC design without reaching phase, a few desired results have been investigated by many authors (Bartoszewicz and Nowacka-Leverton, 2007; Khan et al., 2011; Mobayen and Tchier, 2016; Pervaiz et al., 2014; Pervaiz et al., 2015; Xi and Hesketh, 2010). Of the study (Pervaiz et al., 2015), a sliding mode control (SMC) eliminated the reaching phase has been addressed by augmenting an integral sliding mode (ISM) with the dynamic adaptive backstepping procedure for nontriangular uncertain systems. Thanks to this approach, the authors (Pervaiz et al., 2014) synthesized the dynamics controller having provided the development of sliding mode without reaching phase for nonlinear uncertain systems. The authors in (Khan et al., 2011) carried out the dynamic integral controller where against uncertainties increases from the initial time of the process for a class of multi-input and multi-output nonlinear uncertain systems. The authors of research (Xi and Hesketh, 2010) established the integral controller to ensure the existence of sliding mode in term of matched and mismatched uncertainties for discrete time systems. The SMC algorithm, which has been investigated by eliminating the reaching phase, has adopted in (Bartoszewicz and Nowacka-Leverton, 2007) for the third-order time-varying dynamic system with extraneous disturbances. Besides, the LMI technique (Boyd et al., 1994) is one of the most useful approaches in SMC applications; it has some benefits over conventional methods. By utilizing the LMI Toolbox of the MATLAB software (Gahinet et al., 1995), LMI problems can be performed and solved easily. Based on this technique, the control law has recently established in (Mobayen and Tchier,

2016) for the robust tracking problem of a class of the uncertain nonlinear systems under strict Lipschitz condition without reaching phase. Nevertheless, a serious limitation of the controller is that states have to be accessible. In practice, some the state variables of control systems cannot be measured directly or the available measuring devices are very expensive. Thus, the development of novel VSC design using output feedback is extremely necessary and reaching phase elimination is currently indispensable.

Although a lot of important results have been implemented by sliding mode control techniques without reaching phase, the approaches in (Bartoszewicz and Nowacka-Leverton, 2007; Khan et al., 2011; Mobayen and Tchier, 2016; Pervaiz et al., 2014; Pervaiz et al., 2015; Xi and Hesketh, 2010) cannot be easily performed in large-scale interconnected systems where output information is measured only. Particularly, some existing achievements have been obtained in (Hu and Zhao, 2014; Ma et al., 2015; Spurgeon and Yan, 2014; Sun et al., 2014; Wu, 2016; Xue et al., 2015; Yan et al., 2012) as the matched and mismatched conditions were carefully considered. For example, the robust state feedback controller was constructed in (Wu, 2016) for a class of large-scale uncertain interconnected dynamical systems with the state feedback of system. Also, in (Sun et al., 2014), a state feedback mode dependent controller was achieved by solving a set of parameterized bilinear matrix inequalities for a class of networked control system with Markovian parameter. In addition, for purpose of using output information, a dynamic output feedback H_∞ control law was synthesised in (Hu and Zhao, 2014) for uncertain interconnected systems of neutral type. In (Ma et al., 2015), a decentralized memory static output feedback controller was presented for a class of nonlinear time-delayed interconnected systems with similar structure. By using the Razumikhin-Lyapunov approach, a decentralized delay-dependant static output feedback sliding mode controller was synthesised in (Spurgeon and Yan, 2014) to stabilise a class of large-scale interconnected uncertain systems with time delay. In (Yan et al., 2012), the Razumikhin-Lyapunov method was continued to be applied for designing an output feedback variable structure control which is dependent on time delay for a class of interconnected time-varying delay systems. In (Xue et al., 2015), a decentralized adaptive integral sliding mode controller was established for a class of nonlinear uncertain large-scale systems in the presence of quantization parameter mismatch. In the newest research (Huynh et al., 2018), an adaptive second order sliding mode controller was proposed for a class of complex interconnected systems in term of unknown perturbations and mismatched interconnections. Unfortunately, there is serious limitation which requires that an unmeasurable state variable of system is bounded by constant. Due to this restriction, the output feedback controller is difficult to be satisfied in most cases. Therefore, all the above work can be clearly observed that systems include external perturbations and the effects of interconnections have been attained by applying the conventional variable structure control theory. This also indicates that the existing SMC technique, which has only yielded the desired motion after sliding motion has occurred, may be used to deal with the uncertain interconnected systems. In other words, the system's robustness is not guaranteed during whole intervals of control action. As a result, the global stability of system may not be assured or dangerously corrupted. This necessitates the development of novel VSC that eliminates the reaching phase. In new VSC, the sliding mode dynamics of the complex interconnected system is asymptotically stable in the zero reaching time.

Motivated by all the published studies and the mentioned restrictions above, we will address a decentralized SPRVSC based on a ROE for uncertain systems with extraneous disturbances and interconnections. The purpose is to contribute to the development of single phase robustness variable structure control without reaching phase and the performance analysis for large scale uncertain interconnected systems. The main contributions include three key aspects. Firstly, a decentralized switching surface is specifically proposed for interconnected systems without reaching phase such that the robustness performance against external disturbances is precisely guaranteed at the beginning of process. Secondly, the ROE with lower-dimensional systems is explored to estimate the systems' states which are not measured. Thirdly, based on estimation of the ROE and Moore-Penrose inverse technique, a decentralized robustness variable structure control is designed for complex systems with mismatched interconnections and unknown disturbances without reaching phase. In addition, by employing LMI technique and Razumikhin–Lyapunov approach, sufficient condition is derived for guaranteeing the robust stability of motion dynamics in sliding mode. Finally, a numerical example is also demonstrated by simulation for showing the effectiveness of the proposed method.

The arrangement of this paper is organized as follows: The considered mismatched uncertain systems with unknown disturbances and mismatched interconnections in this study are showed in Section 2. The important contributions of this paper are described in Section 3, where for the interconnection systems with mismatched uncertainties and extraneous disturbances input, a novel SPRVSC is established with the help of ROE tool. Concurrently, the single phase sliding mode stability analysis is discussed in this Section. Section 4 comprises a numerical simulation and finally Section 5 presents the paper ends with several concluding remarks.

Throughout the work, $\|\bullet\|$ denotes the *Euclidean* norm of a vector and the induced spectral norm of a matrix.

2. PROBLEM FORMULATIONS AND PRELIMINARIES

In this paper, a general description of mismatched uncertain systems subjected to external perturbations and mismatched interconnections. The system's state-space form for each subsystem is modelled as follows:

$$\begin{aligned} \dot{x}_i &= \left[A_i + M_i \Delta F_i(x_i,t) N_i\right] x_i + B_i\left[u_i + \xi_i(x_i,t)\right] \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{L} \left(G_{ij} + D_{ij}\Delta G_{ij}(x_j,t) E_{ij}\right) x_j, \\ y_i &= C_i x_i, \end{aligned} \tag{1}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, and $y_i \in R^{p_i}$ are the system state variables, control inputs of the plant, and controlled outputs of the i^{th} subsystem, respectively. The character j is the index of interconnection subsystem, L is the number of subsystems and Σ is the sum of interconnection subsystems. The matrices A_i, M_i, N_i, B_i, and C_i are known constant matrices with appropriate dimensions. The matrix $M_i \Delta F_i(x_i,t) N_i x_i$ show the mismatched parameter uncertainty of system in each isolated subsystem while $\Delta F_i(x_i,t)$ are unknown matrices. The terms $\sum_{j=1, j\neq i}^{L} G_{ij} x_j$ and $\sum_{j=1, j\neq i}^{L} D_{ij}\Delta G_{ij}(x_j,t)E_{ij}x_j$ are respectively known and uncertain interconnections in the dynamic equations of the ith subsystem. Note that G_{ij}, D_{ij}, and E_{ij}, are known constant matrices, and $D_{ij}\Delta G_{ij}(x_j,t)E_{ij}$ are the mismatched interconnections. The symbol $\xi_i(x_i,t)$ stand for the exogenous disturbance input in the i^{th} subsystem. The symbol $\xi_i(x_i,t)$ stand for the exogenous disturbance input in the i^{th} subsystem. To design the SPRVSC for the system (1), the following introduced assumptions will be needed:

Assumption 1: The number of inputs is smaller than or equal to the number of output channels, that is, $m_i \le p_i < n_i$. The input matrices B_i and C_i are full rank matrix and $rank(C_i B_i) = m_i$.

Assumption 2: The pair (A_i, B_i) is completely controllable, and the pair (A_i, C_i) is completely observable.

Assumption 3: The mismatched parameter uncertainties in the state matrix of each isolated subsystem have to satisfy $M_i \Delta F_i(x_i,t) N_i$, where $\Delta F_i(x_i,t)$ is bounded by $\|\Delta F_i(x_i,t)\| \le 1$ for all $(x_i,t) \in R^{n_i} \text{ x } R$.

Assumption 4: The mismatched interconnections have to satisfy $D_{ij}\Delta G_{ij}(x_j,t)E_{ij}$, where $\Delta G_{ij}(x_j,t)$ is unknown but bounded by $\|\Delta G_{ij}(x_j,t)\| \le 1$ for all $(x_j,t) \in R^{n_i} \text{ x } R$.

Assumption 5: There exists known non-negative constants γ_{1_i} and γ_{2_i} such that for $i, j = 1,2,....,L$, $\|\xi_i(x_i,t)\| \le \gamma_{1_i} + \gamma_{2_i}\|x_i(t)\|$.

Remark 2.1 Assumptions 1-4 have been employed in the most existing work (Choi, 2003; Shyu et al., 2001; Xue et al., 2015; Zhang and Xia, 2010) for the output feedback sliding mode controls and assumption 5 can be found in (Huynh et al., 2018; Nguyen and Tsai, 2017; Shyu et al., 2001).

In order to create a novel SPRVSC with single phase, choose the new single phase switching surface which is initially designed to pass arbitrary initial conditions for each subsystem of the uncertain system with mismatched interconnections as follows

$$\begin{aligned} \sigma_i(y_i(t),t) &= \bar{\sigma}_i(y_i,t) - \bar{\sigma}_i(y_i,0) \\ &\times \exp(-\mu_i t) = 0,\ i = 1,2,...,L. \end{aligned} \tag{2}$$

where μ_i is positive constant and $\bar{\sigma}_i(y_i,t) = S_i x_i = P_i y_i$ with the sliding matrix $S_i = P_i C_i$.

Remark 2.2 In the existing sliding mode control techniques, desired motion of the system has only yielded after sliding motion has occurred. It leads the robustness of system is not guaranteed during whole intervals of control action. To overcome this disadvantage, the decentralized switching surface is proposed in equation (2) such that the reaching phase is eliminated. The system's state moves into switching surface from the initial time whose the reaching time is equal to zero.

Now, in order to get regular form of interconnection systems (1), Moore-Penrose inverse technique (Choi, 2003) is used as follows

$$\varGamma_i = I_i - N_i^g N_i \text{ and } \varGamma_j = I_j - E_{ij}^g E_{ij}, \tag{3}$$

where $\varGamma_i$ and $\varGamma_j$ are symmetric matrices; N_i^g and E_{ij}^g are the Moore-Penrose inverse of the matrices N_i and E_{ij}, respectively.

Assume that there are symmetric matrices R_{1i} and R_{2i} in the two following LMIs:

$$\begin{aligned} &\varGamma_i R_{1i}\varGamma_i + B_i R_{2i} B_i^T > 0, \\ &B_i^{\perp T}\left(A_i\ \varGamma_i R_{1i}\varGamma_i + \varGamma_i R_{1i}\varGamma_i A_i^T\right)B_i^{\perp} < 0, \end{aligned} \tag{4}$$

Thus, the sliding matrix (2) is parameterized by

$$P_i C_i = K_i B_i^T H_i^{-1}, \tag{5}$$

where K_i is any $m_i \text{x} m_i$ non-singular matrix, and $H_i = \varGamma_i R_{1i}\varGamma_i + B_i R_{2i} B_i^T$. The matrix P_i should be chosen to satisfy $S_i = P_i C_i$.

The original system (1) is transformed into the regular form with the transformation matrix T_i by using the orthogonal complement technique and the state vector as follows

$$T_i \triangleq \begin{bmatrix} B_i^{\perp T} \\ K_i B_i^T H_i^{-1} \end{bmatrix}, \text{ and } \begin{bmatrix} z_i \\ \bar{\sigma}_i \end{bmatrix} = T_i x_i, \tag{6}$$

where $B_i^{\perp}$ is any basic of the null space of the matrix B_i^T, the switching variable $\bar{\sigma}_i(y_i,t) \in R^{m_i}$ is measurable. Otherwise, the dynamic variable $z_i \in R^{n_i - m_i}$ is unmeasurable.

Note that the matrix T_i is an invertible matrix and its inverse form can be described by the following model

$$T_i^{-1} \triangleq \left[H_i B_i^{\perp}\left(B_i^{\perp T} H_i B_i^{\perp}\right)^{-1}\ \ B_i\left(S_i B_i\right)^{-1} \right]. \tag{7}$$

Assumption 6: The existence of matrix S_i and the equation $S_i = K_i B_i^T H_i^{-1}$ is solvable.

The combination of (1), (6), and (7) can result in the regular form below

$$\begin{bmatrix} \dot{z}_i \\ \dot{\bar{\sigma}}_i \end{bmatrix} = \begin{bmatrix} \bar{A}_{i11} + \Delta\bar{A}_{i11} & \bar{A}_{i12} + \Delta\bar{A}_{i12} \\ \bar{A}_{i21} + \Delta\bar{A}_{i21} & \bar{A}_{i22} + \Delta\bar{A}_{i22} \end{bmatrix} \begin{bmatrix} z_i \\ \bar{\sigma}_i \end{bmatrix} + \begin{bmatrix} 0 \\ S_i B_i \end{bmatrix} u_i + \begin{bmatrix} 0 \\ S_i B_i \end{bmatrix} \xi_i(x_i, t) + \sum_{\substack{j=1 \\ j\neq i}}^{L} \begin{bmatrix} \bar{G}_{ij11} + \Delta\bar{G}_{ij11} & \bar{G}_{ij12} + \Delta\bar{G}_{ij12} \\ \bar{G}_{ij21} + \Delta\bar{G}_{ij21} & \bar{G}_{ij22} + \Delta\bar{G}_{ij22} \end{bmatrix} \begin{bmatrix} z_j \\ \bar{\sigma}_j \end{bmatrix}, \tag{8}$$

where

$$\begin{aligned} \bar{A}_{i11} + \Delta\bar{A}_{i11} &= B_i^{\perp T}\left[A_i + M_i \Delta F_i N_i\right] H_i B_i^{\perp}\left(B_i^{\perp T} H_i B_i^{\perp}\right)^{-1}, \\ \bar{A}_{i12} + \Delta\bar{A}_{i12} &= B_i^{\perp T}\left[A_i + M_i \Delta F_i N_i\right] B_i \left(S_i B_i\right)^{-1}, \\ \bar{A}_{i21} + \Delta\bar{A}_{i21} &= K_i B_i^T H_i^{-1}\left[A_i + M_i \Delta F_i N_i\right] H_i B_i^{\perp}\left(B_i^{\perp T} H_i B_i^{\perp}\right)^{-1}, \\ \bar{A}_{i22} + \Delta\bar{A}_{i22} &= K_i B_i^T H_i^{-1}\left[A_i + M_i \Delta F_i N_i\right] B_i \left(S_i B_i\right)^{-1}, \\ \bar{G}_{ij11} + \Delta\bar{G}_{ij11} &= B_i^{\perp T}\left[G_{ij} + D_{ij}\Delta G_{ij} E_{ij}\right] H_i B_i^{\perp}\left(B_i^{\perp T} H_i B_i^{\perp}\right)^{-1}, \\ \bar{G}_{ij12} + \Delta\bar{G}_{ij12} &= B_i^{\perp T}\left[G_{ij} + D_{ij}\Delta G_{ij} E_{ij}\right] B_i \left(S_i B_i\right)^{-1}, \\ \bar{G}_{ij21} + \Delta\bar{G}_{ij21} &= S_i\left[G_{ij} + D_{ij}\Delta G_{ij} E_{ij}\right] H_i B_i^{\perp}\left(B_i^{\perp T} H_i B_i^{\perp}\right)^{-1}, \\ \bar{G}_{ij22} + \Delta\bar{G}_{ij22} &= S_i\left[G_{ij} + D_{ij}\Delta G_{ij} E_{ij}\right] B_i \left(S_i B_i\right)^{-1}, \end{aligned} \tag{9}$$

$z_i = B_i^{\perp T} x_i$, and $\bar{\sigma}_i = S_i x_i$.

On the order hand, the equation (8) can be rewritten by

$$\begin{aligned} \dot{z}_i &= \left(\bar{A}_{i11} + \Delta\bar{A}_{i11}\right) z_i + \left(\bar{A}_{i12} + \Delta\bar{A}_{i12}\right)\bar{\sigma}_i + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left[\left(\bar{G}_{ij11} + \Delta\bar{G}_{ij11}\right) z_j + \left(\bar{G}_{ij12} + \Delta\bar{G}_{ij12}\right)\bar{\sigma}_j\right], \\ \dot{\bar{\sigma}}_i &= \left(\bar{A}_{i21} + \Delta\bar{A}_{i21}\right) z_i + \left(\bar{A}_{i22} + \Delta\bar{A}_{i22}\right)\bar{\sigma}_i + \left(S_i B_i\right)\left[u_i + \xi_i(x_i, t)\right] + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left[\left(\bar{G}_{ij21} + \Delta\bar{G}_{ij21}\right) z_j + \left(\bar{G}_{ij22} + \Delta\bar{G}_{ij22}\right)\bar{\sigma}_j\right]. \end{aligned} \tag{10}$$

From equation (3) and the obtained results of (Choi, 2003), we can easily get

$$\begin{aligned} B_i^{\perp T} H_i B_i^{\perp} &= B_i^{\perp T}\left(I_i - N_i^g N_i\right) R_{1i}\left(I_i - N_i^g N_i\right) B_i^{\perp} > 0, \\ N_{ii} H_i B_i^{\perp} &= N_i\left[\left(I_i - N_i^g N_i\right) R_{1i}\left(I_i - N_i^g N_i\right) + B_i R_{2i} B_i^T\right] B_i^{\perp} = 0, \\ B_i^{\perp T} H_j B_i^{\perp} &= B_i^{\perp T}\left(I_j - E_{ij}^g E_{ij}\right) R_{1i}\left(I_j - E_{ij}^g E_{ij}\right) B_i^{\perp} > 0, \\ E_{ij} H_j B_i^{\perp} &= E_{ij}\left[\left(I_j - E_{ij}^g E_{ij}\right) R_{1i}\left(I_j - E_{ij}^g E_{ij}\right) + B_i R_{2i} B_i^T\right] B_i^{\perp} = 0. \end{aligned} \tag{11}$$

From eq. (9), (10), and (11), a new model can be showed by

$$\begin{aligned} \dot{z}_i &= \bar{A}_{i11} z_i + \left(\bar{A}_{i12} + \Delta\bar{A}_{i12}\right)\bar{\sigma}_i + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left[\bar{G}_{ij11} z_j + \left(\bar{G}_{ij12} + \Delta\bar{G}_{ij12}\right)\bar{\sigma}_j\right], \\ \dot{\bar{\sigma}}_i &= \bar{A}_{i21} z_i + \left(\bar{A}_{i22} + \Delta\bar{A}_{i22}\right)\bar{\sigma}_i + \left(S_i B_i\right)\left[u_i + \xi_i(x_i, t)\right] + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left[\bar{G}_{ij21} z_j + \left(\bar{G}_{ij22} + \Delta\bar{G}_{ij22}\right)\bar{\sigma}_j\right]. \end{aligned} \tag{12}$$

Now, we are ready for development of observer and controller design.

3. MAIN RESULTS

3.1 Determine an Upper Bound of a Novel Reduced Order Estimator for Systems

In this work, to estimate the unmeasurable state variables for the complex interconnected systems, we will propose a novel ROE tool. Then, the upper bound of estimation error of the ROE will be found for purpose of controller design.

To provide the estimates of the unmeasurable variables for the complex interconnected systems, the decentralized state ROE is proposed as follows

$$\dot{\hat{z}}_i = \bar{A}_{i11}\hat{z}_i + \bar{A}_{i12}\bar{\sigma}_i + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ij11}\hat{z}_j + \bar{G}_{ij12}\bar{\sigma}_j\right). \tag{13}$$

Since

$$\sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ij11}\hat{z}_j + \bar{G}_{ij12}\bar{\sigma}_j\right) = \sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ji11}\hat{z}_i + \bar{G}_{ji12}\bar{\sigma}_i\right). \tag{14}$$

Thus, the equation (13) can be rewritten as

$$\dot{\hat{z}}_i = \bar{A}_{i11}\hat{z}_i + \bar{A}_{i12}\bar{\sigma}_i + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ji11}\hat{z}_i + \bar{G}_{ji12}\bar{\sigma}_i\right), \tag{15}$$

where $\hat{z}_i$ shows the estimation of the plant state z_i.

Now, introduce an estimation error as $e_i(t) = \hat{z}_i(t) - z_i(t)$. By combining the first equation (12) and (15), the prevailing estimation error system is attained:

$$\dot{e}_i(t) = \bar{A}_{i11} e_i(t) - \Delta\bar{A}_{i11} z_i - \Delta\bar{A}_{i12}\bar{\sigma}_i + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ij11} e_j(t) - \Delta\bar{G}_{ij11} z_j - \Delta\bar{G}_{ij12}\bar{\sigma}_j\right). \tag{16}$$

Since

$$\sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ij11} e_j(t) - \Delta\bar{G}_{ij11} z_j - \Delta\bar{G}_{ij12}\bar{\sigma}_j\right) = \sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ji11} e_i(t) - \Delta\bar{G}_{ji11} z_i - \Delta\bar{G}_{ji12}\bar{\sigma}_i\right), \tag{17}$$

we can show that

$$\dot{e}_i(t) = \bar{A}_{i11} e_i(t) - \Delta\bar{A}_{i11} z_i - \Delta\bar{A}_{i12}\bar{\sigma}_i + \sum_{\substack{j=1 \\ j\neq i}}^{L}\left(\bar{G}_{ji11} e_i(t) - \Delta\bar{G}_{ji11} z_i - \Delta\bar{G}_{ji12}\bar{\sigma}_i\right). \tag{18}$$

Based on equation (11), the estimation error dynamics (18) is reduced as follows

$$\dot{e}_i(t)=\bar{A}_{i11}e_i(t)-B_i^{\perp T}M_i\Delta F_i(x_i,t)N_iB_i\left(S_iB_i\right)^{-1}\bar{\sigma}_i$$
$$+\sum_{\substack{j=1\\j\neq i}}^{L}\left[\bar{G}_{ij11}e_j(t)-B_i^{\perp T}D_{ij}\Delta G_{ij}(x_j,t)E_{ij}B_i\left(S_iB_i\right)^{-1}\bar{\sigma}_j\right]. \quad (19)$$

Since

$$\sum_{\substack{j=1\\j\neq i}}^{L}\left[\bar{G}_{ij11}e_j(t)-B_i^{\perp T}D_{ij}\Delta G_{ij}(x_j,t)E_{ij}B_i\left(S_iB_i\right)^{-1}\bar{\sigma}_j\right]$$
$$=\sum_{\substack{j=1\\j\neq i}}^{L}\left[\bar{G}_{ji11}e_i(t)-B_j^{\perp T}D_{ji}\Delta G_{ji}(x_j,t)E_{ji}B_j\left(S_jB_j\right)^{-1}\bar{\sigma}_i\right]. \quad (20)$$

Thus, the above equation (19) can be represented as follows

$$\dot{e}_i(t)=\bar{A}_{i11}e_i(t)-B_i^{\perp T}M_i\Delta F_i(x_i,t)N_iB_i\left(S_iB_i\right)^{-1}\bar{\sigma}_i$$
$$+\sum_{\substack{j=1\\j\neq i}}^{L}\left[\bar{G}_{ji11}e_i(t)-B_j^{\perp T}D_{ji}\Delta G_{ji}(x_j,t)E_{ji}B_j\left(S_jB_j\right)^{-1}\bar{\sigma}_i\right].$$

(21)

For intention of controller design, we now find the upper bound of estimation error by establishing the following theorem.

Theorem 3.1 Let $\lambda_{\max_i}$ be the maximum eigenvalue of $\bar{A}_{i11}$. The estimation error norm $\|e_i(t)\|$ in error dynamics equation (21) is bounded by $\Theta_i(t)$ which is the answer of

$$\dot{\Theta}_i(t)=\rho_i\Theta_i(t)+k_i\left[\left\|B_i^{\perp T}M_i\right\|\left\|N_iB_i(S_iB_i)^{-1}\right\|\right.$$
$$\left.+\sum_{\substack{j=1\\j\neq i}}^{L}\left(\left\|B_j^{\perp T}D_{ji}\right\|\left\|E_{ji}B_j(S_jB_j)^{-1}\right\|\right)\right]\left\|\bar{\sigma}_i(t)\right\| \quad (22)$$

with $\rho_i=\lambda_{\max_i}+k_i\sum_{\substack{j=1\\j\neq i}}^{L}\left\|\bar{G}_{ji11}\right\|<0,$ k_i is positive scalar, and an initial estimation error condition $\Theta_i(0)\geq k_i\left\|e_i(0)\right\|$.

Proof. By means of the recent research (Nguyen and Tsai, 2017), the stable matrix $\bar{A}_{i11}$ implies that $\left\|\exp\left(\bar{A}_{i11}t\right)\right\|\leq k_i\exp\left(\lambda_{\max_i}t\right)$ for some scalars $k_i>0$, and solving (21) to yield

$$\|e_i(t)\|\leq\left\|\exp\left(\bar{A}_{i11}t\right)\right\|\left\|e_i(0)\right\|+\int_0^t\left\|\exp\left[\bar{A}_{i11}\left(t-\tau\right)\right]\right\|$$
$$\times\left[\sum_{\substack{j=1\\j\neq i}}^{L}\left(\left\|\bar{G}_{ji11}\right\|\left\|e_i(\tau)\right\|+\left\|B_j^{\perp T}D_{ji}\right\|\left\|E_{ji}B_j\left(S_jB_j\right)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right)\right.$$
$$\left.+\left\|B_i^{\perp T}M_i\right\|\left\|N_iB_i(S_iB_i)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right]d\tau,$$
$$\leq k_i\left\|e_i(0)\right\|\exp\left(\lambda_{\max_i}t\right)+\int_0^t k_i\exp\left(\lambda_{\max_i}(t-\tau)\right)$$
$$\times\left[\sum_{\substack{j=1\\j\neq i}}^{L}\left(\left\|\bar{G}_{ji11}\right\|\left\|e_i(\tau)\right\|+\left\|B_j^{\perp T}D_{ji}\right\|\left\|E_{ji}B_j(S_jB_j)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right)\right. \quad (23)$$
$$\left.+\left\|B_i^{\perp T}M_i\right\|\left\|N_iB_i(S_iB_i)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right]d\tau.$$

We multiply the expression $\exp\left(-\lambda_{\max_i}t\right)$ to both sides of the above inequality (23), and then

$$\|e_i(t)\|\exp(-\lambda_{\max_i}t)\leq k_i\left\|e_i(0)\right\|+\int_0^t k_i\exp\left(-\lambda_{\max_i}\tau\right)$$
$$\times\sum_{\substack{j=1\\j\neq i}}^{L}\left\|\bar{G}_{ji11}\right\|\left\|e_i(\tau)\right\|d\tau+\int_0^t k_i\exp\left(-\lambda_{\max_i}\tau\right)$$
$$\times\left[\sum_{\substack{j=1\\j\neq i}}^{L}\left(\left\|B_j^{\perp T}D_{ji}\right\|\left\|E_{ji}B_j(S_jB_j)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right)\right. \quad (24)$$
$$\left.+\left\|B_i^{\perp T}M_i\right\|\left\|N_iB_i(S_iB_i)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right]d\tau,$$

To determine the upper bound of estimation error dynamics equation (24), we now apply lemma (Shyu et al., 2001), the above inequality (24) can be rewritten

$$\|e_i(t)\|\leq k_i\left\|e_i(0)\right\|\exp\left[\left(\lambda_{\max_i}+k_i\sum_{\substack{j=1\\j\neq i}}^{L}\left\|\bar{G}_{ji11}\right\|\right)t\right]$$
$$+\int_0^t k_i\exp\left[\left(\lambda_{\max_i}+k_i\sum_{\substack{j=1\\j\neq i}}^{L}\left\|\bar{G}_{ji11}\right\|\right)(t-\tau)\right] \quad (25)$$
$$\times\left[\sum_{\substack{j=1\\j\neq i}}^{L}\left(\left\|B_j^{\perp T}D_{ji}\right\|\left\|E_{ji}B_j(S_jB_j)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right)\right.$$
$$\left.+\left\|B_i^{\perp T}M_i\right\|\left\|N_iB_i(S_iB_i)^{-1}\right\|\left\|\bar{\sigma}_i(\tau)\right\|\right]d\tau,$$
$$\leq\Theta_i(0)\exp\left[\left(\lambda_{\max_i}+k_i\sum_{\substack{j=1\\j\neq i}}^{L}\left\|\bar{G}_{ji11}\right\|\right)t\right]$$
$$+\int_0^t k_i\exp\left[\left(\lambda_{\max_i}+k_i\sum_{\substack{j=1\\j\neq i}}^{L}\left\|\bar{G}_{ji11}\right\|\right)(t-\tau)\right] \quad (26)$$
$$\times\left[\sum_{\substack{j=1\\j\neq i}}^{L}\left(\left\|B_j^{\perp T}D_{ji}\right\|\left\|E_{ji}B_j(S_jB_j)^{-1}\right\|\right)+\left\|B_i^{\perp T}M_i\right\|\right.$$
$$\left.\times\left\|N_iB_i(S_iB_i)^{-1}\right\|\right]\left\|\bar{\sigma}_i(\tau)\right\|d\tau,$$

$\leq\Theta_i(t)$, if $\Theta_i(0)\geq k_i\left\|e_i(0)\right\|$, where $\Theta_i(t)$ gratifies (22). Hence, we can see that $\|e_i\|\leq\Theta_i(t)$ for all time. Thus, the proof of Theorem 3.1 is finished.

3.2 ROE-based Single-Phase Robustness Variable Structure Controller

In the above section, we have established Theorem 3.1 for estimation error upper bound of the ROE. Now, by applying this Theorem, we design a decentralized SPRVSC to keep the plant's state trajectory moving along switching surface from the zero reaching time as our main contribution. Also, the Lyapunov function $V(\sigma_i(x_i(t),t))>0$ and $\dot{V}(\sigma_i(x_i(t),t))<0$ have to satisfy all non-positive constants, the single phase sliding mode scheme is proposed as

$$u_i = -\hat{\varepsilon}_{1_i}\sigma_i - (S_iB_i)^{-1}\Big[\hat{\varepsilon}_{2_i}\left(\|\hat{z}_i\| + \Theta_i(t)\right) + \hat{\varepsilon}_{3_i}\|\bar{\sigma}_i\|$$
$$+\gamma_{1_i}\|S_iB_i\| + \mu_i\|P_i\|\|y_i(0)\|\exp(-\mu_i t)\Big]\frac{\sigma_i}{\|\sigma_i\|}, \tag{27}$$

where the constants $\hat{\varepsilon}_{1_i}$, $\hat{\varepsilon}_{2_i}$ and $\hat{\varepsilon}_{3_i}$ are the control gains and will be determined later for $i = 1, 2,..., L$.

The overall block diagram of the proposed scheme in Figure 1 covers three main blocks, the controlled plant (1), ROE (15) and the controller (27). The control input includes two parts, signals estimated by ROE and measured output signal $y(t)$.

Theorem 3.2 Consider the closed-loop the complex interconnected systems (1) with the decentralized SPRVSC (27), suppose that the assumptions 1-5 are satisfied. Then, the system's states stay in the composite switching surface (2) and maintain motion on it if in some the constant control gains gratify

$$\hat{\varepsilon}_{1_i} > (S_iB_i)^{-1}\alpha_i,$$
$$\hat{\varepsilon}_{2_i} > \|\bar{A}_{i21}\| + \|K_iB_i^TH_i^{-1}M_i\|\|N_iH_iB_i^{\perp}(B_i^{\perp T}H_iB_i^{\perp})^{-1}\|$$
$$+\gamma_{2_i}\|S_iB_i\|\|H_iB_i^{\perp}(B_i^{\perp T}H_iB_i^{\perp})^{-1}\|$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\|\bar{G}_{ji21}\| + \|S_iD_{ji}\|\|E_{ji}H_jB_j^{\perp}(B_j^{\perp T}H_jB_j^{\perp})^{-1}\|\Big], \tag{28}$$
$$\hat{\varepsilon}_{3_i} > \|\bar{A}_{i22}\| + \|K_iB_i^TH_i^{-1}M_i\|\|N_iB_i(S_iB_i)^{-1}\| + \gamma_{2_i}\|S_iB_i\|$$
$$\times\|B_i(S_iB_i)^{-1}\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\|\bar{G}_{ji22}\| + \|S_jD_{ji}\|\|E_{ji}B_j(S_jB_j)^{-1}\|\Big].$$

Proof. Now, we choose the Lyapunov function as follows

$$V(\sigma_i(x_i(t),t)) = \sum_{i=1}^{L}\|\sigma_i\|. \tag{29}$$

By differentiating the equation (29) and combining the second equation of (10), we have

$$\dot{V}(\sigma_i(x_i(t),t)) = \sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}\Big\{\left(\bar{A}_{i21} + \Delta\bar{A}_{i21}\right)z_i + \left(\bar{A}_{i22} + \Delta\bar{A}_{i22}\right)\bar{\sigma}_i$$
$$+\left(S_iB_i\right)\left[u_i + \xi_i(x_i,\mathrm{t})\right] + \sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\left(\bar{G}_{ij21} + \Delta\bar{G}_{ij21}\right)z_j$$
$$+\left(\bar{G}_{ij22} + \Delta\bar{G}_{ij22}\right)\bar{\sigma}_j\Big] + \mu_iP_iy_i(0)\exp(-\mu_i t)\Big\}$$
$$\leq \sum_{i=1}^{L}\left(\|\bar{A}_{i21}\| + \|\Delta\bar{A}_{i21}\|\right)\|z_i\| + \sum_{i=1}^{L}\left(\|\bar{A}_{i22}\| + \|\Delta\bar{A}_{i22}\|\right)\|\bar{\sigma}_i\|$$
$$+\sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}\left(S_iB_i\right)u_i + \sum_{i=1}^{L}\|S_iB_i\|\|\xi_i(x_i,\mathrm{t})\|$$
$$+\sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\left(\|\bar{G}_{ij21}\| + \|\Delta\bar{G}_{ij21}\|\right)\|z_j\| + \Big(\|\bar{G}_{ij22}\| \tag{30}$$
$$+\|\Delta\bar{G}_{ij22}\|\Big)\|\bar{\sigma}_j\|\Big] + \sum_{i=1}^{L}\mu_i\|P_i\|\|y_i(0)\|\exp(-\mu_i t).$$

Based on the state transformation T_i in (6) implies that $x_i = H_iB_i^{\perp}\left(B_i^{\perp T}H_iB_i^{\perp}\right)^{-1}z_i + B_i\left(S_iB_i\right)^{-1}\sigma_i$. Because of $z_i(t) = \hat{z}_i(t) - e_i(t)$ and $\|e_i(t)\| \leq \Theta_i(t)$, we obtain

$$\|x_i\| \leq \|H_iB_i^{\perp}(B_i^{\perp T}H_iB_i^{\perp})^{-1}\|\left(\|\hat{z}_i\| + \Theta_i(t)\right)$$
$$+\|B_i(S_iB_i)^{-1}\|\|\bar{\sigma}_i\|. \tag{31}$$

From assumption 5, inequality (30), (31) and property

$$\sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\left(\|\bar{G}_{ij21}\| + \|\Delta\bar{G}_{ij21}\|\right)\|z_j\| + \left(\|\bar{G}_{ij22}\| + \|\Delta\bar{G}_{ij22}\|\right)\|\bar{\sigma}_j\|\Big]$$
$$= \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\left(\|\bar{G}_{ji21}\| + \|\Delta\bar{G}_{ji21}\|\right)\|z_i\| \tag{32}$$
$$+\left(\|\bar{G}_{ji22}\| + \|\Delta\bar{G}_{ji22}\|\right)\|\bar{\sigma}_i\|\Big].$$

It generates

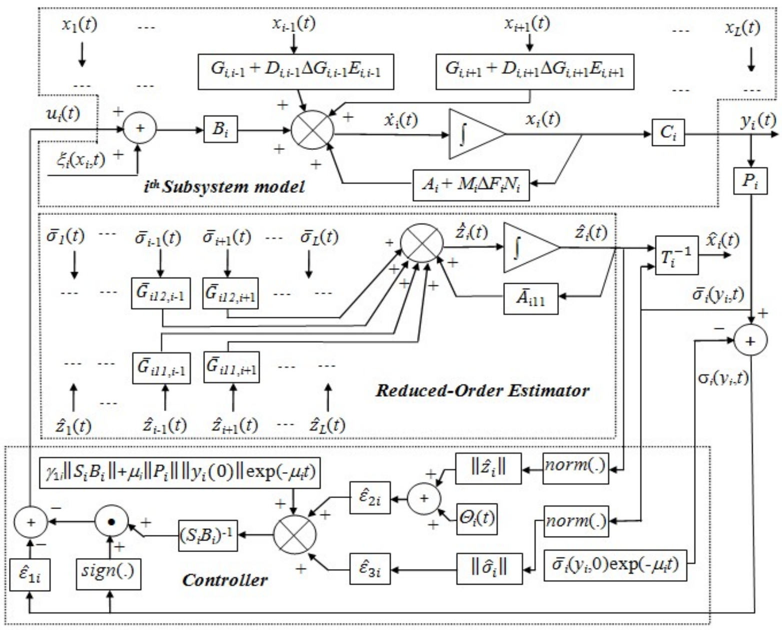

Fig. 1. Block diagram of the i^{th} subsystem

$$\dot{V}(\sigma_i) \le \sum_{i=1}^{L}\Big[\left\|\bar{A}_{i21}\right\| + \left\|\Delta\bar{A}_{i21}\right\| + \gamma_{2_i}\left\|S_iB_i\right\|\left\|H_iB_i^{\perp}(B_i^{\perp T}H_iB_i^{\perp})^{-1}\right\|$$

$$+\sum_{\substack{j=1\\ j\ne i}}^{L}\left(\left\|\bar{G}_{ji21}\right\| + \left\|\Delta\bar{G}_{ji21}\right\|\right)\Big]\left(\left\|\hat{z}_i\right\| + \Theta_i\right) + \sum_{i=1}^{L}\Big[\left\|\bar{A}_{i22}\right\| + \left\|\Delta\bar{A}_{i22}\right\|$$

$$+\gamma_{2_i}\left\|S_iB_i\right\|\left\|B_i(S_iB_i)^{-1}\right\| + \sum_{\substack{j=1\\ j\ne i}}^{L}\left(\left\|\bar{G}_{ji22}\right\| + \left\|\Delta\bar{G}_{ji22}\right\|\right)\Big]\left\|\bar{\sigma}_i\right\|$$

$$+\sum_{i=1}^{L}\left[\gamma_{1_i}\left\|S_iB_i\right\| + \mu_i\left\|P_i\right\|\left\|y_i(0)\right\|\exp(-\mu_i t) + \frac{\sigma_i^T}{\left\|\sigma_i\right\|}\left(S_iB_i\right)u_i\right],$$

$$\le -\sum_{i=1}^{L}\alpha_i\left\|\sigma_i\right\| < 0, \qquad (33)$$

which the system's states stay the switching surface from the zero reaching time for all $t \ge 0$. Hence, Theorem 3.2 is proved.

Remark 3.1 By extending the concept without reaching phase of papers (Bartoszewicz and Nowacka-Leverton, 2007; Bejarano et al., 2009; Khan et al., 2011; Pervaiz et al., 2014; Pervaiz et al., 2015; Xi and Hesketh, 2010) and improving the obtained results of paper (Huynh et al., 2018), the decentralized SPRVSC (27) is developed for complex interconnected systems with mismatched interconnections and unknown disturbances.

Remark 3.2 In the newest research (Huynh et al., 2018), an unmeasurable state variable of system is bounded by constant. This is unrealistic in practice. For this paper, the decentralized SPRVSC has been proposed through the estimated variables, and a new dynamic variable which must be greater or equal to estimation error. Thus, our proposed SPRVSC has removed this shortcoming.

3.3 Stability Analysis in Single-Phase Sliding Mode

In the section above, we have just represented the decentralized SPRVSC for complex interconnected systems. This controller will keep the state trajectories of the system moving along the switching surface towards the origin with desired performance for all time. Next, the task of this section is to find the appropriate LMI conditions which based on the Lyapunov method such that the closed-loop system (10) ensures the asymptotical stability.

Now, we will consider the following LMI as

$$\begin{bmatrix} \Xi_i & Q_i\bar{M}_i & \bar{N}_i^T \\ \bar{M}_i^TQ_i^T & -\varphi_iI_i & 0 \\ \bar{N}_i & 0 & -\varphi_i^{-1}I_i \end{bmatrix} < 0, \qquad (34)$$

where $\Xi_i = \bar{A}_{i11}^TQ_i + Q_i\bar{A}_{i11} + \sum_{\substack{j=1\\ j\ne i}}^{L}\left(\bar{G}_{ji11}^TQ_i + Q_i\bar{G}_{ji11} + \varphi_j^{-1}Q_i\bar{D}_{ji}\bar{D}_{ji}^T \times Q_i^T + \varphi_j\bar{E}_{ji}^T\bar{E}_{ji}\right)$, $Q_i \in R^{(n_i-m_i)(n_i-m_i)}$ is any positive matrix and φ_i, φ_j are positive constants.

To prove the closed-loop system (10) that ensures the asymptotical stability, we propose the following theorem.

Theorem 3.3 Suppose that the LMI (34) has a feasible solution $Q_i > 0$, the switching surface is given by the equation (2). Then, the resulting $(n_i - m_i)$ reduced-order dynamics of the interconnection systems (10) to the switching surface is asymptotically stable.

Proof. When the system is in the sliding mode, $\bar{\sigma}_i = 0$, the first equation of (10) can be described by the following sliding mode dynamics

$$\dot{z}_i = \left(\bar{A}_{i11} + \bar{M}_i\Delta F_i(x_i,t)\bar{N}_i\right)z_i + \sum_{\substack{j=1\\ j\ne i}}^{L}\left(\bar{G}_{ij11} + \bar{D}_{ij}\Delta G_{ij}(x_j,t)\bar{E}_{ij}\right)z_j, \qquad (35)$$

where $\bar{M}_i = B_i^{\perp T}M_i, \bar{N}_i = N_iH_iB_i^{\perp}\left(B_i^{\perp T}H_iB_i^{\perp}\right)^{-1}$, $\bar{D}_{ij} = B_i^{\perp T}D_{ij}$, and $\bar{E}_{ij} = E_{ij}H_iB_i^{\perp}\left(B_i^{\perp T}H_iB_i^{\perp}\right)^{-1}$.

To analyze the stability of the above sliding motion (35), we consider the following Lyapunov positive definition function

$$V = \sum_{i=1}^{L} z_i^TQ_iz_i, \qquad (36)$$

where the positive matrix $Q_i \in R^{(n_i-m_i)(n_i-m_i)}$ is defined by LMI (34). Then, by taking the time derivative of V, combining equation (35), and $\sum_{\substack{j=1\\ j\ne i}}^{L}\left(\bar{G}_{ij11} + \bar{D}_{ij}\Delta G_{ij}(x_j,t)\bar{E}_{ij}\right)z_j = \sum_{\substack{j=1\\ j\ne i}}^{L}\left(\bar{G}_{ji11} + \bar{D}_{ji}\Delta G_{ji}(x_i,t)\bar{E}_{ji}\right)z_i$, we get

$$\dot{V} = \sum_{i=1}^{L} z_i^T\Big(\bar{A}_{i11}^TQ_i + Q_i\bar{A}_{i11} + Q_i\bar{M}_i\Delta F_i(x_i,t)\bar{N}_i + \bar{N}_i^T\Delta F_i^T(x_i,t)\bar{M}_i^TQ_i\Big)z_i + \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\ne i}}^{L} z_i^T\Big(\bar{G}_{ji11}^TQ_i + Q_i\bar{G}_{ji11} + Q_i\bar{D}_{ji}\Delta G_{ji}(x_i,t)\bar{E}_{ji} + \bar{E}_{ji}^T\Delta G_{ji}^T(x_i,t)\bar{D}_{ji}^TQ_i\Big)z_i. \qquad (37)$$

By applying Lemma (Zhang and Xia, 2010) to equation (37), we obtain

$$\dot{V} \le \sum_{i=1}^{L} z_i^T\Big[\bar{A}_{i11}^TQ_i + Q_i\bar{A}_{i11} + \varphi_i^{-1}Q_i\bar{M}_i\bar{M}_i^TQ_i^T + \varphi_i\bar{N}_i^T\bar{N}_i + \sum_{\substack{j=1\\ j\ne i}}^{L}\left(\bar{G}_{ji11}^TQ_i + Q_i\bar{G}_{ji11} + \varphi_j^{-1}Q_i\bar{D}_{ji}\bar{D}_{ji}^TQ_i^T + \varphi_j\bar{E}_{ji}^T\bar{E}_{ji}\right)\Big]z_i, \qquad (38)$$

where φ_i, φ_j are positive scalars.

In addition, applying Lemma (Boyd et al., 1994) by the Schur complement, the LMI (34) is equivalent to the following inequality

$$\bar{A}_{i11}^TQ_i + Q_i\bar{A}_{i11} + \sum_{\substack{j=1\\ j\ne i}}^{L}\left(\bar{G}_{ji11}^TQ_i + Q_i\bar{G}_{ji11} + \varphi_j^{-1}Q_i\bar{D}_{ji}\bar{D}_{ji}^TQ_i^T + \varphi_j\bar{E}_{ji}^T\bar{E}_{ji}\right) + \varphi_i^{-1}Q_i\bar{M}_i\bar{M}_i^TQ_i^T + \varphi_i\bar{N}_i^T\bar{N}_i < 0. \qquad (39)$$

From (38) and (39), we have $\dot{V} < 0$ means that if LMI (34) is satisfied, then the closed loop interconnected systems (10) are asymptotically stable. Theorem 3.3 is completed.

4. NUMERICAL SIMULATION

In the section above, we have provided the appropriate LMI condition that has proved the asymptotically stable sliding motion dynamics (35). In this section, the simulation of a numerical example is performed for showing the effectiveness of the theory developed in this paper. Now, we modify the complex problem of paper (Yan et al., 2012) for the uncertain systems with mismatched interconnections consisting of two subsystems described as follows:

Subsystem I: $n_1 = 3,\ m_1 = 2,\ i = 1,\ j = 2,$ and the dynamics is given as

$$\dot{x}_1 = \left[A_1 + M_1 \Delta F_1(x_1,t) N_1\right] x_1 + B_1\left[u_1 + \xi_1(x_1,t)\right] + \left(G_{12} + D_{12}\Delta G_{12}(x_2,t)E_{12}\right)x_2, \tag{40}$$
$$y_1 = C_1 x_1,$$

where $A_1 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 1 & -0.75 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $G_{12} = \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}$, and $C_1 = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. To demonstrate the robustness of the proposed decentralized SPRSVC, it is assumed that the extraneous disturbance input is $\|\xi_1(x_1,t)\| \le \gamma_{m_1}\|x_1\|$ where $\gamma_{m_1} = 0.3,$ and the mismatched uncertainty in state matrix is $M_1 F_1(x_1,t) N_1,$ where $M_1 = [0\ 0\ 1]^T$, $N_1 = [1\ 1\ 0]$, $\Delta F_1(x_1,t) = 0.1\sin(0.1t),$ and the mismatched interconnection state matrix is $D_{12}\Delta G_{12}(x_2,t)E_{12}$ where $D_{12} = [0\ 1\ 0]^T$, $E_{12} = [1\ 1\ 0]$, and $\Delta G_{12}(x_2,t) = 0.3\sin(0.1t)$. The detailed block diagram of the subsystem I including the proposed ROE and SPRVSC is given in Figure 2.

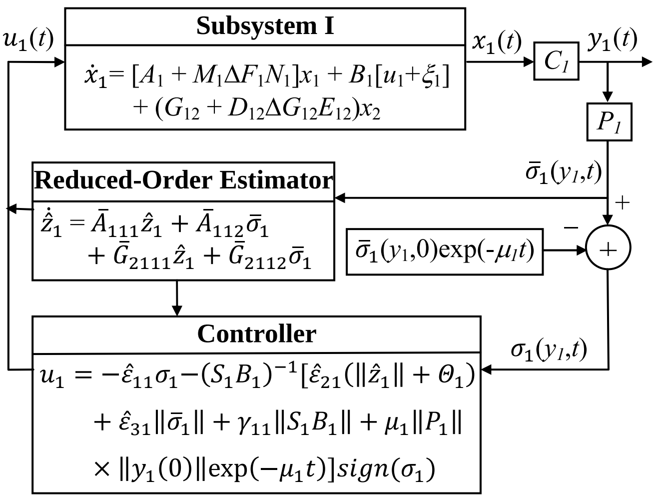

Fig. 2. Block diagram of the subsystem I.

Subsystem II: $n_2 = 3,\ m_2 = 2,\ i = 2,\ j = 1,$ and the dynamics is given as

$$\dot{x}_2 = \left[A_2 + M_2 \Delta F_2(x_2,t) N_2\right] x_2 + B_2\left[u_2 + \xi_2(x_2,t)\right] + \left(G_{21} + D_{21}\Delta G_{21}(x_1,t)E_{21}\right)x_1, \tag{41}$$
$$y_2 = C_2 x_2,$$

where

$A_2 = \begin{bmatrix} -0.1 & 1 & 0.2 \\ 1 & 1 & -1 \\ 0.5 & 1 & 0.1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix}$, $G_{21} = \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}$, and $C_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. To demonstrate the robustness of the proposed decentralized SPRSVC, it is assumed that the extraneous disturbance input is $\|\xi_2(x_2,t)\| \le \gamma_{m_2}\|x_2\|$ where $\gamma_{m_2} = 0.3,$ and the mismatched uncertainty in state matrix is $M_2\Delta F_2(x_2,t)N_2$ where $M_2 = [1\ 0\ 0]^T$, $N_2 = [1\ 1\ 0]$, $\Delta F_2(x_2,t) = 0.1\sin(0.1t)$ and the mismatched interconnection state matrix is $D_{21}\Delta G_{21}(x_1,t)E_{21}$ where $D_{21} = [0\ 1\ 0]^T$, $E_{21} = [1\ 1\ 0]$, and $\Delta G_{21}(x_1,t) = 0.4\sin(0.3t)$. The initial values of two subsystems are chosen as $x_1(0) = [2\ \ -1\ \ 1]^T$ and $x_2(0) = [1.5\ \ -0.5\ \ 0.2]^T$, respectively. The block diagram of subsystem II being regulated by SPRVSC is displayed in Figure 3.

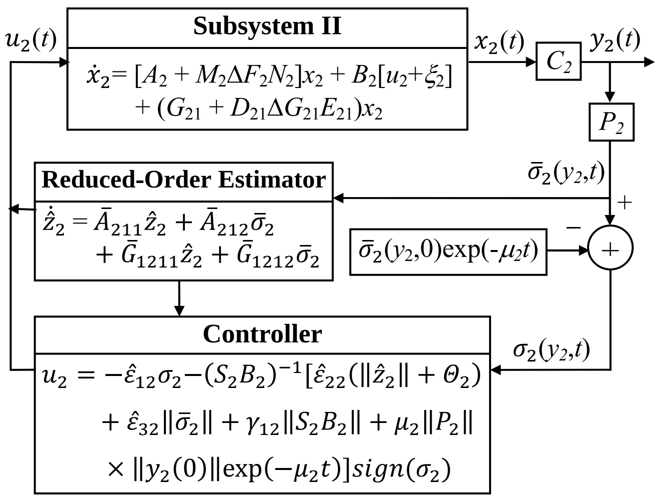

Fig. 3. Block diagram of the subsystem II.

For this study, the settings of parameters are given: $K_1 = K_2 = I,\ \ \mu_1 = 2.56$ and $\mu_2 = 3.74$. Additionally, the selected simulation time is 10 sec for all the obtained figures. Now, based on MATLAB's LMI Control Toolbox, the feasible solutions of the LMIs constraints (4) for two subsystems are found as follows

$$R_{11} = \begin{bmatrix} 37.1298 & -132.2244 & 15.2150 \\ -132.2244 & -300.4981 & 15.2879 \\ 15.2150 & 15.2879 & 1.3491 \end{bmatrix},\ R_{21} = 0.6238, \tag{42}$$

and

$$R_{12} = \begin{bmatrix} 197.7259 & 132.5157 & -149.3973 \\ 132.5157 & 69.4260 & -149.5495 \\ -149.3973 & -149.5495 & 1.4154 \end{bmatrix}, R_{22} = 0.3796. \quad (43)$$

From LMI's results (42) and (43), the single phase switching surfaces of subsystems (2) for the initial time $t = 0$ are

$$\begin{aligned} \sigma_1(y_1(t),t) &= [1.6031 \ \ \text{-}1.6031]y_1 \\ &\quad -[1.6031 \ \ \text{-}1.6031]y_1 e^{-2.56t} = 0, \end{aligned} \quad (44)$$

and

$$\begin{aligned} \sigma_2(y_2(t),t) &= [2.6344 \ \ \text{-}2.6344]y_2 \\ &\quad -[2.6344 \ \ \text{-}2.6344]y_2 e^{-3.74t} = 0. \end{aligned} \quad (45)$$

Now, the decentralized state ROEs (15) corresponding to the researched subsystems are, respectively, established as

$$\dot{\hat{z}}_1 = \begin{bmatrix} -2.3509 & -0.7297 \\ 0.2542 & -0.1491 \end{bmatrix} \hat{z}_1 + \begin{bmatrix} -1.0473 & 1.0473 \\ -0.0849 & 0.0849 \end{bmatrix} y_1, \quad (46)$$

and

$$\dot{\hat{z}}_2 = \begin{bmatrix} -0.8658 & -0.0257 \\ 0.6675 & -0.6842 \end{bmatrix} \hat{z}_2 + \begin{bmatrix} -0.1172 & 0.1172 \\ 1.9307 & -1.9307 \end{bmatrix} y_2. \quad (47)$$

The upper bounds of estimation error dynamics of subsystems are designed, respectively

$$\dot{\Theta}_1(t) = -0.2470\Theta_1(t) + 0.0166\|y_1\|, \quad (48)$$

and

$$\dot{\Theta}_2(t) = -0.6981\Theta_2(t) + 0.0245\|y_2\|. \quad (49)$$

From Theorem 3.2, the single phase switching surfaces (44) and (45), decentralized state ROE tool (46) and (47), upper bound of estimation error dynamics (48) and (49), the single phase sliding mode control laws for the first and the second subsystems are specified as below

$$\begin{aligned} u_1 &= -0.0094\sigma_1 - \Big[3.9157\big(\|\hat{z}_1\| + \Theta_1(t)\big) + 4.4760\|y_1\| \\ &\quad + 0.2100 + 4.2427e^{-2.56t}\Big]\frac{\sigma_1}{\|\sigma_1\|}, \end{aligned} \quad (50)$$

and

$$\begin{aligned} u_2 &= -0.0057\sigma_2 - \Big[2.6906\big(\|\hat{z}_2\| + \Theta_2(t)\big) + 7.1630\|y_2\| \\ &\quad + 0.2100 + 4.2427e^{-3.74t}\Big]\frac{\sigma_2}{\|\sigma_2\|}. \end{aligned} \quad (51)$$

The obtained results are depicted in Figures 4-11, which demonstrated the effectiveness of the proposed method. In addition, these results are also carefully analysed and explained in Remarks 4.1-4.5.

Remark 4.1 The state trajectories of subsystem I and subsystem II are displayed in Figure 4 and Figure 5, respectively. The initial conditions are different from the simulation when the proposed method is employed. From these figures, we can see that the state variables for each subsystem are rapidly convergent to zero after about 2.0 seconds and 3.2 seconds, respectively. In other words, when the designed controllers (50) and (51) are used, the trajectories reach the switching surface $\sigma_i(y_i(t),t) = 0$ at the beginning of the motion $(t = 0)$ where the published work (Hu and Zhao, 2014; Huynh et al., 2018; Ma et al., 2015; Spurgeon and Yan, 2014; Sun et al., 2014; Wu, 2016; Xue et al., 2015; Yan et al., 2012) could not get the achievements.

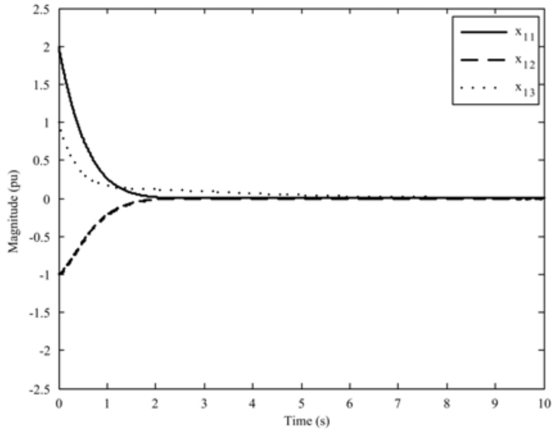

Fig. 4. Time responses of state x_{11} (solid), x_{12} (dashed), and x_{13} (dotted) of subsystem I.

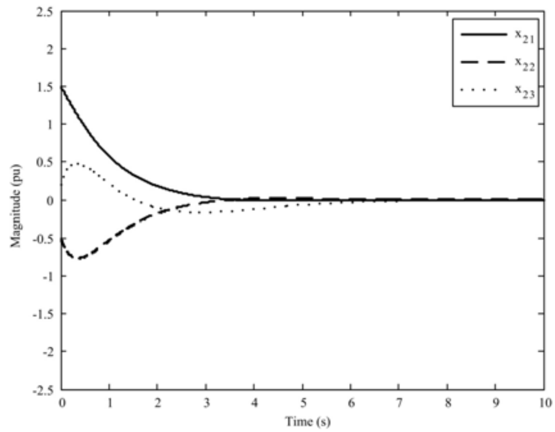

Fig. 5. Time responses of state x_{21} (solid), x_{22} (dashed), and x_{23} (dotted) of subsystem II.

Remark 4.1 The state trajectories of subsystem I and subsystem II are displayed in Figure 4 and Figure 5, respectively. The initial conditions are different from the simulation when the proposed method is employed. From these figures, we can see that the state variables for each subsystem are rapidly convergent to zero after about 2.0 seconds and 3.2 seconds, respectively. In other words, when the designed controllers (50) and (51) are used, the trajectories reach the switching surface $\sigma_i(y_i(t),t) = 0$ at the beginning of the motion $(t = 0)$ where the published work (Hu and Zhao, 2014; Huynh et al., 2018; Ma et al., 2015; Spurgeon and Yan, 2014; Sun et al., 2014; Wu, 2016; Xue et al., 2015; Yan et al., 2012) could not get the achievements.

Remark 4.2 The switching surfaces $\sigma_1(t)$ (44) for subsystem I and $\sigma_2(t)$ (45) for subsystem II shown in Figure 6 and Figure 7 are indicated the removal of reaching phase, respectively. From these figures, it is clear to observe that the system's state moves into switching surface from the initial time whose the reaching time is equal to zero $(t=0)$. Consequently, the robustness has been improved. This is an obvious advantage of the proposed technique over the traditional variable structure control theory and it is the first main contribution of our work for the control field.

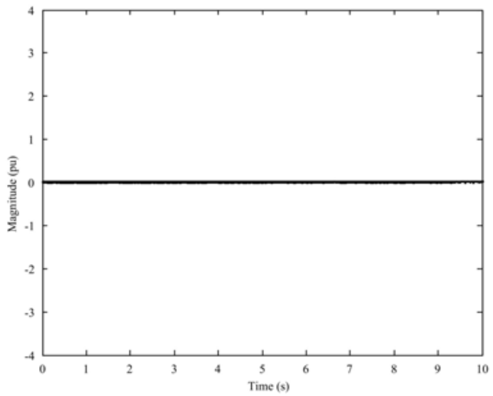

Fig. 6. The trajectory of the sliding function $\sigma_1(t)$ (44) of subsystem I.

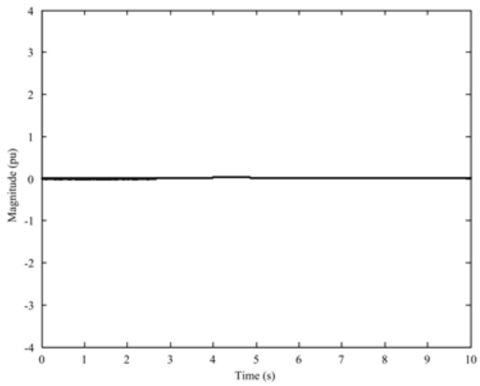

Fig. 7. The trajectory of the sliding function $\sigma_2(t)$ (45) of subsystem II.

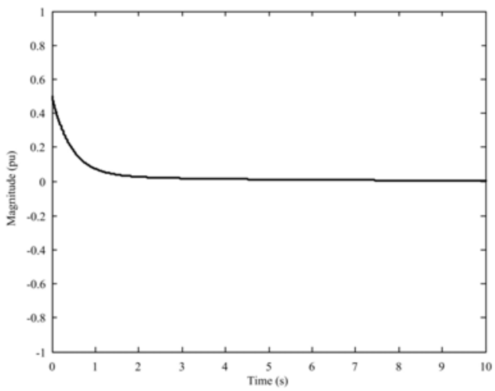

Fig. 8. Time response of error dynamics $e_1(t)$ (21) of subsystem I.

Remark 4.3 The second major achievement consists of the estimation errors of the ROE, $e_1(t)=\hat{z}_1(t)-z_1(t)$ and $e_2(t)=\hat{z}_2(t)-z_2(t)$, that are exposed in Figure 8 and Figure 9, respectively. It can be seen that the errors are exactly regulated to zero by the suggested state observers (46) and (47). The response time of error dynamics reaches zero after about 2.0 seconds. These figures confirm that the estimated variables $\hat{z}_i(t)$ approach to the actual variables of systems $z_i(t)$. Furthermore, the upper bounds of these estimation errors are obtained as equations (48) and (49), whose dynamics are stable.

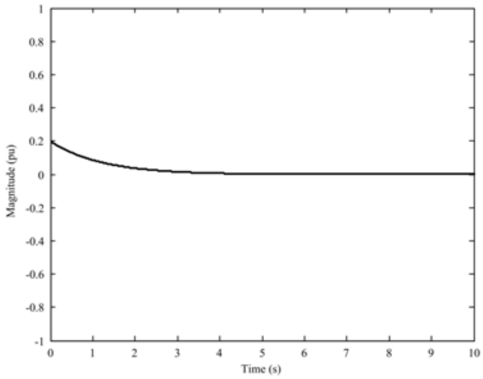

Fig. 9. Time response of error dynamics $e_2(t)$ (21) of subsystem II.

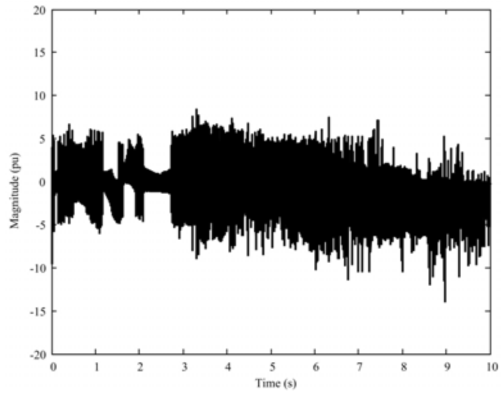

Fig. 10. The time history of the control $u_1(t)$ (50) of subsystem I.

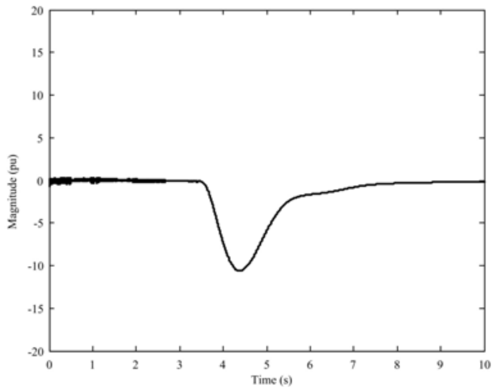

Fig. 11. The time history of the control $u_2(t)$ (51) of subsystem II.

Remark 4.4 The trajectories of control signals $u_1(t)$ and $u_2(t)$ are illustrated in Figure 10 and Figure 11, respectively. For subsystems I and II, the magnitudes of control signals are high but acceptable. This is due to the control input depends on the constant gains $\hat{\varepsilon}_{1_i}$, $\hat{\varepsilon}_{2_i}$, $\hat{\varepsilon}_{3_i}$ and magnitude of $(\|\hat{z}_1\|+\Theta_1(t))$. Clearly, it can be seen that the proposed control law is able to solve complex systems with unknown disturbances and mismatched interconnections without reaching phase. This approach provides better performance than the other methods published in (Huynh et al., 2018; Mobayen and Tchier, 2016; Pervaiz et al., 2014; Pervaiz et al., 2015). This is also the third key task of this study.

Remark 4.5 Although the sliding mode controller guarantees robustness performance, its control signals in Figure 7 and Figure 8 show high frequency oscillations called chattering phenomenon occurs from the initial time of process. The trajectories of state rapidly oscillate around the switching surface. Chattering is caused due to the discontinuous switching function feeding to the system directly. The amplitude of discontinuous control signal is larger when the uncertainty is too large and hence chattering oscillation is increased. This chattering is one of negative effects in real world applications (e.g., backlash in mechanical systems, high heat losses in electrical power circuits); it can lead to low control accuracy, abortive depreciation or even degradation of the system performances being applied to. To get rid of this problem, there are several approaches to prevent the effects of undesirable chattering in the system such as using a saturation function approximation (Chen and Peng, 2006; Zheng et al., 2014) instead of the shifted sigmoid function, inserting a boundary layer (Bartoszewicz and Nowacka-Leverton, 2007) near the sliding plane, a second order sliding mode (Huynh et al., 2018; Din et al., 2017), higher order sliding mode control (Estrada and Fridman, 2010; Estrada and Fridman, 2011) and a fuzzy logic technique (Yau and Chen, 2006). These approaches, however, may not guarantee the stability to the sliding mode, the robustness properties of controller, the performance of system and only apply to standard sliding mode control which involves reaching phase. In addition, chattering elimination is outside the scope of this study. The key objective of this paper only is to design the controller for complex uncertain interconnected systems without reaching phase. Therefore, this disadvantage of chattering will be solved in future work.

From the aforementioned discussion of simulated results, it is clear to observe that the sliding mode exists from the initial time of process $(t = 0)$. Also, it can be indicated that the enhanced robustness and the desired dynamic response are obtained by eliminating reaching phase that has reduced the limitations required in other studies (Bartoszewicz and Nowacka-Leverton, 2007; Bejaranoet al., 2009; Huynh et al., 2018; Khan et al., 2011; Pervaiz et al., 2014; Pervaiz et al., 2015; Xi and Hesketh, 2010).

5. CONCLUSIONS

This paper represents the novel VSC scheme, which eliminates completely the reaching phase and uses output variables only, for the mismatched uncertain systems with extraneous disturbances and interconnections. We have proposed the new switching surface such that the reaching time is equal to zero and the system is insensitive to the external perturbation and uncertainty. The ROE has been established to estimate the unmeasurable states for supporting the controller design. The novel SPVSC has been constructed by employing the ROE and Moore-Penrose inverse technique for mismatched uncertain interconnected systems. Enhanced robustness and the desired dynamic response are obtained by the elimination of reaching phase that has reduced the limitations required in other work. In addition, the sufficient condition has been given by utilizing Razumikhin–Lyapunov approach such that the motion dynamics in sliding mode possess the property of asymptotical stability. Besides, the numerical simulation has been performed by Matlab software for showing the feasibility of the proposed method. From proved theorems and obtained simulation results, it is clearly shown that the proposed SPRVSC is effective in handling mismatched uncertainties, mismatched interconnections, and unknown disturbances without reaching phase. Hence, the extension of the presented approaches to other more complex uncertain systems such as the developed system in discrete-time could be the future trend.

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