Predictive Sliding Mode Control Based on Laguerre Functions ⋆

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Abstract: This paper deals with Predictive Sliding Mode Control (PSMC) that uses Laguerre functions in the design of a control input signal. Two types of PSMC algorithms are considered: one originating from the digital equivalent control method approach, and another containing an additional sliding mode control component that provides the robustness and determines the system dynamics in reaching mode. A one-step-delayed disturbance estimator is introduced to account for system nonlinearities and unknown disturbances, as well as to ensure better system steady-state accuracy. The proposed algorithms are demonstrated by conducting several real-time experiments on a modular DC servo system. Robustness of the closed loop, affected by tuning parameter values, is demonstrated as well.

Keywords: Model Predictive Control, Sliding Mode Control, Predictive Sliding Mode Control, DC servo motor, one-step-delayed disturbance estimator

1. INTRODUCTION

Sliding mode control (SMC) is a particular class of variable structure control, characterized by robustness to external disturbances and parameter variations (Utkin, 1978; Young et al., 1999; Yu and Kaynak, 2009). SMC drives a system state along predefined sliding surface, determined by a SMC switching function. Thus, the effective system dynamics is of lower order and robust performance is achieved. When SMC is realized by using micro-controllers or digital signal processors, a quasi-sliding motion (Utkin, 1977; Milosavljević, 1985) arises in an $O(T)$ vicinity of the sliding surface, where T is a sampling period. A review of digital SMC algorithms can be found in (Milosavljević, 2004; Yu et al., 2011). The discretization process may produce chattering that excites non-modelled system dynamics and increases maintenance costs. To eliminate the chattering phenomenon, several approaches are recommended in (Bartoszewicz, 1998; Golo and Milosavljević, 2000; Bartoszewicz and Leśniewski, 2016; Leśniewski and Bartoszewicz, 2016).

One simple way to design a chattering-free digital SMC is to use a digital equivalent control algorithm that forces the system state to reach the sliding surface at the very next sampling instant (Su et al., 2000). This digital SMC provides $O(T^2)$ sliding mode accuracy when the system disturbances are either known or when a one-step delayed disturbance estimator is used (Milosavljević et al., 2004). It belongs to the class of deadbeat control algorithms (Ignaciuk and Bartoszewicz, 2012), characterized by the use of large control input signals, often beyond the control saturation limits. Furthermore, if the model is inaccurate (which is often the case), the effective disturbance may depend on the model input. In such cases one may experience chattering or oscillations, or even instability. Similar system behaviour also occurs if traditional SMC with a relay control component is applied. Therefore, model predictive control (MPC) seems to be a good candidate for overcoming these SMC imperfections i.e. to deal with constraints and to stabilize the system in the presence of the control input dependent disturbances.

MPC has been a great industrial success, particularly in the process industries (Qin and Badgwell, 2003) and can be also implemented in the control of other complex systems (Sgaverdea et al., 2015; Bojan-Dragos et al., 2015; Duţescu et al., 2017; Chelladurai et al., 2017). Still, robustness of the MPC controllers continues to be an active research issue. There are many approaches to robust MPC, including dynamic programming, optimization over feedback policies, min-max MPC, Tube MPC (Mayne et al., 2006; Rawlings and Mayne, 2009; Benlaoukli et al., 2008), as well as the combination of SMC and MPC.

The SMC based on generalized predictive control (GPC) is considered in (Corradini and Orlando, 1997; Mitić et al., 2013b). In (Garcia-Gabin et al., 2009), the SMC component is not treated as a part of the optimization problem

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at all, and the reaching and existence conditions of the sliding mode, as well as system stability are not discussed. Only the predictive part of controller is derived by optimizing the cost function. Hierarchical control schemes, consisting of a high level MPC and a low level SMC, are considered in (Incremona et al., 2015; Benattia et al., 2015; Rubagotti et al., 2011a,b), where the SMC component rejects the matched disturbances acting the plant, and reduces uncertainty for the MPC design in that way. In (Spasić et al., 2016), SMC is chosen to be the auxiliary controller in Tube MPC, due to its trivial computational requirements and good robustness properties. To cope with the problem of system instability, when the control input dependent disturbances act, two different approaches are given in (Neelakantan, 2005), which integrate SMC and MPC concepts. In the first one, direct optimization of the cost function with respect to the digital equivalent control is proposed. In the second control method, the cost function is optimized with respect to the reaching control part that guides the system towards the sliding surface. Once the system state reaches the sliding surface, it stays on it thanks to the equivalent control term.

In this paper, the predictive sliding mode control (PSMC) is based on using Laguerre functions, unlike the control approaches presented in (Neelakantan, 2005). The implementation of these functions for the approximation of control input signal is justified due to faster solving of the optimization problem. Laguerre functions are chosen among other orthogonal functions due to their smoothness and easiness for tuning of closed-loop system performance by only two parameters (N - the number of the Laguerre terms, and a – the pole of the Laguerre network). There are no fast and steep changes in the control signal and the decay rate of the incremental control signal is determined by the parameter a. The traditional MPC is a particular case of this control approach when $a = 0$. (Wang, 2009, 2001; Barry and Wang, 2004; Bavili et al., 2015). Laguerre functions have been already used in the design of Tube MPC presented in (Spasić et al., 2017) where MPC controls the nominal system, and SMC is introduced as an auxiliary controller.

Two types of PSMC are considered herein. The first one is based on the switching function predictive model derived from (Su et al., 2000). The second one uses the predictive model of the switching function dynamics obtained from the chattering free reaching law method for digital SMC (Golo and Milosavljević, 2000). In both cases, the Laguerre functions compose the control components that are the analogues to the digital equivalent control of SMC, and the cost functions are optimized with respect to the coefficients of Laguerre functions. In the latter control approach, the system dynamics in reaching mode is fully determined and the system state attains the sliding mode in finite number of steps comparing to the second control method presented in (Neelakantan, 2005). The system robustness and steady-state accuracy are improved as well and even become better after introducing the one-step delayed disturbance estimator. The disturbance estimator can be avoided by using the reaching laws with higher relative degree switching functions (Bartoszewicz and Leśniewski, 2016; Bartoszewicz and Latosinski, 2018) at the price of lower accuracy.

The paper is organized as follows. The problem formulation is given in Section 2, first the augmented plant model is defined, then two conventional digital sliding mode control approaches are briefly described, and at the end the Laguerre functions based MPC is explained. The proposed PSMCs based on Laguerre functions are developed in Section 3. The experimental results are presented and discussed in Section 4 where the proposed algorithms are applied to a real DC servo system (Inteco, 2011). Section 5 contains some concluding remarks.

2. PROBLEM FORMULATION

2.1 Mathematical Model of Plant

Consider a discrete-time state-space model of plant given by

$$x_{k+1} = Ax_k + Bu_k + d_k, \tag{1}$$

$$y_k = Cx_k \tag{2}$$

where $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, and $d_k \in \mathbb{R}^{n_x}$ represent vectors of system state, control input signals and disturbances, respectively. To introduce integral action in the controller, the following augmented state-space model is obtained

$$x_{e,k+1} = A_e x_{e,k} + B_e \Delta u_k + \delta_k \tag{3}$$

$$y_k = C_e x_{e,k} \tag{4}$$

where the control increment $\Delta u_k = u_k - u_{k-1}$ is used as an optimization variable and

$$x_{e,k} = \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix}; \delta_k = \begin{bmatrix} I \\ 0 \end{bmatrix} d_k; \tag{5}$$

$$A_e = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}; B_e = \begin{bmatrix} B \\ I \end{bmatrix}; C_e = [C \ \ 0]. \tag{6}$$

It is assumed that the pair (A_e, B_e) is controllable.

2.2 Sliding Mode Control

The design procedure for the sliding mode control of discrete-time systems can be carried out in two steps. The first one involves the selection of a switching function

$$S_k = Gx_{e,k} \tag{7}$$

where $rank(G) = n_u$ and n_u is a number of control inputs. Note that $S_k = 0$ denotes a so-called sliding surface, also known as a sliding manifold, defining the system behaviour in a desired manner. In the second step, the discrete-time control signal Δu_k is selected such that the reaching and existence conditions of sliding mode are satisfied.

In order to determine the system dynamics in sliding mode, the equivalent control method is used (Draženović, 1969; Utkin, 1978). Starting with $S_{k+1} = Gx_{e_{k+1}} = 0$, taking into account (3), the equivalent control input signal is obtained in the form of

$$\Delta u^{eq}{}_k = -(GB_e)^{-1}G(A_e x_{e,k} + \delta_k). \tag{8}$$

By implementing (8) in (3), the system dynamics in sliding mode is defined by

$$x_{e,k+1} = (A_e - B_e(GB_e)^{-1}GA_e)x_{e,k}. \quad (9)$$

in the absence of any disturbance. If the system state is close to the sliding manifold, the equivalent control can drive the system along the sliding surface. However, since δ_k is not available, a one-step-delayed estimator obtained from (3) as

$$\hat{\delta}_k = \delta_{k-1} = x_{e,k} - A_e x_{e,k-1} + B_e \Delta u_{k-1} \quad (10)$$

is usually utilized in design of Δu_k

$$\Delta u_k = -(GB_e)^{-1}G(A_e x_{e,k} + \hat{\delta}_k). \quad (11)$$

Substituting (11) in (3), using (7), the switching function dynamics is described by

$$S_{k+1} = G(\delta_k - \delta_{k-1}). \quad (12)$$

This control approach belongs to the class of deadbeat controllers with some drawbacks. If the control input signal (11) is not saturated, it will drive the system state to the sliding manifold at one sampling period T. When the saturation happens, caused by high calculated values of the control signal, the system dynamics in reaching mode and reaching time will not be predefined. The oscillatory motion may also occur in perturbed system (when the model used is in error) due to such large control input values, since the system is overcompensated and the system state crosses the sliding surface at the very next time instants. Finally, if the disturbance depends on the control input

$$\delta_k = \delta^*{}_k + (\Delta B_e)\Delta u_k \quad (13)$$

where ΔB_e represents the error in B_e and $\delta^*{}_k$ is independent of the control input, the system can either go unstable or produce chattering (Su et al., 1993, 1996). In the latter case, the switching function dynamics is defined by

$$S_{k+1} = -G\Delta B_e(GB_e)^{-1}[2S_k - S_{k-1}] + O(T^2) \quad (14)$$

and for significant error in the control matrix ΔB_e, the poles of the equation (14) can be outside the unit disk in the z-plane. However, it is typically neither necessary nor optimal to force the system to reach the sliding mode at the very next instant. Therefore, in order to avoid high control input signals and to determine switching function dynamics in reaching mode, the additional control component is introduced into the control algorithm (Golo and Milosavljević, 2000) yielding

$$\begin{aligned}\Delta u_k = & -(GB_e)^{-1}G(A_e x_{e,k} + \hat{\delta}_k - x_{e,k} \\ & + min(S_k, diag(K)sign(S_k))). \quad (15)\end{aligned}$$

Substituting (15) in (3), using (7), the switching function dynamics is now defined as

$$\begin{aligned}S_{k+1} = S_k & - min(S_k, diag(K)sign(S_k)) \\ & + G(\delta_k - \delta_{k-1}) \quad (16)\end{aligned}$$

where $K \in \mathbb{R}^{n_u}$ is a gain vector. The control signal (15) will drive the system state to the sliding manifold in a finite number of sampling instants if

$$K > \Delta \quad (17)$$

where

$$|G(\delta_k - \delta_{k-1})| < \Delta \quad (18)$$

and Δ is a positive vector (Mitić et al., 2013a).

Unfortunately, if the disturbance has the form of (13), the system may become unstable in this case, as well. In order to cope with the later issue and to incorporate and solve the problem of control signal saturation in SMC design, model predictive control (MPC) is used due to its ability to deal with constraints and stability problem caused by model error. In this manuscript, unlike the algorithm described in (Neelakantan, 2005), Laguerre functions based MPC is proposed to allow faster solving of the optimization problem.

2.3 Laguerre functions based MPC

The Laguerre functions can be expressed as

$$L_k(z) = \frac{(z^{-1} - a)}{(1 - az^{-1})} L_{k-1}(z) \quad (19)$$

with

$$L_{k-1}(z) = \frac{\sqrt{(1-a^2)}}{1 - az^{-1}} \quad (20)$$

where $0 \leq a < 1$ is the pole of the Laguerre network. A difference equation, defining discrete-time Laguerre functions, is given by

$$L_{k+1} = A_l L_k \quad (21)$$

where

$$L_k = [l_{1k} \quad l_{2k} \quad ... \quad l_{Nk}]^T. \quad (22)$$

is a vector of discrete-time Laguerre functions representing the inverse z-transformations of $L_k(z)$ for $i = 1, N$, where N is the number of Laguerre terms, and A_l is a matrix defined by

$$A_l = \begin{bmatrix} a & 0 & 0 & ... & 0 \\ \beta & a & 0 & ... & 0 \\ -a\beta & \beta & a & ... & 0 \\ -a^{N-2}\beta & -a^{N-3}\beta & ... & -a^{N-N}\beta & a \end{bmatrix} \quad (23)$$

where a and $\beta = 1 - a^2$ are adjustable parameters of the Laguerre functions, and $L(0)$ is determined by

$$L(0)^T = \sqrt{\beta}[1 \quad -a \quad a^2 \quad -a^3 \quad ... \quad (-1)^{N-1}a^{N-1}]. \quad (24)$$

Equation (21) is obtained by implementing the inverse z-transformation on the Laguerre networks defined by (19) and (20). The control increment Δu_{k+m} can be composed of a set of Laguerre functions (Wang, 2009), $l_{1k}, l_{2k}, l_{3k}, ..., l_{N_p k}$, as

$$\Delta u_{k+m} = \sum_{j=1}^{N} c_{j\,k} l_{j\,m}. \quad (25)$$

Here the index m denotes a future sampling instant and $c_{j\,k} (j = \overline{1, N})$ are the Laguerre coefficients at time instant k. Eq. (25) can be written in vector form

$$\begin{aligned}\Delta u_{k+m} &= L_m{}^T \eta_k; \quad (26) \\ \eta_k &= [c_{1k} \quad c_{2k} \quad ... \quad c_{Nk}].\end{aligned}$$

In order to obtain the control increment Δu_k, the optimal Laguerre parameter vector η_k has to ensure that constraints on the input and its rate of change are fulfilled. These are defined by

$$\Theta\eta_k \leqslant \Pi \tag{27}$$

where

$$\Theta = \begin{bmatrix} M_1 \\ -M_1 \\ M_2 \\ -M_2 \end{bmatrix}; \quad \Pi = \begin{bmatrix} \Delta u^{max} \\ -\Delta u^{min} \\ u^{max} - u_{k-1} \\ -u^{min} + u_{k-1} \end{bmatrix} \tag{28}$$

and

$$M_1 = \begin{bmatrix} {L_{m_1}}^T & {0_2}^T & ... & {0_m}^T \\ {0_1}^T & {L_{m_2}}^T & ... & {0_m}^T \\ \vdots & \vdots & \ddots & \vdots \\ {0_1}^T & {0_2}^T & ... & {L_{m_{n_u}}}^T \end{bmatrix}; \tag{29}$$

$$M_2 = \begin{bmatrix} \sum_{j=0}^{i-1} {L_{j_1}}^T & {0_2}^T & ... & {0_m}^T \\ {0_1}^T & \sum_{j=0}^{i-1} {L_{j_2}}^T & ... & {0_m}^T \\ \vdots & \vdots & \ddots & \vdots \\ {0_1}^T & {0_2}^T & ... & \sum_{j=0}^{i-1} {L_{j_{n_u}}}^T \end{bmatrix}$$

The time instants at which the constraints are imposed on Δu_k are represented by $m_1, \cdots, m_{n_u}$.

The implementation of Laguerre functions in the design procedure of PSMC will be described in the next section.

3. PREDICTIVE SMC

The PSMC design based on the equivalent control method is considered first. The future values of the switching function can be obtained by extending (7) within the prediction horizon as follows

$$\begin{aligned} S_{k+1} &= Gx_{e,k+1} = G(A_e x_{e,k} + B_e \Delta u_k + \delta_k) \\ S_{k+2} &= Gx_{e_{k+2}} = G(A_e x_{e,k+1} + B_e \Delta u_{k+1} + \delta_{k+1}) \\ &= G({A_e}^2 x_{e,k} + A_e B_e \Delta u_k + A_e \delta_k + B_e \Delta u_{k+1}) \\ &. \\ &. \\ &. \\ S_{k+N_p} &= Gx_{e_{k+N_p}} \\ &= G(A_e x_{e,k+N_p-1} + B_e \Delta u_{k+N_p-1} + \delta_{k+N_p-1}) \\ &= G({A_e}^{N_p} x_{e,k} + \sum_{i=0}^{N_p-1} {A_e}^i B_e \Delta u_{k+N_p-i-1}) \\ &+ G \sum_{i=0}^{N_p-1} {A_e}^i \delta_{k+N_p-i-1} \end{aligned} \tag{30}$$

By substituting (26) into (30) one obtain the prediction of the future switching function as

$$\begin{aligned} S_{k+m} &= G{A_e}^m x_{e,k} + G(\sum_{i=0}^{m-1} A_e^{m-i-1} B_e {L_i}^T)\eta_k \\ &+ G(\sum_{i=0}^{m-1} {A_e}^i \delta_{k+m-i-1}). \end{aligned} \tag{31}$$

The design goal is to find optimal η_k by minimising a cost function

$$J = \sum_{m=1}^{N_p} {S_{k+m}}^T S_{k+m} + {\eta_k}^T R \eta_k \tag{32}$$

subject to the constraints (27), where $R > 0$ is a weighting matrix. The control input increment Δu_k is then calculated using (26). Unfortunately, the latter control input does not define the system dynamics in reaching mode. That is why PSMC, based on the control approach described by (15), is considered in the sequel. One should expand the difference $\Delta S_{k+m} = S_{k+m} - S_{k+m-1}$ in the prediction horizon ($m = \overline{1, N_p}$) first. This gives

$$\begin{aligned} \Delta S_{k+1} &= Gx_{e,k+1} - Gx_{e,k} + G\delta_k \\ &= G(A_e - I)x_{e,k} + GB_e \Delta u_k + G\delta_k \\ \Delta S_{k+2} &= Gx_{e_{k+2}} - Gx_{e,k+1} \\ &= G(A_e x_{e,k+1} + B_e \Delta u_{k+1} + \delta_{k+1}) \\ &- G(A_e x_{e,k} + B_e \Delta u_k + \delta_k) \\ &= G(A_e - I)A_e x_{e,k} + G(A_e - I)B_e \Delta u_k \\ &+ GB_e \Delta u_{k+1} + G(A_e - I)\delta_k + G\delta_{k+1} \\ &. \\ &. \\ &. \\ \Delta S_{k+N_p} &= Gx_{e_{k+N_p}} - Gx_{e_{k+N_{p-1}}} \\ &= G(A_e - I){A_e}^{N_{p-1}} x_{e,k} \\ &+ G \sum_{i=0}^{N_p-1} {A_e}^i B_e \Delta u_{k+N_p-i-1} \\ &- G \sum_{i=0}^{N_p-2} {A_e}^i B_e \Delta u_{k+N_p-i-2} \\ &+ G \sum_{i=0}^{N_p-1} {A_e}^i \delta_{k+N_p-i-1} \\ &- G \sum_{i=0}^{N_p-2} {A_e}^i \delta_{k+N_p-i-2} \end{aligned} \tag{33}$$

By substituting (26) into (33), the later equation can be rewritten as

$$\Delta S_{k+m} = G(A_e - I){A_e}^{m-1}x_{e,k} + G\sum_{i=0}^{m-1} A_e^{m-i-1}B_e{L_i}^T\eta_k - G\sum_{i=0}^{m-2} A_e^{m-i-2}B_e{L_i}^T\eta_k + G\sum_{i=0}^{m-1} {A_e}^i\delta_{k+m-i-1} - G\sum_{i=0}^{m-2} {A_e}^i\delta_{k+m-i-2}. \quad (34)$$

within the prediction horizon. Now, the desired control should be calculated by minimizing the cost function

$$J = \sum_{m=1}^{N_p}(\Delta S_{k+m} + min(S_{k+m}, diag(K)sign(S_{k+m}))^T (\Delta S_{k+m} + min(S_{k+m}, diag(K)sign(S_{k+m})) + {\eta_k}^T R\eta_k \quad (35)$$

with respect to η_k subject to the constraints (27). Again, the control input increment Δu_k is calculated using (26).

Notice that, if $R = 0$, the minimization of (35) will lead to the predictive model of switching function dynamics (16), represented by the following form

$$S_{k+m+1} = S_{k+m} - min(S_{k+m}, diag(K)sign(S_{k+m})) + G(\delta_{k+m} - \delta_{k+m-1}) \quad (36)$$

for $m = \overline{2, N_p}$. If K is selected according to (17), the sliding manifold will be reached in a finite time within the prediction horizon (Spasić et al., 2016). By choosing an input weight $R > 0$ of significant magnitude, the stability problem is solved in presence of the disturbance depending on the control input signal.

The both optimization problems, (32) and (35), can be handled by a number of optimization routines (Boyd and Vandenberghe, 2004). The one used herein, which includes the Kuhn-Tucker conditions (Luenberger, 2003), together with Hildreth's algorithm (Hildreth, 1957), is described in detail in (Wang, 2009).

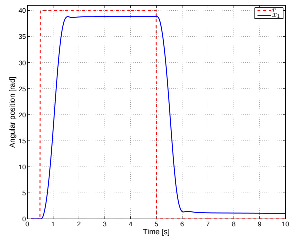
4. EXPERIMENTAL RESULTS

The modular servo system (Inteco, 2011) is used for the demonstration of proposed control algorithms. The transfer function of the DC servo system is

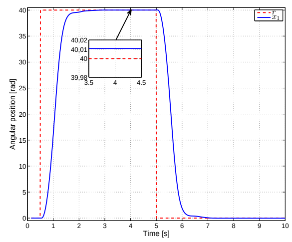
$$G(s) = \frac{\theta(s)}{u(s)} = \frac{K_s}{s(T_s s + 1)} \quad (37)$$

where $K_s = 184.95$ rad/s, $T_s = 0.9$ s, and the system states are angular position $\theta = x_1$, and angular velocity $\omega = x_2$. It is assumed that the control signal is dimensionless scaled input voltage, $u(t) = v(t)/v_{max}$ where $v_{max} = 12$ V which satisfies $|u(t)| \leq 1$.

The discretization is done using Matlab function *c2d.m*, with the sampling time $T = 0.01s$, and the following augmented state-space model is obtained



(a) The angular position x_1 for PSMC based on equivalent control method (without the one-step-delayed estimator).



(b) The angular position x_1 for PSMC based on equivalent control method (with the one-step-delayed estimator).

Fig. 1. The angular position x_1.

$$A_e = \begin{bmatrix} 1 & 0.0099 & 0.0102 \\ 0 & 0.9890 & 2.0437 \\ 0 & 0 & 1 \end{bmatrix}; B_e = \begin{bmatrix} 0.0102 \\ 2.0437 \\ 1 \end{bmatrix}; C_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T.$$

The parameters for design of the Laguerre functions based PSMC described in Section 3 are: the prediction horizon $Np = 30$, the number of Laguerre terms $N = 5$, Laguerre functions parameter $a = 0.15$, the switching function parameter $G = [-0.0358 \quad -0.0071 \quad 0]$. Notice that the proposed approach uses 4 times less parameters in comparison to traditional MPC (Spasić et al., 2016) and, consequently, the optimization problem is solved faster. The reference signal r is defined by

$$r = \begin{cases} 0 \text{ if Time} < 0.5s \\ 40 \text{ if } 0.5s \leq \text{Time} \leq 5s \\ 0 \text{ if Time} > 5s \end{cases}. \quad (38)$$

The constraints on the control signal and its increment are defined as

$$-1 \leq u_k \leq 1; \quad -0.2 \leq \Delta u_k \leq 0.2. \quad (39)$$

The first type of proposed PSMC based on equivalent control is applied to the DC servo system and two experiments are conducted in order to compare the performance of the system with and without the one-step-delayed estimator. The results are shown in Figure 1. It can be seen that the steady state accuracy is better when the one-step-delayed estimator is used in the design of the proposed control law.

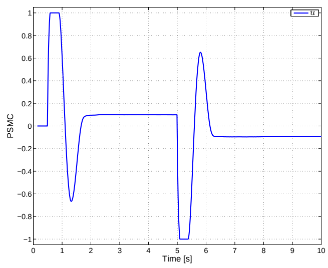
The control signals, Figure 2, and the corresponding control increments, Figure 3, respect the constraints defined by (39) in both cases. One can notice that in the steady state, the control signal is not equal to zero. That happens because of the Coulomb friction and it is in the range of control signal from -0.15 to 0.15. From the previous set of experiments, it is concluded that the one-step-delayed disturbance estimator is needed to improve the system response accuracy.

Then, the second type of PSMC, with the additional SMC term in the form of

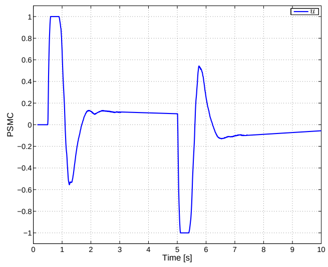
$$min(S_k, diag(K)sign(S_k)),$$

is implemented. In the first experiment, the second type of PSMC without the estimator is used to demonstrate the ability of additional control term to reject the disturbance as good as in the case when the PSMC based on the digital equivalent control with the estimator is implemented. The additional control component parameter value was $K = 0.1$. The system output response is depicted in Figure 4(a). In the next experiment, the second type of PSMC is used with the estimator providing the zero steady-state error, which is presented in Figure 4(b). The constraints, (39), are respected by the control signal and its increment which is shown in Figures 5 and 6, respectively.

In order to show the effectiveness of the proposed PSMC, two additional experiments are performed. It is demonstrated how the choice of the tuning parameter R affects the robustness of the closed loop. The PSMC is using a model where the value of B_d is 40 % larger than the true value. Figure 7(a) shows that the system is unstable with the tuning factor $R = 0.1$, while in Figure 7(b) it is stable and has good performance when choosing $R = 10$. Figure 8 illustrates system robustness when the additional SMC term is used. Stable system with the oscillations in the output response with $R = 0.001$ is presented in Figure 8(a). In order to suppress the oscillations, tuning factor is chosen to be $R = 0.1$, which is demonstrated in Figure 8(b).

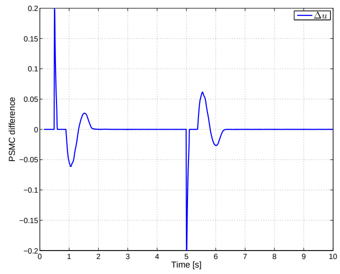


(a) PSMC u based on equivalent control method (without the one-step-delayed estimator).

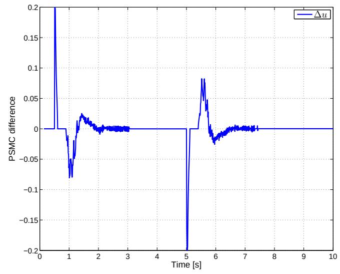


(b) PSMC u based on equivalent control method (with the one-step-delayed estimator).

Fig. 2. PSMC signal u.

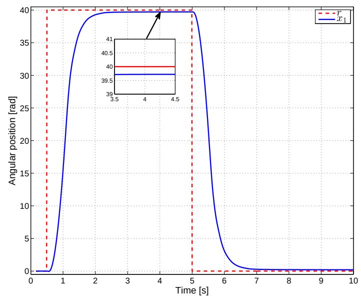


(a) PSMC increment Δu based on equivalent control method (without the one-step-delayed estimator).

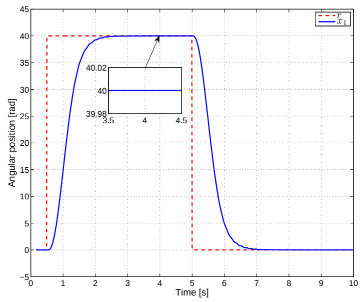


(b) PSMC increment Δu based on equivalent control method (with the one-step-delayed estimator).

Fig. 3. PSMC signal increment Δu.

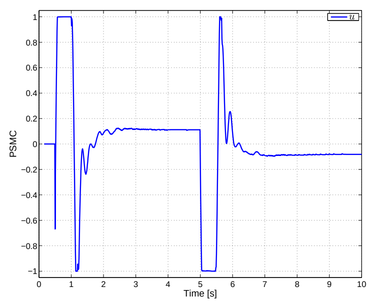


(a) The angular position x_1 for PSMC with the additional SMC term (without the one-step-delayed estimator).

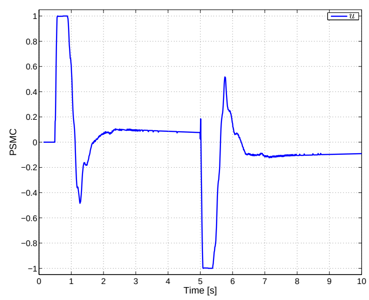


(b) The angular position x_1 for PSMC with the additional SMC term (with the one-step-delayed estimator).

Fig. 4. The angular position x_1.

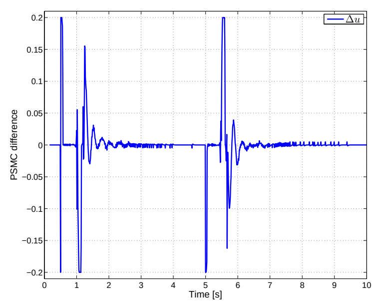


(a) PSMC with the additional SMC term (without the one-step-delayed estimator).

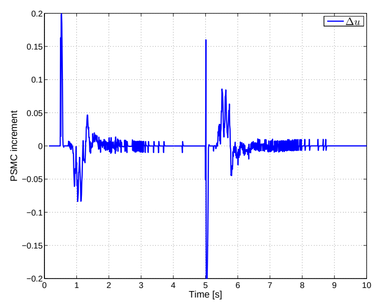


(b) PSMC with the additional SMC term (with the one-step-delayed estimator)

Fig. 5. PSMC signal u.

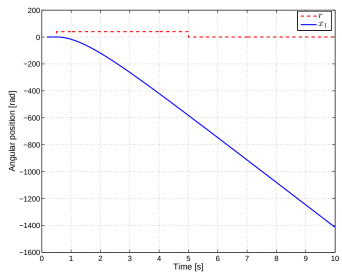


(a) PSMC increment with the additional SMC term (without the one-step-delayed estimator).

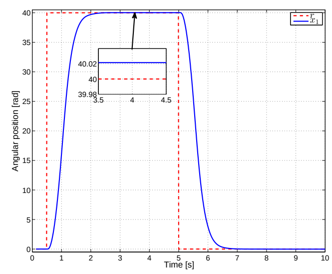


(b) PSMC increment with the additional SMC term (with the one-step-delayed estimator).

Fig. 6. PSMC signal increment Δu.

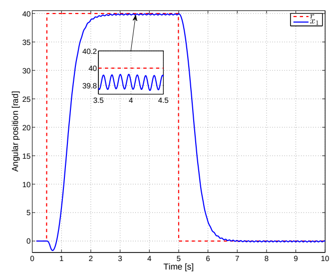


(a) The angular position x_1 for PSMC based on equivalent control, $R = 0.1$ (perturbed system; with the one-step-delayed estimator).

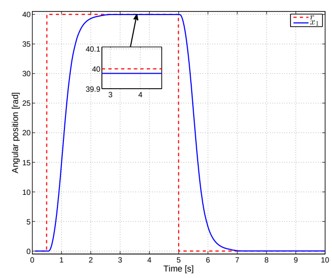


(b) The angular position x_1 for PSMC based on equivalent control method, $R = 10$ (perturbed system; with the one-step-delayed estimator).

Fig. 7. The angular position x_1.



(a) The angular position x_1 for PSMC with the additional SMC term, $R = 0.001$ (perturbed system; with the one-step-delayed estimator)



(b) The angular position x_1 for PSMC with the additional SMC term, $R = 0.1$ (perturbed system; with the one-step-delayed estimator).

Fig. 8. The angular position x_1.

5. CONCLUSION

In this manuscript, an approach to design of Predictive Sliding Mode Control (PSMC) based on Laguerre functions has been studied. Two PSMC algorithms are presented. The first one came from the digital equivalent control method approach, which belongs to the class of deadbeat control laws and implies that the reaching law is not defined at all. The second one is based on the chattering free reaching law method resulting in the control signal with an additional Sliding Mode Control (SMC) component as proposed by (Golo and Milosavljević, 2000). In that way, the system dynamics in reaching law is thoroughly determined and the system robustness is improved. By using the Laguerre functions for PSMC, the onlline optimization problem is solved faster compared to the traditional Model Predictive Control (MPC) approach. The constraints on control input signal and its increment are incorporated, which is impossible to achieve with traditional SMC. The stability problem of SMC, which may arise when the disturbance depends on the control input signal, is overcome by using a sufficiently large weight R. Improved system steady-state accuracy is achieved by introducing the one-step-delayed disturbance estimator in the proposed control algorithms, in order to reduce the effects of system nonlinearities and disturbances further. This combination of MPC and SMC is demonstrated by experimental results.

REFERENCES

Barry, T. and Wang, L. (2004). A model-free predictive controller with laguerre polynomials. In *2004 5th Asian Control Conference (IEEE Cat. No.04EX904)*, volume 1, 177–184.

Bartoszewicz, A. (1998). Discrete-time quasi-sliding-mode control strategies. *IEEE Transactions on Industrial Electronics*, 45(4), 633–637. doi:10.1109/41.704892.

Bartoszewicz, A. and Latosinski, P. (2018). Generalization of gao's reaching law for higher relative degree sliding variables. *IEEE Transactions on Automatic Control*, 1–1. doi:10.1109/TAC.2018.2797193.

Bartoszewicz, A. and Leśniewski, P. (2016). New switching and nonswitching type reaching laws for smc of discrete time systems. *IEEE Transactions on Control Systems Technology*, 24(2), 670–677. doi:10.1109/TCST.2015.2440175.

Bavili, R.E., Khosrowjerdi, M.J., and Vatankhah, R. (2015). Active fault tolerant controller design using model predictive control. *Journal of Control Engineering and Applied Informatics*, 17(3), 68–76.

Benattia, S.E., Tebbani, S., and Dumur, D. (2015). Hierarchical control strategy based on robust mpc and integral sliding mode - application to a continuous photobioreactor. In *Proceedings of 5th IFAC Conference on Nonlinear Model Predictive Control*, 212–217. doi:10.1016/j.ifacol.2015.11.285.

Benlaoukli, H., Hovd, M., and Olaru, S. (2008). Geometrical characterization of robust predictive control strategies. *Journal of Control Engineering and Applied Informatics*, 10(4), 13–19.

Bojan-Dragos, C.A., Stinean, A.I., Precup, R.E., Preitl, S., and Petriu, E.M. (2015). Model predictive control solution for magnetic levitation systems. In *2015 20th International Conference on Methods and Models in Automation and Robotics (MMAR)*, 139–144. doi:10.1109/MMAR.2015.7283861.

Boyd, S. and Vandenberghe, L. (2004). *Convex Optimization.* Cambridge University Press.

Chelladurai, J., Vinod, B., and Brislin, J.J. (2017). Finite control set based optimized model predictive current control of fourleg shunt active power filter. *Journal of Control Engineering and Applied Informatics*, 19(1), 67–76.

Corradini, M.L. and Orlando, G. (1997). A vsc algorithm based on generalized predictive control. *Automatica*, 33(5), 927–932. doi:10.1016/S0005-1098(96)00229-4.

Draženović, B. (1969). The invariance conditions in variable structure systems. *Automatica*, 5(3), 287 – 295. doi:http://dx.doi.org/10.1016/0005-1098(69)90071-5.

Duţescu, D.A., Radac, M.B., and Precup, R.E. (2017). Model predictive control of a nonlinear laboratory twin rotor aero-dynamical system. In *2017 IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI)*, 000037–000042. doi:10.1109/SAMI.2017.7880339.

Garcia-Gabin, W., Zambrano, D., and Camacho, E.F. (2009). Sliding mode predictive control of a solar air conditioning plant. *Control Engineering Practice*, 17(6), 652–663. doi:10.1016/j.conengprac.2008.10.015.

Golo, G. and Milosavljević, Č. (2000). Robust discrete-time chattering free sliding mode control. *Systems & Control Letters*, 41(1), 19–28. doi:10.1016/S0167-6911(00)00033-5.

Hildreth, C. (1957). A quadratic programming procedure. *Naval Research Logistics Quarterly*, 4(1), 79–85. doi:10.1002/nav.3800040113.

Ignaciuk, P. and Bartoszewicz, A. (2012). Discrete sliding-mode control of inventory systems with deteriorating stock and remote supply source. *Journal of Control Engineering and Applied Informatics*, 14(1), 14–21.

Incremona, G.P., Ferrara, A., and Magni, L. (2015). Hierarchical model predictive/sliding mode control of nonlinear constrained uncertain systems. In *Proceedings of 5th IFAC Conference on Nonlinear Model Predictive Control*, 102–109. doi:10.1016/j.ifacol.2015.11.268.

Inteco (2011). *Modular Servo System - Manual.* Inteco.

Leśniewski, P. and Bartoszewicz, A. (2016). Generalized non-switching reaching laws for smc of discrete time dynamical systems. *Journal of Control Engineering and Applied Informatics*, 18(4), 26–35.

Luenberger, D.G. (2003). *Linear and Nonlinear Programming.* Springer US, USA.

Mayne, D., Raković, S., Findeisen, R., and Allgöwer, F. (2006). Robust output feedback model predictive control of constrained linear systems. *Automatica*, 42(7), 1217 – 1222. doi:10.1016/j.automatica.2006.03.005.

Milosavljević, Č. (1985). General conditions for the existence of a quasisliding mode on the switching hyperplane in discrete variable structure systems. *Automation and Remote Control*, 46(3), 307–314.

Milosavljević, Č. (2004). Discrete-time vss. In A. Sabanovic, L. Fridman, and S. Spurgeon (eds.), *Variable Structure Systems: from Principles to Implementation*, chapter 5, 99–128. The Institution of Engineering and Technology, London. doi:10.1049/PBCE066E.

Milosavljević, Č., Mitić, D., and Veselić, B. (2004). Sliding mode based digital servo-system design. *Journal of Control Engineering and Applied Informatics*, 6(4), 37–42.

Mitić, D., Perić, S., Antić, D., Jovanović, Z., Milojković, M., and Nikolić, S. (2013a). Digital sliding mode control of anti-lock braking system. *Advances in Electrical and Computer Engineering*, 13(1), 33 – 40. doi: http://dx.doi.org/10.4316/AECE.2013.01006.

Mitić, D., Spasić, M., Hovd, M., and Antić, D. (2013b). An approach to design of sliding mode based generalized predictive control. In *Proceedings of IEEE 8th International Symposium on Applied Computational Intelligence and Informatics (SACI) 2013*, 347–351. doi: 10.1109/SACI.2013.6608996.

Neelakantan, V.A. (2005). *Modeling, design, testing and control of a two-stage actuation mechanism using piezoelectric actuators for automotive applications.* Ph.D. thesis, Ohio State University, Department of Mechanical Engineering.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 733–764. doi:10.1016/S0967-0661(02)00186-7.

Rawlings, J.B. and Mayne, D.Q. (2009). *Model Predictive Control: Theory and Design.* Nob Hill Publishing, Madison, Wisconsin, USA.

Rubagotti, M., Castanos, F., Ferrara, A., and Fridman, L. (2011a). Integral sliding mode control for nonlinear systems with matched and unmatched perturbations. *IEEE Transactions on Automatic Control*, 56(11), 2699–2704. doi:10.1109/TAC.2011.2159420.

Rubagotti, M., Raimondo, D., Ferrara, A., and Magni, L. (2011b). Robust model predictive control with integral sliding mode in continuous-time sampled-data nonlinear sysemts. *IEEE Transactions on Automatic Control*, 56(3), 556–570. doi:10.1109/TAC.2010.2074590.

Sgaverdea, S., Bojan-Dragos, C.A., Precup, R.E., Preitl, S., and Stinean, A.I. (2015). Model predictive controllers for magnetic levitation systems. In *2015 IEEE 10th Jubilee International Symposium on Applied Computational Intelligence and Informatics*, 171–176. doi: 10.1109/SACI.2015.7208193.

Spasić, M., Hovd, M., Mitić, D., and Antić, D. (2016). Tube model predictive control with an auxiliary sliding mode controller. *Modeling, Identification and Control*, 37, 181–193. doi:10.4173/mic.2016.3.4.

Spasić, M., Mitić, D., Hovd, M., and Antić, D. (2017). Tube model predictive control based on laguerre functions with an auxiliary sliding mode controller. In *2017 IEEE 15th International Symposium on Intelligent Systems and Informatics (SISY)*, 243–248. doi: 10.1109/SISY.2017.8080561.

Su, W.C., Drakunov, S.V., and Özgüner, Ü. (2000). An o(t2) boundary layer in sliding mode for sampled-data systems. *IEEE Transactions on Automatic Control*, 45(3), 482–485. doi:10.1109/9.847728.

Su, W.C., Drakunov, S.V., and Özgüner, Ü. (1996). Implementation of variable structure control for sampled-data systems. In F. Garofalo and L. Glielmo (eds.), *Robust Control via Variable Structure and Lyapunov Techniques*, 87–106. Springer, Berlin, Heidelberg. doi: 10.1007/BFb0027562.

Su, W.C., Drakunov, S., and Özgüner, Ü. (1993). Sliding mode control in discrete time linear systems. *{IFAC} Proceedings Volumes*, 26(2, Part 2), 267 – 270. doi: https://doi.org/10.1016/S1474-6670(17)48941-5. 12th Triennal Wold Congress of the International Federation of Automatic control. Volume 2 Robust Control, Design and Software, Sydney, Australia, 18-23 July.

Utkin, V. (1977). Variable structure systems with sliding modes. *IEEE Transactions on Automatic Control*, 22(2), 212–222. doi:10.1109/TAC.1977.1101446.

Utkin, V.I. (1978). *Sliding Modes and Their Applications in Variable Structure Systems.* MIR, Moscow, USSR.

Wang, L. (2009). *Model Predictive Control System Design and Implementation Using MATLAB.* Springer, London, UK.

Wang, L. (2001). Discrete time model predictive control design using laguerre functions. In *Proceedings of the 2001 American Control Conference. (Cat. No.01CH37148)*, volume 3, 2430–2435 vol.3. doi: 10.1109/ACC.2001.946117.

Young, K.D., Utkin, V.I., and Özgüner, Ü. (1999). A control engineer's guide to sliding mode control. *IEEE Transactions on Control Systems Technology*, 7(3), 328–342. doi:10.1109/87.761053.

Yu, X. and Kaynak, O. (2009). Sliding-mode control with soft computing: A survey. *IEEE Transactions on Industrial Electronics*, 56(9), 3275–3285. doi: 10.1109/TIE.2009.2027531.

Yu, X., Wang, B., and Li, X. (2011). Computer-controlled variable structure systems: the state-of-the-art. *IEEE Transactions on Industrial Informatics*, 8(2), 197–205. doi:10.1109/TII.2011.2178249.