Fractional Approaches Based on Fractional Prefilters in MIMO Path Tracking Design

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Abstract: This paper is devoted to the study of robust path tracking and motion control. A method based on fractional prefilter synthesis was already advanced and extended to multivariable systems. In previous study, an approach using fractional prefilter of type Davidson Cole is advanced and tested on MIMO systems. The contribution of this work is to extend the precedent work using a fractional prefilter of type Frequency Band Limited Fractional Differentiator (FBLFD), whose important characteristics are to eliminate overshoots and to have maximum control value at starting time, through two techniques of optimization of prefilter parameters. Firstly, only the prefilter parameters are optimized and in a second stage the global transfer function is assimilated to a FBLFD prefilter and a general optimization is considered. The MIMO-QFT (Multiple Input Multiple Output Quantitative Feedback theory) design mixed with CRONE control methodology is used to select the feedback controller. Thereafter, the procedure of prefilter parameters optimization is explained. A comparative study is discussed through a simulation results applied on a SCARA robot model.

Keywords: DC fractional prefilter, FBLFD fractional prefilter, Multivariable systems, Path tracking.

1. INTRODUCTION

The large majority of industrial processes are multivariable systems (MIMO: Multiple-Input Multiple-Output). The Quantitative Feedback Theory (QFT) (Choi, 2009; Horowitz, 2001; Qudrat, 2015; Mohammad, 2005) is one of the most used approaches in the problem of MIMO systems. The QFT method is a two-degree-of freedom (2DOF) feedback control approach used for uncertain MIMO plants. The aim of the QFT technique is how to design the feedback controller and the prefilter such that obtaining satisfactory performances in the presence of plant uncertainties.

The MIMO QFT problem is usually replaced by an adequate number of MISO (Multiple-Input Single-Output) problems. A combination of the feedback controller and prefilters which are designed for every MISO system is the solution of the initial MIMO QFT problem. Various approaches have been developed in order to obtain the appropriate compensator (Aissa, 2015; Melchior, 2009; Yousfi, 2012b, 2013, 2014c) for the equivalent MISO systems.

The CRONE control (Oustaloup, 1975, 1991, 1995a, 1995b) can determine, by a frequency design, a robust controller for highly uncertain plants. Desired specifications like precision, rapidity and overshoots are assured by the determination of the open loop transfer function. After that, the ratio of the open loop transfer function by the nominal plant gives the controller expression. The CRONE control approach contains three generations depending on the type of plant frequency uncertainty domain. The CRONE control design is used in this work to obtain the suitable plant controller.

In path tracking situation, a prefilter is generally applied due to its simplicity to adapt and implement to reduce overshoots and overvoltages. Prefilter decreases at high frequency the energy of path planning signal. Nevertheless, the overshoots reduction automatically produces a reduction of dynamic performances for classic linear prefilter approaches. An approach based on fractional (or non-integer order) derivative (Melchior, 2005; Oustaloup, 1995; Samko, 1993) is used in multivariable path tracking design. This approach has been developed with a Davidson-Cole (DC) prefilter (Yousfi, 2012a, 2012b, 2013, 2014c; Melchior, 2013).

A continuous variation on the two constitutive parameters of a DC prefilter (η and τ) can limit the resonance of the feedback control loop. An optimal movement which leads to a minimum path completion time is generated by this prefilter while considering both closed-loop system bandwidth and actuators physical constraints. Also, another fractional prefilter of type frequency band limited fractional differentiator (FBLFD) is considered in multivariable path tracking design (Yousfi, 2014a).

Now, the purpose is to present two approaches based on FBLFD prefilter to optimize both prefilter or plant outputs and to extend it to multivariable systems in the case of trajectory tracking. The main properties of a FBLFD prefilter is eliminating plant overshoots and obtaining maximum control value at starting time by the way of adding a numerator. Minimizing the settling time of the output and maximizing the bandwidth energy between input / output, the parameters of the FBLFD prefilter (η, τ_1, τ_2) can be optimized. Firstly, a comparative study between these two methodologies is considered. In a second stage, these approaches which consider the FBLFD prefiter was also compared with these ones based on DC prefilter.

This work is structured as follows. Section 2 briefly describes background about MIMO-QFT design and CRONE control methodology. FBLFD prefilter approaches based on prefilter and I/O transfer function optimization are presented in section 3. The two methodologies of fractional prefiter optimization based on DC prefilter are given in section 4. In section 5, the pertinence of this approach is discussed by simulations on a SCARA robot model, while the principal conclusions are established in the final section of this work.

2. PRELIMINARIES ABOUT MIMO-QFT AND CRONE CONTROL

2.1 Quantitative Feedback Theory (QFT)

A Linear Time Invariant two Degrees Of Freedom system configuration is considered (see Fig 1), where $P=\left[p_{ij}\right]_{m\times m}$ is a square and minimum-phase uncertain plant to be controlled, *G* and *F* are the controller and the prefilter to be designed respectively.

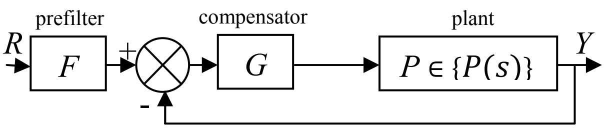


Fig. 1. Two-degrees-of-freedom control MIMO system structure.

The transfer matrix *T* is :

$$T=\frac{Y}{R}=\left[I+PG\right]^{-1}PGF \tag{1}$$

P is a nonsingular matrix, P^{-1} is divided into:

$$P^{-1}=\Lambda+B \tag{2}$$

Where:

- Λ : the diagonal part of P^{-1}
- B : the balance of P^{-1}

The transfer matrix *T* obeys:

$$T=\left[\Lambda+G\right]^{-1}\left[GF-BT\right] \tag{3}$$

For a MIMO system, a 2×2 system is equivalent to 4 subsystems (MISO structure) (Fig.2) (Melchior, 2009).

Where:

$$q_{ij}=\frac{\det(P)}{\operatorname{adj}(P_{ij})} \tag{4}$$

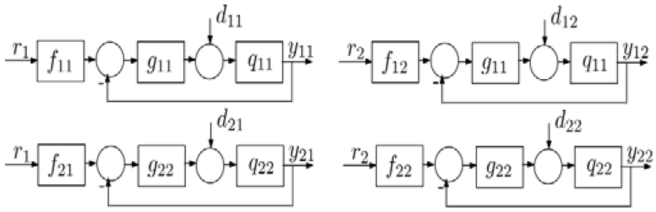


Fig. 2. Two-degrees-of-freedom control system MIMO structure.

The transfer function matrix *T* elements have the following expression:

$$t_{ij}=\omega_{ii}\left(\upsilon_{ij}+d_{ij}\right)=t_{rij}+t_{dij} \tag{5}$$

Where:

$$\omega_{ii}=\frac{q_{ii}}{1+g_{ii}q_{ii}}$$

$$\upsilon_{ij}=g_{ii}f_{ij}$$

And

$$d_{ij}=-\sum_{k\neq i}^{m}\frac{t_{kj}}{q_{ik}},\ k=1,2,\ldots,m$$

When d_{ij} is considered as disturbances, the feedback structure turns into a SISO-QFT design.

t_{dij} must be lower than a small positive function in order to eliminate disturbances.

2.2 Crone Control

In 1970, The CRONE (French acronym: Commande Robuste d'Ordre Non Entier) control approach were synthesized by Oustaloup (0ustaloup, 1975). The base of this methodology is the use of fractional differentiation in the design of the open loop transfer function which is parameterized with few parameters. The CRONE control technique is characterized by three generations. By the reason of various plant frequency uncertainty types, the third generation of CRONE Control approach is recommended. For more details you can see (Lanusse, 2000; CRONE research group, 2010; Nelson Gruel, 2009).

3. FREQUENCY BAND LIMITED FRACTIONAL DIFFERENTIATOR PREFILTER

3.1 Principle and definition

A frequency band-limited fractional differentiator system is a fundamental system characterized by the transmittance F_{FBLFD}:

$$F_{FBLFD}(s)=\left(\frac{1+\tau_2 s}{1+\tau_1 s}\right)^{\eta} \tag{6}$$

Equation (6) is the expression of an FBLFD filter where η is a real.

The FBLFD prefilter described by (6), at high frequencies, decreases the energy of the signal through selecting bandwidth (τ_1, τ_2) and with the continuous nature of the real order η. The briny characteristics of the FBLFD prefilter are as follow:

- Eliminating the plant output overshoots,
- having the maximum value of control at starting time and,
- maximum bandwidth energy.

3.2 Method 1: Fractional prefilter of type F_{FBLFD}

For a SISO path tracking design (see Fig 3), the prefilter *F(s)* separates the regulation and the dynamics behaviors in position control. This approach uses fractional prefilter and only the control loop reference input is optimized.

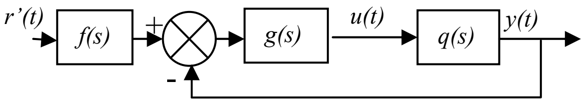


Fig. 3. SISO control structure.

The transfer functions $T_{y/r'}(s)$:

$$T_{y/r'}(s) = \frac{y(s)}{r'(s)} = f(s)\frac{g(s)q(s)}{1+g(s)q(s)} \quad (7)$$

where, *f(s), g(s)* and *q(s)* are respectively the transfer functions of the filter, the controller and the plant.

In order to allow a good tracking performances it is required that:

- The sensitivity transfer function *S(s)* must have a small magnitude for small frequencies which attenuate the disturbances effects.

$$S(s) = \frac{1}{1+g(s)q(s)} \quad (8)$$

Also, the complementary sensitivity transfer function *T(s)* must, for high frequencies, be small in magnitude, and for small frequencies it must be equal to unit to follow asymptotically the input.

$$T(s) = \frac{g(s)q(s)}{1+g(s)q(s)} \quad (9)$$

The reference sensitivity function S_{ref} between the control and the input signal is given by:

$$S_{ref}(s) = \frac{u(s)}{r'(s)} = f(s)\frac{g(s)}{1+g(s)q(s)} \quad (10)$$

$$= \frac{T_{y/r'}(s)}{q(s)} = f(s)\frac{T(s)}{q(s)}$$

In order to avoid overshoots, the frequency constraint is (Melchior, 2007):

$$\forall\ \omega>0, \left|S_{ref}(j\omega)\right| < \frac{u_{max}}{e_{max}} \quad (11)$$

Where u_{max} is the maximum static constant value of the control signal and e_{max} is a constant signal applied on the prefilter input.

From expressions (10) and (11), it is deduced that the frequency constraint which puts the signals to be controlled under its maximum limit, is:

$$\forall\ \omega>0, \left|\frac{1+j\tau_2\omega}{1+j\tau_1\omega}\right|^{\eta} \times \left|\frac{g(\omega)}{1+g(\omega)q(\omega)}\right| < \frac{u_{max}}{e_{max}} \quad (12)$$

Using the initial value theorem and the equation (12), we can deduce the following expression of the parameter τ_2

$$\tau_2 = \left[\left(\lim_{s\to\infty}\frac{1+g(s)q(s)}{g(s)}\right)\frac{u_{max}}{e_{max}}\right]^{\frac{1}{\eta}} \times \tau_1 \quad (13)$$

Thanks to the expression (13), we are asked to find only the two parameters (η; τ_1). The desired range of the closed-loop transfer function is described by two frequency bounds specified bellow:

$$\forall\ \omega>0,\tau>0, \left|T_{RL}(j\omega)\right| \leq \left|T_{y/r'_{ii}}(j\omega)\right| \leq \left|T_{RU}(j\omega)\right| \quad (14)$$

This equation (14) becomes:

$$\forall\ \omega>0,\tau>0, \left|T_{RL}(j\omega)\right| \leq \left|T_{y/r'_{ii}}(j\omega)\right|_{min} \quad (15)$$

$$\left|T_{y/r'_{ii}}(j\omega)\right|_{max} \leq \left|T_{RU}(j\omega)\right| \quad (16)$$

with the closed loop transfer function:

$$T_{y/r'_{ii}}(j\omega) = \frac{f_{ii}(j\omega)g_{ii}(j\omega)q_{ii}(j\omega)}{1+g_{ii}(j\omega)q_{ii}(j\omega)} \quad (17)$$

3.3 Method 2: I/O transfer function for SISO systems of type F_{FBLFD}

This approach is an extension of the previous method based on FBLFD type transfer function which does optimizes the output of the plant. In path tracking design, for a SISO system, the filtered unity feedback control loop is given by Fig 4 :

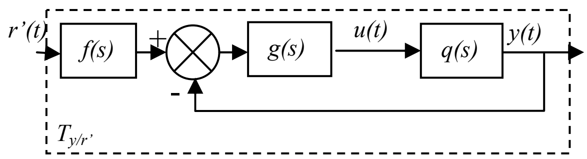


Fig. 4. SISO control structure.

$T_{y/r'}(s)$ is the transfer function of the filtered unity feedback control:

$$T_{y/r'}(s) = \frac{y(s)}{r'(s)} = f(s)\frac{g(s)q(s)}{1+g(s)q(s)} \quad (18)$$

The plant transfer function q(s) is divided into this form:

$$q(s) = \frac{K_0}{s^r}q_1(s) = \frac{K_0}{s^r} \times \frac{1+a_1s+\ldots+a_ps^p}{1+b_1s+\ldots+b_qs^q} \quad (19)$$

Where

- r : is the number of integrations in the plant and
- K_0 : is the static gain in the plant
- and $q \geq p$ and $m=q-p$

The I/O transfer function $T_{y/r'}(s)$ is assimilated to an FBLFD prefilter in accordance with the following expression (Melchior, 2005):

$$T_{y/r'}(s) = \left(\frac{1+\tau_2s}{1+\tau_1s}\right)^{\eta}\frac{1}{(1+\tau_2s)^m} \quad (20)$$

The low pass term $\frac{1}{(1+\tau_2 s)^m}$ is added to the transfer function $T_{y/r'}(s)$ so as to resolve the analytic problem related to the initial value theorem.
Combining equations (10), (18) and (20), the frequency constraint is:

$$\forall\ \omega>0, \frac{1}{|q(j\omega)|}\left|\frac{1+j\tau_2\omega}{1+j\tau_1\omega}\right|^{\eta}\frac{1}{|1+j\tau_2\omega|^m}<\frac{u_{max}}{e_{max}} \quad (21)$$

Where u_{max} is the maximum static constant value control signal and e_{max} is a constant signal applied on the prefilter input.

Also, the optimization of the prefilter parameters (τ_1, τ_2 and η) can be simplified to the optimization of only τ_1 and η. Indeed, from the FBLFD prefilter characteristic which is having maximum control signal at starting time we have:

$$\lim_{t\to 0} u(t)=\lim_{s\to+\infty} sU(s)=\lim_{s\to+\infty} S_{ref}(s)e_{max}=u_{max} \quad (22)$$

So, using (10):

$$\lim_{s\to+\infty} S_{ref}(s)=\frac{u_{max}}{e_{max}} => \lim_{s\to+\infty}\frac{T_{y/r'}(s)}{q(s)}=\frac{u_{max}}{e_{max}} \quad (23)$$

Thereby, the τ_2 expression is as:

$$\tau_2=\left[\lim_{s\to\infty}\left(\frac{q(s)}{T_{y/r'}(s)}\right)\frac{u_{max}}{e_{max}}\right]^{\frac{1}{\eta}}\times\tau_1 \quad (24)$$

On the other hand, according to the FBLFD prefilter definition:

$$\tau_2<\tau_1 => \left(\frac{a_p}{b_q}\tau_1^{\eta}\right)^{\frac{1}{\eta-m}}<\tau_1 => \left(\frac{a_p}{b_q}\right)^m<\tau_1 \quad (25)$$

For better explained details we can consult (Melchior, 2007; Melchior, 2012; Yousfi, 2014b).

Finally, the frequency constraint of the FBLFD I/O transfer function is as follow:

$$\forall\,\omega>0, \frac{1}{|q(j\omega)|}\left|\frac{1+j\tau_2\omega}{1+j\tau_1\omega}\right|^{\eta}\frac{1}{|1+j\tau_2\omega|^m}<\frac{u_{max}}{e_{max}}$$

$$\left(\frac{a_p}{b_q}\right)^m<\tau_1$$

$$\tau_2=\left(\frac{a_p}{b_q}\tau_1^{\eta}\right)^{\frac{1}{\eta-m}} \quad (26)$$

$$q_1(s)=\frac{1+a_1 s+\ldots+a_p s^p}{1+b_1 s+\ldots+b_q s^q}\ and\ q\geq p\ and\ m=q-p$$

$$q(s)=\frac{K_0}{s^r}q_1(s)$$

The desired range of the closed-loop transfer function is characterized by two frequency constraints given by (14), (15) and (16) with the closed loop transfer function:

$$T_{y/r'_{ii}}(j\omega)=\left(\frac{1+j\tau_{2i}\omega}{1+j\tau_{1i}\omega}\right)^{\eta_i}\frac{1}{(1+j\tau_{2i}\omega)^m} \quad (27)$$

3.4 FBLFD prefilter optimization

The frequency constraints of both approach are used to avoid the saturation of the control input signal. In order to extract a step response without overshoot, the integral gap criteria is often used. For the F_{FBLFD} step response, the analytic expression of the integral gap is (Melchior, 2007):

$$I_e \leq \eta(\tau_1-\tau_2) \quad (28)$$

This expression for $k\times k$ MIMO system becomes:

$$I_e \leq \eta_1(\tau_{11}-\tau_{21})+\eta_2(\tau_{12}-\tau_{22})+\ldots+\eta_k(\tau_{1k}-\tau_{2k}) \quad (29)$$

For the second method, the prefilter expression can be deduced from the FBLFD I/O transfer function ((18) and (20)):

$$f(s)=\frac{1+g(s)q(s)}{g(s)q(s)}\left(\frac{1+\tau_2 s}{1+\tau_1 s}\right)^{\eta}\frac{1}{(1+\tau_2 s)^m} \quad (30)$$

4. DAVIDSON COLE PREFILTER

4.1. Principle and definition

The fractional prefilter of type Davidson Cole (DC) $F_{DC}(s)$ is described by the following expression:

$$F_{DC}(s)=\frac{1}{(1+\tau s)^{\eta}} \quad (31)$$

This prefilter permits to reduce the energy of the signal at high frequencies. It is simply used to reduce overshoots like analog or digital filter (Melchior, 2009; Yousfi, 2013).

4.2 Method 1: Fractional prefilter of type DC

Considering Fig.3, in order to avoid overshoots, the frequency constraint is:

$$\forall\omega\succ 0, |S_{ref}(\omega)|\prec\frac{u_{max}}{e_{max}} \quad (32)$$

From (10), it is deduced that:

$$\forall\ \omega>0, \left|\frac{1}{1+j\tau\omega}\right|^{\eta}\times\left|\frac{g(\omega)}{1+g(\omega)q(\omega)}\right|<\frac{u_{max}}{e_{max}} \quad (33)$$

The Closed loop transfer function must be into the desired bounds T_{RL} and T_{RU} (see (14), (15), (16) and (17)).

4.3 Method 2: I/O transfer function for SISO systems of type DC

In this section, the I/O transfer function $T_{y/r'}$ structure takes the following form of a Davidson-Cole prefilter multiplied by a number of integration like the plant q(s):

$$T_{y/r'}(s)=\frac{y(s)}{r'(s)}=\frac{1}{s^r(1+\tau s)^\eta} \tag{34}$$

Indeed, using the final value theorem:

$$\lim_{s\to 0} S_{ref}(s)=\frac{u_{max}}{e_{max}} \Rightarrow \lim_{s\to 0} T_{y/r'}(s)=\lim_{s\to 0} q(s)\frac{u_{max}}{e_{max}}=\frac{K_0}{s^r}\frac{u_{max}}{e_{max}} \tag{35}$$

From this equation (35) the integration number of the plant and the transfer function $T_{y/r'}$ is the same so the structure of $T_{y/r'}$ given in equation (34) is deduced.

To avoid overshoots the reference sensitivity transfer function S_{ref} must verify the inequality (11). From (9), (10) and (26) it is deduced the following frequency constraint:

$$\forall\ \omega>0, \left|\frac{1}{1+j\tau\omega}\right|^\eta \times \left|\frac{1}{q_1(\omega)}\right| < \frac{u_{max}}{e_{max}} \tag{36}$$

The acceptable variation range of the closed loop systems is defined by (14), (15), (16) where:

$$T_{y/r'_{ii}(j\omega)}=\left(\frac{1}{1+j\tau_1\omega}\right)^{\eta_i}\frac{1}{(j\omega)^r} \tag{37}$$

4.4 DC prefilter optimization

For a step response of a DC transfer function, the integral gap criterion is given by the following expression:

$$I_e \le \eta\tau \tag{38}$$

For a multivariable system of order *m*, this expression (38) becomes:

$$I_e \le \eta_1\tau_1+\eta_2\tau_2+\ldots+\eta_m\tau_m \tag{39}$$

The prefilter expression for the second approach is obtained from the I/O DC type transfer function:

$$f(s)=\frac{1+g(s)q(s)}{g(s)q(s)}\left(\frac{1}{1+\tau s}\right)^\eta\frac{1}{s^r} \tag{40}$$

5. NUMERICAL SIMULATION

5.1 SCARA robot model

The adept one SCARA robot (Garcia-sanz, 2002; Yousfi, 2013) is considered to verify the effectiveness of our approach of MIMO-QFT using a diagonal controller with FBLFD and DC prefilters.

The plant outputs are the two angles δ_1 and δ_2. However the input signals are $u1$ and $u2$. These two signals u_1 and u_2 will be applied to the motors robot. The Lagrange equation method has been used to obtain the SCARA robot model (Garcia-sanz, 2002).

$$\begin{pmatrix}\delta_1\\ \delta_2\end{pmatrix}=P(s)\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}p_{11} & p_{12}\\ p_{21} & p_{22}\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} \tag{41}$$

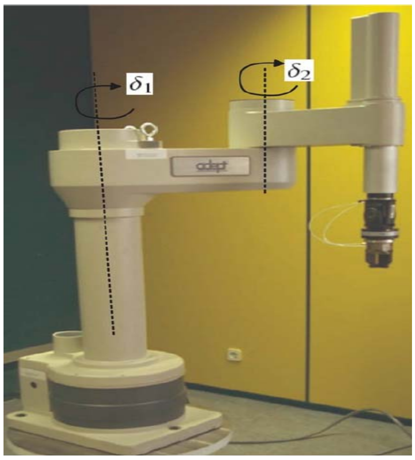


Fig. 5. SCARA robot.

$$P(s)=\begin{pmatrix}\frac{\alpha_2 s+\nu_2}{s\Delta(s)}\frac{1}{k} & \frac{-(\alpha_2+\alpha_3 h)}{\Delta(s)}\frac{1}{k}\\ \frac{-(\alpha_2+\alpha_3 h)}{\Delta(s)}\frac{1}{k} & \frac{(\alpha_1+2\alpha_3 h)s+\nu_1}{s\Delta(s)}\frac{1}{k}\end{pmatrix} \tag{42}$$

Where

$$\Delta(s)=\sigma_2 s^2+\sigma_1 s+\sigma_0 \tag{43}$$

$\alpha_{i,i=1,2,3}$ and $\sigma_{j,j=0,1,2}$ are as follow:

- $\sigma_0=\nu_1\nu_2$
- $\sigma_1=\alpha_2\nu_1+\nu_2(\alpha_1+2\alpha_3 h)$
- $\sigma_2=\alpha_2(\alpha_1+2\alpha_3 h)-(\alpha_2+\alpha_3 h)^2$
- $\alpha_1=I_1+I_2+m_1x_1^2+m_2(l_1^2+x_2^2)$
- $\alpha_2=I_2+m_2x_2^2$
- $\alpha_3=m_2l_1x_2$

With:

k Power amplifier gain; ν_i Coefficients of viscous friction; I_i Moment of inertia; m_i Mass; x_i i^{th} link; l_1 Length of link 1; h Cosine value of q_2.

The plant coefficient uncertainties are given in table 1:

Table 1. MIMO plant conditions.

	Minimum	Nominal	Maximum
$\alpha_1 k$	729	766	813
$\alpha_2 k$	186	193	200
$\alpha_3 k$	154	182	200
$\nu_1 k$	200	290	381
$\nu_2 k$	61.6	71.75	91.9
h	-1	0	1

5.2 Performance specifications

For both methods, the diagonal closed loop transfer function $T_{y/r'}$ must verify the following inequalities.

$$\forall\ \omega>0, \left|T_{RL}\right| \le \left|T_{y/r'_{ii}}\right| \le \left|T_{RU}\right| \quad (44)$$

Where:

$$T_{RL}(s)=\frac{0.45}{0.1s^4+1.47s^3+5.015s^2+3.195s+0.45}$$

$$T_{RU}(s)=\frac{0.4083s+12.25}{0.1s^3+1.525s^2+6.475s+12.25}$$

5.3 RGA matrix

Various parameters are analyzed to verify the pairing rule using the RGA (Relative Gain Array) technique. So, based on this approach we can deduce that δ_1 and δ_2 are respectively controlled by u_1 and u_2.

5.4 Controller design

Using the CRONE control toolbox, the CRONE controller $G(s)$ of SCARA robot is obtained while respecting these specifications:

- For both outputs, we must have a zero steady state error.
- We are looking for smallest settling time.
- Also, robustness against uncertainties and disturbances is required.
- Finally, we must obtain an overshoots less than 5%.

Respecting all these rigorous specifications and after optimization the following controller is obtained:

$$G(s)=\begin{pmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{pmatrix} \quad (45)$$

With:

$$g_{11}(s)=\frac{5.214s^2+219.9s+82.6}{0.0003157s^2+0.03577s+1}$$

$$g_{12}(s)=0$$

$$g_{21}(s)=0$$

$$g_{22}(s)=\frac{0.2821s^3+41.15s^2+233.4s+87.8}{1.257e\text{-}07s^3+7.777e\text{-}05s^2+0.01252s+1}$$

5.5 Fractional prefilter

The prefilter matrix is as follow:

$$F(s)=\begin{pmatrix} f_{11}(s) & 0 \\ 0 & f_{22}(s) \end{pmatrix} \quad (46)$$

Firstly, it is recommended to select the maximum and minimum of plants. After that, the ratio $\frac{u_{max}}{e_{max}}$ is fixed. In all cases of fractional prefilter expressions, the Oustaloup approximation is used to get the rational prefilter equations (Oustaloup, 1995).

5.5.1 Prefilter of type *FBLFD*

a. *method 1:* Minimizing (29) and making into consideration the frequency constraints (12 and 13) and the performance tracking specifications (14,15 and 16), the optimized parameters are:

$$\eta_1=2.783;\ \tau_{11}=0.786;\ \tau_{21}=0.615$$
$$\eta_2=1.5;\ \tau_{12}=4.762;\ \tau_{22}=0.1 \quad (47)$$

A rational approximation of the fractional prefilter $F(s)$ can be found through a graphic design using the CRONE software:

$$f_{11}(s)=\left(\frac{1+\tau_{21}s}{1+\tau_{11}s}\right)^{\eta_1} =\frac{6.9302(s+567.8)(s+143)(s+9.62)}{(s+770.8)(s+252)(s+16.2)(s+1.55)(s+1.11)} \quad (48)$$

$$f_{22}(s)=\left(\frac{1+\tau_{22}s}{1+\tau_{12}s}\right)^{\eta_2} =\frac{0.0084507(s+762)(s+130)(s+26.28)(s+12.31)(s+3.14)}{(s+617.9)(s+104.1)(s+26.27)(s+2.691)(s+0.187)} \quad (49)$$

b. method 2: Minimizing the integral gap criterion (29), and taking into account the frequency constraints (26) and the performance tracking specifications (14,15 and 16), we can obtain the optimized parameters:

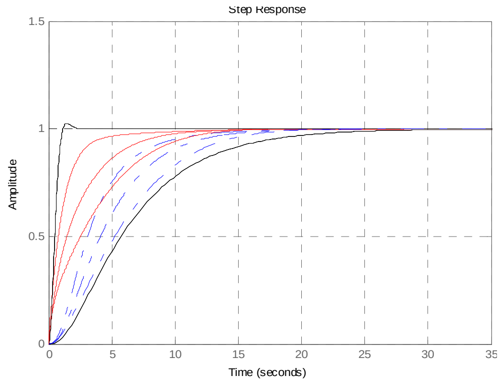
$$\eta_1=1.266;\ \tau_{11}=0.984;\ \tau_{21}=0.558$$
$$\eta_2=1.914;\ \tau_{12}=0.714;\ \tau_{22}=0.12 \quad (50)$$

The prefilter expression $F(s)$ is deduced from the I/O transfer function (30). A rational approximation is determined of each element of the prefilter *F(s):*

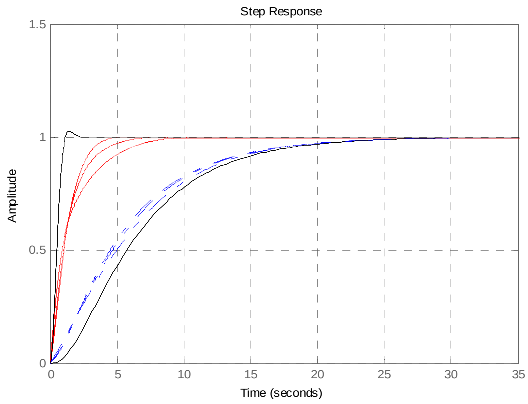
$$f_{11}(s)=\frac{1+g_{11}(s)q_{11}(s)}{g_{11}(s)q_{11}(s)}\left(\frac{1+\tau_{21}s}{1+\tau_{11}s}\right)^{\eta_1}\frac{1}{(1+\tau_{21}s)} =\frac{33.4999(s+505.7)(s+272.2)(s+81.47)(s+0.3801)}{(s+260.8)(s^2+1830s+8.382e05)(s+0.6532)} \quad (51)$$

$$f_{22}(s)=\frac{1+g_{22}(s)q_{22}(s)}{g_{22}(s)q_{22}(s)}\left(\frac{1+\tau_{22}s}{1+\tau_{12}s}\right)^{\eta_2}\frac{1}{(1+\tau_{22}s)} =\frac{0.029106(s+404.4)(s^2+62.71s+984.2)(s+234.4)(s^2+335.3ss+2.837e04)}{(s+878)(s+630.5)(s+90.07)(s+54.12)(s+13.1)(s+2.18)} \quad (52)$$

c. Simulation results: The resultant time domain closed-loop tracking response under all plant cases are given by Fig 6. The closed-loop tracking responses under all operating cases are exposed in Fig. 6, where the tracking requirements are plotted in black line. As observed, the proposed methodology leads to satisfactory performances and all plant cases are under the required lower and upper bounds. A comparison of the results obtained via the first methodology with thus of the closed- loop tracking responses obtained using the second method clearly presents the benefits of the second approach which optimizes the global I/O closed loop transfer function. The first and second main transfer function responses shows that the second methodology gives a faster response as compared the approach using the first one.



(a)



(b)

Fig. 6. (a),(b): Closed loop tracking response (in degree) with fractional FBLFD prefilter using method 1(dashed line) and method 2(Continued red line), tracking references (black)

5.5.2 DC prefilter

a. Method 1: Minimizing the integral gap criterion (39) under the frequency constraint (33) and the performance tracking specifications (14,15 and 16),the optimized parameters are:

$$\eta_1=1.1;\ \tau_1=1.052 \qquad \eta_2=1.592;\ \tau_2=0.815 \tag{53}$$

Using the CRONE software, the rational approximation of the fractional prefilter obeys:

$$f_{11}(s)=\left(\frac{1}{1+\tau_1 s}\right)^{\eta_1} =\frac{633.546(s+340.1)(s+57.8)(S+2.44)}{(s+931)(s+415.1)(s+41.89)(s^2+2.74s+1.877)} \tag{54}$$

$$f_{22}(s)=\left(\frac{1}{1+\tau_2 s}\right)^{\eta_2} =\frac{32.8014(s+269.1)(s+47.86)(S+9.516)}{(s+805.9)(s+89.02)(s+21.6)(s+2.378)(s+1.091)} \tag{55}$$

b. Method 2: Minimizing the integral gap criterion (39) under the frequency constraint (36) and the performance tracking specifications (14,15 and 16),the optimized parameters are:

$$\eta_1=1.8;\ \tau_1=1.2 \qquad \eta_2=1.599;\ \tau_2=1.299 \tag{56}$$

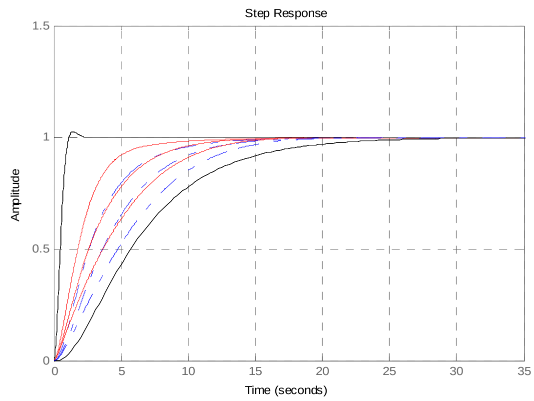
From (37), we can deduce each elements of the prefilter matrix $F(s)$:

$$f_{11}(s)=\frac{1+g_{11}(s)q_{11}(s)}{g_{11}(s)q_{11}(s)}\left(\frac{1}{1+\tau_1 s}\right)^{\eta_1}\frac{1}{s} =\frac{0.085424(s+328.9)(s+54.1)(s+6.679)(s+0.392)}{(s+585)(s+10.7)(s+0.9836)(s+0.6468)} \tag{57}$$

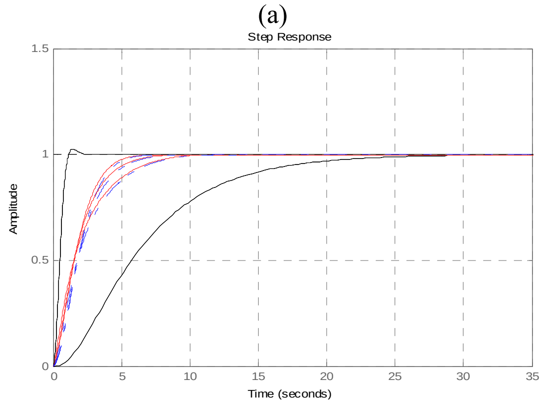
$$f_{22}(s)=\frac{1+g_{22}(s)q_{22}(s)}{g_{22}(s)q_{22}(s)}\left(\frac{1}{1+\tau_2 s}\right)^{\eta_2}\frac{1}{s} =\frac{0.0011696(s+346)(s+204)(s+149)(s+1.45)}{(s+796.9)(s+15.8)(s^2+2.38s+1.416)} \tag{58}$$

c. Simulation results: The comparison between the two approaches of path tracking design by the fractional prefilters of type DC is illustrated in Fig. 7.

As shown in Fig 7, both methods using DC prefilter give a satisfactory performance tracking specifications (all operating plant conditions are under specified bounds (black line)). But, it is clearly observed that the second method used with DC or FBLFD prefilter gives a better closed loop rise time and settling time. So, it is deduced that the I/O closed loop transfer function optimization proves a sufficient results.



(a)

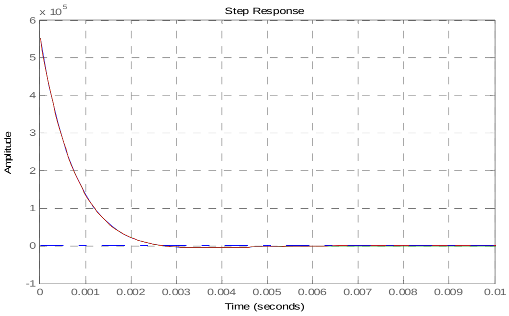


(b)

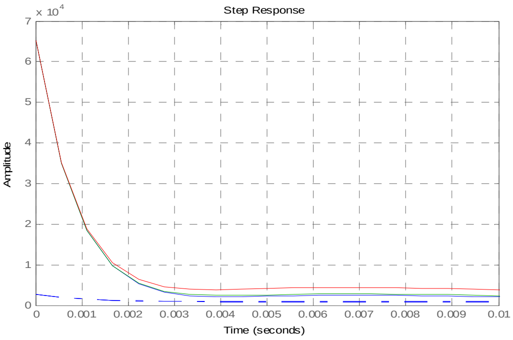
Fig. 7. (a),(b): Closed loop tracking response (in degree) with fractional DC prefilter using method 1(dashed line) and method 2(Continued red line), tracking references (black).

It is the time to deduce the utility of the second approach which is based on the assimilation of the global I/O transfer function to a fractional prefilter of type DC or FBLFD.

The maximum value of the control signal using FBLFD prefilter is kept (Fig 8). This property allows the generation of optimal reference input movement which leads to a minimum path completion time which is observed in Fig.9 and table 2.



(a)



(b)

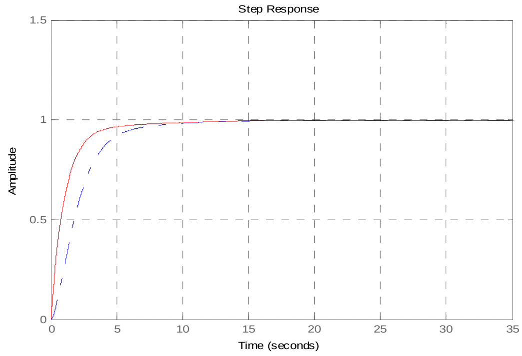
Fig. 8. (a),(b): First and second main control signals(in counts) (method 2) with DC (dashed line) and FBLFD (continued line) prefilters.

In Fig.9, the closed loop responses given by the I/O transfer function of type FBLFD is faster than the response obtained using the DC prefilter. These results are confirmed by the values given in table 2.

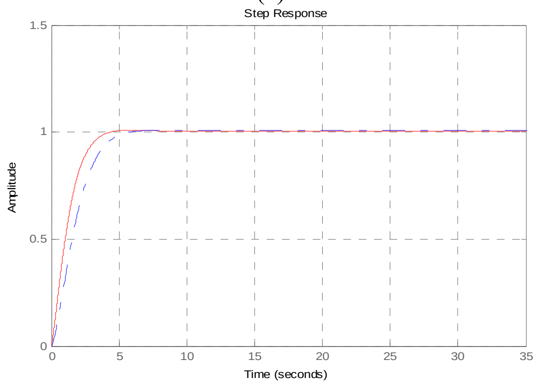
The characteristic of FBLFD prefilter of having maximum control loop at starting time is approved in the control loop curves given by figure 8. This characteristic allows the system to be more quickly in response than the fractional prefilter of type DC.

Table 2. comparison between FBLFD and DC prefilter using method 2 (case 1).

Charac.	$T_{y/r'1} : DC$	$T_{y/r'1} : FBLFD$
Rise time (s)	4.0429	2.6242
Settling time(s)	9.3074	7.3874
Charac.	$T_{y/r'2} : DC$	$T_{y/r'2} : FBLFD$
Rise time (s)	3.0739	2.3239
Settling time(s)	4.8146	3.6163



(a)



(b)

Fig. 9. (a), (b): First and second main closed loop responses (in degree) (method 2) with DC (dashed line) and FBLFD (continued line) prefilters.

These results are demonstrated in table 2 which represent a comparison between settling time and rise time while using the two types of fractional prefilters referring to figure 9.

6. CONCLUSION

In this paper, an extension of path tracking design applied to multivariable systems based on a Davidson-Cole prefilter is explored. A MIMO QFT approach is used with CRONE control approach to extract the controller expression of the MIMO processes. Thereafter, the prefilter parameters have to be determined. The Frequency Band-Limited Fractional Differentiator (FBLFD) prefilter whose general properties are eliminating overshoot on the plant and having maximum control value at start time is used in this paper to resolve the problem of multivariable motion control. Two methodologies are developed. The first one is based on a local optimization of the fractional prefilter parameters and the object of the second approach is to assimilate the I/O closed loop transfer function to a fractional prefilter and to optimize the I/O transfer function parameters. A comparative study between both methods using DC or FBLFD prefilters is illustrated through a simulation on a SCARA robot model which validates the proposed methodology. However, further works are needed to apply these developed approaches in real systems.

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