An improvement on the PI controller for a class of high-order unstable delayed systems: Application to a thermal process

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Abstract: This work addresses the stabilization and control problem of a class of input-output delayed unstable linear systems, i.e. n-order systems with one unstable pole and possibly one minimum phase zero. The problem is solved employing a modified version of the traditional PI, called the PI_f controller, which incorporates a low-pass first order filter. This new scheme allows improving the existing results using the PI controller for high-order systems with time delay. The proposed control strategy is experimentally assessed through the temperature control of an unstable heat flow process.

Keywords: Linear Systems, time–delay, modified PI control, unstable process, thermal process.

1. INTRODUCTION

Time-delays often arise in a variety of dynamical systems such as chemical processing systems (Richard, 2003), transportation systems, communication systems, and power systems, among others (Franklin et al., 1995; Wang et al., 1999). In some cases, time-delays are introduced by sensor and actuator devices (Xian et al., 2005; Liu et al., 2005). Moreover, time delay may be produced due to heat and mass transfer in chemical processes, high computing burden and hardware restrictions in digital processing systems, high inertia in heavy machinery systems, and communication lags in spacecraft and remote operation (Azorin et al., 2003).

Control system designers may sometimes neglect relatively small delays in which the systems still satisfy design requirements. But time delay may cause instability, poor performance, and unwanted behavior in real applications and its effects cannot be underestimated. Therefore, it is necessary to take special attention to stability and controller design issues to handle systems with this feature. For the above reasons, there exists an increasing motivation for the study of the effects produced by time-delays in closed-loop dynamical systems (Hu and Lin, 2001; Trentelman et al., 2001; Shamsuzzoha et al., 2007). It is also worth remarking that the design of a feedback control law becomes more complicated when, besides the time-delay, the system is open-loop unstable (Sipahi et al., 2011; Gu et al., 2003; Tiao-Yang et al., 2016), among others.

The control problem of time-delay systems has been studied from different perspectives. The simplest approach is to ignore the term linked to the delay τ and to design a control strategy for the system without delay. It is important to note that the result will be satisfactory as long as the delay is small enough. When the delay magnitude is considerable, the operator $e^{-\tau s}$ related to the time delay in the transfer function of the original system, can be approximated by a Taylor series expansion or through a Pade's approximation, so the exponential term is represented by a rational function in the complex variable s (Munz et al., 2009; Baranowski, 2016). However, when these techniques are used, the designed control law may not properly work when applied to the original system due to an inaccurate approximation.

Another approach is to compensate for the effect of the time delays by removing the exponential term from the characteristic equation of the process. This technique was introduced by Smith (Smith, 1957) and is well-known as the Smith Predictor (SP). The idea is to estimate the future value of an internal signal before being affected by the input delay. This technique does not have a stabilization step, restricting its application to open-loop stable plants. To deal with this disadvantage, some modifications of the original SP structure have been proposed, see for instance, (Palmor, 1996; Seshagiri et al., 2007; Kawnish and Choudhury, 2012), and for some particular family of unstable plants (Novella et al., 2013; Marquez et al., 2012).

On the other hand, the most common controllers in industrial applications are the P, PI, PD and PID controllers due to their simplicity (Xue et al., 2007). There have been a significant amount of researches on this kind of controllers in the literature, for example in (Xiu-Wei and Jian-Yue, 2013), based on Pontryagin's results and using a generalization of the Hermite-Behler theorem construct the stabilizing PID region of a retarded-type time-delay systems. A complete analysis of these kind of controllers have been carried out in

(Silva et al., 2005), where the limits for the stabilization of first-order systems with time-delay are provided. A further-step is given by (Yeroglu, 2015), where a methodology is provided to compute all stabilizing robust PI and PID controllers for multiple time delay systems with parametric uncertainty structure, also all values of the controller parameters in the proposed stability region guarantee the robust stability of multiple time delay systems. In (Xiang et al., 2007), a frequency approach for the design of $P/PI/PD/PID$ controllers focused on a specific class of second order systems with an unstable pole was introduced. Under a similar approach, in (Lee et al., 2010) a generalization for higher order systems with an unstable pole is provided. The generalization of this result is provided in (Hernandez-Perez et al., 2015) where delayed systems with possible complex conjugate poles are addressed. Recent works focuses on a more particular class of systems: for instance in (Novella et al., 2017), PD/PID controllers for the stabilization of high order delayed systems with two unstable poles are considered. It is important to note that, all previously cited works consider the use of P, PI, PD or PID in their classical form (proporcional, integral and derivative term). In this sense, for example, in (Vazquez et al., 2017; Vazquez et al., 2017), a delayed recycle plants is considered. It is addressed the control problem of a class of linear high-order unstable recycling systems with time delay in the direct and the recycling paths by means of a $P/PI/PD/PID$-like structure that produces a time delayed feedback for the stabilization of the corresponding closed-loop system.

Therefore, although classical PID controllers are the most used in the industry, they have two disadvantages even without the presence of delays such as: the derivative term causes the controller to have implementation problems because from a transfer function point of view the controller is an improper function. Moreover, the derivative action is sensitive to noise produced by high frequencies, which also implies a relatively large gain. The amplified noise may cause the saturation of the actuator that may lead to an unexpected control action. It is desirable to limit the value of the gain or to leave aside the derivative action. Likewise, in the case of processes with large time-delays, the anticipatory action of the derivative term is no longer working since the linear approximation $e(t+T_d)=e(t)+T_d \frac{d}{dt}e(t)$ is only valid for small values of the derivative time constant T_d. This causes that the control action reacts an instant of time later on the variable of interest, which affects the system performance. For this reason, it is common to avoid the derivative term of the control strategy and just keep using the proportional-integral action.

Under this approach and in order to improve the results given in the literature concerning the stabilization of delayed systems using a conventional PI, in this work a modified version of the traditional PI control scheme, called the PI_f controller, is proposed. The PI_f control is composed of a traditional PI but adding as a third term a simple first order filter, i.e., $k_f/(s+\phi)$, instead of the derivative term used in the traditional PID. From this modification it is possible to have some advantages such as help to reduce the noise that may be present in the system since it does not resort on a derivative term, also it allows the stabilization and control of the same family of systems considered in (Lee et al., 2010), but with the advantage that the delays tolerated by the PI_f are larger than those tolerated by the traditional PI. Additionally, the PI_f controller can also deal with systems having one minimum phase zero (which are common in chemical processes such as Continuously Stirred Tank Reactors (CSTR) Bequette, 2003), a result which seems difficult to prove using a standard PI controller. Moreover, the proposed control law keep the basic properties of a conventional PI controller such as disturbance rejection and tracking of step references. This work reports necessary and sufficient conditions to stabilize a particular class of high-order unstable systems with time delay by using a PI_f controller and sufficient conditions when the system include one minimum phase zero. The stability conditions are expressed in terms of the maximum allowable time-delay magnitude. Moreover, a procedure is also provided for determining the parameter ranges of the PI_f controller. Finally, a remark is provided considering the case when the system contains complex poles.

The proposed control strategy is experimentally assessed through the temperature control of an unstable heat flow process which is implemented by using a laboratory prototype. It is worth to note that the unstable system was emulated in the laboratory prototype by a recycle loop. This phenomenon also produces a delay time in the measurement of the sensors for the recording of the temperature which further complicates the problem.

The paper is organized as follows: Section 2 presents the problem statement. Section 3 addresses the proposed control strategy, and establishes the necessary and sufficient condition to stabilize high-order unstable systems with time delay and one minimum phase zero using a PI_f controller. In Section 4, the proposed strategy is applied to a laboratory prototype. Some conclusions are given in Section 5.

2. PROBLEM STATEMENT

Consider the following class of single input-single output (SISO) linear time invariant systems (LTI) with delay in the input-output path,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{\alpha}{(s-\gamma)\prod_{m=1}^{q}(s+\delta_m)}e^{-\tau s}, \tag{1}$$

with γ, δ_m and $\tau > 0$. Note that the above model has one unstable pole, q stable poles and a time delay τ. The objective is to provide necessary and sufficient conditions to stabilize this class of systems by means of a PI_f controller. Fig. 1 depicts a block diagram of the closed-loop system.

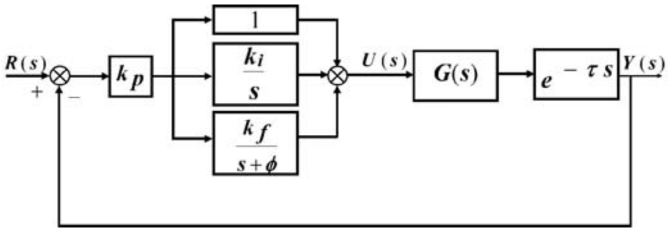

Fig. 1. Proposed control strategy PI_f in closed-loop with the system (1).

The following expression define the proposed PI_f control law,

$$H(s)=k_p\left(1+\frac{k_i}{s}+\frac{k_f}{s+\phi}\right) \tag{2}$$

with k_p, k_i and $\phi>0$. Note that the PI_f control law produces an open-loop transfer function $Q(s)$ represented by,

$$Q(s)=H(s)G(s)e^{-\tau s} \tag{3}$$

Notice that instead of adding a derivative term to the classical PI, in order to get a PID controller, in this work it is considered a first order filter where its cutoff frequency is not determined by the approximation of the derivative term. This is the reason why the control strategy has been called PI_f controller. However, it is interesting to note that even if the derivative term or its approximation is not explicitly included in (2), after elementary algebra, this expression can be rewritten as: $k(s+a)(s+b)/[s(s+\phi)]$, which indeed, is equivalent to a filtered PID controller $[PID * c/(s+\phi)]$, or if it is necessary, it can be seen as a PID with the derivative term numerically approximated.

The proposed control scheme preserves the basic properties of the traditional PI/PID; however, it will be shown later that the considered strategy improves the stability conditions provided by the classical PI controller and that has the ability to deal with systems having one minimum phase zero.

3. PROPOSED CONTROL STRATEGY

In this section necessary and sufficient conditions for stabilizing the close-loop system (1)-(2) are presented. The proof of the main result of this work is based on the well-know Nyquist stability criterion that for the sake of completeness is recalled here.

Theorem 1. (Nyquist stability criterion). A linear system is stable if and only if $N+P=0$, where P is the number of poles in the right half complex plane and N is the number of clockwise rotations to the point $(-1,0j)$ in the Nyquist diagram.

Notice that if N is negative, the rotations should be in the counterclockwise direction. In order to design the PI_f control strategy, the following theorem is stated.

Theorem 2. Consider the class of high-order time-delay systems with one unstable pole (1). There exists a PI_f controller given by (2) such that the corresponding closed-loop system is stable if and only if,

$$\tau<\frac{1}{\gamma}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}. \tag{4}$$

Proof. This result can be demonstrated under the frequency domain framework. From (3), the open-loop frequency response is given by,

$$Q(j\omega)=k_p\alpha\frac{\left((j\omega)^2+(k_f+k_i+\phi)j\omega+k_i\phi\right)}{j\omega(j\omega-\gamma)(j\omega+\phi)\prod_{m=1}^{q}(j\omega+\delta_m)}e^{-j\omega\tau}. \tag{5}$$

For the sake of simplicity consider the following definitions,

$$\begin{aligned}\bar{k}_p&=k_p(k_f+k_i+\phi),\\ \bar{k}_f&=\frac{1}{k_f+k_i+\phi},\\ \bar{k}_i&=\frac{k_i\phi}{k_f+k_i+\phi}.\end{aligned} \tag{6}$$

The above equalities allow rewriting the open-loop transfer function (5) as follows,

$$Q(j\omega)=\bar{k}_p\bar{k}_f\alpha\frac{\left((j\omega)^2+\frac{1}{\bar{k}_f}j\omega+\frac{\bar{k}_i}{\bar{k}_f}\right)}{j\omega(j\omega-\gamma)(j\omega+\phi)\prod_{m=1}^{q}(j\omega+\delta_m)}e^{-j\omega\tau}. \tag{7}$$

To begin with the analysis, let us consider first the particular case where $\bar{k}_i=0$. The next expressions give the magnitude $M_Q(j\omega)$ and phase $\angle Q(j\omega)$ for (7)

$$M_{Q_{\bar{k}_i=0}}(j\omega)=\bar{k}_p\alpha\sqrt{\frac{1+\bar{k}_f^{\,2}\omega^2}{(\omega^2+\gamma^2)(\omega^2+\phi^2)\prod_{m=1}^{q}(\omega^2+\delta_m^2)}} \tag{8}$$

$$\begin{aligned}\angle Q_{\bar{k}_i=0}(j\omega)&=-\left(\pi-\arctan\left(\frac{\omega}{\gamma}\right)\right)-\omega\tau+\\ &\arctan(\bar{k}_f\omega)-\arctan\left(\frac{\omega}{\phi}\right)-\sum_{m=1}^{q}\arctan\left(\frac{\omega}{\delta_m}\right)\end{aligned} \tag{9}$$

(Necessity) Suppose that a PI_f controller in closed-loop with (1) produces an asymptotically stable system. Then, the Nyquist criterion is satisfied and therefore, there exists a counterclockwise rotation to the point $(-1,0j)$ in the complex plane.

From (9) notice that,

$$\begin{aligned}\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)&=-\tau+\frac{\gamma}{\omega^2+\gamma^2}+\frac{\bar{k}_f}{\bar{k}_f^2\omega^2+1}-\\ &\frac{\phi}{\omega^2+\phi^2}-\sum_{m=1}^{q}\frac{\delta_m}{\omega^2+\delta_m^2}\end{aligned} \tag{10}$$

It is clear that (10) could be a positive or negative function depending on the value of the involved parameters, however, if $d\left(\angle Q_{\bar{k}_i=0}\right)/d\omega<0$ then equation (9) is a decreasing function for all ω, and if $d\left(\angle Q_{\bar{k}_i=0}\right)/d\omega>0$ the function has only one change of sign as $\omega\to\infty$. The existence of a counterclockwise rotation to the $(-1,0j)$ point allows concluding the next inequality

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0}>0 \tag{11}$$

or equivalently,

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0}=-\tau+\frac{\gamma}{\omega^2+\gamma^2}+\frac{\bar{k}_f}{\bar{k}_f^2\omega^2+1}-\frac{\phi}{\omega^2+\phi^2}-\sum_{m=1}^{q}\frac{\delta_m}{\omega^2+\delta_m^2}>0.$$

Therefore, evaluating (10) at $\omega=0$ yields,

$$-\tau+\frac{1}{\gamma}+\bar{k}_f-\frac{1}{\phi}-\sum_{m=1}^{q}\frac{1}{\delta_m}>0$$

from which the next inequality holds,

$$\tau<\frac{1}{\gamma}-\frac{1}{\phi}+\bar{k}_f-\sum_{m=1}^{q}\frac{1}{\delta_m}. \tag{12}$$

From the counterclockwise rotation assumption and the fact that (11) is satisfied for $\omega=0$, it is clear that for this frequency value the magnitude (8) should be a decreasing function. The above is equivalent to ask that,

$$\frac{d}{d\omega}\left(\frac{M^2_{Q_{\bar{k}_i=0}}(j\omega)}{\bar{k}_p^2\alpha^2}\right)\Bigg|_{\omega=0}<0. \tag{13}$$

After some manipulatons, inequality (13) produces,

$$\frac{2\omega\bar{k}_f^2-2\omega\left(1+\bar{k}_f^2\omega^2\right)\left[\frac{1}{\omega^2+\gamma^2}+\frac{1}{\omega^2+\phi^2}+\sum_{m=1}^{q}\frac{1}{\omega^2+\delta_m^2}\right]}{\left(\omega^2+\gamma^2\right)\left(\omega^2+\phi^2\right)\prod_{m=1}^{q}\left(\omega^2+\delta_m^2\right)}<0.$$

In the interval $\omega\in(0,\infty)$ the above inequality is equivalent to,

$$\bar{k}_f^2-\left(1+\bar{k}_f^2\omega^2\right)\left[\frac{1}{\omega^2+\gamma^2}+\frac{1}{\omega^2+\phi^2}+\sum_{m=1}^{q}\left(\frac{1}{\omega^2+\delta_m^2}\right)\right]<0.$$

Evaluating the right-hand-side of the above inequality for $\omega=0$ leads to the next condition,

$$\bar{k}_f<\sqrt{\frac{1}{\gamma^2}+\frac{1}{\phi^2}+\sum_{m=1}^{q}\left(\frac{1}{\delta_m^2}\right)}. \tag{14}$$

Consider now the case $\bar{k}_i\neq 0$ that corresponds to the analysis of the transfer function (7). The corresponding magnitude $M_Q(j\omega)$ and phase expressions $\angle Q(j\omega)$ are given by the next equations,

$$M_Q(j\omega)=\bar{k}_p\alpha\sqrt{\frac{1+\left(\bar{k}_f\omega-\frac{\bar{k}_i}{\omega}\right)^2}{\left(\omega^2+\gamma^2\right)\left(\omega^2+\phi^2\right)\prod_{m=1}^{q}\left(\omega^2+\delta_m^2\right)}} \tag{15}$$

$$\begin{aligned}\angle Q(j\omega)=&-\left(\pi-\arctan\left(\frac{\omega}{\gamma}\right)\right)-\omega\tau+\\&\arctan\left(\bar{k}_f\omega-\frac{\bar{k}_i}{\omega}\right)-\arctan\left(\frac{\omega}{\phi}\right)-\sum_{m=1}^{q}\arctan\left(\frac{\omega}{\delta_m}\right).\end{aligned} \tag{16}$$

In the above expressions, if $\bar{k}_i\to 0$, then $M_{Q_{\bar{k}_i=0}}(j\omega)\to M_Q(j\omega)$ and $\angle Q(j\omega)\to\angle Q_{\bar{k}_i=0}(j\omega)$.

Hence, applying a continuity argument on $\bar{k}_i$ it is always possible to choose a gain $\bar{k}_i$ small enough such that the inequalities (11) and (13) are fulfilled. From this fact, the expression (12) can be rewritten by using (14), such that the following relation,

$$\tau<\frac{1}{\gamma}-\frac{1}{\phi}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\frac{1}{\phi^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}} \tag{17}$$

is true.

Due to the freedom in selecting the parameter ϕ of the PI_f controller given by (2), with $\forall\phi>0$, the following expression is always fulfilled,

$$\begin{aligned}&\frac{1}{\gamma}-\frac{1}{\phi}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\frac{1}{\phi^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}<\\&\frac{1}{\gamma}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}.\end{aligned} \tag{18}$$

and allows obtaining the next bound on the time delay τ in (17),

$$\tau<\frac{1}{\gamma}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}.$$

which corresponds to the bound (4).

(Sufficiency) Suppose that (4) holds, then there exists an ε such that,

$$\left[\frac{1}{\gamma}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}\right]-\tau=\varepsilon. \tag{19}$$

Let us introduce a parameter ϕ such that $\forall\phi>0$ the following inequality holds

$$\frac{1}{\gamma}-\frac{1}{\phi}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\frac{1}{\phi^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}<\frac{1}{\gamma}-\sum_{m=1}^{q}\frac{1}{\delta_m}+\sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}.$$

Therefore, there exists $\bar{\varepsilon} > 0$ such that,

$$\left[\frac{1}{\gamma} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}\right] - \left[\frac{1}{\gamma} - \frac{1}{\phi} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}\right] = \bar{\varepsilon}. \tag{20}$$

On the other hand, since

$$\lim_{\phi\to\infty}\left[\frac{1}{\gamma} - \frac{1}{\phi} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}\right] = \left[\frac{1}{\gamma} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}\right] \tag{21}$$

it is possible to conclude that,

$\bar{\varepsilon} < \varepsilon.$

From (19), it follows that,

$$\frac{1}{\gamma} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}} = \varepsilon + \tau. \tag{22}$$

Substituting (22) into (20) produces,

$$-\tau + \left[\frac{1}{\gamma} - \frac{1}{\phi} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}\right] = \varepsilon - \bar{\varepsilon} \tag{23}$$

Since $\bar{\varepsilon} < \varepsilon$, then $\varepsilon - \bar{\varepsilon} > 0$ and it is allows obtaining the next inequality,

$$\tau < \frac{1}{\gamma} - \frac{1}{\phi} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}. \tag{24}$$

Notice that the Nyquist diagram for a stable system has a counterclockwise rotation around the point $(-1,0j)$. Note also that,

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0} = -\left(\tau - \frac{1}{\gamma} + \frac{1}{\phi} + \sum_{m=1}^{q}\frac{1}{\delta_m}\right) + \bar{k}_f \tag{25}$$

$$\frac{d}{d\omega}\left(\frac{M^2_{Q\bar{k}_i=0}(j\omega)}{\bar{k}_p^2\alpha^2}\right) = 2\omega\left[\frac{\bar{k}_f{}^2 - \left(1+\bar{k}_f{}^2\omega^2\right)\left[\dfrac{1}{\omega^2+\gamma^2} + \dfrac{1}{\omega^2+\phi^2} + \sum_{m=1}^{q}\dfrac{1}{\omega^2+\delta_m^2}\right]}{\left(\omega^2+\gamma^2\right)\left(\omega^2+\phi^2\right)\prod_{m=1}^{q}\left(\omega^2+\delta_m^2\right)}\right] = \frac{2\omega\varphi(\omega)}{\left(\omega^2+\gamma^2\right)\left(\omega^2+\phi^2\right)\prod_{m=1}^{q}\left(\omega^2+\delta_m^2\right)}. \tag{26}$$

Since, $\omega > 0$, the sign of (26) is determined equivalently by the sign of the next function,

$$\varphi(\omega) = \bar{k}_f^2 - \left(1 - \bar{k}_f^2\omega^2\right)\left[\frac{1}{\omega^2+\gamma^2} + \frac{1}{\omega^2+\phi^2} + \sum_{m=1}^{q}\frac{1}{\omega^2+\delta_m^2}\right]$$

then, evaluating around $\omega = 0$, it is produced the following expression,

$$\varphi(\omega)\big|_{\omega=0} = \bar{k}_f^2 - \left(\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}\right). \tag{27}$$

Moreover, rewriting inequality (24) yields,

$$\tau - \frac{1}{\gamma} + \frac{1}{\phi} + \sum_{m=1}^{q}\frac{1}{\delta_m} < \sqrt{\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}. \tag{28}$$

From this last expression it is possible to choose $\bar{k}_f$ such that,

$$\tau - \frac{1}{\gamma} + \frac{1}{\phi} + \sum_{m=1}^{q}\frac{1}{\delta_m} < \bar{k}_f < \sqrt{\frac{1}{\gamma^2} + \frac{1}{\phi^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}. \tag{29}$$

The above inequality allows obtaining the next inequalities,

$$\frac{d}{d\omega}\left(\angle Q(j\omega)\right)\Big|_{\omega=0} > 0$$

$$\frac{d}{d\omega}\left(\frac{M_Q^2(j\omega)}{\bar{k}_p^2\alpha^2}\right)\Bigg|_{\omega=0} < 0.$$

If the magnitude $M_Q(j\omega)$ is monotonically decreasing and that the phase $\angle Q(j\omega)$ has a change of sign, then the existence of an counterclockwise rotation is established in the Nyquist diagram and the closed-loop stability is ensured.

In a similar way as in the necessity part, notice that if $\bar{k}_i \neq 0$, it is always possible to choose a gain $\bar{k}_i$ small enough such that $M_Q(j\omega)$ is monotonically decreasing and $\angle Q(j\omega)$ is an increasing function for $\omega = 0$.

3.1 Stabilizing control parameters

Suppose that the condition (4) in Theorem 2 is satisfied. Therefore, there exists a PI_f controller that stabilizes systems (1). The PI_f parameters $\bar{k}_f$, $\bar{k}_i$ and $\bar{k}_p$ are obtained from the preceding developments. Note that from (17) it is possible to get,

$$\phi > \frac{\prod_{m=1}^{q}\delta_m(\gamma\tau - 1) + \gamma}{\prod_{m=1}^{q}\delta_m\left(\gamma\tau^2 - 2\tau\right) + 2(\gamma\tau - 1)} \tag{30}$$

Taking into account (21), the parameter $\bar{k}_f$ can be obtained from (29) as

$$\tau - \frac{1}{\gamma} + \sum_{m=1}^{q}\frac{1}{\delta_m} < \bar{k}_f < \sqrt{\frac{1}{\gamma^2} + \sum_{m=1}^{q}\frac{1}{\delta_m^2}}. \tag{31}$$

Once the parameters $\bar{k}_f$ are chosen, the gain $\bar{k}_i$ should be selected small enough such that inequalities (11) and (13) are satisfied. Finally, to ensure the counterclockwise rotation to the point $(-1,0j)$, the parameter $\bar{k}_p$ must satisfy,

$$\bar{k}_p\left(\omega_{c_1}\right) < \bar{k}_p < \bar{k}_p\left(\omega_{c_2}\right),$$

where ω_{c_1}, ω_{c_2} are the first two phase crossover frequencies solutions of the next equation,

$$\arctan\left(\frac{\omega_{c_i}}{\gamma}\right) - \omega_c\tau + \arctan\left(\bar{k}_f\omega_{c_i} - \frac{\bar{k}_i}{\omega_{c_i}}\right) - \sum_{m=1}^{q}\arctan\left(\frac{\omega_{c_i}}{\delta_m}\right) = 0. \tag{32}$$

and $\bar{k}_p\left(\omega_{c_{i=1,2}}\right)$ are given by,

$$\bar{k}_p\left(\omega_{c_{i=1,2}}\right) = \frac{1}{\alpha_1}\sqrt{\frac{\left(\omega_{c_i}^2+\gamma^2\right)\prod_{m=1}^{q}\left(\omega_{c_i}^2+\delta_m^2\right)}{1+\left(\bar{k}_f\omega_{c_i} - \frac{\bar{k}_i}{\omega_{c_i}}\right)^2}}. \tag{33}$$

Remark 1. It is worth noting that the proposed PI_f controller not only maintains the basic properties of conventional PI/PID controllers regarding constant disturbance rejection and tracking of step references; besides, it exhibits two advantages. First, the stability condition is improved by the term $\sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m{}^2}}$ compared with P or PI controllers, i.e., the PI_f allows stabilizing systems with larger delays than the ones allowed by the conventional P/PI controllers. Compared with PID controller a second advantage is that the PI_f controller does not resort on derivative terms; the above feature eases its practical implementation. Finally, the PI_f controller is able to stabilize systems with one minimum phase zero, a problem that has not been addressed in past work by means of $P/PI/PID$ controllers.

3.2 Particular case: Systems with one minimum phase zero

Another advantage of the proposed PI_f controller is that it can deal with unstable delayed systems having one minimum phase zero. The following corollary gives bounds on the maximum allowable time delay for a stable closed-loop system.

Corollary 1. Consider the class of high-order unstable time-delayed systems with one minimum phase zero plane given by,

$$\frac{Y(s)}{U(s)} = \frac{\alpha(s+\beta)}{(s-\gamma)\sum_{m=1}^{q}(s+\delta_m)}e^{-\tau s}, \tag{34}$$

with $\beta > 0$. There exists a PI_f controller defined by (2) such that the corresponding closed-loop system is stable if,

$$\tau < \frac{1}{\gamma} - \sum_{m=1}^{q}\frac{1}{\delta_m} + \sqrt{\frac{1}{\gamma^2}+\sum_{m=1}^{q}\frac{1}{\delta_m^2}}.$$

Proof. Suppose that condition (4) is satisfied. Choose the cut-off frequency of the filter in the PI_f controller as $\phi = \beta$. Therefore, the open-loop response in the frequency domain corresponds to the next expression,

$$\bar{Q}(j\omega) = k_p\alpha\frac{(j\omega)^2 + \left(k_f + k_i + \phi\right)j\omega + k_i\phi}{j\omega(j\omega-\gamma)\prod_{m=1}^{q}(j\omega+\delta_m)}e^{-j\omega\tau}. \tag{35}$$

Using the definitions (6) allow writing (35) as follows,

$$\bar{Q}(j\omega) = \bar{k}_p\bar{k}_f\alpha\frac{(j\omega)^2 + \left(\frac{1}{\bar{k}_f}\right)j\omega + \frac{\bar{k}_i}{\bar{k}_f}}{j\omega(j\omega-\gamma)\prod_{m=1}^{q}(j\omega+\delta_m)}e^{-j\omega\tau}. \tag{36}$$

First, consider the case when $\bar{k}_i = 0$, the phase and magnitude of (36) are given by,

$$\angle\bar{Q}_{k_{i=0}}(j\omega) = -\left(\pi - \arctan\left(\frac{\omega}{\gamma}\right)\right) - \omega\tau + \arctan\left(\bar{k}_f\omega\right) - \sum_{m=1}^{q}\arctan\left(\frac{\omega}{\delta_m}\right). \tag{37}$$

$$M\bar{Q}_{\bar{k}_i=0}(j\omega) = \bar{k}_p\alpha\sqrt{\frac{1+\bar{k}_f^2\omega^2}{\left(\omega^2+\gamma^2\right)\prod_{m=1}^{q}\left(\omega^2+\delta_m^2\right)}} \tag{38}$$

From this point, the proof of the corollary continues along the same path that the one followed in the the proof of the Theorem 2.

Remark 2. It is worth of mention that the procedure for finding the stabilizing parameters of the PI_f controller is the same as the one stated in Subsection 3.1.

Remark 3. It is also worth studying the case where the pole-zero cancellation proposed in Corollary 1 is not exact, i.e., $\phi \approx \beta$. Under this condition the resulting phase expression $\angle\bar{Q}^*(j\omega)$ is represented as,

$$\angle\bar{Q}^*(j\omega) = \angle\bar{Q}(j\omega) + \arctan\left(\frac{\omega}{\beta}\right) - \arctan\left(\frac{\omega}{\phi}\right). \tag{39}$$

Since $\phi \approx \beta$ then,

$$\arctan\left(\frac{\omega}{\beta}\right) - \arctan\left(\frac{\omega}{\phi}\right) \approx 0$$

and $\angle\bar{Q}^*_{\bar{k}_i=0}(j\omega) \approx \angle\bar{Q}_{\bar{k}_i=0}(j\omega)$ and inequality (11) is satisfied. It is also possible to prove that under the condition $\phi \approx \beta$, the decreasing property (13) holds. Therefore, if condition (4) is satisfied, then there exist $\bar{k}_f$, $\bar{k}_i$ and $\bar{k}_p$ gains that stabilize system (1) in closed-loop with a PI_f controller.

3.3 *PI_f: A general approach to deal with delayed systems including possible complex conjugate poles.*

In order to generalize the result presented in this work, it is proposed to extend the control strategy to addressed delayed systems with possible complex conjugate poles and one minimum phase zero.

Corollary 2. Considering a high-order unstable delayed system with possible complex conjugate poles and a single zero given by,

$$\frac{Y(s)}{U(s)} = \frac{\alpha(s+\beta)e^{-\tau s}}{(s+\gamma)\prod_{m=1}^{q}\left(s^2 + 2\zeta_m\,\omega_{n_m} s + \omega_{n_m}^2\right)}, \tag{40}$$

with $\gamma > 0$, $q = (n-1)/2$, ζ the damping relation and ω_n the undamped natural frequency. Then, there exists a PI_f controller defined by (2) such that the corresponding close-loop system is stable if and only if

$$\tau < \frac{1}{\gamma} + \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q}\frac{2\zeta_m^2 - 1}{\omega_{n_m}^2}} - 2\sum_{m=1}^{q}\frac{\zeta_m}{\omega_{n_m}}. \tag{41}$$

Notice that when $0 < \zeta < 1$, it is dealing with a couple of complex conjugate poles.

The proof of Corollary 2 can easily be inferred from the results presented in (Hernandez-Perez et al., 2015) and applying the same methodology as the one exposed in Theorem 2.

4. EXPERIMENTAL EVALUATION

The proposed PI_f control law is experimentally evaluated through the temperature control of an unstable heat flow process consisting of a delayed recycle thermal process. The process consist of the QUANSER Heat-Flow Experiment (HFE) laboratory prototype, which is modified by adding a recycle pipe segment; the later introduces positive feedback that leads to instability.

4.1 QUANSER Heat-Flow Experimental (HFE)

The QUANSER HFE consists of a fiberglass chamber whose measures are roughly $50cm \times 15cm \times 10cm$. An electric heating resistance and a fan are located at one side of the chamber. The prototype has three temperature platinum transducers; two of them located in the fiberglass chamber and one in recycle tube. Moreover, the HFE has an amplifier that provides the energy required by the heating electric resistance and the fan . A control signal $(0V - 5V)$ feeds the amplifier and regulates the energy provided to the heating electric resistance; a second control signal $\pm 6V$ feed another section of the amplifier and allows regulating the fan speed. The algorithms are coded using MatLab/Simulink and implemented in a Personal Computer (PC) endowed with a Sensoray data acquisition card model 626. Fig. 2 depicts the modified QUANSER HFE prototype with the recycling tube extension.

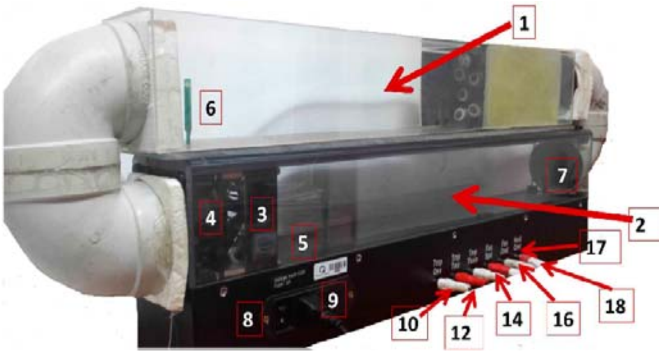

Fig. 2. QUANSER Heat-Flow Experiment HFE.

The tube extension has one heater and one fan that work with a constant voltage. The tube extension renders the whole thermal process unstable since the recycle trajectory adds a positive feedback to the original thermal process. The components in Fig. 2 are described in Table 1.

Table 1. System Components of HFE.

#ID	Component	#ID	Component
1	Recycle tube	10	Tmp one Connector $(^\circ C)$
2	Fiberglass chamber	11	Temperature sensor 1 offset
3	Blower	12	Tmp two Connector $(^\circ C)$
4	Heater coil	13	Temperature sensor 2 offset
5	Temperature sensor 1	14	Tmp three Connector $(^\circ C)$
6	Temperature sensor 2	15	Temperature sensor 3 offset
7	Temperature sensor 3	16	Fan spd Connector
8	Switch	17	Fan Cmd Connector
9	Power on/off	18	Heat Cmd Connector

It is worth mentioning that the temperature sensors provide analog signals from $0-5V$ that are proportional to the temperature with a calibration constant of $20^\circ C$. Moreover, the sensitivity of the sensors is $\pm 0.1^\circ C$. Note also that the heat transport phenomenon introduces a time delay in the measurement provided by temperature sensor 3. The control goal is to regulate the temperature along of the fiberglass chamber by means of the heat dissipated by the electric resistance and assuming a constant air flow rate. Sensor 3 measures this temperature. The control signal generated by means of the PI_f control law regulates the power provided to the electric resistance. The fan speed is kept constant at 1350 RPM.

4.2 Thermal HFE process modeling

The model of the modified HFE process is considered as the feedback interconnection of two first order systems. The corresponding block diagram is shown in Fig. 3, where the transfer functions functions $G_h(s)$ and $G_r(s)$ models respectively the forward and the feedback paths, where $\tau > 0$ corresponds to the time-delay generated by the heat-transfer phenomenon.

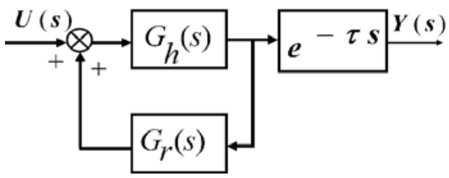

Fig. 3. Thermal HFE scheme with recycle and time-delay.

The modified HFE process is identified by using the bumptest method. See reference (QUANSER-Innovate, 2014) for further details. The forward path transfer function $G_h(s)$ is identified according to the next procedure:

1.- Turn on the HFE and let the blower run for two minutes to ensure the temperature sensors are settled down and any excess heat is flushed out. If the HFE was recently run, then extend this time.

2.- Set the HFE temperature to the actual room temperature, which is considered at $20^{\circ}C$. If the temperature sensors do not match the actual temperature, then adjust its offset by using the temperature sensor offset connectors (see Fig. 2: 11, 13 and 15 connectors).

3.- Set the fan speed at $1350 RPM$.

4.- The voltage supplied to the heater resistance is a step signal from 0 to $3V$.

The temperature time response and the voltage input signals obtained using the aforementioned procedure are illustrated in Fig. 4.

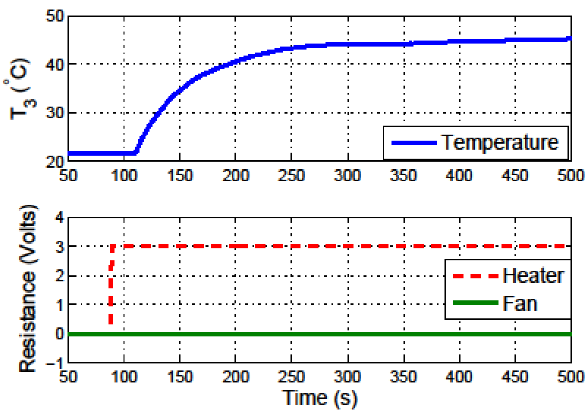

Fig. 4. Output and input signal used for the bumptest method.

The model obtained from this experiment corresponds to the next transfer function,

$$G_h(s) = \frac{0.284}{s + 0.024}. \tag{42}$$

In order to identify the recycle phenomenon $G_r(s)$, it is assumed that the the dynamics of the direct path $G_h(s)$ reaches its stationary response (Fig. 4). At that time the external recycle tube is connected to the process. It is important to mention that in order to identify $G_r(s)$, only one side of the recycle tube extension is connected to the fiberglass chamber; the sensor 3 is attached to this side. The temperature at the free side of the tube is measured using the sensor 2. The time evolution of the recycle temperature and the input voltage of the fan and heater are shown in Fig. 5.

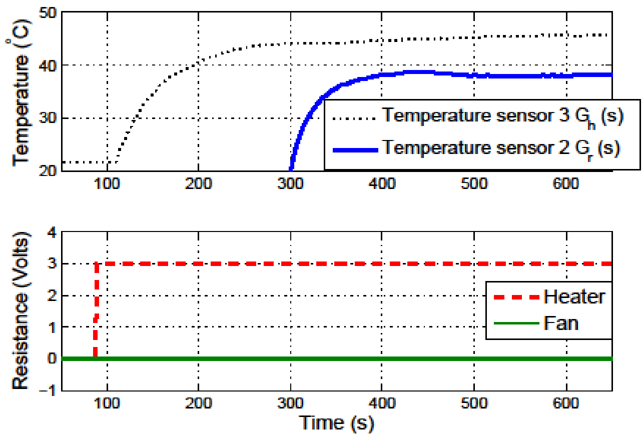

Fig. 5. Input and output signals for the recycle transfer function.

The transfer function $G_r(s)$ corresponds to the next expression,

$$G_r(s) = \frac{0.353}{s + 0.014} \tag{43}$$

To illustrate the problem statement given in Section 2, a blocks reduction of Fig. 3 between $G_h(s)$ and $G_r(s)$ is carried out, resulting in a single transfer function $G_T(s)$ given by,

$$G_T(s) = \frac{0.284(s + 0.014)}{(s - 0.296)(s + 0.334)}.$$

Besides the recycle generated during the thermal process, a time delay of 2 seconds is considered in the temperature register of sensor 3. This delay takes place because the heat flow is transported along the fiberglass chamber of the process. This phenomena causes that the sensor measures the generated temperature a time period after the variation in the input voltage of the resistance appears. Therefore, from the identified transfer functions $G_h(s)$ and $G_r(s)$, it is possible to obtain the open loop transfer function of the modified HFE process including the time delay,

$$G_T(s) = \frac{0.284(s + 0.014)}{(s - 0.296)(s + 0.334)} e^{-2s}. \tag{44}$$

4.3 Experimental Results: Temperature control

Consider the model of the modified HFE process (44). The block diagram for the PI_f controller is shown in Fig. 6.

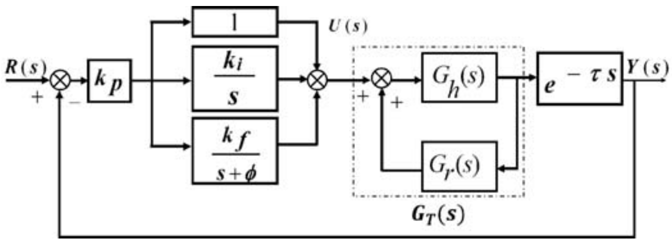

Fig. 6. Proposed PI_f control strategy.

From Corollary 1, the stability condition is satisfied since,

$$\tau = 2 < \frac{1}{0.296} - \frac{1}{0.334} + \sqrt{\frac{1}{(0.296)^2} + \frac{1}{(0.334)^2}} = 4.89.$$

As a first step, the free pole provided by the PI_f transfer function $H(s)$ is located in the same position of the β zero of the modified HFE process such that $\phi = 0.014$. According to (31), a stabilizing gain $\bar{k}_f = 4$ is obtained in the range $1.6 < \bar{k}_f < 4.5$. The value $\bar{k}_i = 0.009$ ensures that the magnitude and phase conditions are satisfied. Finally, solving (32) allows obtaining the range $0.47 < \bar{k}_p < 0.84$ from which the value $\bar{k}_p = 0.48$ is considered. Taking into acount (6) permits obtainig the transfer function of the proposed PI_f controller,

$$H(s) = 1.92\left(1 + \frac{0.16}{s} + \frac{0.076}{s + 0.014}\right).$$

Fig. 7 shows the time evolution of the temperature of the modified HFE process considering a reference signal of $60^\circ C$.

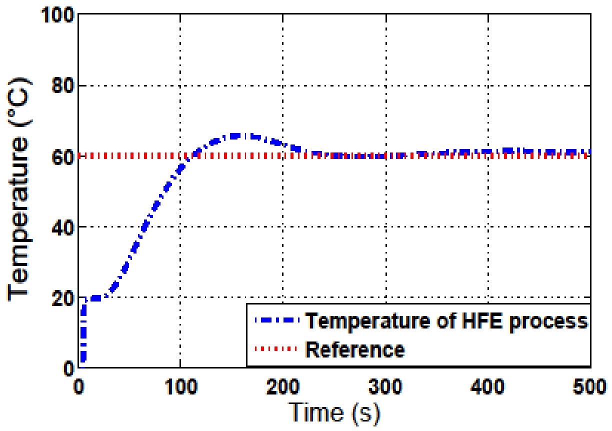

Fig. 7. Temperature evolution of the thermal process HFE.

To evaluate the robustness of the control strategy against parametric uncertainties that are allways present on a real process, Fig. 8 shows a comparison between the simulation (mathematical model describing the HFE process given in (44)) and the modified HFE prototype laboratory. As it can be observed, the experimental response is similar to that of the numerical simulation, since both initiate with a similar initial conditions around $20^\circ C$. Moreover, Fig. 8 also illustrates how the modified HFE process tracks a reference signal step change from $60^\circ C$ to $75^\circ C$.

Fig. 9 illustrates how the PI_f controller rejects a step disturbance applied at 450 seconds. This disturbance is produced by an obstruction of 70% of the heat flow passage in the fiberglass chamber. The process temperature returns to the desired reference at 800 seconds approximately. Also, Fig. 9 presents a comparison between the real process and the simulation of the mathematical model (44). It is possible to observe that in the simulation signal an oscillation occurs when the perturbation acts, phenomenon that is not present on the real-time experiment. Both signals converge around 750 seconds.

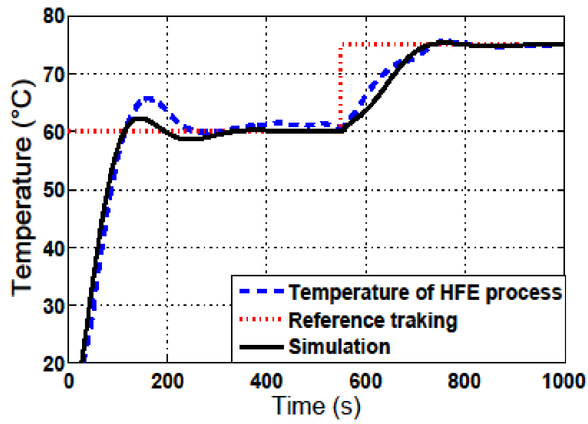

Fig. 8. Reference tracking of the control temperature.

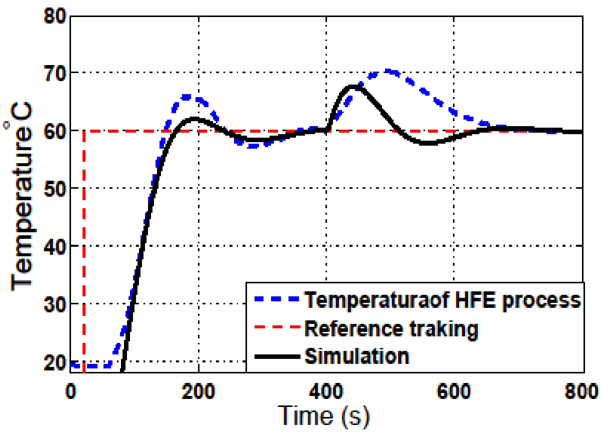

Fig. 9. Disturbance rejection property of the process.

Finally, the control signal related to this experiment is shown in Fig. 10.

On the other hand, in order to prove the efficiency of the proposed control strategy, a comparison between the PI_f controller and a classic PI controller with a filtered derivative action (PI_{FDA}) was done to show how the proposed controller provides a more appropriate transients response than the classical one.

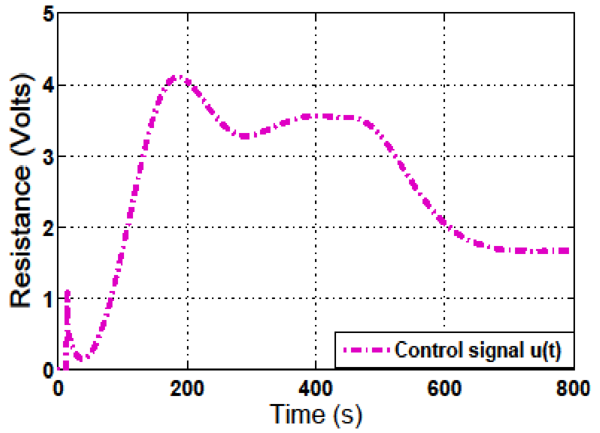

Fig. 10. Control action.

Fig. 11 illustrates the time evolution of both controllers (dotted line vs solid line) when they are applied to the mathematical model of the HFE thermal process in a numercial simulation. Both responses are compared with the laboratory prototype response (dashed line).

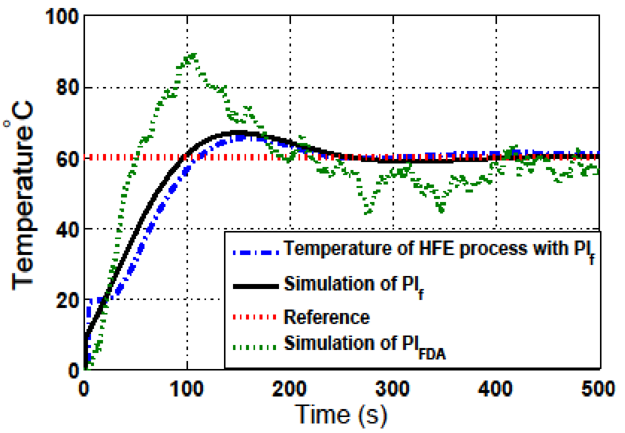

Fig. 11. Disturbance rejection property of the process.

Fig. 11 shows that the PI_f controller presents a better performance at the transient response since not only eliminates the noise but also provides a smoother response which is a desirable feature in any controller.

Remark 4: The considered PI_{FDA} controller was design as,

$$H^*(s)=k_p\left(1+\frac{k_i}{s}+\frac{s\,k_d T_d}{1+\frac{sT_d}{N}}\right), \tag{45}$$

where the control parameters are given as $k_d = 0.00342$, $T_d = 571.429$, $N = 8$, $k_i = 0.023$ and $k_p = 0.2352$. It is important to highlight that the calculation of both, the tuning parameters as well as the stability conditions necessary to apply the classic PI controller with filter, are not obvious. Therefore, further analysis would be required to specify the most accurate range of values for the tuning parameters.

5. CONCLUSION

This work studies the stabilization problem of a class of high-order unstable systems with time delay by using a novel control law called the PI_f controller. It consists of a standard PI controller plus a first order low-pass filter. Necessary and sufficient conditions to stabilize a system consisting of one unstable pole, $n-1$ stable poles, time delay and possible one minimum phase zero are stated, and a procedure to tune the stabilizing gains k_p, k_i and k_f is given. It is shown that the proposed PI_f controller improves the stability conditions of a standard PI controller. Moreover, the proposed PI_f allows stabilizing the same family of unstable time delay systems than those tackled in (Lee et al., 2010) with a traditional PID controller, however, it is worth noting that the proposed PI_f controller does not resort on a derivative term and then, it can easily be implemented as a proper transfer function. This allows to reduce the noise caused by the derivative term. Additionally, the PI_f controller is able to stabilize plants having one minimum phase zero, which is not a trivial task when using a standard PID controller. The performance of the control strategy is evaluated by considering an unstable laboratory prototype, which consist in a thermal flow process endowed with a recycle path and a time-delay. These outcomes show an adequate performance of the PI_f controller in the implementation.

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