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H_∞ Fault-Tolerant Control for Dynamic Positioning Ships based on Sampled-data

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Abstract: This study addresses the H_∞ fault-tolerant control issue for sampled-data dynamic positioning (DP) ships with actuator fault. Firstly, the actuator-failure mode is established. Then, the input delay approach combined with Lyapunov-Krasovskii functional are used for guaranteeing the stability of the system and achieving H_∞ tracking performance. Thus the H_∞ fault-tolerant sampled-data controller is designed. Finally, simulation results show that the fault-tolerant controller is effective so that the required output can track the given signal without steady-state error under the external disturbances and actuator faults.

Keywords: sampled-data control, actuator fault, fault-tolerant control, dynamic positioning ships

1. INTRODUCTION

A DP ship is a computer-controlled vessel (see Sørensen, 2011). Different from the conventional motion control methods such as anchoring or mooring, the DP ship's motion is controlled by DP systems which are widely used in many types of vessels, for example, survey, shuttle tankers, offshore drilling units and so on. In the past few years, many papers have focused on the DP system (Strand et al., 1999; Xiong et al., 2017; Donnarumma et al., 2018; Du et al., 2018), such as proportional integral derivative (PID) (Fossen, 1994); Kalman filtering technology (Balchen et al. , 1976; A. J. Sørensen et al., 1996); Vectorial backstepping techonology (Fossen et al., 1998; Snijders, 2005); passive nonlinear observer (Fossen et al., 1999; Lin et al., 2013; Lindegaard, 2003) and so on. Recently, many technical papers have focused on H_∞ control technology (Doyle, 2013; Francis, 1987; Zhou K et al., 1996) for DP ships (Katebi, 2011). In (Wang et al., 2012), based on mixed sensitivity, a robust controller is designed for the DP ships with uncertain model. (Ngongi et al., 2015) has focused on the issue about optimal H_∞ control for DP ships with **a** T-S fuzzy model (Zheng et al., 2018; Wang et al., 2018a); In (You, 2017), the issues about the DP ship's robustness are discussed using mixed μ-synthesis and H_∞ technology.

In reality, the electromagnetic interference, zero shift and actuators ageing exist in the DP ship system, so actuator failures are unavoidable. Therefore, designing a controller to tolerate actuator faults is necessary and important. Recently fault-tolerant control (FTC) has received considerable attention for DP ships. In (Benetazzo et al., 2015), by combining the Luenberger observer with the parity space approach, a reconfigurable discrete-time variable-structure controller is provided for a DP ship with actuator faults. In (Fang et al., 2015), by using the structural reliability-based approach, the two faults of buoyancy element loss and line breakage for a/the DP ship are dealt with efficiently. In (Cristofaro et al., 2014), in order to detect and isolate the actuator and faults, the FTC allocation is presented for the over-actuated DP ship system by using unknown input observers. In (Su et al., 2016), an FTC controller is developed for the DP ship to avoid actuator saturation and the partial loss of actuator effectiveness fault. In (Lin et al., 2016), by using an iterative learning observer, a FTC scheme is developed for DP ships under the unknown environment disturbance and thruster fault. In (Jing et al., 2017), by using a hyper-bolic tangent function, an active disturbance rejection controller is proposed for depth-pitch control of a DP ship.

It is noted that the above mentioned results about the DP ship system are mainly on a continuous-time system. However, now that digital computers are developing rapidly, the analogue controllers are replaced by digital controllers and the available literatures have focused on that digital system called sampled-data system (see Delchev et al., 2014; Abedi, 2015; Wang et al., 2018b). DP ship system is also a sampled-data system (see Fig.1), which adopts digital controllers to collect all types of sensor information, such as the position reference information (mostly using differential GPS (DGPS), the heading (mostly using the electric compass), the wind speed and direction, the acceleration, the vertical motion information and so on. These sensors provide continuous-time signals of ship's motion and environment, which will become discrete signals after being sampled and quantized by the computer, and then the signals will be converted to continuous-time signals by zero-order holder, which will be send to the propeller to produce the continuous control thrust to determine a ship's position at sea. Therefore, the continuous signals and discrete signals coexist in this system, which is the characteristic of the sampled-data system and it has more important theoretical significance than continuous-time system. Up to now, there has been less research focusing on the issue of sampled-data DP ships system. In (Katayama et al., 2010), the SPA sampled-data controller has been designed for DP ship's sampled-data. In (Zheng et al., 2017a,b), the robust H_∞ sampled-data stabilization problem for DP ships has been discussed.. However, there is almost no literature that has considered the fault-tolerant control problem for DP ships based on sampled-data despite its practical importance, which is the motivation for the research work.

In this brief, the problem about H_∞ fault-tolerant control for a sampled-data DP ship with actuator fault is discussed. By using input delay approach, the sampled-data DP ship's system is transformed to time-varying delay system. Then the stability criterion is provided to guarantee that the system is asymptotically stable and to establish H_∞ tracking performance when actuator faults happen. And a H_∞ fault-tolerant sampled-data controller is designed. One practical example is given to show that the DP ship system tracks the given signal with a lower tracking performance under the actuator faults and external disturbance.

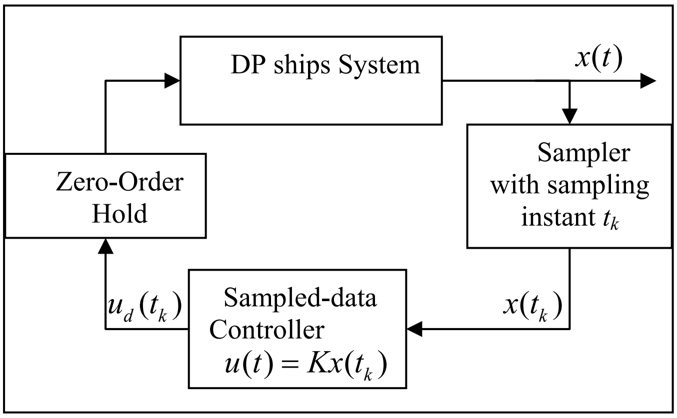

Fig. 1. DP ships control system.

2. PROBLEM FORMULATION

Considered the following DP ships model (see T. I. Fossen, 2002):

$$\begin{aligned} M\dot{\upsilon} + D\upsilon &= \tau + w \\ \dot{\eta} &= J(\psi)\upsilon \end{aligned} \tag{1}$$

Where $\eta = \begin{bmatrix} x & y & \psi \end{bmatrix}^T$ represents the earth-fixed position x, y and heading ψ of the ship, $\upsilon = \begin{bmatrix} p & v & r \end{bmatrix}^T$ is linear velocities (i.e. surge p, sway v and yaw r) of ship in the body-fixed frame (see Fig.2). τ is the control input vector of forces and moments; w is the environment disturbance. $J(\psi)$ represents the rotation matrix between the two coordinates defined as

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

M and D are the inertia and damping matrices respectively, and they are defined as

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix},$$

$$D = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix}.$$

Where m is the ship mass; I_z is the inertia moment. $X_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{r}}, N_{\dot{r}}$ are the accessional mass.

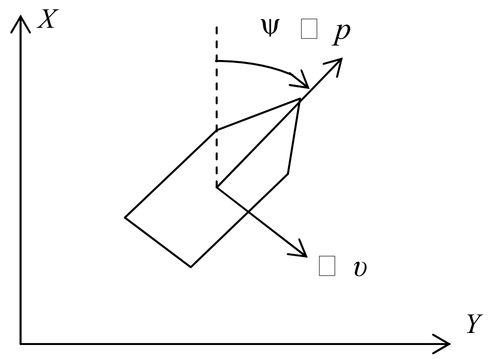

Fig. 2. Body-fixed coordinate systems.

Under the assumption that the ship is stable with small ψ, so that $J(\psi)$ can be rewritten

$$J(\psi) \cong I_{3\times 3}. \tag{2}$$

then the linearized DP ship's model becomes:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

where

$$x(t) = \begin{bmatrix} \eta & \upsilon \end{bmatrix}^T = \begin{bmatrix} x & y & \psi & p & v & r \end{bmatrix}^T,$$

$$A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Where $y(t) = \psi$ represents the control system output, $u(t)$ represents the control input, $w(t)$ represents the exogenous disturbance, C is a constant matrix, assumed to be known.

The control object is to consider the tracking control problem when actuator faults exist in the DP ships. The following actuator fault model is adopted as follows.

$$u^f(t) = \theta u(t), \tag{4}$$

where $\theta = diag\{\theta_1, \theta_2, \dots \theta_i\}$ is actuator efficiency factor, θ_i is the i-th actuator fault which takes value from the interval $[\underline{\theta}, \overline{\theta}]$. $\underline{\theta}$, $\overline{\theta}$ are the lower and upper bound of θ_i

respectively. When $\underline{\theta}=\overline{\theta}=1$, it means the actuator $u^f(t)$ has no fault. The fault modes of actuator failures are listed in table1. In the paper, the loss of actuator effectiveness is considered as the type of fault.

Table 1. Fault mode.

Fault mode	$\underline{\theta}$	$\overline{\theta}$
Healthy	1	1
Outage/Stuck	0	0
Loss of effectiveness	>0	<1

Define the tracking error as follow

$$e(t) = r(t) - y(t). \tag{5}$$

Where $r(t)$ represents the reference signal. Noted that a controller's tracking error integral plays a key role to eliminate the tracking error. Then, similar to (Liao et al., 2002), an augmented state-space description for the DP ships system with actuator faults (4) is considered as follows.

$$\begin{aligned} \dot{\varsigma}(t) &= \overline{A}\varsigma(t) + \overline{B}\theta u(t) + \overline{E}\overline{w}(t) \\ z(t) &= \overline{C}\varsigma(t) \end{aligned} \tag{6}$$

Where

$$\varsigma(t) = \begin{bmatrix} x^T(t) & \left(\int_0^t e(t)dt\right)^T \end{bmatrix}^T, \overline{w}(t) = \begin{bmatrix} w^T(t) & r^T(t) \end{bmatrix}^T$$

$$\overline{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \overline{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} 0 & I \end{bmatrix}$$

The paper's object is to design the a sampled-data controller to ensure that:

1) The system (6) is asymptotically stable when actuator failure occurs, and the output $y(t)$ can track the reference signal $r(t)$ without steady-state error;

2) To reject the wave, wind and current disturbances, it is required that $\| z(t) \|_2 \le \gamma \| w(t) \|_2$ for any non-zero $w(t) \in L_2[0,\infty)$ under zero condition which satisfies the assumption below, where $\gamma > 0$;

Assumption

1) The exogenous disturbance vector $w(t) = [w_1(t), w_2(t), w_3(t)]^T$ is unknown and time-variant. Where $w_1(t)$ and $w_2(t)$ represent the disturbance forces in surge and sway respectively, and $w_3(t)$ represents the disturbance moment in yaw.

2) $w(t)$ is bounded and satisfies that

$$|w_i(t)| \le p_i, \quad i = 1,2,3$$

where $p_i > 0, i = 1,2,3$ are unknown constants.

It is assumed that the state variables of DP ships are measured in $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$, where $t_k, k = 1,2,\ldots$ is sampling instant. The sampling period is assumed that

$$t_{k+1} - t_k \le d, \forall k \ge 0, d > 0, \tag{7}$$

Considered the state-feedback control law as follow

$$u(t) = u(t_k) = K\varsigma(t_k) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x(t_k) \\ \int_0^{t_k} e(t)dt \end{bmatrix}, \quad t_k \le t < t_{k+1}, \tag{8}$$

where K is the sampled-data controller gain. Substituting (8) into (1), then

$$\begin{aligned} \dot{\varsigma}(t) &= \overline{A}\varsigma(t) + \overline{B}\theta K\varsigma(t_k) + \overline{E}\overline{w}(t) \\ z(t) &= \overline{C}\varsigma(t) \end{aligned} \tag{9}$$

Remark 1: The system (9) includes not only discrete signals but also continuous signals, and it is different from reference (Li et al., 2017), in which the control signals are continuous, and it is more difficult to analyse. Besides, compared to the existing literatures, the sampled-data control method for DP ship systems in the paper is different and has more practical significance.

By input delay approach, the system (9) can be converted to the system as follows.

$$\begin{aligned} \dot{\varsigma}(t) &= \overline{A}\varsigma(t) + \overline{B}\theta K\varsigma(t-\tau(t)) + \overline{E}\overline{w}(t) \\ z(t) &= \overline{C}\varsigma(t) \end{aligned} \tag{10}$$

where

$$\tau(t) = t - t_k, \quad t_k \le t < t_{k+1}, \tag{11}$$

$\tau(t)$ is a time-varying delay and satisfies

$$0 \le \tau(t) \le d, \quad \dot{\tau}(t) = 1, \quad t \ne t_k \tag{12}$$

Remark 2: Input delay approach, which is introduced by (Fridman et al., 2004), is one of the major approaches to analyse the sampled-data system, in which the sampling period could be non-constant. Besides, compared with the traditional lifting techniques, it can easily deal with the control problem for the system with uncertain parameters (Gao et al., 2010).

The following lemma will be used in the paper:

Lemma 1 (Leibniz-Newton formula): If $f(x)$ is a differentiable function which is defined in the interval $[a,b]$, then

$$\int_a^b f'(t)dt = f(b) - f(a)$$

3. MAIN RESULTS

In this section, the fault-tolerant sampled-data control problem for system (10) will be solved. The Lyapunov functionals are constructed to guarantee that the system is asymptotically stable and establish H_∞ tracking performance.

And the design methods of fault-tolerant sampled-data controllers are provided.

Theorem 1: Given scale $d>0, \gamma>0$, the system (10) achieves the H_∞ tracking performance γ, if there exist matrices $Z>0, P>0, Q>0,\ M_i, N_i,\ i=1,2,3,4$ satisfying

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \sqrt{d}M_1 & \sqrt{d}N_1 & \sqrt{d}\bar{A}^T \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \sqrt{d}M_2 & \sqrt{d}N_2 & \sqrt{d}\theta K^T \bar{B}^T \\ * & * & \Omega_{33} & -N_4^T & \sqrt{d}M_3 & \sqrt{d}N_3 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{d}M_4 & \sqrt{d}N_4 & \sqrt{d}\bar{E}^T \\ * & * & * & * & -Z & 0 & 0 \\ * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \tag{13}$$

Where

$$\Omega_{11} = P\bar{A} + \bar{A}^T P + Q + M_1 + M_1^T + \bar{C}^T \bar{C},$$
$$\Omega_{12} = P\bar{B}K - M_1 + M_2^T + N_1$$
$$\Omega_{13} = M_3^T - N_1$$
$$\Omega_{14} = P\bar{E} + M_4^T$$
$$\Omega_{22} = -M_2 - M_2^T + N_2 + N_2^T$$
$$\Omega_{23} = -M_3^T - N_2 + N_3^T$$
$$\Omega_{24} = -M_4^T + N_4^T$$
$$\Omega_{33} = -Q - N_3 + N_3^T$$

Proof. First, consider the following Lyapunov-Krasovskii functional.

$$V(t) = \sum_{i=1}^{3} V_i(t), \quad t \in [t_k, t_{k+1}) \tag{14}$$

$$V_1(t) = \varsigma(t)^T P \varsigma(t) \tag{15}$$

$$V_2(t) = \int_{t-d}^{t} \varsigma(s)^T Q \varsigma(s) ds \tag{16}$$

$$V_3(t) = \int_{-d}^{0}\int_{t+\alpha}^{t} \dot{\varsigma}(s)^T Z \dot{\varsigma}(s) ds d\alpha \tag{17}$$

Calculating the derivative of $V(t)$, it can be obtained that

$$\begin{aligned} \dot{V}_1(t) &= 2\varsigma(t)^T P \dot{\varsigma}(t) \\ \dot{V}_2(t) &= \varsigma(t)^T Q \varsigma(t) - \varsigma(t-d)^T Q \varsigma(t-d) \\ \dot{V}_3(t) &= d\dot{\varsigma}(t)^T Z \dot{\varsigma}(t) - \int_{t-d}^{t} \dot{\varsigma}(s)^T Z \dot{\varsigma}(s) ds \end{aligned} \tag{18}$$

Using the Leibniz-Newton formula, it can be obtained that

$$\varsigma(t) - \varsigma(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\varsigma}(s) ds = 0 \tag{19}$$

$$\varsigma(t-\tau(t)) - \varsigma(t-d) - \int_{t-d}^{t-\tau(t)} \dot{\varsigma}(s) ds = 0 \tag{20}$$

That is, for any appropriately dimensioned matrices $M_i, N_i,\ i=1,2,3,4$, the equations can be obtained as follow:

$$\begin{aligned} &2[\varsigma^T(t)M_1 + \varsigma^T(t-\tau(t))M_2 + \varsigma^T(t-d)M_3 + \bar{w}^T(t)M_4] \\ &\times[\varsigma(t) - \varsigma(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\varsigma}(s) ds] = 0 \end{aligned} \tag{21}$$

$$\begin{aligned} &2[\varsigma^T(t)N_1 + \varsigma^T(t-\tau(t))N_2 + \varsigma^T(t-d)N_3 + \bar{w}^T(t)N_4] \\ &\times[\varsigma(t-\tau(t)) - \varsigma(t-d) - \int_{t-d}^{t-\tau(t)} \dot{\varsigma}(s) ds] = 0 \end{aligned} \tag{22}$$

Similarly, it can be obtained that

$$\int_{t-d}^{t} \dot{\varsigma}(s) Z \dot{\varsigma}(s) ds = \int_{t-\tau(t)}^{t} \dot{\varsigma}(s) Z \dot{\varsigma}(s) ds + \int_{t-d}^{t-\tau(t)} \dot{\varsigma}(s) Z \dot{\varsigma}(s) ds \tag{23}$$

Adding Eqs.(21)-(23) into (18) yield

$$\begin{aligned} \dot{V}(t) &= 2\varsigma(t)^T P \dot{\varsigma}(t) + \varsigma(t)^T Q \varsigma(t) - \varsigma(t-d)^T Q \varsigma(t-d) \\ &+ d\dot{\varsigma}(t)^T Z \dot{\varsigma}(t) - \int_{t-d}^{t} \dot{\varsigma}(s)^T Z \dot{\varsigma}(s) ds \\ &+ 2\left[\varsigma^T(t)M_1 + \varsigma^T(t-\tau(t))M_2 + \varsigma^T(t-d)M_3\right. \\ &\left.+ \bar{w}^T(t)M_4\right] \times \left[\varsigma(t) - \varsigma(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\varsigma}(s) ds\right] \\ &+ 2\left[\varsigma^T(t)N_1 + \varsigma^T(t-\tau(t))N_2 + \varsigma^T(t-d)N_3\right. \\ &\left.+ \bar{w}^T(t)N_4\right] \times \left[\varsigma(t-\tau(t)) - \varsigma(t-d) - \int_{t-d}^{t-\tau(t)} \dot{\varsigma}(s) ds\right] \\ &\le \zeta^T(t)(\Theta + d\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix}^T \\ &\times Z\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix} \\ &+ dMZ^{-1}M^T + dNZ^{-1}N^T)\zeta(t) \\ &- \int_{t-\tau(t)}^{t}\left[\zeta^T(t)M + \dot{\varsigma}^T(s)Z\right]Z^{-1}\left[M^T\zeta(t) + Z\dot{\varsigma}(s)\right] ds \\ &- \int_{t-d}^{t-\tau(t)}\left[\zeta^T(t)N + \dot{\varsigma}^T(s)Z\right]Z^{-1}\left[N^T\zeta(t) + Z\dot{\varsigma}(s)\right] ds \end{aligned} \tag{24}$$

where

$$\begin{aligned} \zeta(t) &= \begin{bmatrix}\varsigma^T(t) & \varsigma^T(t-\tau(t)) & \varsigma^T(t-d) & \bar{w}(t)\end{bmatrix}^T, \\ M &= \begin{bmatrix}M_1^T & M_2^T & M_3^T & M_4^T\end{bmatrix}^T, \\ N &= \begin{bmatrix}N_1^T & N_2^T & N_3^T & N_4^T\end{bmatrix}^T, \\ \Theta &= \begin{bmatrix} \Omega_{11} & \Omega_{12} & M_3^T - N_1 & PB + M_4^T \\ * & \Omega_{22} & -M_3^T - N_2 + N_3^T & M_4^T + N_4^T \\ * & * & -Q - N_3 - N_3^T & -N_4^T \\ * & * & * & -\gamma^2 I \end{bmatrix} \end{aligned} \tag{25}$$

Since $Z>0$, then the last two terms in (24) are negative. According to the Schur complement, (13) implies that

$$\begin{aligned} &\Theta + d\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix}^T Z\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix} \\ &+ dMZ^{-1}M + dNZ^{-1}N < 0 \end{aligned} \tag{26}$$

Thus, from Eq. (26), it can be obtained that

$$z^T(t)z(t)-\gamma^2\overline{w}^T(t)\overline{w}(t)+\dot{V}(t)<0 \tag{27}$$

Under zero conditions, there are $V(0)=0$ and $V(\infty)\geq 0$. Integrating (27) yields $\|z(t)\|_2\leq\gamma\|w(t)\|_2$ for any non-zero $w(t)\in L_2[0,\infty)$, then the H_∞ tracking performance is guaranteed, which completes the proof.

Remark 3. In order to achieve a H_∞ tracking performance for system (10), a free-weighting matrix approach combined with the Lyapunov-Krasovskii functional are used in Theorem 1.

A free-weighting matrix approach was introduced in (He, et.al, 2004), in which the relationships between the terms of the Leibniz–Newton formula are established by free-weighting matrices. Since matrix M_i and N_i, $i=1,2,3,4$ are free respectively, their optimal values can be obtained if the corresponding LMI is solved. Compared with the fixed weight matrix, it can overcome the conservatism. This approach can also avoid the restriction on the derivative of time-varying delays. Therefore, it plays a role key in reducing conservatism for the delay-dependent results.

Then the fault-tolerant sampled-data controller (8) will be designed.

Theorem 2: Given scales $d>0,\gamma>0$, the system (10) is asymptotically stable under the assumption that the sampling period is bounded, that is, $t_{k+1}-t_k\leq d,\forall k\geq 0$, if there exist matrices $\tilde{P}>0,\tilde{Q}>0,\tilde{Z}>0,\tilde{M}_j,\tilde{N}_j$, $j=1,2,3,4$, satisfying

$$\begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} & \sqrt{d}\tilde{M}_1 & \sqrt{d}\tilde{N}_1 & \sqrt{d}\tilde{P}\overline{A}^T & \tilde{P}C^T \\ * & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24} & \sqrt{d}\tilde{M}_2 & \sqrt{d}\tilde{N}_2 & \sqrt{d}\theta\tilde{K}^T\overline{B}^T & 0 \\ * & * & \tilde{\Omega}_{33} & -\tilde{N}_4^T & \sqrt{d}\tilde{M}_3 & \sqrt{d}\tilde{N}_3 & 0 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{d}\tilde{M}_4 & \sqrt{d}\tilde{N}_4 & \sqrt{d}\tilde{P}\overline{E}^T & 0 \\ * & * & * & * & -\tilde{Z} & 0 & 0 & 0 \\ * & * & * & * & * & -\tilde{Z} & 0 & 0 \\ * & * & * & * & * & * & \tilde{Z}-2\tilde{P} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix}<0, \tag{28}$$

Where

$$\tilde{\Omega}_{11}=\tilde{P}\overline{A}+\overline{A}^T\tilde{P}+\tilde{Q}+\tilde{M}_1+\tilde{M}_1^T$$
$$\tilde{\Omega}_{12}=\overline{B}\tilde{K}-\tilde{M}_1+\tilde{M}_2^T+\tilde{N}_1$$
$$\tilde{\Omega}_{13}=\tilde{M}_3^T-\tilde{N}_1$$
$$\tilde{\Omega}_{14}=\overline{E}+\tilde{M}_4^T$$
$$\tilde{\Omega}_{22}=-\tilde{M}_2-\tilde{M}_2^T+\tilde{N}_2+\tilde{N}_2^T$$
$$\tilde{\Omega}_{23}=-\tilde{M}_3^T-\tilde{N}_2+\tilde{N}_3^T$$
$$\tilde{\Omega}_{24}=-\tilde{M}_4^T+\tilde{N}_4^T$$
$$\tilde{\Omega}_{33}=-\tilde{Q}-\tilde{N}_3+\tilde{N}_3^T$$

and the sampled-data controller gain matrix K is obtained as

$$K=\begin{bmatrix}K_1 & K_2\end{bmatrix}=\tilde{K}\tilde{P}^{-1} \tag{29}$$

Proof: Noting that $\tilde{Z}>0$, it can be obtained that

$$(\tilde{Z}-\tilde{P})\tilde{Z}^{-1}(\tilde{Z}-\tilde{P})>0, \tag{30}$$

which is equivalent to

$$-\tilde{P}\tilde{Z}^{-1}\tilde{P}\leq\tilde{Z}-2\tilde{P} \tag{31}$$

Let $\eta=diag\left\{P^{-T},P^{-T},P^{-T},I,P^{-T},P^{-T},I,I\right\}$.

Denoting

$$\tilde{P}=P^{-1},\tilde{K}=KP^{-1},\tilde{Q}=P^{-T}QP^{-1},$$
$$\tilde{M}_1=P^{-T}M_1P^{-1},\tilde{M}_4=M_4P^{-1},\tilde{Z}=P^{-T}ZP^{-1}$$

Pre- and post-multiplying (13) by η and η^T respectively, the following inequality can be obtained:

$$\begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} & \sqrt{d}\tilde{M}_1 & \sqrt{d}\tilde{N}_1 & \sqrt{d}\tilde{P}\overline{A}^T \\ * & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24} & \sqrt{d}\tilde{M}_2 & \sqrt{d}\tilde{N}_2 & \sqrt{d}\theta\tilde{K}^T\overline{B}^T \\ * & * & \tilde{\Omega}_{33} & -N_4^T & \sqrt{d}\tilde{M}_3 & \sqrt{d}\tilde{N}_3 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{d}\tilde{M}_4 & \sqrt{d}\tilde{N}_4 & \sqrt{d}\tilde{P}\overline{E}^T \\ * & * & * & * & -\tilde{Z} & 0 & 0 \\ * & * & * & * & * & -\tilde{Z} & 0 \\ * & * & * & * & * & * & -\tilde{Z}^{-1} \end{bmatrix}<0, \tag{32}$$

where

$$\tilde{\Pi}_{11}=\tilde{P}\overline{A}+\overline{A}^T\tilde{P}+\tilde{Q}+\tilde{M}_1+\tilde{M}_1^T+\tilde{P}C^TC\tilde{P}$$

by the Schur complement, Eq. (28) can be obtained. This completed the proof.

4. NUMERICAL EXAMPLES

To validate the performance of the proposed methods, a simulation example about output tracking control for DP ship is given. The main parameters are that: the ship's length is 76.2m, tonnage is 4.591×10^6 kg. Similar with (Tannuri et al., 2010), the M and D in model (1) are given by:

$$M=\begin{bmatrix} 5.3122\times10^6 & 0 & 0 \\ 0 & 8.2831\times10^6 & 0 \\ 0 & 0 & 3.7454\times10^9 \end{bmatrix}$$

$$D=\begin{bmatrix} 5.0242\times10^4 & 0 & 0 \\ 0 & 2.7229\times10^5 & -4.3933\times10^6 \\ 0 & -4.3933\times10^6 & 4.1894\times10^8 \end{bmatrix}$$

Similar to (Li et al., 2017), the environment disturbance w(t) is considered as

$$w(t)=J^T(\psi)b \tag{33}$$

where $J(\psi)$ is defined in (1), and $b\in\Re^3$ represents the vector describing the un-modeled disturbance force and moment. For simulation, this paper assumed that

$b=\begin{bmatrix}0.3 & 0.5 & -0.1\end{bmatrix}$, the ship's initial states $x_s(t)=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$. In addition, considering the actuator fault, the lower and upper bound of θ_i are 0.1 and 1 respectively. Then, the actuator efficiency factor θ is assumed to be $\{1,1,0.35\}$. sampling interval is assumed to be $d=1$, then the H_∞ tracking performance is obtained that $\gamma_{\min}=0.7652$. And the controller gain matrices are given by

$$K_1=\begin{bmatrix}-0.0165 & 0 & 0 & -0.0482 & 0 & 0\\ 0 & -0.0283 & -0.0031 & -.0001 & -0.0805 & 0.0031\\ -0.0040 & 3.0321 & -7.7750 & -0.969 & 3.5593 & -28.3172\end{bmatrix}$$

$$K_2=\begin{bmatrix}0\\ 0.0031\\ 7.7752\end{bmatrix}$$

When $r(t)=0.5$ and $r(t)=0.5\sin 4t$, the ship's output $y(t)$ and the reference signal $r(t)$ are given in Figs. 3 and 5 respectively, and the error between $y(t)$ and $r(t)$ are shown in Fig. 4 and 6 respectively, from which it can be seen that whether $r(t)$ is a constant or a variable, when there exist actuator fault (at about $t=0.3$ in this case) and external disturbances, $y(t)$ can still track $r(t)$ well. This indicates that the DP ship has an acceptable tracking performance when the external disturbances and actuator fault happen.

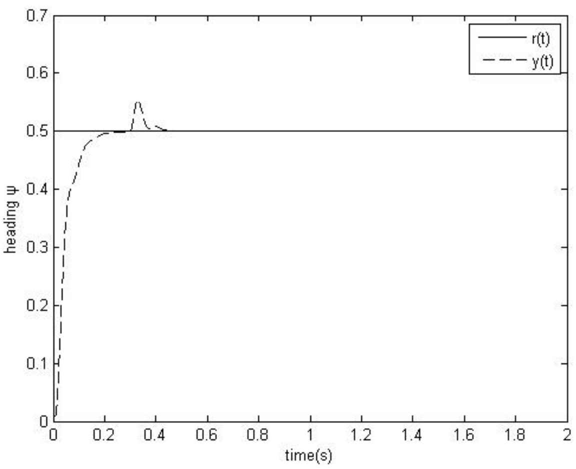

Fig. 3. $y(t)$ and $r(t)$ when $r(t)=0.5$.

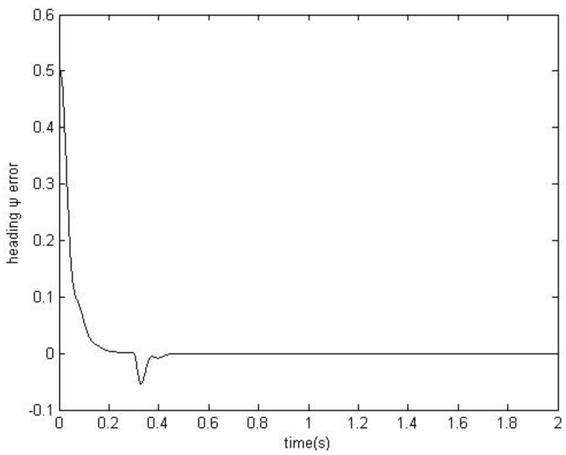

Fig. 4. The error between $y(t)$ and $r(t)$ when $r(t)=0.5$

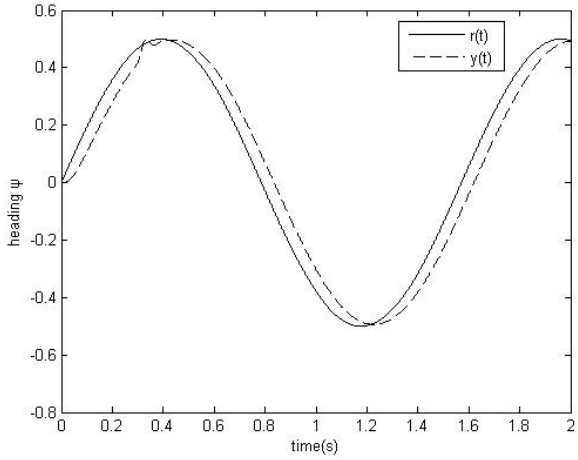

Fig.5 $y(t)$ and $r(t)$ when $r(t)=0.5\sin 4t$.

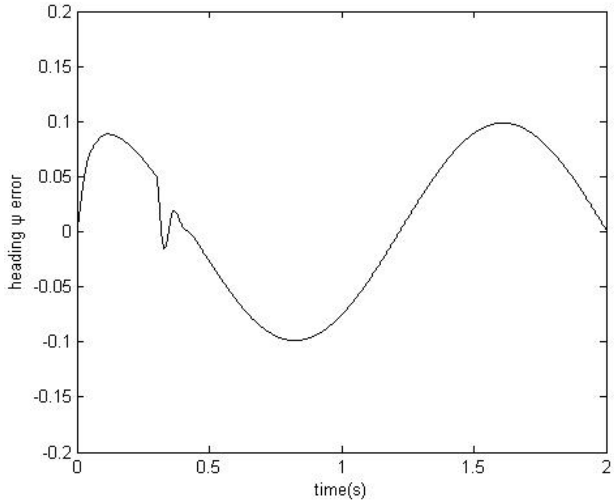

Fig. 6. The error between $y(t)$ and $r(t)$ when $r(t)=0.5\sin 4t$

5. CONCLUSIONS

In this brief, the issue about H_∞ fault-tolerant control for sampled-data DP ships with actuator fault is discussed. Firstly, the DP ship's linearized model with sampled-data is converted to a time-varying delay system. Then a H_∞ fault-tolerant sampled-data controller is designed to guarantee stability and establish H_∞ tracking performance. Finally, the effectiveness of the obtained method is demonstrated by a DP ship example. The method proposed in the paper is also suitable for the other type of ship. In future, the new sliding mode control method (see Wang et.al, 2018c,d) for sampled-data nonlinear DP ship systems will be studied.

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