Robust H_∞ Control for Sampled-data Dynamic Positioning Ships

Minjie Zheng*, **, Yujie Zhou*, Shenhua Yang, Yongfeng Suo**, Lina Li****

** Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai,200240, China;*
(Tel: 86-135-9950-9986; e-mail: jimi_1205@163.com, zhou863@vip.sina.com)
*** Navigation College, Jimei University, Jimei, Xiamen 361021, China*
(e-mail:yangshh@163.com, yfsuo@qq.com,lln668@163.com)

Abstract: This study investigates the robust H_∞ control problem for dynamic positioning (DP) ships based on sampled-data. By using input delay approach, the DP ships system is converted to a time-varying delay system. Sufficient conditions are derived to make the system exponentially stable and achieve the H_∞ performance using Lyapunov stability theorems. Then, the H_∞ sampled-data controller is obtained by analyzing the admissible condition to guarantee that the DP ships can maintain the desired position, heading and velocities. Simulation result is shown that the proposed method and the designed controller for DP ships are effective in the existence of varying environment disturbance.

Keywords: dynamic positioning ships; H_∞ Control; sampled-data control; Lyapunov-Krasovskii functional; time delays

1. INTRODUCTION

The dynamic positioning (DP) is a system which is controlled by computer, can keep a ship's desired position and heading or sail an exact track by using of active thrusters and propellers(see T. I. Fossen, 2002), has been employed in various vessel types, such as offshore support vessels, Semi-submersible crane vessels, cruise ships, oceanographic research vessels and so on. Compared with the conventional anchor moored position-keeping methods such as jack-up barge and anchoring, the DP mode has its own advantages: High positioning accuracy and flexibility; Easy to be changed position; No anchor handling tugs needed; Not dependent on water depth and so on. With the expansion of ocean to the deeper water, the DP ships technology has attracted considerable attention and become an important research topic.

In the beginning, proportional integral derivative (PID) controller was used and today are still used in DP system (see Fossen, 1994). Subsequently, Kalman filtering technology and optimal control were discussed (see Balchen et al., 1976; A. J. Sørensen et al., 1996; Xie, 2014). Recently, more advanced control algorithms have been proposed for DP ships. The estimators and nonlinear controllers were applied to deal with the vessel dynamics' intrinsic nonlinear characteristics (see Strand et al., 1999; Zhao et al., 2010; Wang et al., 2014). Vectorial backstepping techonology, which is used to guarantee the exponentially stable of DP ships system is presented in (Fossen et al., 1998; Snijders, 2005). In (Loria et al., 2000), separation principle combined with PD-type control law is used to prove the globally asymptotically stable of DP ships. In (Fossen et al., 1999; Lin et al., 2013; Wang, 2012; Lindegaard, 2003), a passive nonlinear observer was designed to estimate the vessel velocity using Lyapunov methods.

Robust H_∞ control theory, since its inception by Zames, has received considerable attentions and made remarkable achievements during the past few decades (see Doyle, 2013; Francis, 1987; Zhou K et al., 1996). Recently, the H_∞ robust control strategy has been proposed for DP ships (see Katebi, 2011). In (Wang et al., 2012), based on mixed sensitivity, a robust controller is designed for the DP ships with uncertain model. In (Ngongi et al., 2015), the fuzzy controller for DP ships is designed using optimal H_∞ control techniques based on T-S fuzzy model (see Wang et al., 2017a,b); In (You et al., 2017), based on the mixed H_∞ and μ-synthesis framework, the robust control problem for DP ships is discussed.

In the last decades, sampled-data system has been an important topic, because modern control systems widely used the digital computers to control continuous-time systems. For example, in a DP ship (see Fig. 1), a variety of sensors, such as Differential Global Positioning System (DGPS) for obtaining higher accuracy and reliability position of ships; gyrocompasses for determining heading of ships; Vertical Reference Sensor (VRS), for determining the ship's roll pitch and heave; wind sensors for anticipating wind gusts etc. are used by digital computers to determine the ship's motion state. The sensors produce continuous-time signals which are transformed into discrete-time control signals by being sampled and quantized with a microcontroller, and they will be transformed into continuous-time signals again by the zero-order holder. Until now, considerable research methods have been proposed for analysing these sampled-data systems (see Fridman, 2006; Wang et al., 2013; Rubagotti et al., 2011; Lee, 2012; Wu, 2013; Chen et al., 2014; Abedi, 2015; Chen, 2015; Wang et al., 2016a,b; Moarref, 2016; Liaquat, 2016; Wang et al., 2017). Input delay approach (see Fridman et al., 2004; Wang, 2016; Liu, 2015; Krishnasamy, 2015; Chen, 2015; Wu, 2014), is one of the main methods proposed by (Fridman et al., 2004). In the approach, the sampling period can be converted to a bounded time-varying delay. Moreover, the sampling distances are not required to be constant, which is its most significant advantage. Besides, the approach can be applied to uncertain sampling systems (see Gao et al., 2010), which is difficult for traditional lifting techniques to deal with.

It should be pointed out that, the control methods existing in literatures for DP ships are mainly focused on the design of continuous-time controller for continuous-time model. However, the DP ships system, commonly utilizes embedded computers for producing discrete-time control signal. So, the results obtained from the existing methods can't be applied directly for DP ships control systems. Hence, in view of the advantage of the input delay approach mentioned above, how to design sampled-data controllers directly for DP ships using the input delay approach is an important and meaningful question. To the best of our knowledge, literature cannot be found to consider the issue, which motivated the paper to handle the sampled-data control problem for DP ships, and it has significance both in practice and theory.

In this brief, the issue about H_∞ control for DP ships based on sampled-data is discussed. The DP control system is transformed to a time-delay system by input delay approach. In terms of LMI approach, adequate conditions are established to make the system exponentially stable with prescribed H_∞ performance using Lyapunov stability theorems. Then, the H_∞ sampled-data controller is obtained to guarantee that the DP ships can maintain the desired position, heading and velocities under the external disturbances like waves, wind, ocean currents. Finally, a simulation of DP ships is provided to demonstrate that the proposed method is effective.

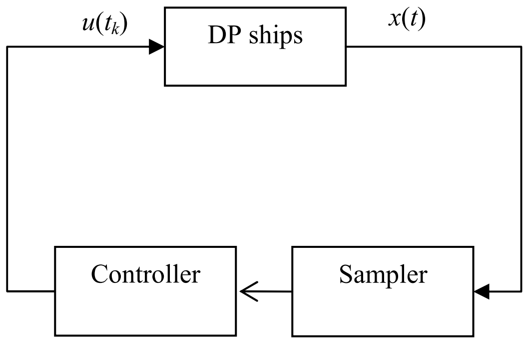

Fig. 1. DP ships control system.

2. PROBLEM FORMULATION

The mathematical model of DP ships is described as follows:

$$\begin{aligned} M\dot{v} + Dv &= u + w \\ \dot{\eta} &= J(\psi)v \end{aligned} \tag{1}$$

where

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix}$$
$$D = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix} \tag{2}$$
$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\eta = [x \quad y \quad \psi]^T$ represents the ship's position x, position y and heading ψ related to the earth-fixed frame. $\upsilon = [p \quad v \quad r]^T$ represents the ship's velocity of the body-fixed frame, where p represents surge velocity, v represents the sway velocity and r represents the yaw velocity. Similar to (Wang et al., 2014), the body-fixed coordinate system is shown in Fig.2. $J(\psi)$ represents the transformation matrix of the two frames mentioned above; u represents the vector of control force and moment; w represents a disturbance vector including waves, wind, ocean currents; M and D represent the inertia and linear damping matrix respectively.

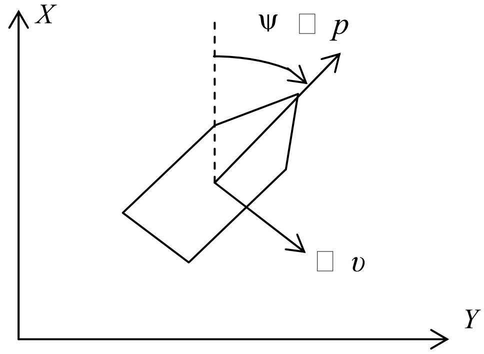

Fig. 2. Coordinate systems of DP ships.

As the heading ψ is very small, and it can be obtained that

$$J(\psi) \cong I_{3\times 3}. \tag{3}$$

Under the condition (3), the linearized low frequency motion of the model is got as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

where

$$x(t) = [\eta \quad \upsilon]^T = [x \quad y \quad \psi \quad p \quad v \quad r]^T,$$

$$A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$
$$C = [I \quad 0] \tag{5}$$

In this paper, the state variables of DP ships are assumed to be measured at the sampling instant $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$, that is, in the interval $t_k \le t \le t_{k+1}$, only $x(t_k)$ is available. The sampling period follow the assumption that it is bounded by a constant d, that is,

$$t_{k+1} - t_k \le d, \forall k \ge 0, d > 0, \quad (6)$$

Considered the state-feedback control law as follow,

$$u(t) = u(t_k) = Kx(t_k), \quad t_k \le t < t_{k+1}, \quad (7)$$

where t_k represents the sampling instant, $u(t_k)$ represents the discrete-time control signal, K is a state-feedback gain matrix. By substituting (7) into (1), then

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t_k) + Ew(t), & t > 0 \\ y(t) = Cx(t) \end{cases} \quad (8)$$

Remark 1: Note that both discrete and continuous signals is included in system (8), which is more different and practical than the existing continuous-time control method for DP ships system. Besides, because the parameter uncertainties exist in the system, it is difficult for the traditional lifting technique to deal with the problem.

The paper's purpose is to find the state feedback gain to satisfy the following requirements:

1) The system (8) with $w(t) = 0$ is exponential stable, if there exist scalars $\alpha > 0$ and $\beta > 0$,then

$$\| x(t) \| \le \beta \sqrt{e^{-\alpha t}} \| x_0 \|_c, \qquad \forall t \ge 0 \quad (9)$$

where $\| x_0 \|_c = \sup_{-d \le \theta \le 0} \| x(\theta) \|$

2) To reject the varying environment disturbances like waves, ocean currents and wind, the closed-loop system is assumed to satisfy that $\| y(t) \|_2 \le \gamma \| w(t) \|_2$ for all nonzero $w(t) \in L_2 [0, \infty)$ under zero condition, where $\gamma > 0$.

Then, by input delay approach, the state-feedback controller $u(t)$ is rewritten as

$$\begin{aligned} & u(t) = u(t_k) = u(t - (t - t_k)) = u(t - \tau(t)), \\ & t_k \le t < t_{k+1}, \quad \tau(t) = t - t_k, \end{aligned} \quad (10)$$

where the time-varying delay $\tau(t)$ is piecewise-linear satisfying

$$\begin{aligned} & 0 \le \tau(t) \le d, \\ & \dot{\tau}(t) = 1, \quad t \ne t_k \end{aligned} \quad (11)$$

Thus, the sampled-data system in (8) is converted to a system with time-varying delay as follow:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)) + Ew(t), & t > 0 \\ y(t) = Cx(t) \\ x(t) = \phi(t), & t \in [-d, 0] \end{cases} \quad (12)$$

3. MAIN RESULTS

The H_∞ control problem for sampled-data DP ships is studied in this section. By establishing Lyapunov-Krasovskii functional, adequate condition is obtained to guarantee the exponentially stability of the system. Then the sampled-data controller is designed by analysing the stabilization condition.

Theorem 1: Given scale $d > 0$, $\alpha > 0$, $\gamma > 0$, the closed-loop system (12) is exponentially stable with H_∞ performance γ, if there exist symmetric positive-definite matrices Z, P, Q, M_i, N_i, i=1,2,3,4 such that

$$\begin{bmatrix} \Pi & \sqrt{\frac{e^{\alpha d} - 1}{\alpha}} \Omega & \sqrt{d} M & \sqrt{d} N \\ \sqrt{\frac{e^{\alpha d} - 1}{\alpha}} \Omega^T & -Z^{-1} & 0 & 0 \\ \sqrt{d} M^T & 0 & -Z & 0 \\ \sqrt{d} N^T & 0 & 0 & -Z \end{bmatrix} < 0, \quad (13)$$

Where

$$\begin{aligned} & \Omega = \begin{bmatrix} A & BK & 0 & E \end{bmatrix}^T, \\ & M = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}^T, \\ & N = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix}^T, \\ & \Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ \Pi_{12}^T & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ \Pi_{13}^T & \Pi_{23}^T & \Pi_{33} & \Pi_{34} \\ \Pi_{14}^T & \Pi_{24}^T & \Pi_{34}^T & -\gamma^2 I \end{bmatrix} \end{aligned} \quad (14)$$

where

$$\begin{aligned} & \Pi_{11} = \alpha P + PA + A^T P + e^{\alpha d} Q + M_1 + M_1^T + C^T C, \\ & \Pi_{12} = PBK - M_1 + M_2^T + N_1, \\ & \Pi_{13} = M_3^T - N_1 \\ & \Pi_{14} = PE + M_4^T \\ & \Pi_{22} = -M_2 - M_2^T + N_2 + N_2^T \\ & \Pi_{23} = -M_3^T - N_2 + N_3^T \\ & \Pi_{24} = -M_4^T + N_4^T \\ & \Pi_{33} = -Q - N_3 + N_3^T \\ & \Pi_{34} = -N_4^T \end{aligned} \quad (15)$$

Proof. First, considered the following Lyapunov-Krasovskii functional

$$V(t)=\sum_{i=1}^{3}V_i(t),\quad t\in[t_k,t_{k+1}) \tag{16}$$

$$\begin{aligned}
V_1(t)&=e^{\alpha t}x(t)^T Px(t)\\
V_2(t)&=\int_{t-d}^{t} e^{\alpha(s+d)}x(s)^T Qx(s)ds\\
V_3(t)&=\int_{-d}^{0}\int_{t+\theta}^{t} e^{\alpha(s-\theta)}\dot{x}(s)^T Z\dot{x}(s)dsd\theta
\end{aligned} \tag{17}$$

Calculating the derivative of $V(t)$, it can be obtained that:

$$\begin{aligned}
\dot{V}_1(t)&=\alpha e^{\alpha t}x(t)^T Px(t)+2e^{\alpha t}x(t)^T P\dot{x}(t)\\
\dot{V}_2(t)&=e^{\alpha(t+d)}x(t)^T Qx(t)-e^{\alpha t}x(t-d)^T Qx(t-d)\\
\dot{V}_3(t)&=e^{\alpha t}\dot{x}(t)^T Z\dot{x}(t)\frac{e^{\alpha d}-1}{\alpha}-e^{\alpha t}\int_{t-d}^{t}\dot{x}(s)^T Z\dot{x}(s)ds
\end{aligned} \tag{18}$$

Using the Leibniz-Newton formula, for any matrices of proper dimension M_i, N_i, i=1,2,3, the equations can be obtained as follow:

$$\begin{aligned}
&2e^{\alpha t}[x^T(t)M_1+x^T(t-\tau(t))M_2+x^T(t-d)M_3]\\
&\times[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds]=0
\end{aligned} \tag{19}$$

$$\begin{aligned}
&2e^{\alpha t}[x^T(t)N_1+x^T(t-\tau(t))N_2+x^T(t-d)N_3]\\
&\times[x(t-\tau(t))-x(t-d)-\int_{t-d}^{t-\tau(t)}\dot{x}(s)ds]=0
\end{aligned} \tag{20}$$

Similarly, it can be obtained that

$$\begin{aligned}
e^{\alpha t}\int_{t-d}^{t}\dot{x}(s)Zx(s)ds &=e^{\alpha t}\int_{t-\tau(t)}^{t}\dot{x}(s)Zx(s)ds\\
&+e^{\alpha t}\int_{t-d}^{t-\tau(t)}\dot{x}(s)Zx(s)ds
\end{aligned} \tag{21}$$

Substituting (19)-(21) into (18) that

$$\begin{aligned}
\dot{V}(t)\le e^{\alpha t}\zeta^T(t)(\Theta+&\frac{e^{\alpha d}-1}{\alpha}[A\quad BK\quad 0]^T\cdot\\
&Z[A\quad BK\quad 0]+dMZ^{-1}M^T+dNZ^{-1}N^T)\zeta(t)\\
&-e^{\alpha t}\int_{t-\tau(t)}^{t}[\zeta^T(t)M+\dot{x}^T(s)Z]Z^{-1}[M^T\zeta(t)+Z\dot{x}(s)]ds\\
&-e^{\alpha t}\int_{t-d}^{t-\tau(t)}[\zeta^T(t)N+\dot{x}^T(s)Z]Z^{-1}[N^T\zeta(t)+Z\dot{x}(s)]ds
\end{aligned} \tag{22}$$

where

$$\zeta(t)=[x^T(t)\quad x^T(t-\tau(t))\quad x^T(t-d)]^T$$

$$M=\begin{bmatrix}M_1\\M_2\\M_3\end{bmatrix},N=\begin{bmatrix}N_1\\N_2\\N_3\end{bmatrix},\Theta=\begin{bmatrix}\Pi_{11}&\Pi_{12}&\Pi_{13}\\\Pi_{12}^T&\Pi_{22}&\Pi_{23}\\\Pi_{13}^T&\Pi_{23}^T&\Pi_{33}\end{bmatrix} \tag{23}$$

Since Z>0, then the last two parts in (22) are all less than 0. According to Schur complement, (10) implies that

$$\begin{aligned}
\Theta+&\frac{e^{\alpha d}-1}{\alpha}[A\quad BK\quad 0]^T Z[A\quad BK\quad 0]\\
&+dMZ^{-1}M^T+dNZ^{-1}N^T<0.
\end{aligned} \tag{24}$$

Hence, $\dot{V}(x_t)<0$ can be obtained.

Thus, it follows that, for $t\in[t_k,t_{k+1})$

$$V(t)\le V(t_k)\le V(t_{k-1})\le\ldots\le V(0) \tag{25}$$

It can be calculated that

$$\begin{aligned}
V(0)&=x(0)^T Px(0)+\int_{-d}^{0} e^{\alpha(s+d)}x(s)^T Qx(s)ds\\
&+\int_{-d}^{0}\int_{\theta}^{0} e^{\alpha(s-\theta)}\dot{x}(s)^T Z\dot{x}(s)dsd\theta\\
&\le\lambda_{\max}(P)\sup_{-d\le\theta\le0}\|x(\theta)\|^2+\\
&\frac{e^{\alpha d}-1}{\alpha}\lambda_{\max}(Q)\sup_{-d\le\theta\le0}\|x(\theta)\|^2\\
&+3\left\{\lambda_{\max}(A^T A)+\lambda_{\max}(K^T B^T BK)\right\}\times\\
&\frac{e^{\alpha d}-1-\alpha d}{\alpha^2}\lambda_{\max}(Z)\sup_{-d\le\theta\le0}\|x(\theta)\|^2\\
&\le\Lambda\sup_{-d\le\theta\le0}\|x(\theta)\|^2
\end{aligned} \tag{26}$$

Where

$$\begin{aligned}
\Lambda=\lambda_{\max}(P)+&\frac{e^{\alpha d}-1}{\alpha}\lambda_{\max}(Q)+\\
&3\left\{\lambda_{\max}(A^T A)+\lambda_{\max}(K^T B^T BK)\right\}\times\\
&\frac{e^{\alpha d}-1-\alpha d}{\alpha^2}\lambda_{\max}(Z)
\end{aligned} \tag{27}$$

On the other hand, there is

$$V(t)\ge e^{\alpha t}x(t)^T Px(t)\ge e^{\alpha t}\lambda_{\min}(P)\|x(t)\|^2 \tag{28}$$

Using the (27) and (28), it can be got that

$$\begin{aligned}
\|x(t)\|^2\le e^{-\alpha t}\frac{1}{\lambda_{\min}(P)}V(t)&\le e^{-\alpha t}\frac{1}{\lambda_{\min}(P)}V(0)\\
&\le e^{-\alpha t}\frac{\Lambda}{\lambda_{\min}(P)}\sup_{-d\le\theta\le0}\|x(\theta)\|^2
\end{aligned} \tag{29}$$

then

$$\|x(t)\|\le\sqrt{\frac{\Lambda}{\lambda_{\min}(P)}e^{-\alpha t}}\sup_{-d\le\theta\le0}\|x(\theta)\| \tag{30}$$

Thus, according to Definition 1, the system (12) is exponential stability. The proof is completed.

Now, the H_∞ performance will be considered for system (12). By choosing the same Lyapunov-Krasovskii functional given in (11), using the following equations to replace the (19) and (20),

$$2e^{\alpha t}[x^T(t)M_1 + x^T(t-\tau(t))M_2 + x^T(t-d)M_3 + w(t)M_4]\times[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds]=0 \quad (31)$$

$$2e^{\alpha t}[x^T(t)N_1 + x^T(t-\tau(t))N_2 + x^T(t-d)N_3 + w(t)N_4]\times[x(t-\tau(t))-x(t-d)-\int_{t-d}^{t-\tau(t)}\dot{x}(s)ds]=0 \quad (32)$$

and following similar proof, it can be got that

$$y^T(t)y(t)-\gamma^2 w^T(t)w(t)+e^{-\alpha t}\dot{V}(t)\le \begin{bmatrix}\zeta(t)\\ w(t)\end{bmatrix}^T \{\Pi+\frac{e^{\alpha d}-1}{\alpha}[A \quad BK \quad 0 \quad E]^T Z\cdot [A \quad BK \quad 0 \quad E]+dMZ^{-1}M^T+dNZ^{-1}N^T\}\begin{bmatrix}\zeta(t)\\ w(t)\end{bmatrix} \quad (33)$$

where

$$M=\begin{bmatrix}M_1\\M_2\\M_3\\M_4\end{bmatrix}, N=\begin{bmatrix}N_1\\N_2\\N_3\\N_4\end{bmatrix} \quad (34)$$

Π is defined in (15).

By Schur complement, (13) guarantee

$$\Pi+\frac{e^{\alpha d}-1}{\alpha}[A \quad BK \quad 0 \quad E]^T Z\cdot [A \quad BK \quad 0 \quad E]+dMZ^{-1}M+dNZ^{-1}N<0 \quad (35)$$

So, it can be obtained from (33) and (35) that

$$y^T(t)y(t)-\gamma^2 w^T(t)w(t)+e^{-\alpha t}\dot{V}(t)<0 \quad (36)$$

Therefore, from (36), $\| y(t)\|_2 \le \gamma \| w(t)\|_2$ can be got for all nonzero $w(t)\in L_2[0,\infty)$, then the H_∞ performance is established.

Remark 2: Theorem 1 gives a sufficient condition to solve the exponential stability problem for system (12). Besides, it should be pointed out that using the free-weighting matrices can reduce conservativeness of proposed delay-dependent results (see (Fridman et al., 2004b, 2005c; Suplin et al., 2007). In addition, it is worth noting that in this work the value of decay rate α is free and can be chosen for different situations, which is more excellent and significant than the result existing in sampled-data systems.

Now, the H_∞ sampled-data controller (7) will be designed based on Theorem 2.

Theorem 2: Given scales $d>0$, $\alpha>0$, $\gamma>0$, the system (12) is exponentially stable under the assumption that the sampling period is bounded by a constant d, that is, $t_{k+1}-t_k\le d, \forall k\ge 0$,if there exist symmetric positive-definite matrices Z, P ,Q , H_i, R_i, i=1,2,3,4, such that the LMIs hold as follow :

$$\begin{bmatrix}\bar{\Pi} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{\Omega} & \sqrt{d}\bar{M} & \sqrt{d}\bar{N} & \bar{P}\bar{C}^T\\ \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{\Omega}^T & \bar{Z}-2\bar{P} & 0 & 0 & 0\\ \sqrt{d}\bar{M}^T & 0 & -\bar{Z} & 0 & 0\\ \sqrt{d}\bar{N}^T & 0 & 0 & -\bar{Z} & 0\\ \bar{C}\bar{P} & 0 & 0 & 0 & -I\end{bmatrix}<0, \quad (37)$$

Where

$$\bar{\Omega}=\begin{bmatrix}A\bar{P} & \bar{K}B & 0 & E\end{bmatrix}^T,$$
$$\bar{M}=\begin{bmatrix}\bar{M}_1^T & \bar{M}_2^T & \bar{M}_3^T & \bar{M}_4^T\end{bmatrix}^T,$$
$$\bar{N}=\begin{bmatrix}\bar{N}_1^T & \bar{N}_2^T & \bar{N}_3^T & \bar{N}_4^T\end{bmatrix}^T,$$
$$\bar{C}=\begin{bmatrix}C & 0 & 0 & 0\end{bmatrix},$$
$$\bar{\Pi}=\begin{bmatrix}\bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14}\\ \bar{\Pi}_{12}^T & \bar{\Pi}_{22} & \bar{\Pi}_{23} & \bar{\Pi}_{24}\\ \bar{\Pi}_{13}^T & \bar{\Pi}_{23}^T & \bar{\Pi}_{33} & \bar{\Pi}_{34}\\ \bar{\Pi}_{14}^T & \bar{\Pi}_{24}^T & \bar{\Pi}_{34}^T & -\gamma^2 I\end{bmatrix}$$

$$\begin{aligned}&\bar{\Pi}_{11}=\alpha\bar{P}+\bar{P}A+A^T\bar{P}+e^{\alpha d}\bar{Q}+\bar{M}_1+\bar{M}_1^T,\\ &\bar{\Pi}_{12}=B\bar{K}-\bar{M}_1+\bar{M}_2^T+\bar{N}_1,\\ &\bar{\Pi}_{13}=\bar{M}_3^T-\bar{N}_1\\ &\bar{\Pi}_{14}=E+\bar{M}_4^T\\ &\bar{\Pi}_{22}=-\bar{M}_2-\bar{M}_2^T+\bar{N}_2+\bar{N}_2^T\\ &\bar{\Pi}_{23}=-\bar{M}_3^T-\bar{N}_2+\bar{N}_3^T\\ &\bar{\Pi}_{24}=-\bar{M}_4^T+\bar{N}_4^T\\ &\bar{\Pi}_{33}=-\bar{Q}-\bar{N}_3+\bar{N}_3^T\\ &\bar{\Pi}_{34}=-\bar{N}_4^T\end{aligned} \quad (38)$$

Moreover, a suitable controller with H_∞ performance γ in forms of (7) is designed to satisfy the proposed conditions. And the control gain matrix K is obtained as

$$K=\bar{K}\bar{P}^{-1} \quad (39)$$

Proof: By noticing that $-\bar{P}\bar{Z}^{-1}\bar{P}\le\bar{Z}-2\bar{P}$

Let £$=diag\{P^{-T},P^{-T},P^{-T},I,I,P^{-T},P^{-T}\}$. Denoting

$$\bar{P}=P^{-1}, \bar{K}=KP^{-1}, \bar{Q}=P^{-T}QP^{-1}, \bar{M}_1=P^{-T}M_1P^{-1}, \bar{M}_4=M_4P^{-1}, \bar{Z}=P^{-T}ZP^{-1} \quad (40)$$

and pre-multiplying and post-multiplying (13) by $£$ and $£^T$ respectively, (37) and the following inequality can be obtained:

$$\begin{bmatrix} \tilde{\Pi} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\tilde{\Omega} & \sqrt{d}\bar{M} & \sqrt{d}\bar{N} \\ \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\tilde{\Omega}^T & -\bar{P}Z^{-1}\bar{P} & 0 & 0 \\ \sqrt{d}\bar{M}^T & 0 & -Z & 0 \\ \sqrt{d}\bar{N}^T & 0 & 0 & -Z \end{bmatrix} < 0, \quad (41)$$

where

$$\tilde{\Omega} = \begin{bmatrix} A\bar{P} & \bar{K}B & 0 & E\bar{P} \end{bmatrix}^T,$$

$$\tilde{\Pi} = \begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14} \\ \bar{\Pi}_{12}^T & \bar{\Pi}_{22} & \bar{\Pi}_{23} & \bar{\Pi}_{24} \\ \bar{\Pi}_{13}^T & \bar{\Pi}_{23}^T & \bar{\Pi}_{33} & \bar{\Pi}_{34} \\ \bar{\Pi}_{14}^T & \bar{\Pi}_{24}^T & \bar{\Pi}_{34}^T & -\gamma^2 I \end{bmatrix}$$

$$\tilde{\Pi}_{11} = \alpha\bar{P} + \bar{P}A + A^T\bar{P} + e^{\alpha d}\bar{Q} + \bar{M}_1 + \bar{M}_1^T + \bar{P}C^TC\bar{P} \quad (42)$$

By Schur complement, (41) is equivalent to (37). The proof is completed.

Remark 3: According to the Theorem 2, sufficient conditions are provided to deal with the robust H_∞ control problem for DP ships based on sampled-data, and the proper sampled-data controller is proposed. Noted that the conditions are formulated by means of LMI, which is easy to be determined by standard numerical software. The tool provided by LMI technique is effective for designing the sampled-data controller of DP ships. Moreover, the proposed methods of the work can be extended to other ships easily.

4. NUMERICAL EXAMPLES

For validating the effectiveness of the proposed methods, a numerical simulation is carried out for a DP ship with environment disturbance in the section. The DP ship's main parameters are referenced to the Ship Handling Simulator designed by the Institute of Navigation Jimei University. The length and beam of vessel are respectively 170m and 25.8m, tonnage is 2.4×10^7 kg, draft is 9.5m. The M and D in model (1) are given by:

$$M = \begin{bmatrix} 2.6415\times10^7 & 0 & 0 \\ 0 & 3.345\times10^7 & 1.492\times10^7 \\ 0 & 1.492\times10^7 & 6.52\times10^9 \end{bmatrix}$$

$$D = \begin{bmatrix} 2.22\times10^4 & 0 & 0 \\ 0 & 2.22\times10^5 & -1.774\times10^6 \\ 0.2 & -1.774\times10^6 & 7.151\times10^8 \end{bmatrix}$$

Using the parameters given above and formula (3), the system matrices for DP model can be obtained. The extern disturbances like waves, wind, ocean currents are considered in the simulation. The direction and significant height of wave are respectively 150° and 5.5 m. The direction and speed of current are respectively 100° and 0.25 m/s. The direction and speed of wind are respectively 225° and 10 m/s. The ship's initial position and heading is η_i = [0m 0m 0°], and body-fixed velocities v_i = [2m/s -2m/s 0.4°/s]. The final desired state is η_f = [10 10 4°], v_f = [0m/s 0m/s 0°/s]. Assuming the sampling interval d=1.0s, the minimum value achieved by Theorem 2 to guarantee H_∞ performance γ_{min}=1.630. Then, the value of feedback gain matrix is obtained that

$$K = \begin{bmatrix} -0.0332 & 0 & 0 & -2.1871 & 0 & 0 \\ 0 & 0.8548 & -0.178 & 0 & 1.5367 & -1.308 \\ 0 & -0.178 & 0.652 & 0 & 0.0303 & -0.2838 \end{bmatrix}$$

The simulation results are shown in Fig. 3-5. Fig. 3 and Fig. 4 show the ship's x direction and y direction, Fig. 5 shows the ship's heading ψ, which are respectively response of the proposed controller.

From the Fig. 3-5, it can be seen that the settling time for the x position 2s; the settling time for the y position is 3.5s, and there is no overshoot and steady state error for x and y position; the settling time for heading ψ is 8s, though there exist an overshoot for heading ψ, it is no more than 20%, and the steady state error is nearly to zero. The main performance parameters (overshoot, rise time settling time) are shown in table1.

This indicates that the proposed sampled-data H_∞ controller can stabilize the ship and keep the ship at the desired target position and heading.

Table 1. The main performance parameters of x, y and ψ.

Parameters	x	y	ψ
Rise time	1.2s	2.7s	2.1s
Overshoot time	0	0	18%
Settling time	2s	3.5s	8s

To assess the system's robustness performance, the sensitivity function S, control sensitivity function C is described respectively as follow.

$$S = \left(I + GK\right)^{-1}$$
$$C = K\left(I + GK\right)^{-1} \quad (43)$$

where G represents the ships nominal mode, S represents the effect of the input disturbances. C represents the control signals' frequency content which determines the modulation of thruster.

In order to reduce the influence of the low-frequency disturbances such as current and wind, and the thruster modulation to a lower level, weighting functions W_s and W_c, which are used to weight sensitivity function S, control sensitivity function C respectively, are selected as follow

$$W_S = \frac{s/1.5+10}{s+10^{-3}}$$
$$W_C = \frac{s+0.7}{0.1s+1} \tag{44}$$

where the amplitude of W_s at low frequency should be large to achieve a good tracking performance and disturbance rejection ability; a high pass filter should be selected for W_c to ensure that the desired controller rolls off at high frequency.

The singular value plots for the sensitivity function and control sensitivity functions are described in Fig. 6-7. They are all bounded by the weightings respectively.Fig.6 shows that the sensitivity function *S* rolls off at low frequency range which indicates that the performance of disturbance rejection is good. Fig.7 shows that the control sensitivity function *C* rolls off at high frequency which indicates that the high thruster modulation is avoided and the ware for thruster is minimized. So, from Fig. 6-7, it is shown that the proposed sampled-data H_∞ controller has a good performance.

Velocities [*u v r*] response of the controller is shown in Fig. 8-10, which indicates that the velocities [*u v r*] tend to zero during a short time (about 4.5s). That is, the designed sampled-data H_∞ controllers can stabilize the ship's velocities and guarantee robust H_∞ control performance under the external disturbance like waves, wind, ocean currents.

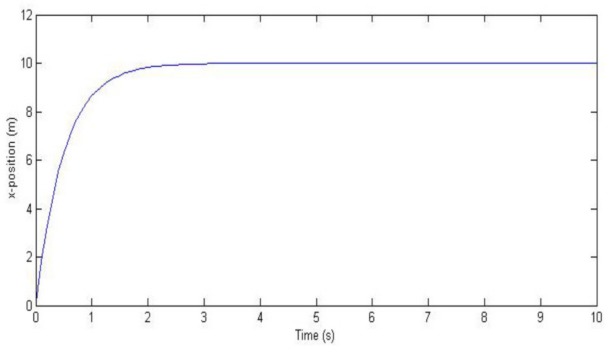

Fig. 3. Position of the *x* direction of the ship.

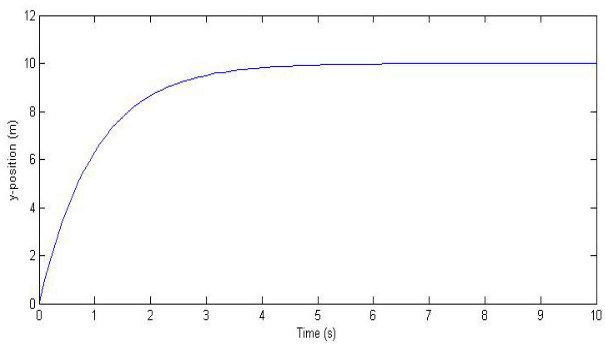

Fig. 4. Position of the y direction of the ship.

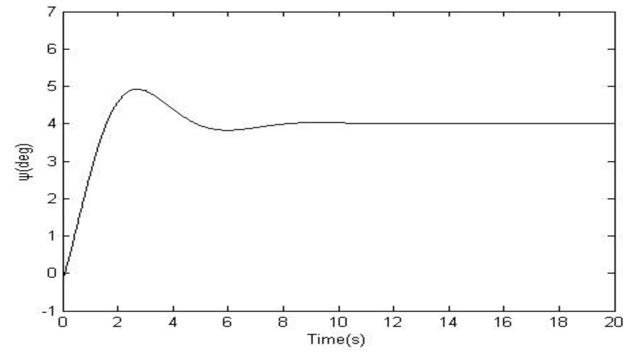

Fig. 5. Heading ψ of the ship.

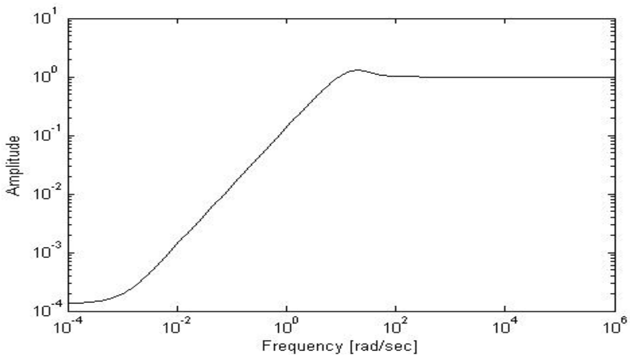

Fig. 6. Singular value plots of sensitivity functions.

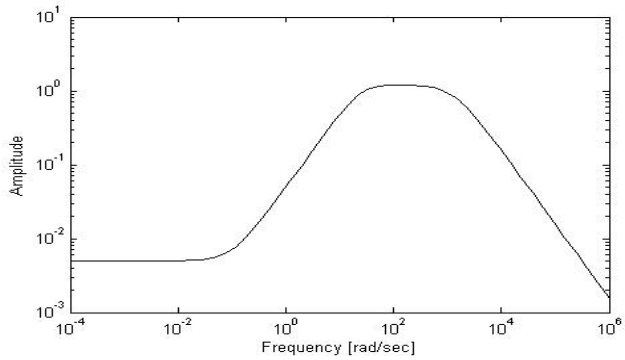

Fig. 7. Singular value plots of control sensitivity functions.

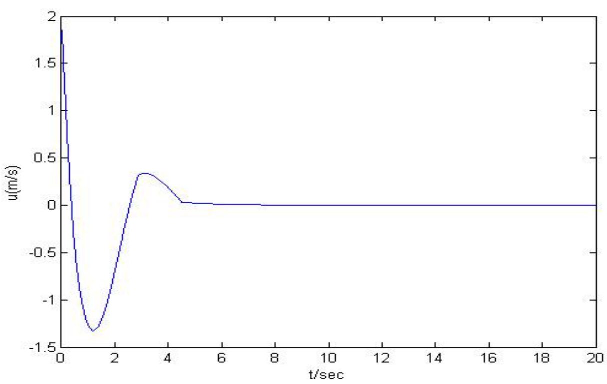

Fig. 8. Surge velocity u of the ship.

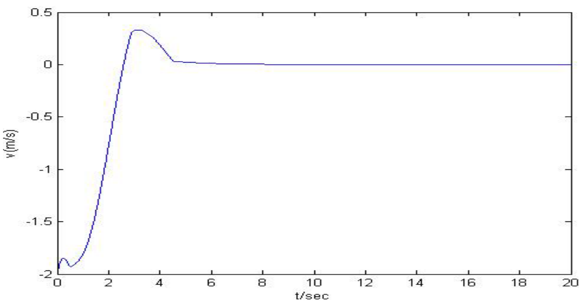

Fig. 9. Sway velocity v of the ship.

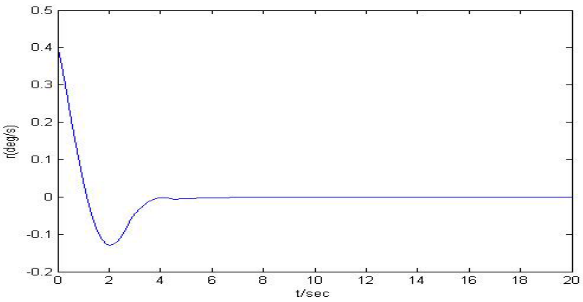

Fig. 10. Yaw rate r of the ship.

5. CONCLUSIONS

In the paper, the H_∞ robust control technique for sampled-data DP ships is proposed. Using the input delay approach, the ship DP system is transformed to a time-varying delay system. By using the Lyapunov approach, sufficient condition is derived to guarantee the exponential stability of DP ships system. Finally, from a DP ship simulation, the connection between simulations results and H_∞ control is shown that :

1) Under the extern environment disturbance, the designed sampled-data H_∞ controller can make the ship's position achieve the desired target value in a short settling time and without overshoot.
2) Though there exists an overshoot for heading ψ and the settling time is longer than that of the ship's position, it can be stabilized by the proposed sampled-data H_∞ controller after a short time.
3) To guarantee that the closed-loop system is robust, the performances in terms of sensitivity functions are introduced. Then the H_∞ controller performance is shown using frequency response and time-domain simulations. Simulations results show that the ship's velocities are stabilized by the sampled-data H_∞ controller under the external disturbance, which indicated that the proposed sampled-data H_∞ controller for the DP ships is effective and robust. And the proposed methods can be extended to other ships.

In future, the new sliding mode control method (see Wang et al., 2017a,b) will be studied for sampled-data DP ships system.

ACKNOWLEDGEMENT

This work was supported by National Natural Science Foundation of China (51579114); The Project of New Century Excellent Talents of Colleges and Universities of Fujian Province (JA12181)；Natural Science Foundation of Fujian Province (2015J05103); The Scientific Research Foundation of Jimei University and The Project of Young and Middle-aged Teacher Education of Fujian Province(JAT170309)

REFERENCES

Abedi, S.M. (2015). An Optimal Sample-Data holds by using a Bi-Objective criterion: Trade-off between the Phase Delay and the Stability Robustness, *control engineering and applied informatics*, 17(2), 32-42

Balchen, J. G., Jenssen, N. A., Sælid, S.(1976). Dynamic positioning using kalman filtering and optimal control theory, *Proceedings of the IFAC/IFIP Symposium on Automation in Offshore Oil Field Operation*,183(186).

Chen, F., Garnier, H., Gilson, M. (2015). Robust identification of continuous-time models with arbitrary time-delay from irregularly sampleddata, *Journal of process control* , 19-27

Chen, T., Qiu, L. (2014). H_∞ design of general multirate sampled-data, *J. Cent. South Univ*. 21,1339−1346

Chen, X., Li, Y. J., Xiang, J. (2015). Sampled-data synchronized output regulation of linear system based on input delay approach, *27th Chinese Control and Decision Conference (CCDC)* , 688-693

Doyle, J.C., Francis, B.A., and Tannenbaum, A.R. (2013). *Feedback control theory*, Courier Corporation,

Fossen, T. I. (2002). *Marine Control Systems: Guidance, navigation and control of ships*, Rigsand Underwater Vecicles, Marine Cybernetics, Trondheim, Norway, lst edition.

Fossen, T. I.(1994). *Guidance and control of ocean vehicles*, Wiley.

Fossen, T. I., Grovlen, A.(1998). Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping, *IEEE Transactions on Control Systems Technology*, 6(1),121-128.

Fossen, T. I., Strand, J. P.(1999). Passive nonlinear observer design for ships using lyapunov methods: full-scale experiments with a supply vessel, *Automatica*, 35,3-16.

Francis, B.A. (1987). *A course in H_∞ control theory*, Berlin; New York: Springer-Verlag,

Fridman, E.(2006). Robust sampled-data H_∞ control of linear singularly perturbed systems [J]. *IEEE Transactions on Automatic Control*, 51(3), 470−475.

Fridman, E., Seuret, A. (2004). Richard J P. Robust sampled-data stabilization of linear systems: An input delay approach . *Automatica*, 40(8), 1441−1446.

Gao, H., Sun, W. and Shi, P. (2010). Robust sampled-data H∞ control for vehicle active suspension systems, *IEEE Trans. Control Syst. Tech.*, 18(1), 238-245.

Katebi, M.R., Grimble, M.J., Zhang, Y. (2011). H_∞ Robust Control Design for Dynamic Ship Positioning. *IEE Proceedings of Control Theory and Applications,* 114(2), 110–120

Krishnasamy, R., Balasubramaniam, P. (2015). Stabilisation analysis for switched neutral systems based on sampled-data control, *International journal of systems science* ,46 (14), 2531-2546.

Lee, H. J., Kim, M. H., Lee, S. Y., and Kim, T.Y.(2012). Robust Sampled-Data Fuzzy Control of Nonlinear Systems with Parametric Uncertainties: Its Application to Depth Control of Autonomous Underwater Vehicles, *International journal of control automation and systems*, 10(6),1164-1172

Liaquat, M., and Malik, M. B. (2016). Sampled data output regulation of n-link robotic manipulator using a realizable reconstruction filter, *Robotica*, 34(4), 900-912

Lin, X.G., Xie, Y.H., Zhao, D.W., and Xu, S.S.(2013). Estimation of parameters of observer of dynamic positioning ships, *Mathematical Problems in Engineering*. Vol.2013, Article ID 173603, 7 pages.

Lindegaard, K. P. (2003). Acceleration feedback in dynamic positioning, *Norwegian University of Science and Technology*, Norway.

Liu, Y.J., Lee, S.M. (2015). Sampled-data Synchronization of Chaotic Lur'e Systems with Stochastic Sampling, *circuits systems and signal processing*, 34(12), 3725-3739

Loria, A., Fossen, T.I. and Panteley, E.(2000). A separation principle for dynamic positioning of ships: theoretical and experimental results. *IEEE Trans. on Control Systems Technology*, 8, 332-343.

Moarref, M., Rodrigues, L. (2016). Sensor allocation with guaranteed exponential stability for linear multi-rate sampled-data systems, *International journal of robust and nonlinear control*, 26(7), 1512-1529

Ngongi, W., Du, J. (2015). Linear fuzzy controller design for dynamic positioning system of surface ships. *International Journal of Intelligent Systems Technologies and Applications*, 14(1), 1-26.

Rubagotti, M., Raimondo, D.M., Ferrara A, Magni L. (2011). Robust model predictive control with integral sliding mode in continuous time sampled-data nonlinear systems, *IEEE Transactions on Automatic Control*, 56(3), 556-570.

Snijders, J. G. (2005). M.Sc. Thesis, Wave filtering and Thruster allocation for dynamic positioned ships, *Delft University of Technology*.

Sørensen, A. J., Sagatun, S. I., & Fossen, T. I. (1996). Design of a dynamic positioning system using model-based control, *Control Engineering Practice*, 4(3),359–368.

Strand, J. P. and Fossen, T. I.(1999). Nonlinear passive observer design for ships with adaptive wave filtering, New Directions in Nonlinear Observer Design, *Lecture Notes in Control and Information*. 244,113-134.

Wang, X.H., Wang, X.H., and Xiao, J.M. (2012). Robust Controller for Ship Dynamic Positioning Based on H_∞, *Advanced Materials Research.*,503, 1668-1671.

Wang, Y., Gao, Y. , Karimi, H. R., Shen, H., and Fang, Z. (2017). Sliding mode control of fuzzy singularly perturbed systems with application to electric circuits, *IEEE Trans. Syst. Man, Cybern. Syst.*, in press, DOI: 10.1109/TSMC.2017.2720968.

Wang, Y., Shen, H., Duan, D.P. (2016).On Stabilization of Quantized Sampled-Data Neural-Network-Based Control Systems, *IEEE Transactions on Cybernetics*.

Wang, Y., Shen, H., Karimi, H. R., and Duan, D. (2017). Dissipativity-based fuzzy integral sliding mode control of continuous-time T-S fuzzy systems, IEEE Trans. Fuzzy Syst., in press, DOI: 10.1109/TFUZZ.2017. 2710952.

Wang, Y., Shi, P., Wang, Q., Duan, D. (2013). Exponential H_∞ Filtering for Singular Markovian Jump Systems With Mixed Mode-Dependent Time-Varying Delay, *IEEE Transactions on Circuits and Systems I*, 60(9), 2440-2452

Wang, Y., Xia, Y., Li, H. and Zhou, P. (2017). A new integral sliding mode design for nonlinear stochastic systems, *Automatica*.

Wang, Y., Xia, Y., Li, H. and Zhou, P. (2017). SMC design for robust stabilization of nonlinear Markovian jump singular systems, *IEEE Trans. Autom. Control*, in press, DOI: 10.1109/TAC.2017.2720970

Wang, Y., Xia, Y., Zhou, P. (2016). Fuzzy-Model-Based Sampled-Data Control of Chaotic Systems: A Fuzzy Time-Dependent Lyapunov-Krasovskii Functional Approach, *IEEE Transactions on Fuzzy Systems*.

Wang, Y., Zhu, Y., Karimi, H. R., and Li, X. (2017). Sampled-Data Exponential Synchronization of Chaotic Lur’e Systems. *IEEE Access*, 5, 17834-17840.

Wang, Y.H., Sui, Y.F., Wu, J., and Jiao, J.F.(2012). Research on nonlinear model predictive controltechnology for ship dynamic positioning system, *Proceeding of the IEEE international conference on automation and logistics*, 348–351.

Wang, Y.H., Zou, C.T., Ding, F.G., et al.(2014). Structural reliability based dynamic positioning of turret-moored FPSOs in extreme seas, *Mathematical Problems in Engineering*, vol. 2014, Article ID 302481, 6 pages.

Wang, Z.P., Wu, H.N. (2016). Finite dimensional guaranteed cost sampled-data fuzzy control for a class of nonlinear distributed parameter systems, *information sciences*, 21-39.

Wu, Z.G, Shi, P., Su, H.Y. (2014). Exponential Stabilization for Sampled-Data Neural-Network-Based Control Systems, *IEEE transactions on neural networks and learning systems*, 25(12), 2180-2190

Wu, Z.G., Shi, P., Su, H.Y. (2013). Stochastic Synchronization of Markovian Jump Neural Networks With Time-Varying Delay UsingSampled Data, *IEEE transactions on cybernetics*, 43(6),1796-1806

Xie, Y.H. (2014). Kalman Filter for Dynamic Positioning Ships Using Position and Acceleration Feedback, *Applied Mechanics and Materials*, 2362-2367

You, S.S, Lim, T.W, Kim, J.Y, et al. (2017). Robust control synthesis for dynamic vessel positioning. *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment*.

Zhao, D.W., Bian, X.Q., and Ding, F.G. (2010). Nonlinear controller based ADRC for dynamic posit ioned vessels, *2010 IEEE International Conference on Information and Automation*, 1367-1371.

Zhou, K., Doyle, J. C., Glover K. (1996). *Robust and optimal control*, New Jersey: Prentice hall.