

## An Experimental Study on Model Reference Adaptive Control of TRMS by Error-Modified Fractional Order MIT Rule

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**Abstract:** Model Reference Adaptive Control (MRAC) strategies find application in flight control because of changes in dynamics of flight conditions. This paper demonstrates an application of Fractional Order Adjustment Rule MRAC (FOAR-MRAC) with the modification of model error dead zone for adaptive control of Twin Rotor Multi-input multi-output System (TRMS). Here, we implement FOAR-MRAC structure with feedforward and feedback MIT rules by using a fractional order integrator. Previously, Vinagre et al. have reported that MIT with fractional order integrator can improve tracking performance of MRAC. In the current experimental study, we modified model approximation error by using a piecewise linear, near-zero dead zone function and manage stability of adaptation process in practical application. Accordingly, when the control system response approximates to response of reference model, adaptation process is interrupted. This modification improves quasi-stabilization of updating rule by omitting low level errors and contributes to applicability of MRAC in real applications. An adaptive PID rotor control system is developed by integrating the proposed FOAR-MRAC structure. Simulation and experimental results, obtained for TRMS setup, are presented to show effectiveness of the proposed method.

*Keywords:* Adaptive PID controller, model reference adaptive control, fractional order integrator, DC rotor control, TRMS

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### 1. INTRODUCTION

Model reference adaptive control has been traced back to sixties. Originally, model reference approach was developed around 1960 (Osburn et al., 1961) and considered a decade later by Landau (Landau, 1979). Afterwards, MRAC method has become a fundamental topic of adaptive control and extensively studied in many works (Astrom and Wittenmark, 1995; Vinagre et al., 2002). MRAC is known as an effective and straightforward approach to implement. It was shown that MRAC can improve robust performance in case of parameter variations, noise and uncertain dynamics (Bernardo et al., 2013). For this reasons MRAC has been widely utilized in practical applications. For instance, real-coded genetic algorithm was implemented in tuning of a modified MRAC structure for hybrid tank control application (Mohideen et al., 2013). MRAC was modified for the speed estimation of the induction motor drive (Ravi et al., 2012). Guo and Parsa implemented MRAC for control of five-phase interior-permanent-magnet motor drives (Guo and Parsa, 2012). MRAC structure was also used for distributed control applications: A distributed adaptive protocol was proposed for the system without disturbances by adopting the MRAC (Liu and Jia, 2012). A distributed model reference adaptive control architecture was developed to achieve cooperative tracking of uncertain dynamical multi-agent systems, where the reference model serves as a virtual leader for the group (Peng et al., 2013). There are also comparative case studies

that implement MRAC and compare with the control performance of other methods. (Duka et al., 2007; Abraham et al., 2016). As a consequence, MRAC strategy was considered particularly for the control applications where system dynamics can alter by changing environmental condition. For instance, a solution for fault-tolerant tracking control for near-space-vehicle dynamics was proposed (Jiang et al., 2010). Sadeghzadeh et al. discussed testing trajectory and the experimental flight testing results with both Gain-Scheduled-PID and MRAC for fault/damage (Sadeghzadeh et al., 2011). In order to address the control of an air-breathing hypersonic flight vehicle (AHFV) with actuator saturation, a model reference adaptive switching control approach was suggested (Dong et al., 2010).

In general, flight control systems should deal with nonlinear aerodynamics and variability of flight conditions, altitude, payload and weather conditions. Reference model based adaptive control systems provide self-tuning of control systems and it can contribute to robust performance of flight control systems under varying conditions (Alagoz et al., 2013). Considering instantaneously changing dynamics of flight conditions, MRAC has significance in flight control applications because of a real-time tuning of controller according to a reference model response that allows online adaptation to this condition. Meta-heuristic optimization methods can find satisfactory solutions in simulations (Alagoz et al., 2013; Tran and Wang, 2016), these methods perform repetitive set and trail sessions in order to update

controller coefficients. Hence, meta-heuristic methods need further development to be used in real-time tuning applications. In particular, MRAC structure is useful for real control systems, where online adjustment of control parameters are required during changing control dynamics. A popular implementation of MRAC employs well known MIT rule that is based on descending of model approximation error regarding to gradient directions of cost function. Essentially, this mechanism leads to real-time tuning of control system without a need for distinct set and trail actions during operation.

Nowadays, there is a growing trend for utilization of fractional calculus in solution of science and engineering problems (Petras, 2011). Specifically, the control engineering has been widely benefited from advantages of fractional calculus. Many works have shown that fractional order controllers and system models can enhance control system performance (Oustaloup, 1991a, 1995, 1999; Podlubny 1999a, 1999b; Chen, 2004; Petras, 1999, 2011; Valerio, 2012; Monje, 2004, 2010; Yeroglu and Ates, 2014; Yeroglu and Kavuran, 2014). In this fashion, Vinagre et al. have improved the performance of conventional MRAC structure in a theoretical study by performing fractional order integration in MIT rule (Vinagre et al., 2002). For experimental illustration, MRAC with fractional order feedback MIT rule was implemented on a low-cost ARM microcontroller for coaxial rotor control application (Kavuran et al., 2016). In the current study, we implement forward and backward MIT rules and investigate contribution of dead zone error modification for control of TRMS platform.

This paper presents an experimental study on TRMS, which demonstrates implementation of FOAR-MRAC structure for adaptive rotor control. The study discusses solutions to improve real-time control performance of FOAR-MRAC structure, which implements feedforward and feedback MIT rule. In order to manage approximation of control system response to the response of reference model, we used a near-zero dead zone for model approximation error. This modification enables to establish adaptation process in two states: (i) adaptation active and (ii) adaptation interrupted. When control system does not yield response similar to reference model, the system goes into the adaptation active state and updates gain parameters of the control system. It interrupts the adaptation process, again, in the case that control system response approximates to the reference model. Accordingly, adaptation process activates when responses of the control system and reference model are different from each other. This also prevents complications resulting from continuously updating of parameters. We also show that this modification contributes to updating rule of FOAR-MRAC by omitting low level errors and makes the adaptation process more robust against the noise and disturbance of practical systems. Rotor control generally needs PID control action to obtain a satisfactory time response. The proposed FOAR-MRAC is integrated to closed loop PID control system of DC rotor to develop an adaptive PID control. Conventional FOAR-MRAC structure provides only adaptive proportional control actions, however we observed that control of TRMS

requires the utilization of PID control actions. TRMS simulation and experimental results obtained for FOAR-MRAC, conventional MRAC are compared and results are discussed.

## 2. METHODOLOGY

### 2.1. Preliminaries of Fractional Calculus

Fractional order derivative operator is an extension of integer order one. The first discussions on a fractional-order derivative (FOD) trace back to 17<sup>th</sup> century with a speculation between L'Hospital and Leibniz (Ricardo et al., 2010; Leibnitz, 1962). However, underpinning works in the field were carried out in the last century by suggestion of popular definitions e.g. Grünwald-Letnikov, Riemann-Liouville and Caputo definitions. The Caputo definition was given as, (Petras, 2011; Sierociuk et al., 2013),

$${}_a D_t^\alpha = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad n-1 \leq \alpha < n \quad (1)$$

where, the operator  ${}_a D_t^\alpha$  represents fractional order derivative operator. Parameters  $a$  and  $t$  denote the lower and upper bounds, and parameter  $\alpha \in R$  stands for the fractional-order.  $\Gamma(\cdot)$  is Euler's gamma function. Laplace transform of fractional order derivative was used in transfer function modelling and it was expressed as  $L(D^\alpha f(t)) = s^\alpha F(s)$  for zero initial conditions. According to Equation (1), one can see that fractional order derivative is not localized and it spreads backwards in time to all previous values of function. Due to inclusion of all previous values in function in time, realization of ideal fractional order derivative consumes a great deal of computation resources as time goes by. To reduce computation complexity, integer order approximate models of fractional order models have been developed and widely utilized in realization of fractional order system models in applications. For control applications, due to providing a satisfactory low frequency region approximation to fractional order derivative operators, Continued Fraction Expansion (CFE) approximation method is commonly used in practice. In this study, we implemented the fourth order integer order approximate model of fractional order derivatives by using following CFE expansion (Chen et al., 2004),

$$C(s) = a_0 + \frac{s-s_0}{a_1 +} \frac{s-s_1}{a_2 +} \frac{s-s_2}{a_3 +} \quad (2)$$

### 2.2. Adaptive PID Control by FOAR-MRAC with Error Dead Zone Modification

Conventional MRAC structure was proposed for gain adaptive control applications. The adaptation mechanism of MRAC is based on gradient descent optimization, which was also known as MIT rule. The model approximation error is defined as,

$$e = y_p - y_m \quad (3)$$

where  $y_m$  is the reference model output, and  $y_p$  is the control system output. Approximation of this error to zero enforces the control system adapt itself according to response of reference model. The reference model is commonly chosen as a theoretical model providing a desired control performance (Kavuran et al., 2016). MRAC method can perform adaptation under the assumption of that system parameters deviate more slowly than the adaptation parameter of MRAC because of the quasi-stationary treatment of derivatives (Vinagre et al., 2002). In order to improve quasi-stationary settle of adaptation parameters in practical applications, we employed a dead zone error function as in Fig. 1. This function interrupts adaptation process when an acceptable adaptation to reference model is achieved.

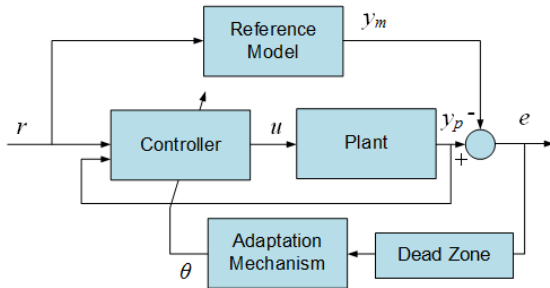


Fig. 1. Schematic diagram of MRAC, with dead-zone error modification.

In practice, fully equalization of control system to a reference model may not be always possible, and it may prevent stabilization of adaptation parameters. Instability of adaptation parameters results in decrease of control performance. On the other hand, dead zone modified error is also needed for the reduction of negative effects of noise and instantaneous parameter fluctuations on the adaptation process (Lavretsky, 2009). The dead zone function to modify model approximation error signal is expressed by means of piecewise linear function as following,

$$e_d(t) = \begin{cases} 0 & , |e(t)| < e_z \\ e(t) & , \text{for others} \end{cases} \quad (4)$$

Fig. 2(a) depicts piecewise linear dead zone function. Parameter  $e_z$  specifies the width of error dead zone, where adaptation process should be interrupted. When the reference model and control system outputs are similar enough to each other,  $e_d$  takes the value of zero because the amplitude of model approximation error  $e(t)$  is less than  $e_z$  threshold. If the reference model output and control system output begin to discriminate,  $e_d$  takes the value of  $e(t)$  at linear part, and FOAR-MRAC system performs adaptation process until the  $e(t)$  decreases into dead zone ranges.

Parameter  $e_z$  defines a threshold for the transition between activation and interruption states of adaptation process. Activation or interruption of adaptation process is depicted

according to the magnitude of model approximation error in Fig. 2 (b) and (c). Adaptation process enforces the decrease of  $e(t)$  by performing MIT rule and hence it is in tendency of driving the system towards interruption of adaptation process in time.

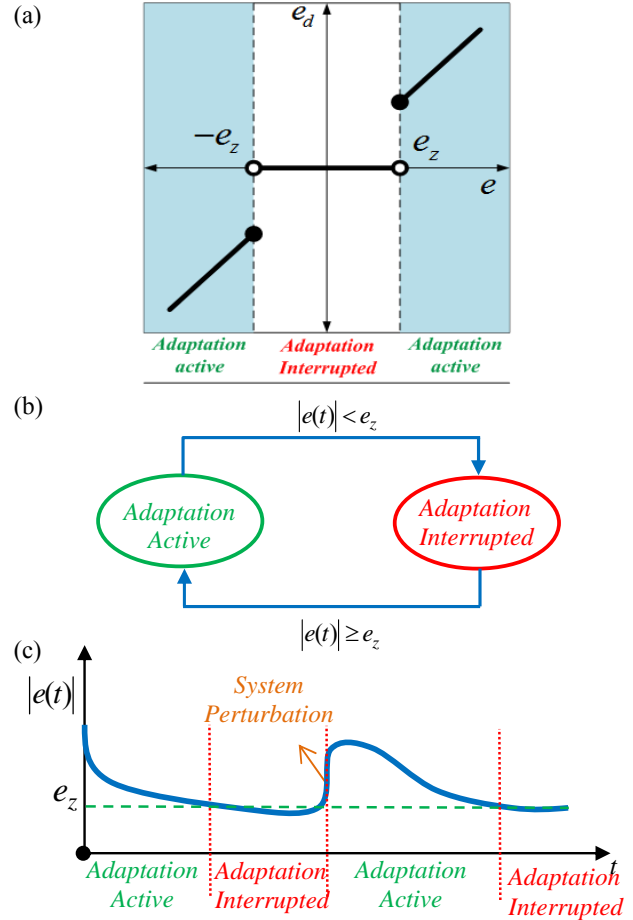


Fig 2. (a) Piecewise linear dead zone function for the modification of model approximation error; (b) State transition of adaptation process; (c) An illustration of activation or interruption of adaptation process according to magnitude of model approximation error.

Whenever responses of reference model and control system differ, it increases amplitude of  $e(t)$  over  $e_z$  and FOAR-MRAC goes in adaptation active state. Thus, it ensures approximation of control system response to reference model response by minimizing a convex cost function ( $J$ ) via MIT rule. Besides, the dead zone around zero reduces misleading effect of random system noise as long as noise magnitude is less than  $e_z$ . Therefore, presetting of  $e_z$  should be done according to requirements of control applications.

Major advantage of error dead zone modification comes from that it is very helpful to overcome complications encountered in real applications. Because, this modification indeed introduces a tolerance range for control system perturbation and thus it prevents disturbance of control performance by continual parameter updating of adaptation process. It

triggers adaptation process only when the discrepancy between real control system and reference model exceeds a tolerable range that is configured by the threshold  $e_z$ . The cost function to be minimized was given as,

$$J(\theta) = \frac{1}{2} e_d^2(\theta) \quad (5)$$

According to (5), MIT adjustment rule was expressed as,

$$\frac{d\theta}{dt} = -\gamma \frac{dJ}{d\theta} = -\gamma e_d \frac{de_d}{d\theta} \quad (6)$$

Equation (6) allows to update adaptation parameter ( $\theta$ ) in the direction reducing the cost function. Later, Vinagre et al. modified the conventional MRAC structure by using fractional order integrator as,

$$\frac{d^\alpha \theta}{d^\alpha t} = -\gamma \frac{dJ}{d\theta} = -\gamma e_d \frac{de_d}{d\theta} \quad (7)$$

By using,  $\frac{\partial e_d}{\partial \theta} = y_m$ , the adaptation parameter  $\theta$  was obtained as,

$$\theta = D^{-\alpha} (-\gamma e_d y_m) \quad (8)$$

Fig. 3 illustrates feedforward and feedback MIT rule implementation of FOAR-MRAC for rotor control application. The figure also includes the dead zone block that is connected to model approximation error. By feedforward and feedback MIT rule, control signal from PID controller is modified as,

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \quad (9)$$

where,  $\theta_1$  parameter is for feedforward adaptation and  $\theta_2$  parameter is for feedback adaptation of the control system. The temporal evaluation of  $\theta_1$  and  $\theta_2$  adaptation parameters were written to minimize the cost function as the following,

$$\frac{\partial^\alpha \theta_1}{\partial^\alpha t} = -\gamma \frac{\partial J}{\partial \theta_1} = -\gamma \frac{\partial e_d}{\partial \theta_1} e_d \quad (10)$$

$$\frac{\partial^\alpha \theta_2}{\partial^\alpha t} = -\gamma \frac{\partial J}{\partial \theta_2} = -\gamma \frac{\partial e_d}{\partial \theta_2} e_d \quad (11)$$

Reference model is assumed to be a first order system in the form of  $\frac{b_m}{s+a_m}$ . When the PID controller is considered in

Fig. 3, one can write sensitivity derivatives as,

$$\frac{\partial e_d}{\partial \theta_1} = \frac{a_m}{s+a_m} PID(s)(u_c - y_p) \quad (12)$$

$$\frac{\partial e_d}{\partial \theta_2} = -\frac{a_m}{s+a_m} y_p \quad (13)$$

Here,  $PID(s)$  represents transfer function of PID controller defined as  $PID(s) = k_p + k_i/s + k_d s$ . The solutions for the update rules of parameters  $\theta_1$  and  $\theta_2$  with dead zone error modification can be written as follows,

$$\theta_1(t) = \begin{cases} \theta_1(t) & , |e(t)| < e_z \\ -\frac{\gamma}{s^\alpha} \left( \frac{a_m}{s+a_m} \right) (u_c - y_p) & , \text{for others} \\ (k_p + k_i/s + k_d s) e(t) & \end{cases} \quad (14)$$

$$\theta_2(t) = \begin{cases} \theta_2(t) & , |e(t)| < e_z \\ \frac{\gamma}{s^\alpha} \left( \frac{a_m}{s+a_m} \right) y_p e(t) & , \text{for others} \end{cases} \quad (15)$$

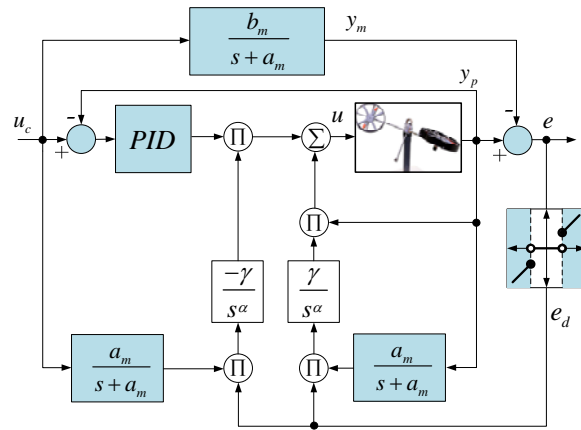


Fig. 3. Block diagram of proposed FOAR-MRAC structure for TRMS rotor control.

Equations (14) and (15) imply the interruption of adaptation process, in the case that real control system response and reference model response are similar enough. Consequently, the condition  $|e(t)| < e_z$  contributes to stabilization of adaptation parameters  $\theta_1(t)$  and  $\theta_2(t)$ . For  $|e(t)| \geq e_z$ , adaptation process takes place according to solutions of

$$\theta_1(t) = -\frac{\gamma}{s^\alpha} \left( \frac{a_m}{s+a_m} \right) (k_p + k_i/s + k_d s) (u_c - y_p) e(t) \quad \text{and}$$

$$\theta_2(t) = \frac{\gamma}{s^\alpha} \left( \frac{a_m}{s+a_m} \right) y_p e(t). \quad \text{In conventional FOAR-MRAC}$$

structure, it was written as  $\theta_1(t) = -\frac{\gamma}{s^\alpha} \left( \frac{a_m}{s+a_m} \right) u_c e(t)$  and it is limited to a proportional gain control (Vinagre et al., 2002). Since, rotor control needs to support of integrator and derivative control actions to present a desired flight control performance, FOAR-MRAC structure should work in conjunction with conventional PID control systems (Kavuran et al., 2016). This integration also brings advantage of easily transformation of conventional PID control system into adaptive PID control systems.

### 3. SIMULATION STUDY

The TRMS is an experimental system that has been developed for flight control simulation. It is a two-rotor system which manoeuvres by vertical and horizontal rotors, (Tao et al., 2010). Fig. 4 shows a picture of TRMS experimental system. The motion of the shaft can be controlled by the input voltages that adjust the rotational speed of these two propellers driven by DC electric motors. The pitch rotor has a control on vertical angle of the shaft, and the yaw rotor controls the horizontal angle of the shaft. A pendulum counterweight is attached to the propeller shaft to adjust the angular momentum of the pitch rotor motion on the vertical plane (Alagoz et al., 2013).

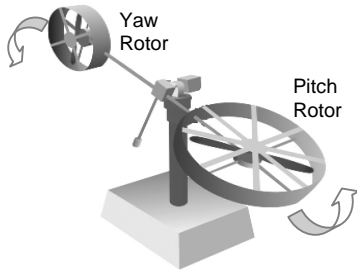


Fig. 4. Representation of TRMS system

The positioning control of vertical rotor speed introduces a nonlinear control problem (Ahammad and Purwar, 2009). In our simulations, we used nonlinear dynamical model of pitch rotor, which considers the friction momentum, the gravity momentum, the gyroscopic momentum and the cross reaction momentum. Dynamic model of the pitch rotor was given as follows, (Bernardo et al., 2013; Mohideen et al., 2013; Song et al., 2013).

$$D \begin{bmatrix} \psi \\ \dot{\psi} \\ \tau_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -B_{1\psi}/I_1 & b_1/I_1 \\ 0 & 0 & -T_{10}/T_{11} \end{bmatrix} \begin{bmatrix} \psi \\ \dot{\psi} \\ \tau_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k_1/T_{11} \end{bmatrix} u_{\psi} + f_{\psi}(x) \quad (16)$$

where  $\psi$  is the pitch angle and  $D\psi$  represents angular velocity around the vertical axis. The parameter  $\tau_1$  is output torque of pitch rotors and  $u_{\psi}$  is voltage applied to the electrical motor. Remaining parameters of the TRMS are given in Table 1 (Song et al., 2013). The function  $f_{\psi}(x)$  represents nonlinearity.

TRMS simulations were performed via Matlab/Simulink environment. Fig. 5 shows pitch angle responses of TRMS system for a multi-sinusoidal input signal given by,

$$u_c = 0.4 + 0.1 \sin(2\pi 0.05t) + 0.1 \sin(2\pi 0.02t) \quad (17)$$

In simulations,  $\gamma = 4$  and  $e_z = 0.03$  were configured for the proposed FOAR-MRAC structure. Fig. 5(a) shows the response obtained by conventional MRAC structure by

setting  $\alpha = 1.0$ . Fig. 5(b) shows FOAR-MRAC structure by setting  $\alpha = 1.55$ . Fractional integrator for  $\alpha = 1.55$  was implemented by CFE method according to (2) as follows,

$$1/s^{1.55} \cong \frac{-0.8792s^4 + 35.4895s^3 + 538.257s^2 + 878.5345s + 228.5983}{228.5983s^4 + 878.5345s^3 + 538.257s^2 + 35.4895s - 0.8792} \quad (18)$$

Table 1. Parameters of TRMS model.

Symbols	Definitions	Values
$B_{1\psi}$	Friction momentum function parameter of vertical axis	6x10-3 Nms/rad
$I_1$	Moment of inertia of pitch rotor	6,8x10-2 kg,m2
$b_1$	Static characteristic parameter	0,0924
$a_1$	Static characteristic parameter	0,0135
$\tau_1$	Momentum of pitch rotor	$\tau_1 = \frac{k_1}{T_{11}s + T_{10}} u_{\psi}$
$k_1$	Gain of pitch motor	1,1
$T_{10}$	Denominator of pitch motor	1,2
$T_{11}$	Denominator of pitch motor	1,1

For PID controller,  $k_p = 0.05$ ,  $k_d = 2$  and  $k_i = 0.9$  coefficients, were used in the simulation. These PID coefficients were not well-tuned coefficients for PID control in order to show performance improvement provided by adaptation process. Results in Fig. 5 reveal that FOAR-MRAC with dead zone error modification can improve reference model tracking performance. Fig. 6 (a) and (b) demonstrate that FOAR-MRAC structure with  $\alpha = 1.55$  can speed up adaptation process. As shown in figures, adaptation parameters of FOAR-MRAC structure ( $\alpha = 1.55$ ) settle faster to higher value than those of conventional MRAC structure ( $\alpha = 1.0$ ). Fig. 7 shows dead zone modified error signal of FOAR-MRAC structure for  $e_z = 0.03$  and  $\alpha = 1.55$ . After acceptable adaptation is achieved, the dead zone function assigns zero value to error signal in order to interrupt adaptation process. This also improves quasi-stationary change of adaptation parameters and reduces negative effects of noise and distortion on the adaptation process. In order to demonstrate this effect, we applied a square wave reference signal to FOAR-MRAC structure. Figs. 8 and 9 compare the simulation results at the presence and absence of dead zone modified error.

As seen in figures, since a desired model approximation to reference model was achieved, FOAR-MRAC with error dead zone interrupts adaptation process at about 300 sec. However, FOAR-MRAC without error dead zone continued adaptation process and it consistently increases adaptation parameters. This leads to overshoots in the response of FOAR-MRAC without error dead zone as seen in Fig. 8(a). Rotor control for multi-rotor systems particularly requires smooth and low overshoot control to maintain flight stability and reduces energy consumption (Alagoz et al., 2013). In this

manner, error dead zone modification to FOAR-MRAC structure allows low overshoot control of adaption process and improves rotor control performance. Fig. 10 shows simulation results when input disturbance was applied to TRMS. Disturbance signal was composed of random signal with a bias level. As seen in figures, since a desired model approximation to reference model was achieved, FOAR-MRAC with error dead zone interrupts adaptation process at about 200 sec. However, after disturbance started at 300 sec, the proposed FOAR-MRAC reactivates adaptation process so that the response of control system slightly diverges from reference model response and it increases the model approximation error  $e(t)$ . After adapting new working condition,  $e(t)$  reduces to below of threshold  $e_z$  and it interrupts again adaptation process and performs in normal PID control mode.

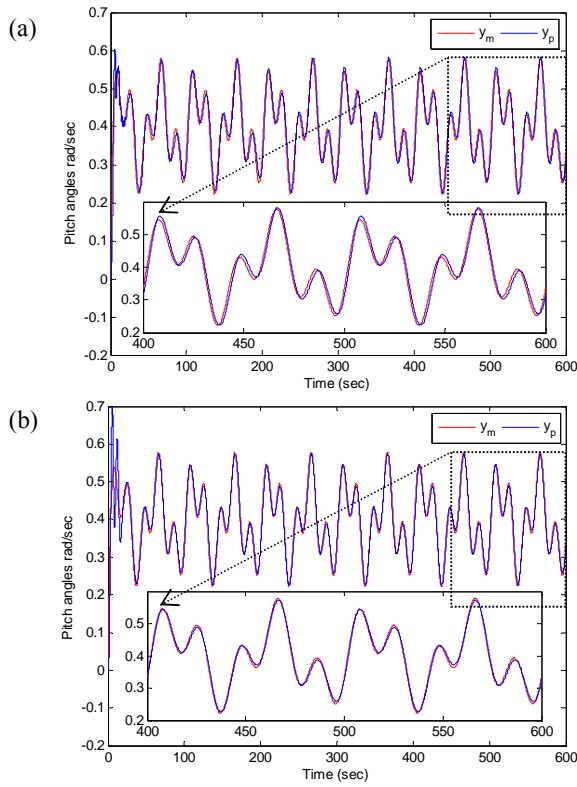


Fig. 5. Pitch angle response of TRMS for (a)  $\alpha = 1.0$  and (b)  $\alpha = 1.55$ .

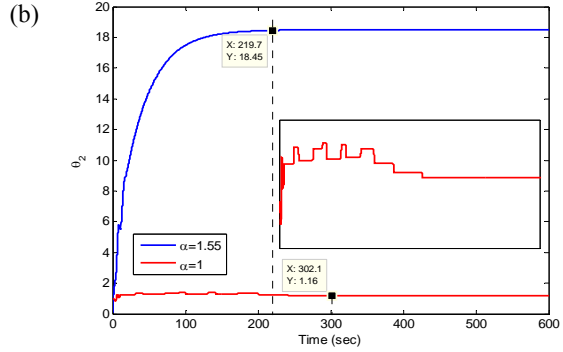
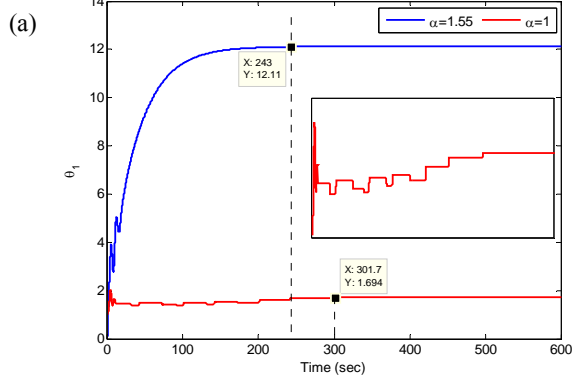


Fig. 6. a) Temporal evolution of  $\theta_1$  for  $\alpha=1.0$  and  $\alpha=1.55$ ; b) Temporal evolution of  $\theta_2$  for  $\alpha=1.0$  and  $\alpha=1.55$ .

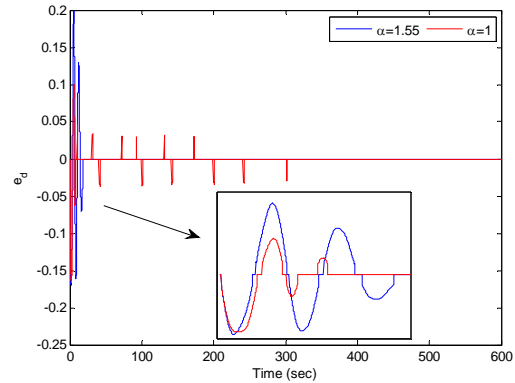


Fig. 7. Error with dead zone for integer and fractional cases.

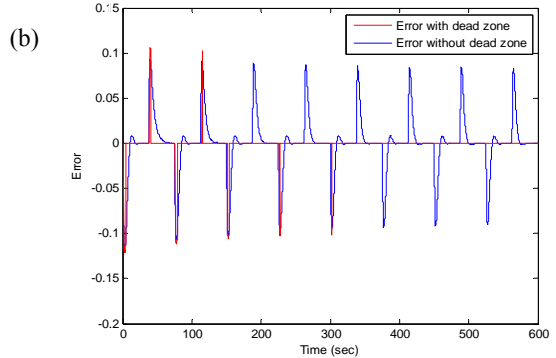
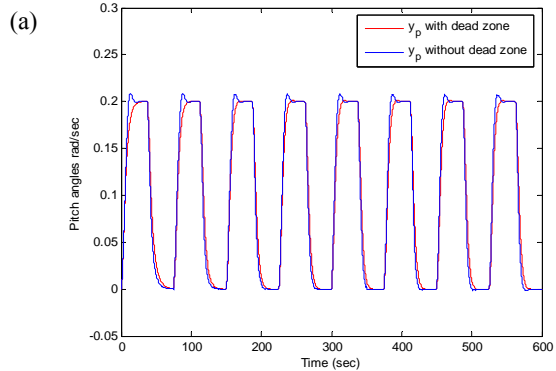


Fig. 8. a) Step response FOAR-MRAC with and without dead zone modified error; b) Model approximation error signal with dead zone and without dead zone.

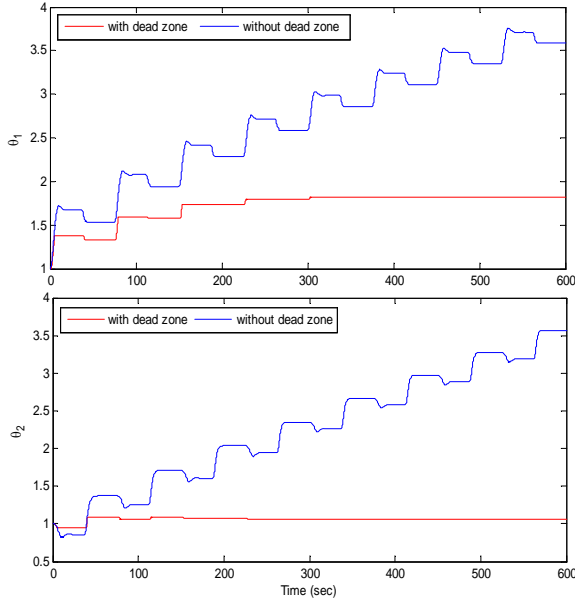


Fig. 9. Temporal evolution  $\theta_1$  and  $\theta_2$  for FOAR-MRAC.

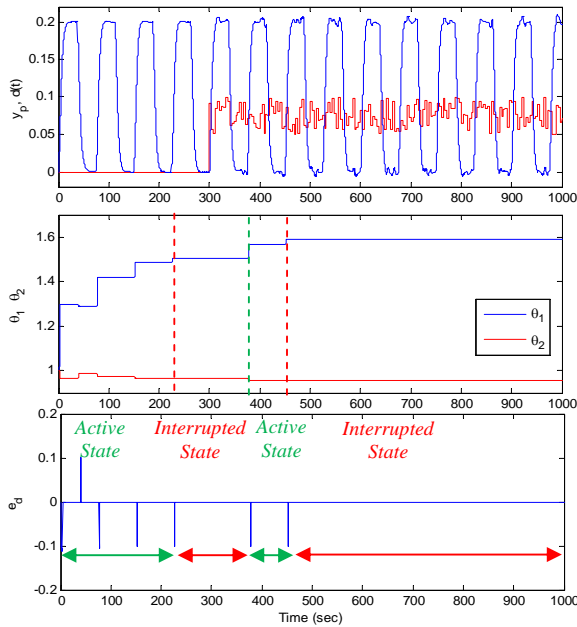


Fig. 10. Simulation results of FOAR-MRAC without dead zone under input disturbance applied to TRMS.

4. EXPERIMENTAL STUDY

For experimental demonstration of proposed FOAR-MRAC structure, we conducted experiment study on the TRMS experimental setup shown in Fig. 11(a), which was produced by Feedback Inc. Fig. 11(b) illustrates real time Simulink control model of TRMS including the proposed FOAR-MRAC structure. The control system was implemented in adaptive controller block and the real time data acquisitions for control operations were performed by “Advantech PCI

1711” interface cards installed in the computer. The experiments were carried out in indoor conditions. Fig. 12 shows pitch angle responses of TRMS system for a multi-sinusoidal input signal given by (17). As in the simulation, we configured  $\gamma = 4$  in the experiment study. However, we set  $e_z = 0.04$  due to the noise and disturbance of experimental setup. A PID controller with  $k_p = 0.1$ ,  $k_d = 2$  and  $k_i = 0.6$  coefficients are used in the experimental study. Fig. 12 shows the responses obtained by conventional MRAC structure for  $\alpha = 1.0$  and the proposed FOAR-MRAC for  $\alpha = 1.3$ . Fractional integrator for  $\alpha = 1.3$  was implemented by CFE method according to (2) as follows,

$$1/s^{1.3} \cong \frac{-0.9639s^4 + 68.1156s^3 + 627.6366s^2 + 812.2356s + 172.9761}{172.9761s^4 + 812.2356s^3 + 627.6366s^2 + 68.1156s - 0.9639} \quad (19)$$

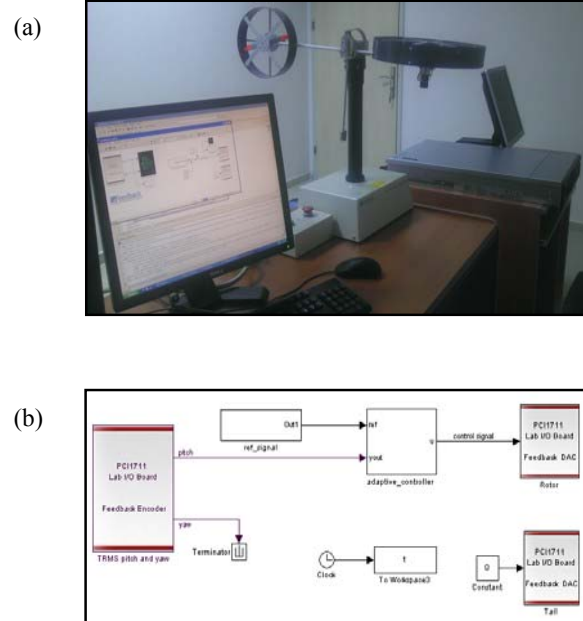
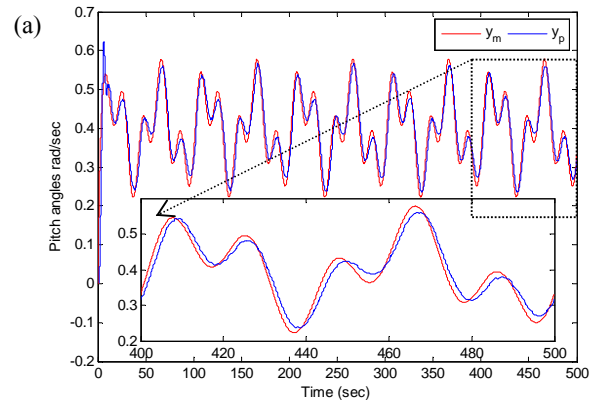


Fig. 11. (a) TRMS experimental setup; (b) Real time control model of TRMS including FOAR-MRAC with error dead zone modification.



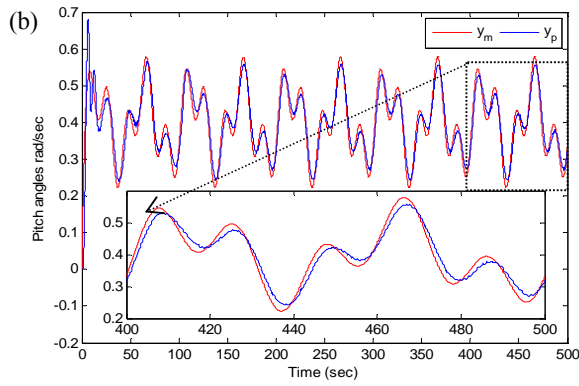


Fig. 12. Pitch angle response of TRMS for (a)  $\alpha = 1.0$  and (b)  $\alpha = 1.3$ .

Fig. 13 demonstrates that FOAR-MRAC structure ( $\alpha = 1.3$ ) can provide faster adaptation than the conventional MRAC ( $\alpha = 1.0$ ) in the same conditions. Fig. 14 shows dead zone modified error signal of FOAR-MRAC structure and conventional MRAC. After acceptable adaptation performance was achieved, the dead zone kept error signal at zero value in order to interrupt adaptation process. Since FOAR-MRAC structure provides faster convergence than conventional MRAC, FOAR-MRAC goes adaptation interrupted state rather earlier than conventional MRAC. In order to demonstrate operation of dead zone modification of model error, Figs. 15, 16 and 17 show response of TRMS with and without dead zone modification. In this experiment, a rectified sinusoidal signal was applied to the system input as the following,

$$u_c(t) = \begin{cases} 0.3 + 0.2 \sin(\omega t) & 0 < \omega t < \pi \\ 0.3 & \pi < \omega t < 2\pi \end{cases}, \quad (20)$$

where  $\omega = 0.1 \text{ rad/sec}$ . As shown in figures, without dead zone modification, adaptation process performs continuously and this can prevent quasi-stabilization of adaptation parameters  $\theta_1$  and  $\theta_2$ . Fig. 17 reveals noisy data of model approximation error that is captured from experimental system.

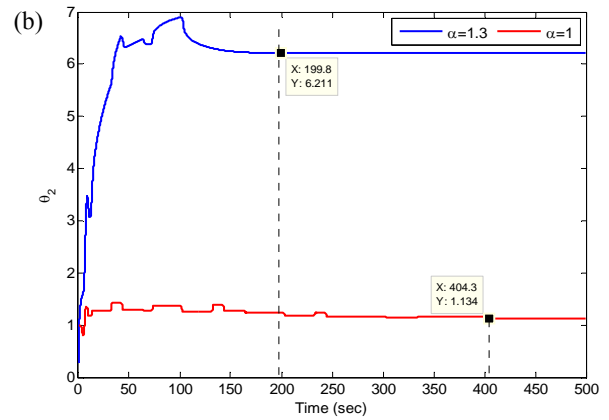
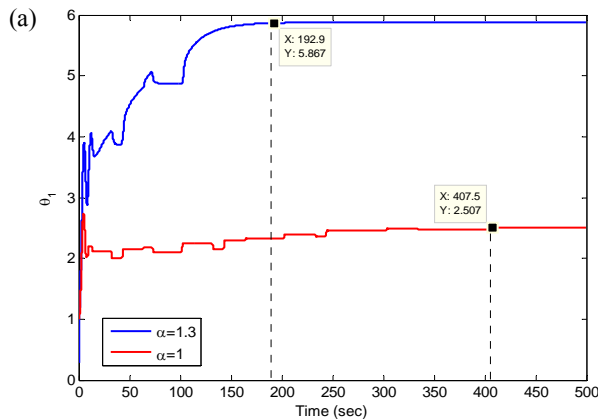


Fig. 13. a) Deviation of  $\theta_1$  for  $\alpha = 1.0$  and  $\alpha = 1.3$ ; b) Deviation of  $\theta_2$  for  $\alpha = 1.0$  and  $\alpha = 1.3$ .

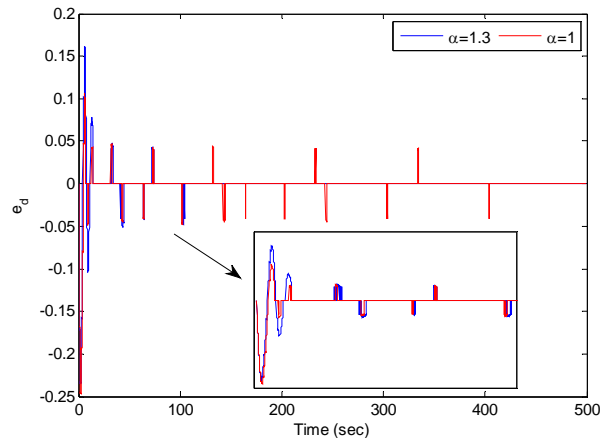


Fig. 14. Error with dead zone for integer and fractional order cases.

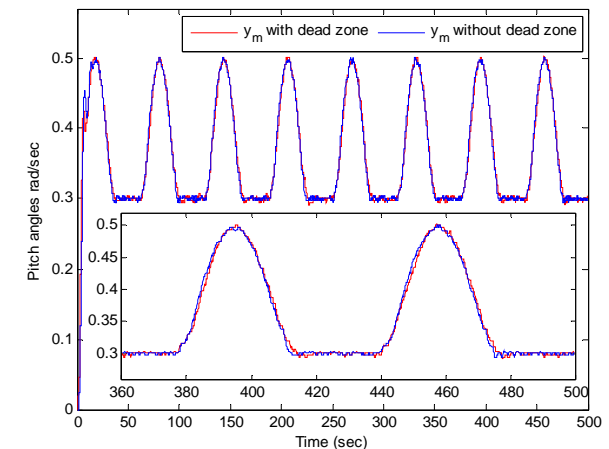


Fig. 15. Response of TRMS with and without dead zone modification for  $\alpha = 1$ .



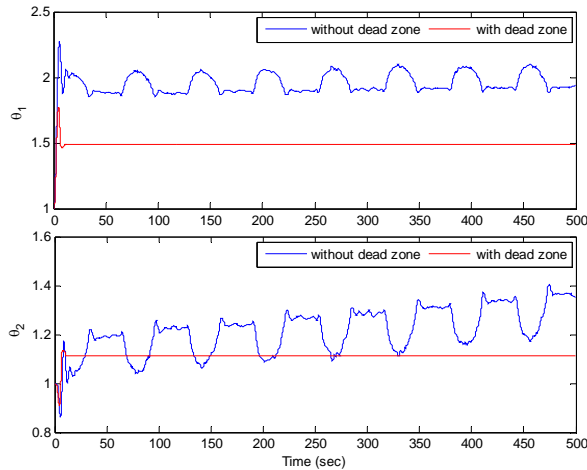


Fig. 16. Evolution of parameters  $\theta_1$  and  $\theta_2$  with and without dead zone modification for  $\alpha = 1$ .

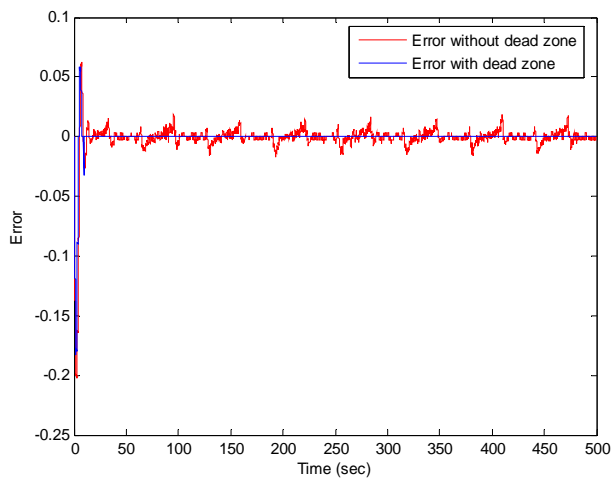


Fig. 17. Evolution of model approximation error signal with and without dead zone modification for  $\alpha = 1$ .

Due to structural noise in system and external disturbances, the model approximation error fluctuates around the zero. This noisy data, which can unnecessarily stimulates parameter updating of MRAC, is suppressed by the dead zone application as shown in the figures.

## 5. DISCUSSIONS AND CONCLUSIONS

Control of flight systems needs adaptive control techniques to deal with changing flight conditions. This study discusses application of FOAR-MRAC method for adaptive rotor control of TRMS. One of the advantages of the proposed adaptive control system, a PID control loop can be transformed into adaptive system only by appending FOAR-MRAC structure. This property makes it easy-to-use in flight control applications, where a PID control loop has been already employed. We implemented feedforward and feedback MIT rules with fractional order integrator and modified FOAR-MRAC structure for requirements of experimental rotor control application. Controlling of multi rotor system requires smooth and low overshoot control

responses. Configuring the theoretical reference model for smooth and low overshooting allows online improvement of PID control system response by support of FOAR-MRAC structure. Contribution of this study for adaptive control has two folds: One is the dead zone modification of model approximation error, which enables management of adaptation process for real applications. The other is development of model reference adaptive PID control system by integrating FOAR-MRAC structure.

Simulation and experimental study shows us that dead zone error modification to MRAC structure can facilitate practical utilization of MRAC structures. It was observed that quasi-stabilization of updating rule is a major problem for real application of FOAR-MRAC structure in flight control application. To deal with this problem in practical applications; we used a model approximation error with a dead zone modification in addition to combine with a PID controller. Piecewise linear function was used to implement an error dead zone around the zero. The dead zone near to zero forms error insensitive region. This region provides interruption of the adaptation process, when the model approximation error magnitude decreases below an acceptable error level. This avoids the over-adaptation of FOAR-MRAC structure or misleading effects of noise and disturbances on the adaptation process. We demonstrate these effects in numerical and experimental results.

For the proposed adaptive rotor control, employment of model approximation error with dead zone modification enables to management of adaptation process in two-state: adaptation active state and adaptation interrupted state. This allows operation of adaptation process only when the response of reference model and real control system diverge undesirably. We concluded that dead zone modification to model error enhances applicability of MRAC based adaptive control methods in real control engineering problems.

Conventional PID control systems are commonly used in industrial control applications. This study demonstrates integration of conventional PID control to FOAR-MRAC structure. This integration enables to transformation of PID control systems into model reference adaptive control systems by simply connecting the proposed FOAR-MRAC structures. Thus facilitates utilization of adaptive control skills in industrial control applications and contributes to improvement of robust performance for practical control systems. For instance, servomotor position control by PID controller may need the adaptation skill in long term because of ageing of mechanical and electrical parts. By using the proposed adaptive PID control structure, conventional static PID control systems can gain adaptation skill that is needed for maintaining control performance. On the other hand, FOAR-MRAC structures can perform a proportional control action. Cooperation with a PID controller improves control performance of FOAR-MRAC structures when integrator and derivative control action are required. Additionally, in our experimental works, we observed that it can be possible to find out fractional order integrators providing faster adaptation to the reference model (Vinagre et al., 2002). This

observation confirms that use of fractional order integrator can speed up the adaptation process.

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