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Synchronization of 4-D Hyperchaotic Rikitake Dynamo System Along With Unknown Parameters Via Adaptive Integral Sliding Mode

Sami ud Din*, Fazal ur Rehman*, Qudrat Khan**

** Sami ud Din and Fazal ur Rehman are with the Department of Electrical Engineering, Capital University of Science and Technology (CUST), Kahuta Road, Express Highway, Islamabad 44000, Pakistan.*
***Qudrat Khan is with the Center for Advance Studies in Telecommunications, COMSATS Institute of Information Technology, Islamabad 44000, Pakistan.*
(Email: engrsamiuddin@gmail.com, frehman@cust.edu.pk, qudratullah@comsats.edu.pk)

Abstract: An adaptive integral sliding mode control approach is proposed for the synchronization of 4-D identical hyperchaotic Rikitake dynamo system which operates with unknown parameters. The error dynamics are first transformed into a special structure which contains a known nominal part as well as unknown terms. The unknown terms are computed adaptively via an adaptive compensator, and the resulted error dynamics are stabilized asymptotically using integral sliding mode control. This stabilizing controller is an algebraic sum of the nominal control and the compensating control. The dangerous chattering phenomenon is suppressed via the designed compensator control. The closed loop stability of the designed compensator controller and the adapted law are ensured via the Lyapunov stability strategy. It is also worthy to report that the posed problem is handled via a reduced number of control inputs as compared the existing literature. In this case, the proposed design becomes a good candidate for chaotic systems with unknown parameters. The simulation results confirms the made claims.

Keywords: Synchronization, 4-D identical Hyperchaotic Chaos, Rikitake, Adaptive Integral Sliding Mode Control

1. INTRODUCTION

Researchers like (Suzuki et al., 2006; Wang et al., 2010; Moskalenko et al., 2010; Abdullah et al., 2013; Feki et al., 2003) are showing keen interest in synchronization as well as in anti-synchronization of chaotic systems for the last two decades due to their rapidly growing applications in secure communications. (Guegan et al., 2009) proves, chaotic systems play a role in population models, finance models and also in economics. (Volos et al., 2012; Guégan et al., 2009; Caraiani et al., 2013) prove chaotic system application in ecology as well as in psychology. (Denton et al., 1990) proves synchronization is also important in cardiology, complex dynamical networks and in robotics, while anti-synchronization of chaotic systems proves its vital importance in alleviating electrical power outage and framework management by (Harb et al., 2003) and (Abbasi et al., 2011).

As a chaotic phenomenon is usually present in nonlinear systems, that is highly sensitive towards disruption (or uncertainty) in their parametric framework leading to volatility in their future advancements. Chaotic systems can be defined as uncertain, nonlinear dynamical systems which are very responsive towards their initial conditions. Lyapunov exponent is the indicator for the sensitivity of chaotic systems. If the summations of Lyapunov exponent are negative, at that instant, the system declares as dissipative systems. Chaos synchronization/anti-synchronization happens when two dissipative chaotic systems are connected such that, in spite of the exponential divergence of their nearby trajectories, synchrony or anti-synchrony is attained in their chaotic behavior as $t \to \infty$. Synchronization of two chaotic systems steps up when, synchronization between heart and lung are required in cardiorespiratory system, such kind of synchronization is also becoming essential in human brain and in cells of paddlefish. Moreover, synchronization of chaotic systems may also be required, when a chaotic attractor commutes another chaotic attractor. Due to its vital importance and wide applications different techniques are proposed for the synchronization or anti-synchronization of chaotic systems, which includes adaptive control proposed by (Vaidyanathan et al., 2013), linear and nonlinear active control by (Chen et al., 2006; Kapitaniak et al., 2006). Fuzzy control by (Sargolzaei et al., 2013), impulsive control by (Yang et al., 1997) and back-stepping method suggested by (Ge et al., 2000; Chen et al., 2013). (Agrawal et al., 2014) recently proposed strategy for fractional order chaotic systems via adaptive synchronization and parameter identification with unknown parameters based on the Lyapunov stability method. Researchers like (Zhou et al., 2008; Li et al., 2008) have also proof interest in 3-D chaotic systems. (Li et al., 2003; Gong et al., 2010) concludes Chaotic theory has applications in chemical reactors. It is also proven that application of chaotic systems is also found in vibration control by (Shi et al., 2013; Yang et al., 2014). Chaotic behavior is also observed in oscillators by (Li et al., 2007; Sharma et al., 2012). (Rhouma et al., 2011; Volos et al., 2013) proves cryptosystems also emerges as application of chaotic systems.

As compared to chaotic systems, hyperchaotic systems appear to be having much complicated dynamical response, as well as they also acquire vital relevance in engineering, like secure communication proved by (Smaoui et al., 2011; Yang et al., 2013). (Buscarino et al., 2009; Zhou et al., 2014; Wei et al., 2012) proves behavior of hyperchaotic systems is also present in electrical circuits. A system with minimal, two positive Lyapunov exponents is referred as hyperchaotic systems. Synchronization of hyperchaotic identical systems seems to be a challenging task due to the butterfly effect. Due to the butterfly effect, the design of control law by which, effective output of respond system tracks the output of the drive system in finite time asymptotically is considered to be a challenging assignment. Firstly, hyperchaotic system was discovered by (Rossler et al., 1979), later on many more hyperchaotic systems are also reported, namely, Wang hyperchaotic system, discovered by (Wang et al., 2008), hyperchaotic Chen system discovered by (Li et al., 2009), hyperchaotic Vaidyanathan systems discovered by (Vaidyanathan et al., 2013, 2014) etc. Due to their complexity and unpredictable behavior, hyperchaotic systems have many applications in areas such as secure communications and cryptosystems (Gao et al., 2006; Xu et al., 2014). Circuit realization and memristive devices are also associated with the study of chaotic and hyperchaotic systems.

To ensure robust control of the aforesaid systems, the sliding mode control (SMC) (Utkin 1992 and Edward 1998) becomes an appealing candidate. However, this strategy suffers from high frequency vibrations in sliding mode which may result in the wear tear of the system. A number of control techniques (see for instance, (Levant 2003, Ferrara et al., 2001, Bartolini et al., 1998 and Fridman et al., 2015), have been proposed to overcome the undesirable phenomena. To confirm robustness from the very start of the processes a variant of SMC the so-called integral sliding mode control (ISMC) was proposed (Utkin 1999, Gao et al., 2013) which eliminate the high frequency vibrations and enhances robustness via the elimination of reaching phase. To establish the sliding mode as well as to converge the states to the origin in finite time the terminal sliding mode control (TSMC) was proposed (Yu et al., 2002). In the existing literature, researchers like (Hou et al., 2012; Zribi et al., 2009) use SMC for the control of chaotic systems to provide robustness against external noise and parametric variation.

The authors in this work proposed an adaptive integral sliding mode control approach for the synchronization of an uncertain 4-D identical hyperchaotic Rikitake dynamo. The first contribution in this work is the compensation of the nonlinear unknown terms and the unknown parameters via the adaptive part of the control law. Second, this adaptive law steers the error dynamics of a transformed special structure to the origin asymptotically. In other words, the master and slave system are synchronized via the proposed control law in the presence of parametric uncertainties. The third contribution is that, unlike the existing work (Vaidyanathan et al., 2015), we assume less number of control inputs. This reduction results in less energy usage and improves the applicability of the control strategy. The closed loop stability of the designed compensator controller and the adapted law are ensured via the Lyapunov stability strategy. The system under study is simulated which demonstrates the benefits of the proposed algorithm.

The remaining paper is arranged as follows. In Section 2, mathematical and graphical description of 4-D hyperchaotic Rikitake dynamo system is presented. In Section 3, a first order SMC is designed for the stabilization of the considered dynamic system subject to known parameters whereas in Section 4 an adaptive ISMC is designed for the given problem in the presence of unknown parameters. The simulation results of the stabilization problems are given in their respective sections. Synchronization of identical 4-D hyperchaotic Rikitake dynamo system with known and unknown parameters is carried out in Section 5 and Section 6, respectively. Section 7 concludes the paper followed by relevant cited references.

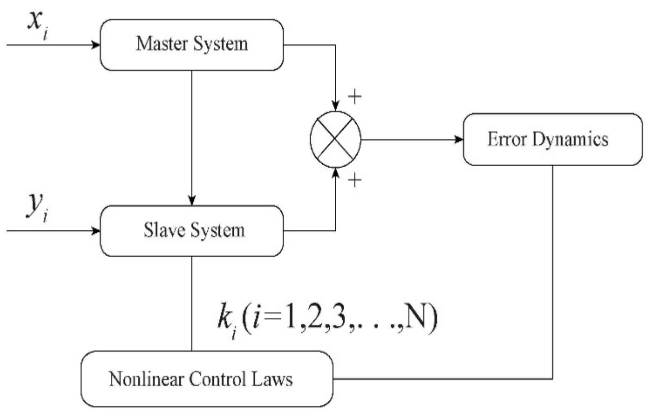


Fig. 1. Architecture for Synchronization and Antisynchronization of two systems.

2. SYSTEM DESCRIPTION OF A 4-D NOVEL HYPERCHAOTIC RIKITAKE DYNAMO SYSTEM

Consider the following 4-D hyperchaotic Rikitake dynamo system (Vaidyanathan et al., 2015).

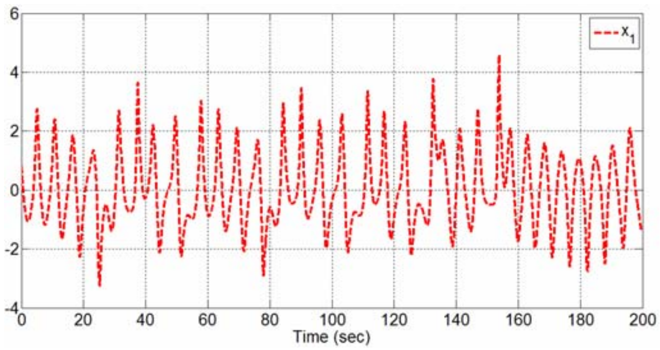
$$\begin{cases} \dot{x}_1 = -ax_1 + x_2x_3 - px_4 \\ \dot{x}_2 = -ax_2 + x_1(x_3 - b) - px_4 \\ \dot{x}_3 = 1 - x_1\, x_2 \\ \dot{x}_4 = cx_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 and x_4 are the state variables, a, b, c and p are considered positive constant parameters. The hyperchaotic Rikati dynamo system (1), illustrate hyperchaotic behavior when the parameter values are chosen as follows:

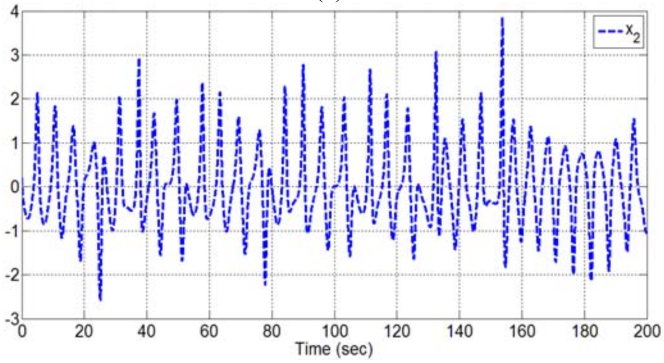
$$a = 1, b = 1, c = 0.7 \text{ and } p = 1.7$$

For numerical simulations, we proceed with initial values of the 4-D system as given below:

$$x_1(0) = 0.8,\; x_2(0) = 0.2, x_3(0) = 0.4 \text{ and } x_4(0) = 0.6$$



(a)



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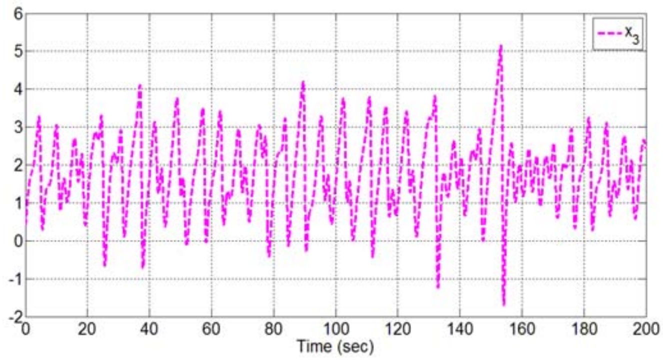

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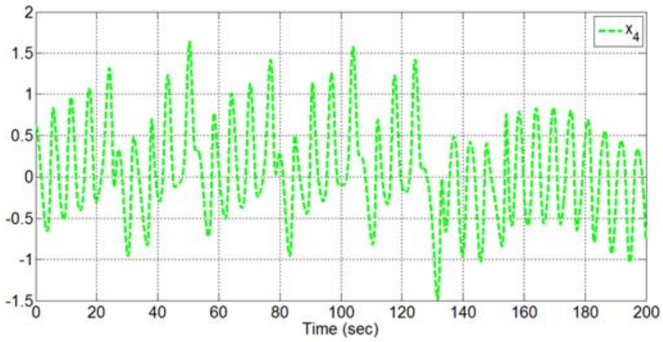
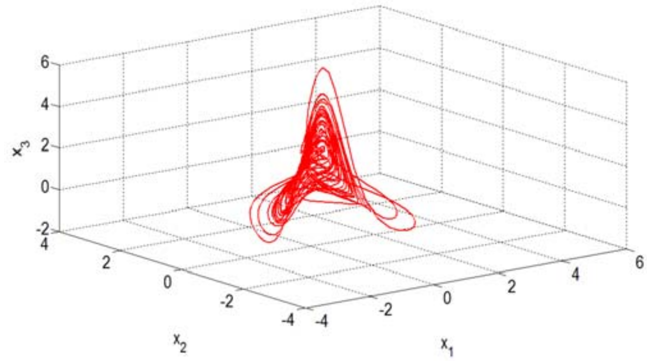


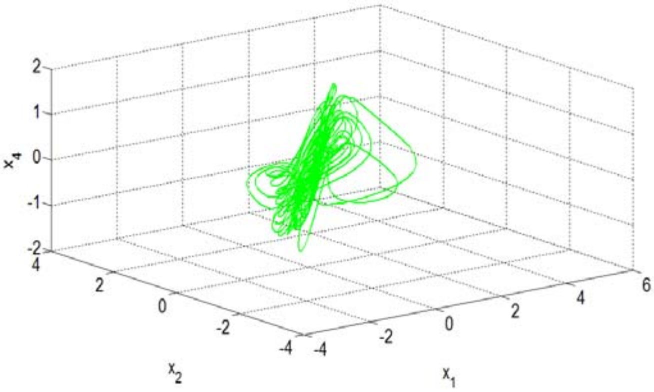
(d)

Fig. 2. States representation of hyperchaotic Rikitake dynamo system, x_1, x_2, x_3, x_4.

In (Fig. 2) a, b, c, d displays the chaotic behavior of states x_1, x_2, x_3, x_4 in two dimensions, while in (Fig. 3) a, b, c, d projects the three dimensional projection of states x_1, x_2, x_3, x_4.



(a)



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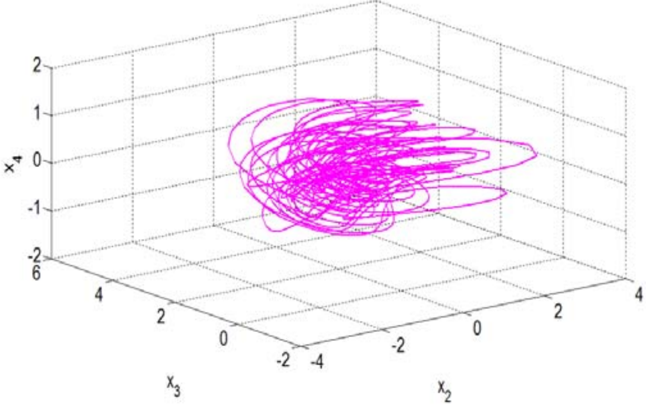

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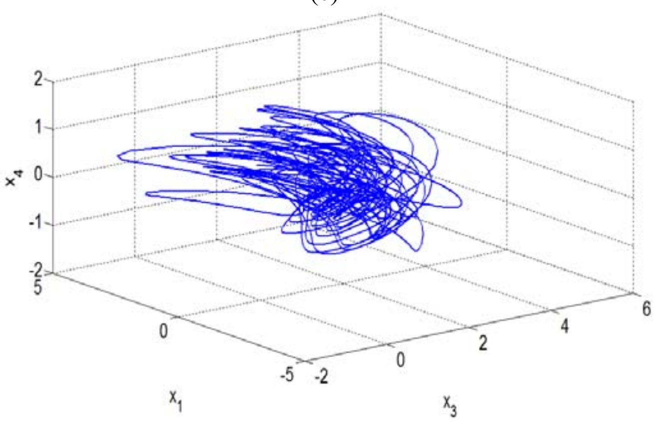


(d)

Fig. 3. 3-D Projection of the hyperchaotic Rikitake dynamo system on the, $x_{i's}$ space, where $i = 1 \rightarrow 4$.

3. SLIDING MODE CONTROL OF THE 4-D HYPERCHAOTIC RIKITAKE DYNAMO SYSTEM WITH KNOWN PARAMETERS

In this Section, authors have designed a first order sliding mode control, for global stabilization of the 4-D hyperchaotic Rikitake dynamo system with known parameters. Following system shows the mathematical representation of the controlled hyperchaotic system

$$\begin{cases} \dot{x}_1 = -ax_1 + x_2x_3 - px_4 + u_1 \\ \dot{x}_2 = -ax_2 + x_1(x_3 - b) - px_4 + u_2 \\ \dot{x}_3 = -1 + x_1x_2 + u_3 \\ \dot{x}_4 = cx_2 \end{cases} \quad (2)$$

In above system (2), u_1, u_2 and $u_3$are the controls to be designed using sliding mode control, whereas x_1, x_2, x_3 and x_4 are state variables along with positive constant parameters as a, b, c and p. If we select u_3, u_2 and u_1 as follows, then (2) can be written as (4)

$$u_3 = -1 + x_1x_2 + x_1$$

$$u_2 = ax_2 - x_1(x_3 - b) + px_4 + x_3$$

$$u_1 = ax_1 - x_2x_3 + Px_4 + v \quad (3)$$

where equation (3) shows new input as v, so system (2) can be rewritten as (4) shown below

$$\begin{cases} \dot{x}_1 = v \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_1 \\ \dot{x}_4 = cx_2 \end{cases} \quad (4)$$

The system shown in the above equation (4) can be rearranged as (5)

$$\begin{cases} \dot{x}_4 = cx_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_1 \\ \dot{x}_1 = v \end{cases} \quad (5)$$

Sliding surface for the system (5) is defined as:

$$\sigma = x_4 + 3cx_2 + 3cx_3 + 3cx_1 \quad (6)$$

Equation (7) can be obtained by taking the time derivative of (6)

$$\dot{\sigma} = \dot{x}_4 + 3c\dot{x}_2 + 3c\dot{x}_3 + c\dot{x}_1 \quad (7)$$

By choosing

$$v = -x_2 - 3x_3 - 3x_1 - \left(\frac{k}{c}\right)(\sigma + sign(\sigma)) \text{ , where } k > 0$$

We have

$$\dot{\sigma} = -k_0\, \sigma - k_1 sign(\sigma)$$

Therefore the system (5) is asymptotically stable.

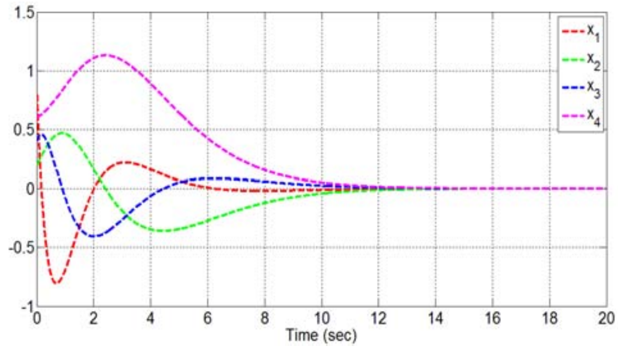


Fig. 4. Stabilization of hyperchaotic Rikitake dynamo system via sliding mode control.

The above simulation shows stabilization of the 4-D chaotic system using sliding mode control in nearly 12 seconds. In the forthcoming Section, authors are going to design and simulate integral sliding mode control for 4-D novel hyperchaotic systems along unknown parameters. The important point to be noted here, the only parameter "c" considered to be known, knowing of "c" will reduce the number of controllers required to control the respond system from 4 to 3.

4. ADAPTIVE INTEGRAL SLIDING MODE CONTROL OF 4-D NOVEL HYPERCHAOTIC RIKITAKE DYNAMO SYSTEM WHEN ALL PARAMETERS ARE UNKNOWN, EXCEPT c

In order to reduce the number of controllers from four, by (Vaidyanathan et al., 2015) to three, we have proposed "c" should be known among all other positive constants, which is also considered as a major contribution of this research article. As integral sliding mode control is proven to be very effective over sliding mode control by (Din et al. 2016). We have considered the controlled hyperchaotic system as shown in (2)

$$\dot{x}_1 = -ax_1 + x_2x_3 - px_4 + u_1$$

$$\dot{x}_2 = -ax_2 + x_1(x_3 - b) - px_4 + u_2$$

$$\dot{x}_3 = -1 + x_1x_2 + u_3$$

$$\dot{x}_4 = cx_2$$

As we aim to vary parameters adaptively and also considering the parameters to be unknown, therefore we have to design the estimates for a, b, p as $\hat{a}, \hat{b}, \hat{p}$ respectively. During the estimation of parameters, error will be appears, which will represented as

$$\tilde{a} = a - \hat{a}$$

$$\tilde{b} = b - \hat{b}$$

and $$\tilde{p} = p - \hat{p}$$

For adaptive estimation of parameters, the system in (2) can be rewritten as (8).

$$\begin{cases} \dot{x}_1 = -\hat{a}x_1 + x_2x_3 - \tilde{a}x_1 - \hat{p}x_4 - \tilde{p}x_4 + u_1 \\ \dot{x}_2 = -\hat{a}x_2 - \tilde{a}x_2 + x_1x_3 - \hat{b}x_1 - \tilde{b}x_1 - \hat{p}x_4 - \tilde{p}x_4 + u_2 \\ \dot{x}_3 = -1 + x_1x_2 + u_3 \\ \dot{x}_4 = cx_2 \end{cases} \quad (8)$$

By putting following values of u_3, u_2 and u_1 in the above equation (8), equation (9) is obtained

$$u_3 = -1 + x_1x_2 + x_1$$

$$u_2 = \hat{a}x_2 - x_1x_3 - \hat{b}x_1 + \hat{p}x_4 + x_3$$

$$u_1 = \hat{a}x_1 - x_2x_3 + \hat{p}x_4 + v$$

where v is the new input in (9), written below

$$\begin{cases} \dot{x}_1 = v - \tilde{a}x_1 - \tilde{p}x_4 \\ \dot{x}_2 = x_3 - \tilde{a}x_2 - \tilde{b}x_1 - \tilde{p}x_4 \\ \dot{x}_3 = x_1 \\ \dot{x}_4 = cx_2 \end{cases} \quad (9)$$

which can be rewritten as:

$$\begin{cases} \dot{x}_4 = cx_2 \\ \dot{x}_2 = x_3 - \tilde{a}x_2 - \tilde{b}x_1 - \tilde{p}x_4 \\ \dot{x}_3 = x_1 \\ \dot{x}_1 = v - \tilde{a}x_1 - \tilde{p}x_4 \end{cases} \quad (10)$$

Choose the nominal system for (10) as:

$$\begin{cases} \dot{x}_4 = cx_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_1 \\ \dot{x}_1 = v_0 \end{cases} \quad (11)$$

The sliding surface σ_0 for nominal system (11) is same as displayed in (5):

$$\sigma_0 = x_4 + 3cx_2 + 3cx_3 + cx_1$$

The time derivative of the above equation is given below

$$\dot{\sigma}_0 = \dot{x}_4 + 3c\dot{x}_2 + 3c\dot{x}_3 + c\dot{x}_1$$
$$= cx_2 + 3cx_3 + 3cx_1 + cv_0 + cv_s$$

By choosing

$v = -x_2 - 3x_3 - 3x_1 - \left(\frac{k}{c}\right)(\sigma_0 + sign(\sigma_0))$, where $k > 0$

We have

$$\dot{\sigma} = -k_0\,\sigma_0 - k_1 sign(\sigma_0)$$

Therefore the nominal system (11) is asymptotically stable. Now we are going to add the integral term "z" at the end of sliding surface σ_0 shown above and assigned the symbol σ.

$$\sigma = \sigma_0 + z \quad (12)$$

$$= x_4 + 3cx_2 + 3cx_3 + cx_1 + z \quad (13)$$

In (13), z is some integral term computed later. To avoid the reaching phase, choose $z(0)$ such that $\sigma(0) = 0$. Select $v = v_0 + v_s$, where v_0 is the nominal input and v_s is compensator term computed later.
Then

$$\dot{\sigma} = \dot{x}_4 + 3c\dot{x}_2 + 3c\dot{x}_3 + c\dot{x}_1 + \dot{z} \quad (14)$$

$$= cx_2 + 3cx_3 - 3c\tilde{a}x_2 - 3c\tilde{b}x_2 - 3c\tilde{p}x_4 + 3cx_1 + cv_0 + cv_s - \tilde{a}cx_1 - \tilde{p}cx_4 + \dot{z} \quad (15)$$

By selecting the Lyapunov candidate function as

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{a}^2 + \frac{1}{2}\tilde{b}^2 + \frac{1}{2}p^2$$

We are able to design the adaptive laws for $\tilde{a}, \hat{a}, \tilde{b}, \hat{b}, \tilde{p}$ and $\hat{p}$, and compute $v_s$such that $\dot{V} < 0$ (Abbasi et al., 2016).

Theorem 1

Consider a Lyapunov function $V = \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{a}^2 + \frac{1}{2}\tilde{b}^2 + \frac{1}{2}p^2$. Then $\dot{V} < 0$, if the adaptive laws for $\tilde{a}, \hat{a}, \tilde{b}, \hat{b}, \tilde{p}$ and $\hat{p}$, and the value of v_s are chosen as:

$$\dot{z} = -cx_2 - 3cx_3 - 3cx_1 - cv_0, v_s = -\left(\frac{k}{c}\right)(\sigma + sign(\sigma)) \quad (16)$$

$$\dot{\tilde{a}} = 3c\sigma x_2 + \sigma cx_1 - k_1\tilde{a} \quad \text{and} \quad \dot{\hat{a}} = -\dot{\tilde{a}} \quad (17)$$

$$\dot{\tilde{b}} = 3c\sigma x_1 - k_2\tilde{b} \quad \text{and} \quad \dot{\hat{b}} = -\dot{\tilde{b}} \quad (18)$$

$$\dot{\tilde{p}} = 4c\sigma x_4 - k_3\tilde{p} \quad \text{and} \quad \dot{\hat{p}} = -\dot{\tilde{p}} \quad (19)$$

where k and $k_i > 0, i = 1, \ldots 3$

Proof: Since

$$\dot{V} = \sigma\dot{\sigma} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{p}\dot{\tilde{p}} \quad (20)$$

$$= \sigma\{cx_2 + 3cx_3 + 3c\tilde{a}x_2 - 3c\tilde{b}x_1 - 3c\tilde{p}x_4 + 3cx_1 + cv_0 + cv_s - \tilde{a}cx_1 - \tilde{p}cx_4 + \dot{z}\} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{p}\dot{\tilde{p}}$$

$$= \sigma\{cx_2 + 3cx_3 + 3cx_1 + cv_0 + cv_s + \dot{z}\} + \tilde{a}\{\dot{\tilde{a}} - 3c\sigma x_2 - \sigma cx_1\} + \tilde{b}\left\{\dot{\tilde{b}} - 3c\sigma x_1\right\} + \tilde{p}\{\dot{\tilde{p}} - 4c\sigma x_4\} \quad (21)$$

By using equation (16) to (19) in (21), we got (22)

$$\dot{V} = -k\sigma^2 - k|\sigma| - k_1\tilde{a}^2 - k_2\tilde{b}^2 - k_3\tilde{p}^2 \quad (22)$$

From this, we conclude that $\sigma, \tilde{a}, \tilde{b}, \tilde{p} \to 0$. As $\to 0$, $x \to 0$.

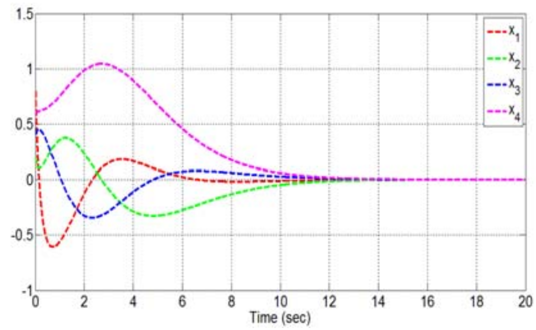


Fig. 5. Stabilization of hyperchaotic Rikitake dynamo system via integral sliding mode control.

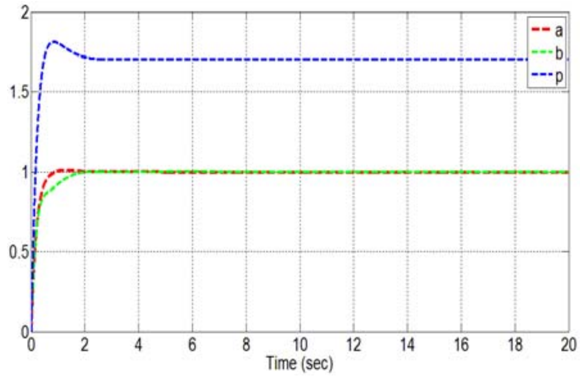


Fig. 6. Parameter estimation of hyperchaotic Rikitake dynamo system via adaptive integral sliding mode control.

Stabilization of states is shown in (Fig. 5), whereas (Fig. 6) presents the estimation of parameters. Forthcoming paragraph displays the very effective synchronization of identical fourth dimensional hyperchaotic system.

5. SYNCHRONIZATION OF IDENTICAL 4-D NOVEL HYPERCHAOTIC RIKITAKE DYNAMO SYSTEMS WITH KNOWN PARAMETERS

In this section, efforts are directed to design a global synchronization control protocol for the identical 4-D hyperchaotic Rikitake dynamo systems along with known parameters. The master and the slave systems are considered with identical dynamics. In the master hyperchaotic Rikitake dynamo system the states are represented by x_1, x_2, x_3 and x_4 whereas for the slave system the states are described by y_1, y_2, y_3 and y_4 as shown in (23).

Master system

$$\dot{x_1} = -ax_1 + x_2x_3 - px_4$$
$$\dot{x_2} = -ax_2 + x_1(x_3 - b) - px_4$$
$$\dot{x_3} = 1 - x_1\,x_2$$
$$\dot{x_4} = cx_2$$

Slave system

$$\dot{y_1} = -ay_1 + y_2y_3 - py_4 + u_1$$
$$\dot{y_2} = -ay_2 + y_1(y_3 - b) - py_4 + u_2$$
$$\dot{y_3} = 1 - y_1\,y_2 + u_3$$
$$\dot{y_4} = cy_2 \quad (23)$$

For synchronization, the error signal is defined as:

$$e_i = y_i - x_i, \text{ where } i = 1 \ldots .4$$

Then the error dynamics become:

$$\dot{e_1} = \dot{y_1} - \dot{x_1}$$
$$= -ay_1 + y_2y_3 - py_4 + u_1 + ax_1 - x_2x_3 + px_4 \quad (24)$$
$$\dot{e_2} = \dot{y_2} - \dot{x_2}$$
$$= -ay_2 + y_1(y_3 - b) - py_4 + u_2$$
$$+ax_2 - x_1(x_3 - b) - px_4 \quad (25)$$
$$\dot{e_3} = \dot{y_3} - \dot{x_3}$$
$$= 1 - y_1\,y_2 + u_3 - 1 + x_1\,x_2 \quad (26)$$
$$\dot{e_4} = \dot{y_4} - \dot{x_4}$$
$$= cy_2 - cx_2 \quad (27)$$

By choosing

$$u_2 = ay_2 - y_1(y_3 - b) + py_4 - ax_2 + x_1(x_3 - b) + px_4 + e_3$$
$$u_3 = -1 + y_1\,y_2 + 1 - x_1\,x_2 + e_1$$
$$u_1 = ay_1 - y_2y_3 + py_4 - ax_1 + x_2x_3 - px_4 + v$$

Equation (24) to (27) can be rewritten as (28)

$$\begin{cases} \dot{e_1} = v \\ \dot{e_2} = e_3 \\ \dot{e_3} = e_1 \\ \dot{e_4} = ce_2 \end{cases} \quad (28)$$

System in (28), can be rewritten as (29)

$$\begin{cases} \dot{e_4} = ce_2 \\ \dot{e_2} = e_3 \\ \dot{e_3} = e_1 \\ \dot{e_1} = v \end{cases} \quad (29)$$

Sliding surface for system shown in (29) is defined as

$$\sigma = e_4 + 3ce_2 + 3ce_3 + ce_1 \quad (30)$$

Time derivative of (30) is shown in (31)

$$\dot{\sigma} = \dot{e_4} + 3c\dot{e_2} + 3c\dot{e_3} + c\dot{e_1}$$
$$= ce_2 + 3ce_3 + 3\,ce_1 + cv \quad (31)$$

The system shown in (29) is considered to be asymptotically stable if

$$\dot{\sigma} = -k_0\sigma - k_1 sign(\sigma)$$

By choosing,

$$v = -e_2 - 3e_3 - 3e_1 - \left(\frac{k}{c}\right)(\sigma + sign(\sigma)) \text{ , for } k > 0$$

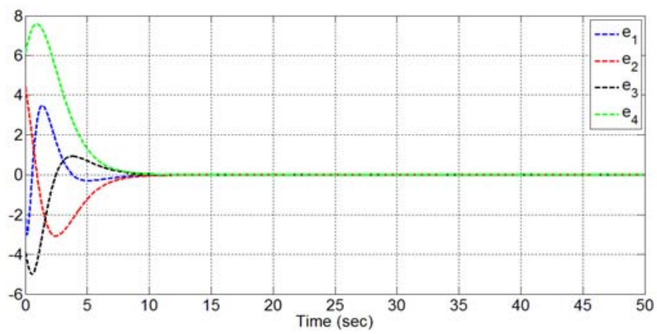


Fig. 7. Error convergence of identical hyperchaotic Rikitake dynamo system

Error convergence profile of hyperchaotic Rikitake dynamo system is displayed above in (Fig.7), whereas a, b, c, d of (Fig. 8), shows the synchronization of states regarding master and slave system.

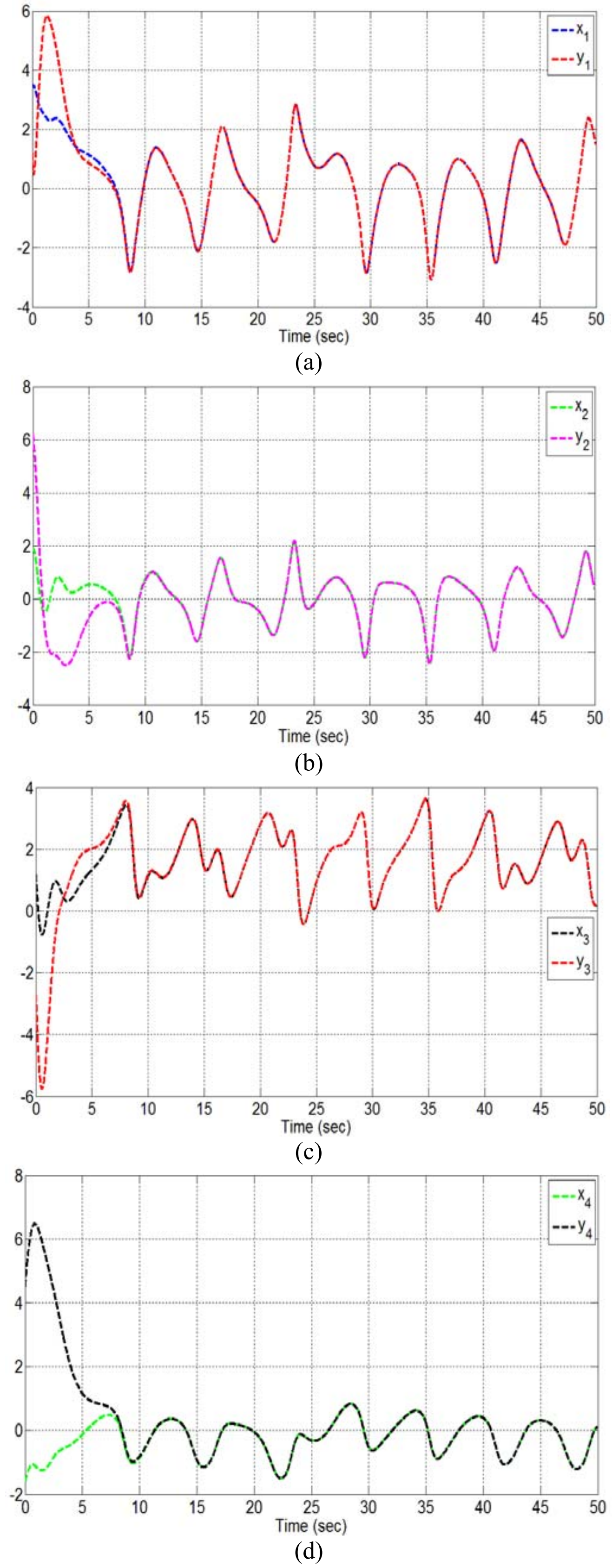


Fig. 8. (a). Synchronization of the states x_1 and y_1, (b). Synchronization of the states x_2 and y_2, (c). Synchronization of the states x_3 and y_3, (d). Synchronization of the states x_4 and y_4.

6. SYNCHRONIZATION OF IDENTICAL 4-D HYPERCHAOTIC RIKITAKE DYNAMO SYSTEMS ALONG UNKNOWN PARAMETERS a, b, p

Similar to Section 5, $\hat{a}, \hat{b}, \hat{p}$ be estimates of a, b, p and $\tilde{a} = a - \hat{a}$, $\tilde{b} = b - \hat{b}$, and $\tilde{p} = p - \hat{p}$ be the errors during the estimation of a, b and p respectively. The system (1) and (23) can be written as (32) and (33):

Drive system

$$\begin{cases} \dot{x}_1 = -\hat{a}x_1 - \tilde{a}x_1 + x_2x_3 - \hat{p}x_4 - \tilde{p}x_4 \\ \dot{x}_2 = -\hat{a}x_2 - \tilde{a}x_2 + x_1x_3 - \hat{b}x_1 - \tilde{b}x_1 \\ \qquad -\hat{p}x_4 - \tilde{p}x_4 \\ \dot{x}_3 = 1 - x_1x_2 \\ \dot{x}_4 = cx_2 \end{cases} \tag{32}$$

Response system

$$\begin{cases} \dot{y}_1 = -\hat{a}y_1 - \tilde{a}y_1 + y_2y_3 - \hat{p}y_4 - \tilde{p}y_4 + u_1 \\ \dot{y}_2 = -\hat{a}y_2 - \hat{a}y_2 + y_1y_3 - \hat{b}y_1 \\ \qquad -\tilde{b}y_1 - \hat{p}y_4 - \tilde{p}y_4 + u_2 \\ \dot{y}_3 = 1 - y_1 y_2 + u_3 \\ \dot{y}_4 = cy_2 \end{cases} \tag{33}$$

Then the error dynamics become:

$$\dot{e}_1 = \dot{y}_1 - \dot{x}_1 \\ = -\hat{a}y_1 - \tilde{a}y_1 + y_2y_3 - \hat{p}y_4 - \tilde{p}y_4 + u_1 + \hat{a}x_1 + \tilde{a}x_1 - x_2x_3 + \hat{p}x_4 + \tilde{p}x_4 \tag{34}$$

$$\dot{e}_2 = \dot{y}_2 - \dot{x}_2 \\ = -\hat{a}y_2 - \tilde{a}y_2 + y_1y_3 - \hat{b}y_1 - \tilde{b}y_1 - \hat{p}y_4 - \tilde{p}y_4 + u_2 + \hat{a}x_2 + \tilde{a}x_2 - x_1x_3 + \hat{b}x_1 + \tilde{b}x_1 + \hat{p}x_4 + \tilde{p}x_4 \tag{35}$$

$$\dot{e}_3 = \dot{y}_3 - \dot{x}_3 \\ = 1 - y_1 y_2 + u_3 - 1 + x_1x_2 \tag{36}$$

$$\dot{e}_4 = \dot{y}_4 - \dot{x}_4 \\ = cy_2 - cx_2 \tag{37}$$

By choosing

$$u_2 = \hat{a}y_2 - y_1 y_3 + \hat{b}y_1 + \hat{p}y_4 - \hat{a}x_2 + x_1x_3 - \hat{b}x_1 \\ -\hat{p}x_4 + e_3$$

$$u_3 = -1 + y_1 y_2 + 1 - x_1 x_2 + e_1$$

$$u_1 = \hat{a}y_1 - y_2y_3 + \hat{p}y_4 - \hat{a}x_1 + x_2x_3 - \hat{p}x_4 + v$$

the system displayed between (34)-(37) can be written as system (38) shown below, where v is the new input

$$\begin{cases} \dot{e}_1 = v - \tilde{a}y_1 - \tilde{p}y_4 + \tilde{a}x_1 + \tilde{p}x_4 \\ \dot{e}_2 = e_3 - \tilde{a}y_2 - \tilde{b}y_1 - \tilde{p}y_4 + \tilde{a}x_2 \\ \qquad +\tilde{b}x_1 - \tilde{p}x_4 \\ \dot{e}_3 = e_1 \\ \dot{e}_4 = ce_2 \end{cases} \tag{38}$$

System (38) can be rewritten as follows:

$$\begin{cases} \dot{e}_4 = ce_2 \\ \dot{e}_2 = e_3 - \tilde{a}\,y_2 - \tilde{b}y_1 - \tilde{p}y_4 + \tilde{a}x_2 \\ \quad +\tilde{b}x_1 + \tilde{p}x_4 \\ \dot{e}_3 = e_1 \\ \dot{e}_1 = v - \tilde{a}y_1 - \tilde{p}y_4 + \tilde{a}x_1 + \tilde{p}x_4 \end{cases} \quad (39)$$

Whereas, (40) represents the nominal system for (39) as (40)

$$\begin{cases} \dot{e}_4 = ce_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = e_1 \\ \dot{e}_1 = v_o \end{cases} \quad (40)$$

Sliding surface for the nominal system shown in (40) is as follows

$$\sigma_0 = e_4 + 3ce_2 + 3ce_3 + ce_1 \quad (41)$$

Then

$$\dot{\sigma_0} = \dot{e_4} + 3c\dot{e_2} + 3c\dot{e_3} + c\dot{e_1}$$
$$= ce_2 + 3ce_3 + 3ce_1 + cv_o$$

By choosing

$$v_0 = -e_2 - 3e_3 - 3e_1 - \left(\frac{k}{c}\right)(\sigma_0 + sign(\sigma_0)),\ \ k > 0,$$

we have

$$\dot{\sigma} = -k_0\,\sigma_0 - k_1 sign(\sigma_0)$$

which shows system in (40) is asymptotically stable. Equation (42) represents the sliding surface choose for the system shown (39). Now choosing the sliding surface for the system shown in (39) as below:

$$\sigma = \sigma_0 + z$$
$$= e_4 + 3ce_2 + 3ce_3 + ce_1 + z \quad (42)$$

where, z is some integral term computed later. To avoid the reaching phase, choose $z(0)$ such that $\sigma(0) = 0$. Choose $v = v_0 + v_s$ where, v_0 is the nominal input and v_s is the compensator term computed later, then the time derivative of (42) becomes

$$\dot{\sigma_0} = \dot{e}_4 + 3c\dot{e_2} + 3c\dot{e_3} + c\dot{e_1} + \dot{z}$$
$$= ce_2 + 3ce_3 - 3c\widetilde{a}\,y_2 - 3c\widetilde{b}\,y_1 - 3c\widetilde{p}\,y_4 + 3c\widetilde{a}\,x_2 + 3c\widetilde{b}\,x_1 + 3c\widetilde{p}\,x_4 + 3ce_1 + cv_o + cv_s - c\widetilde{a}\,y_1 - c\,\widetilde{p}\,y_4 + c\widetilde{a}\,x_1 + c\,\widetilde{p}\,x_4 + \dot{z} \quad (43)$$

By choosing a Lyapunov function:

$V = \frac{1}{2}\sigma^2 + \frac{1}{2}\widetilde{a^2} + \frac{1}{2}\widetilde{b^2} + \frac{1}{2}\widetilde{p^2}$, design the adaptive laws for $\widetilde{a}, \widehat{a}, \widetilde{b}, \widehat{b}, \widetilde{p}, \hat{p}$ and compute v_s such that $\dot{V} < 0$ (Abbasi et al., 2016).

Theorem 2

Consider a Lyapunov function $V = \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{a}^2 + \frac{1}{2}\tilde{b}^2 + \frac{1}{2}p^2$. Then $\dot{V} < 0$, if the adaptive laws for $\tilde{a}, \hat{a}, \tilde{b}, \hat{b}, \tilde{p}$ and $\hat{p}$, and the value of v_s are chosen as:

$$\dot{z} = -ce_2 - 3ce_3 - 3ce_1 - cv_o,\ v_s = -\left(\frac{k}{c}\right)(\sigma + sign(\sigma)) \quad (44)$$

$$\dot{\tilde{a}} = 3c\sigma y_2 - 3c\sigma x_2 + \sigma c y_1 - \sigma c x_1 - k_1\tilde{a} \quad \text{and} \quad \dot{\hat{a}} = -\dot{\tilde{a}} \quad (45)$$

$$\dot{\tilde{b}} = 3c\sigma y_1 - 3c\sigma x_1 - k_2\,\tilde{b} \quad \text{and} \quad \dot{\hat{b}} = -\dot{\tilde{b}} \quad (46)$$

$$\dot{\tilde{p}} = 4c\sigma y_4 - 4c\sigma x_4 - k_3\,\tilde{p} \quad \text{and} \quad \dot{\hat{p}} = -\dot{\tilde{p}} \quad (47)$$

where k and $k_i > 0, i = 1, \dots 4$

Proof: Since

$$\dot{V} = \sigma\dot{\sigma} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{p}\dot{\tilde{p}}$$
$$= \sigma\{ ce_2 + 3ce_3 - 3c\widetilde{a}\,y_2 - 3c\widetilde{b}\,y_1 - 3c\widetilde{p}\,y_4 + 3c\widetilde{a}\,x_2 + 3cb\,x1 + 3cp\,x4 + 3ce1 + cvo + cvs - ca\,y1 - c\,p\,y4 + ca\,x1 + c\,p\,x4 + z_{+aa+bb+pp} \quad (48)$$

$$= \sigma\{ce_2 + 3ce_3 + 3ce_1 + cv_0 + cv_s + \dot{z}\} +$$
$$\{\dot{\tilde{a}} - 3c\sigma y_2 + 3c\sigma x_2 - \sigma c y_1 + \sigma c x_1\} +$$
$$\tilde{b}\{\dot{\tilde{b}} - 3c\sigma y_1 - 3c\sigma x_1\} + \tilde{p}\{\dot{\tilde{p}} - 4c\sigma y_4 - 4c\sigma x_4\} \quad (49)$$

By using equation (44) to (47) in (49), we got following equation.

$$\dot{V} = -k_0\sigma^2 - k|\sigma| - k_1\tilde{a}^2 - k_2\tilde{b}^2 - k_3\tilde{p}^2$$

From above expression, we can conclude that $\sigma, \tilde{a}, \tilde{b}, \tilde{p} \rightarrow 0$. As $\rightarrow 0$, $x \rightarrow 0$.

Fig. 9, shows the error convergence profile regarding synchronization of identical 4-D hyperchaotic Rikitake dynamo systems among unknown parameters, whereas (Fig. 10) displays the synchronization of states x_i and y_i for the master and slave system where $i = 1, \dots .4$

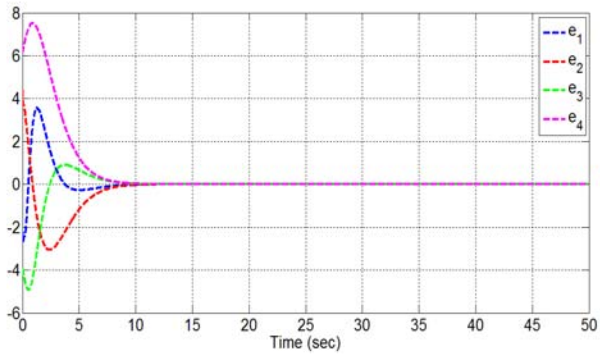
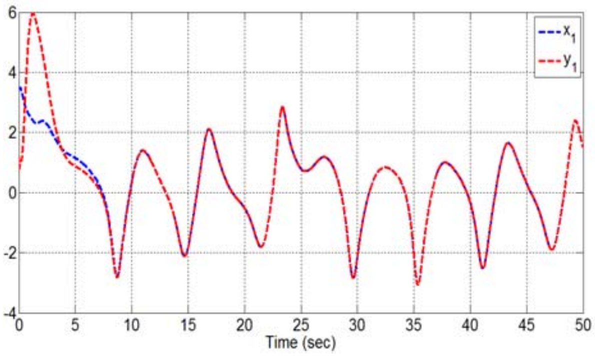
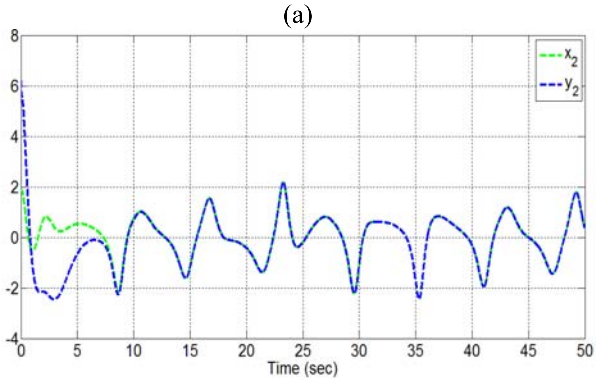


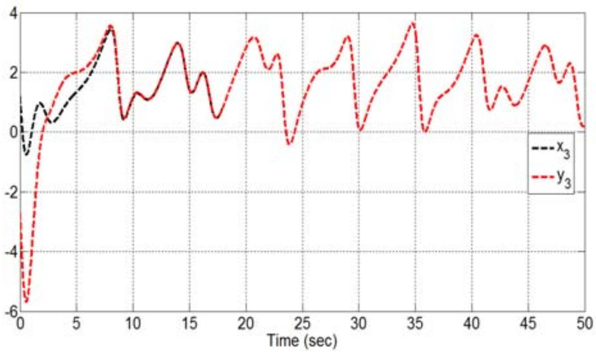
Fig. 9. Error convergence of identical hyperchaotic Rikitake dynamo system.



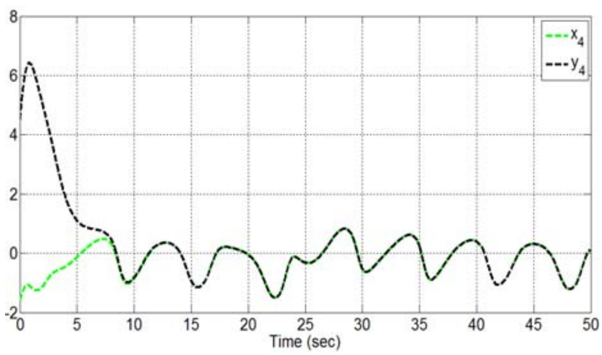
(a)



(b)



(c)



(d)

Fig. 10. (a). Synchronization of the states $x_1$and y_1, (b). Synchronization of the states $x_2$and y_2, (c). Synchronization of the states $x_3$and y_3, (d). Synchronization of the states x_4 and y_4.

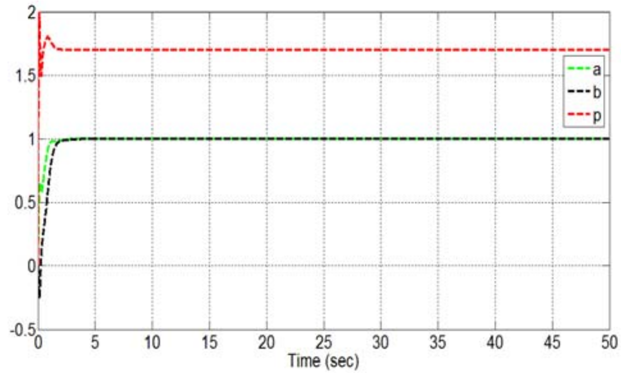


Fig. 11. Parameter estimation of hyperchaotic Rikitake dynamo system via adaptive integral sliding mode control.

In the above (Fig. 11), adaptive parametric estimation shows quite impressive results, as error were converging to zero already shown in (Fig. 9). The fourth coming Section represents the conclusion of this research article.

7. CONCLUSION

An adaptive ISMC approach is proposed for the synchronization of an uncertain 4-D identical hyperchaotic Rikitake dynamo. The nonlinear unknown terms and the unknown parameters are compensated via the adaptive part of the integral sliding mode control law. The synchronization error of the identical master and slave system is steered to origin asymptotically. In other words, the master and slave system are synchronized via the proposed control law in the presence of parametric uncertainties. This overall work is carried out with less number of control inputs as compared to the existing work (Vaidyanathan et al., 2015). This reduction results in less energy usage and improves the applicability of the control strategy. The closed loop stability of the designed compensator controller and the adapted law are ensured via the Lyapunov stability strategy. The system under study is simulated which demonstrates the benefits of the proposed algorithm.

REFERENCES

Abbasi, H.R.,Gholami. A., Rostami.M. and Abbasi,A. (2011).Investigation and control of unstable chaotic behaviour using chaos theory in electrical power systems.*Iranian Journal of Elect.Elect.Engg,* 7(1),42-51.

Abbasi, W. and Rehman, F., (2016). Adaptive Integral Sliding Mode Stabilization of Nonholonomic Drift-Free Systems. *Mathematical Problem in Engineering*. Article ID 9617283, doi.10.1155/2016/9617283.

Abdullah. A. (2013). Synchronization and secure communication of uncertain chaotic systems based on full-order and reduced-order output-affine observers. *Applied Mathematics and Computation*. 219, 10000-10011.

Agrawal, S. K. and S. Das, (2014). Function projective synchronization between four dimensional chaotic systems with uncertain parameters using modified adaptive control method. *Journal of Process Control.* 24(5).

Bartolini, G., Ferrara, A. and Usai, E.(1998) Chattering avoidance by second-order sliding mode control. *IEEE Transactions on Automatic Control*, vol. 43, no. 2, pp. 241–246.

Bullard, E.C. (1955).The stability of homopolar dynamo, *Proc. Cambridge Philosophical Society*, 51, 744.

Caraiani. P. (2013).Testing for nonlinearity and chaos in economic time series with noise titration. *Economics Letters.* 120, 192-194.

Chen, A., Lu, J.,Lü, J. and Yu, S. (2006).Generating hyperchaotic Lü attractor via state feedback control. *Physica A*, 364, 103–110.

Chen, N., Song, F., Li, G., Sun, X. and Ai, C. (2013). An adaptive sliding mode backstepping control for the mobile manipulator with nonholonomic constraints. *Communications in Nonlinear Science and Numerical Simulation,*18(10), 2885–2899.

D. Li. .A three-scroll chaotic attractor. *Physics Letters A.* 372, 387-393.

Denton. T.A., Diamond. G.A., Helfant. R.H., Khan. S. and H. Karagueuzian. (1990). Fascinating rhythm: A primer on chaos theory and its applications to cardiology. *American Heart Journal.* 120, 1419-1440.

Din, S.U., Khan, Q., Rehman, F., and Akmeliawati, R. (2016). Robust control of underactuated systems: Higher order integral sliding mode approach. *Mathematical problems in engineering.* Article ID 5641478, doi:10.1155/2016/5641478.

Edwards. C. and S. K. Spurgeon (1998), Sliding Modes Control: Theory and Applications, *Taylor & Francis,* London, UK.

Edwin A. U., Jacob. O. and Ebozoje.(2014). Anti-Synchronization of the Bullard and Rikitake Dynamo Systems via Nonlinear Active Control.*The International Journal of Engineering And Science (IJES)*, 3(4), 48-53.

Ferrara, A. and Giacomini, L. (2001) "On modular backstepping design with second order sliding modes," *Automatica,* vol. 37, no. 1, pp.129–135.

Gao,Z. and Liao, X. (2013). Integral sliding mode control for fractional-order systems with mismatched uncertainties, *Nonlinear Dynamics,* 72.

Gao. H., Zhang. Y., Liang.S. and Li. D. (2006). A new chaotic algorithm for image encryption.Chaos, *Solitons and Fractals.* 29, 393-399.

Ge, S.S., Wang, C. and Lee, T.H. (2000). Adaptive backstepping control of a class of chaotic systems. *International Journal of Bifurication and Chaos in Applied Sciences and Engineering,* 10(5), 1149–1156.

Gong. Y., Xie. Y., Lin. X., Hao. Y. and Ma.X. (2010).Ordering chaos and synchronization transitions by chemical delay and coupling on scale-free neuronal networks. *Chaos, Solitons and Fractals.* 43, 96-103.

Guegan,D. (2009). Chaos in economics and finance, *Annual Reviews in Control,* 33(1) , 89-93.

Harb A. M. and Abed-Jabar,N. (2003).ControolingHopf bifurcation and chaos in a small power system. *Chaos, Solitons and Fractal,* 18,1055-1063.

Hou, Y.,Liau, B. and Chen, H. (2012).Synchronization of unified chaotic systems using sliding mode controller. *Mathematical problem in engineering.* doi:10.1155/2012/632712.

Kapitaniak, T. and Chua, L. (2006).Hyperchaotic Attractors of Undirectionally-Coupled Chua's Circuits. *International Journal of Bifurication and Chaos in Applied Sciences and Engineering*, 4, 477–482.

Levant, A. (2003). "Higher-order sliding modes, differentiation and output-feedback control. *International Journal of Control*, vol.76, no. 9-10, pp. 924–941.

Li. G.H., Zhou. S.P. and Yang.K. (2007).Controlling chaos in Colpitts oscillator. *Chaos, Solitons and Fractals.* 33, 582-587.

Li. X. (2009). Modified projective synchronization of a new hyperchaotic system via nonlinear control, *Communications in Theoretical Physics.* 52, 274-278. (2009).

Moskalenko. O.I., Koronovskii. A.A. and Hramov. A.E. (2010). Generalized synchronization of chaos for secure communication: Remarkable stability to noise. *Physics Letters A*, 374, 2925-2931.

Rhouma. R. and Belghith.S. (2011).Cryptanalysis of a chaos-based cryptosystem on DSP. *Communications in Nonlinear Science and Numerical Simulation.* 16, 876-884.

Rössler. O.E. (1979). An equation for hyperchaos, *Physics Letters A.* 71, 155-157.

Rössler. O.E. (2008).An equation for continuous chaos. Physics Letters A. 57, 397-398.

Sargolzaei, M., Yaghoobi, M. and Yazdi,R.A.G (2013). Modelling and synchronization of chaotic gyroscope using TS fuzzy approach. *Advances in Electronic and Electric Engineering,* 3(3), 339-346.

Sharma. A., Patidar. V., Purohit.G. and Sud.K.K. (2012). Effects on the bifurcation and chaos in forced Duffing oscillator due to nonlinear damping. *Communications in Nonlinear Science and Numerical Simulation*. 17, 2254-2269.

Shi. J., Zhao. F., Shen. X. and Wang. X. (2013).Chaotic operation and chaos control of travelling wave ultrasonic motor. *Ultrasonics*. 53(6), 1112-1123.

Smaoui. N., Karouma. A. and Zribi.M. (2011). Secure communications based on the synchronization of the hyperchaotic Chen and the unified chaotic systems. *Communications in Nonlinear Science and Numerical Simulation*. 16, 3279-3293.

Suzuki. K. and Imai. Y. (2006).Decryption characteristics in message modulation type chaos secure communication system using optical fiber ring resonators. *Optics Communications,* 259, 88-93.

Utkin, V.,(1992) Sliding Modes in Control Optimization, Springer,Berlin, Germany.

Vaidyanathan, S. (2013). Adaptive controller and synchronizer design for hyperchaotic Zhou system with unknown parameters. *International Journal of Information Technology, Modeling and Computing (IJITMC)*, 1(1), 18–32.

Vaidyanathan. S. (2013).A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control. *International Journal of Control Theory and Applications*. 6 (2), 97-109.

Vaidyanathan. S. (2014). Qualitative analysis and control of an eleventerm novel 4-D hyperchaotic system with two quadratic nonlinearities. *International Journal of Control Theory and Applications*. 7 (1), 35-47.

Vaidyanathan. S., Volos. Ch.K.and Pham. V. T. (2014). Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation. *Archives of Control Sciences*. 24 (4), 409-446.

Vaidyanathan. S., Volos. Ch.K.and Pham. V. T. (2015). Analysis, control, synchronization and SPICE Implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium. *Iestr Journal of Engineering and Science and Technolgy Review.* 8 (2), 232-244.

Volos. Ch. K., Kyprianidis. I.M.and Stouboulos. I.N. (2013). Text encryption scheme realized with a chaotic pseudo-random bit generator. *Journal of Engineering Science and Technology Review.* 6(4), 9-14.

Volos. Ch. K., Kyprianidis. I.M.and Stouboulos. I.N. (2013). Image encryption process based on chaotic synchronization phenomena, *Signal Processing*, 93(5), 1328-1340.

Volos. Ch.K., Kyprianidis. I.M.and Stouboulos. I.N. (2012). Synchronization phenomena in coupled nonlinear systems applied in economic cycles. *WSEAS Trans Systems*. 11(12), 681-690.

W. Zhou, Y. Xu, H.Lu, and L. Pan. (2008). On dynamics analysis of a new chaotic attractor. *Physics Letters A.* 372, 5773-5777.

W. Liu. and G. Chen. (2003). A new chaotic system and its generation.*International Journal of Bifurcation and Chaos*. 13, 261-267.

Wang. J. and Chen. Z. (2008). A novel hyperchaotic system and its complex dynamics. *International Journal of Bifurcation and Chaos*. 18, 3309-3324.

Wang. X.Y. and Gao. Y.F. (2010). A switch-modulated method for chaos digital secure communication based on user-defined protocol.*Communications in Nonlinear Science and Numerical Simulation*, 15, 99-104.

Wei. X., Yunfei. F. andQiang. L. (2012). A novel four-wing hyperchaotic system and its circuit implementation. *Procedia Engineering.* 29, 1264-1269.

Xu. Y., Wang.H., Li. Y. and Pei.B. (2014). Image encryption based on synchronization of fractional chaotic systems. *Communications in Nonlinear Science and Numerical Simulation*. 19(10), 3735-3744.

Yang, T. and Chua,L.O. (1997). Impulsive stabilization for control and synchronization of chaotic systems: Applications to secure communications.*IEEE Transaction on Circuits and Systems-I: Fundamental Theory and Applications*, 44(10), 976-988.

Yang. D. and Zhou. J. (2014). Connections among several chaos feedback control approaches and chaotic vibration control of mechanical systems, *Communications in Nonlinear Science and Numerical Simulation*. 19(11), 3954-3968.

Yang. J. and Zhu.F. (2013).Synchronization for chaotic systems and chaos-based secure communications via both reduced-order and step-by-step sliding mode observers. *Communications in Nonlinear Science and Numerical Simulation.* 18, 926-937.

Yu, X. and Zhihong, M. (2002). Fast Terminal Sliding Mode Control Design for Nonlinear Dynamical Systems. *IEEE Trans. Circuit and Systems*, vol. 49 (2), pp. 261–264.

Zhou. P. and Huang.K. (2014). A new 4-D non-equilibrium fractionalorder chaotic system and its circuit implementation. *Communications in Nonlinear Science and Numerical Simulation*. 19, 2005-2011.

Zribi, M.,Smaoui,N.andSalim,H. (2009).Synchronization of the unified chaotic systems using a sliding mode controller. Chaos, Solitons and Fractals, 42(5), 3197-3209.