CEAI, Vol.19, No.4 pp. 43-51, 2017 Printed in Romania

Multiple Model Bank Selection Based on a New Validity Criterion

Ali Zribi, Mohamed Chtourou, Mohamed Djemel

Control & Energy Management Lab (CEM LAB), National School of Engineering of Sfax, Tunisia, (e-mail: ali_zribi@yahoo.fr, mohamed.chtourou@enis.rnu.tn, mohamed.djemel@enis.rnu.tn).

Abstract: In this paper, a new validity criterion determining the optimal decomposition and the minimum linear model bank was employed to design a controller for a nonlinear system. Based on the gap metric, the proposed validity criterion quantifies the compactness degree for a model-base. For a decomposition giving a low degree of compactness, the optimal linear model bank providing sufficient information for multimodel controller design of the nonlinear system was obtained without models redundancy. To validate the relevance of the proposed method, multimodel H_∞ control algorithm was applied to three nonlinear processes for set point tracking.

Keywords: multiple models, validity criterion, gap metric, nonlinear systems.

1. INTRODUCTION

The multimodel approach is considered as one of the popular methods used to solve modeling problem and complex control, nonlinear and/or ill-defined processes. This approach approximates complex systems based on a set of local linear systems to which classical control design techniques can be easily applied (Zribi et al., 2017; Kangji et al., 2017; Samia and Moufida, 2016; Du et al., 2009, Ciprian et al., 2008; Ciprian, 2003; Sergiu et al., 2005). One of the major problems in the multiple-model technique lies in determining the number of linear models and their locations used to ensure better control performance (Hariprasad et al., 2012; Zribi et al., 2012a). It has been proven that when an excessive number of models are used the composite model captures better the process of nonlinear behavior; but it may lead to a large and a complex computation. Nonetheless, too few models may be unable to get the desired performance.

Recently, (Galán et al., 2003) use the gap metric as a guideline for selecting local models. The models base is obtained by grouping models that have the same closed-loop performances. Afterwards, (Du et al., 2012, 2013, 2014) propose a method, which is an extension of the method proposed by (Galán et al., 2003), to select operating points and the minimum linear model bank for nonlinear systems via gap metric and margin stability. In (Wen et al., 2004), by specifying the desired performance through a pre- and/or post-compensators, it is shown that the H_∞ loop-shaping approach can integrate the procedure of operating points selection as well as the local controller design through gap metric. In (Hosseini et al., 2012), an algorithm incorporating a nonlinearity measure and a modified gap based metric was meant to perform the optimal operating range decomposition.

Clustering algorithms has been also used to create an organized model bank from a dataset where the optimal number of models for the operating regime decomposition is determined by employing cluster validity measures based on geometric distance (Zribi et al., 2012b; Elfelly et al., 2012). Despite it shows good control performance and stability features of closed-loop systems, the optimal partition given by the validity criteria may result in redundant linear models, because the designers use excessive local linear models to guarantee the global stability and robust performance.

In order to avoid linear model redundancy and simplify the structure of a multimodel controller, a new validity criterion for decomposition of the nonlinear space to a number of linear models was proposed. Based on the gap metric, the proposed criterion quantifies the degree of compactness for the decomposition by computing similarity between local models. Thus, the optimal decomposition which can provide sufficient information for controller design is expected to have a low degree of compactness.

This paper was organized according to following outline: In Section 2, the Fuzzy c-means (FCM) for the determination of the models' base for nonlinear systems was first proposed. Then, the gap metric as tool to analyse the relationships among candidate local models was presented. To design a controller for a nonlinear system, a new criterion for systematic determination of an optimal model bank was described in section 3. In Section 4, three examples were considered to check the relevance of the proposed approach. Finally, conclusion was provided in Section 5.

2. FUZZY CLASSIFICATION BASED LOCAL MODEL DEVELOPMENT AND GAP METRIC

2.1 Fuzzy c-means

The first stage of the proposed algorithm consists on developing the local models. Based on the data set, the FCM clustering algorithm was employed to divide behavior in local ones. Then, a given identification procedure was applied to get local models.

FCM clustering algorithm previously developed by (Dunn, 1973) and subsequently improved by (Bezdek, 1981), represents a data clustering technique allowing each data point to belong to more than one cluster with different membership degrees ranging between 0 and 1.

The objective of the FCM method is minimizing the function given by the following equation:

$$J_m(U,V) = \sum_{i=1}^{N}\sum_{j=1}^{C}\left(u_{ji}\right)^m \left(d_{ji}\right)^m \tag{1}$$

where :

m: weighting exponent (real number greater than 1) which is a constant that influences the membership values;

u_{ji}: degree of membership of x_i to cluster *j*, such as $u_{ii} \in [0,1]$, $\sum_{j=1}^{C} u_{ji} = 1$.

d_{ji} is the distance between the point x_i and different centers v_j;

N: number of observations;

C: number of clusters ($2 \le C < N$);

The FCM algorithm that minimizes (1) is described in the following:

Step1: Fix the number of clusters *C* and randomly pick initial set of centers $V=(v_1,v_2,\ldots,v_c)$. Choose the stopping criteria ε.

Step2: Compute membership degrees and the new cluster centers in each iteration *k*.

$$u_{ji,k} = \frac{1/(d(x_i,v_j))^{\frac{2}{m-1}}}{\sum_{j=1}^{C} 1/(d(x_i,v_j))^{\frac{2}{m-1}}} \tag{2}$$

$$v_{j,k} = \frac{\sum_{i=1}^{N}\left(u_{ji,k}\right)^m x_i}{\sum_{i=1}^{N}\left(u_{ji,k}\right)^m} \tag{3}$$

Step3: Test of the convergence: if $error = \left\|V_k - V_{k-1}\right\| \prec \varepsilon$ then stop else go to step 2.

The classification results are then used for the parametric identification of the local models. The multiple linear models will be obtained by linearizing the system around the cluster's center.

2.2 Review of Gap Metric

The gap metric was first introduced into the system theory in (Zames and El-Sakkary, 1980; El-Sakkary, 1985), as a suitable tool for the study of the uncertainty in feedback systems. It provides a measure to calculate the distance between two dynamic systems from closed loop stability view point (Galán et al., 2003; Christopher, 2017).

P(s) is equal to $p\times m$ rational transfer matrix with the normalized right coprime factorization given by the following equation:

$$P = N M^{-1} \quad \text{with} \quad \tilde{M} M + \tilde{N} N = I \tag{4}$$

where $\tilde{(\bullet)}$ denotes complex conjugate, i.e $\tilde{M}(s) = M^T(-s)$.

The graph of *P* is the subspace of H_2 (standard Hardy space in the right half of the complex plane) given by:

$$G(P) = \begin{bmatrix} M \\ N \end{bmatrix} H_2 \tag{5}$$

For two finite-dimensional linear systems P_1 and P_2 with the same number of inputs and outputs, the gap between P_1 and P_2 is defined as:

$$\delta(P_1,P_2) = \left\|\Pi_{G(P_1)} - \Pi_{G(P_2)}\right\| \tag{6}$$

where $\Pi_{G(P)}$ denotes the orthogonal projection onto *G(P)*.

Let $P_1 = N_1 M_1^{-1}$ and $P_2 = N_2 M_2^{-1}$ be the normalized right coprime factorizations of P_1 and P_2, respectively. Typically the gap metric for two dynamic models P_1 and P_2 can be calculated as:

$$\delta\left(P_1,P_2\right) = \max\left\{\vec{\delta}(P_1,P_2), \vec{\delta}(P_2,P_1)\right\} \tag{7}$$

where

$$\vec{\delta}\left(P_1,P_2\right) = \inf_{Q\in H_\infty}\left\|\begin{bmatrix} M_1 \\ N_1 \end{bmatrix} - \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} Q\right\|_\infty \tag{8}$$

The gap metric ($0\le\delta\le1$) represents the notion of distance in the space of linear systems. The tendency of the gap metric of two systems toward 0 indicates that the distance between the two linear systems is small (i.e., the two systems are close) and means the existence of at least one feedback controller stabilizing both systems. On the contrary, if the gap is closer to 1, then the two systems dynamic behaviors are apart and they behave differently when they are placed in a control loop.

3. VALIDITY CRITERION IN OBTAINING A REDUCED MODEL BANK

3.1 Research in the optimal number of models

Indices shown in graph clustering are generally based on the comparison of intracluster connectivity (the clusters compactness) as well as the inter-cluster variability (i.e., the separability between clusters) (Isacenkova et al., 2014; Arbelaitz et al., 2013). To assess the quality of graph clustering results (Boutin and Hascoet, 2004) provide a good review of validity indices and define a new normalized compactness index using the measure of similarity instead of distance.

$$C_p = \frac{\sum_{i=1}^{C-1}\sum_{j=i+1}^{C} sim(v_i, v_j)}{C(C-1)} \qquad (9)$$

where $0 \le sim(v_i, v_j) \le 1$ defines the similarity index between the two nodes v_i and v_j. If $sim(v_i, v_j) = 0$, then v_i and v_j are disconnected.

Based on their intra-connectivity characteristics, the graph compactness (C_p) is a cluster validity crietrion that indicates how 'compact' (or homogeneous) the graphs are. It is bounded by 0 and 1, where a small value indicates that the graph is completely disconnected. Motivated by this observation, a new crietrion for optimal systematic determination of model bank to design a controller for a nonlinear system is proposed. It requires, firstly, the determination of the model bank using clustering algorithm, then, validating the obtained model bank by computing a proposed validity criterion.

In clustering algorithms, the most widely validity criteria used to construct model bank employ a geometric distance. The drawback of the geometric distance is that measures the separation between linear systems in the open-loop sense instead of the closed-loop sense which can easily lead to linear model bank redundancy since designers tend to use more local linear models than needed in order to guarantee the global stability and robust performance. A new similarity measure, based on the gap metric, is proposed to overcome the above-mentioned shortcoming of traditional validity criteria and to avoid linear model redundancy.

For example, consider two local linear models P_1 and P_2 for a non linear system:

$$P_1 = \frac{1}{s+1} \quad , \quad P_2 = \frac{1}{s-1} \qquad (10)$$

The two open-loop models look quite different, since $\|P_1 - P_2\|_\infty = 2000$. However, the gap metric between P_1 and P_2 is $\delta(P_1, P_2) = 0.002$ which shows that P_1 and P_2 are close to one another in the closed loop-sense, only one of them will be needed to provide sufficient information for controller design.

Thus, a new normalized criterion denote D_C that use the gap metric as a similarity measure instead of geometric distance is proposed.

$$D_C = \frac{\sum_{i=1}^{C-1}\sum_{j=i+1}^{C} (1-\delta_{ij})}{C(C-1)} \qquad (11)$$

where δ_{ij} is the gap metric between two linear models P_i and P_j and C is the number of models in the model bank.

The proposed criterion ranges from 0 to 1. A high value of the proposed criterion means that the obtained model bank may includes redundant (similar) models. To avoid linear model redundancy and simplify the structure of a multimodel controller, we propose the minimization of the proposed criterion. These results imply that when the optimal partition is attained, the presented criterion is minimal. The minimization of the criterion allows one to find an optimal model bank that contains no redundant and well-separated models in the closed loop-sense.

3.2 Multimodel control

After obtaining the local models/controllers of a nonlinear system, these local linear controllers will be combined into a global one to act on the nonlinear system. In the proposed control strategy, the controller output is obtained by fusion of generated local controllers pondered by weighting functions. The global control output is presented by the following equations.

$$u(k) = \sum_{j=1}^{C} r_j(k) u_j(k) \qquad (12)$$

where u_j is the control output of the j^{th} local H_∞ controller and the weights r_j are adjusted based on the gap metrics defined by:

$$r_j(k) = \frac{1}{C-1}\left[1 - \frac{\delta_j}{\sum_{j=1}^{C}\delta_j}\right] \qquad (13)$$

where δ_j is the gap metric between linearized model of the system at the current state and the j^{th} local model defined in the model bank.

The weight function r_j returns a number between *0* and *1* that indicates the level of contribution of the local controller at that value of the output. A small gap metric δ_j between the linearized model of the system at the current state and a local model defined in the model bank means that the corresponding weight is high. It also indicates that the corresponding local controller dominates the control action.

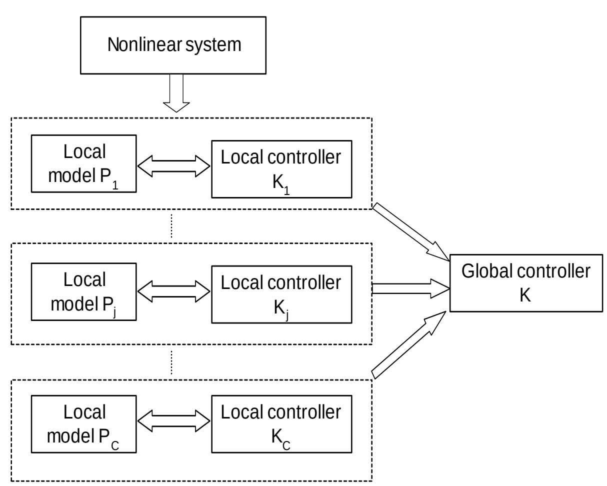

Fig. 1. Schematic diagram of the multimodel control.

The schematic design of the used multimodel control is given in Figure 1. It consists on building a local model in every subspace. Then, a local controller was attributed to each local model. Eventually, the obtained local models and controllers are combined in order to establish a powerful global model and controller.

4. SIMULATIONS RESULTS

To validate the efficiency of the proposed validity criterion, comparison with other traditional indices (Table 1) used in the field of fuzzy clustering algorithm such as partition coefficient, partition entropy and compactness separability (Arnaud et al., 2002) was done.

Table 1. Description of the cluster validity criteria.

Validity criteria	Functional description	Optimal cluster number
Partition coefficient	$PC = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ji}^2$	max(*PC; U; C*)
Partition entropy	$PE = -\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ji} \log\left(u_{ji}\right)$	min(*PE; U; C*)
Compactness separation	$XB = \frac{\sum_{j=1}^{C} \sum_{i=1}^{N} \left(u_{ji}\right)^m \left\Vert x_i - v_j \right\Vert^2}{N \min_{j,k} \left\Vert v_k - v_j \right\Vert^2}$	min(*XB; U; C*)

4.1 Example 1: Isothermal CSTR

The above algorithm has been applied to a simple nonlinear process. The process chosen is an isothermal continuous stirred tank reactor (CSTR) in which a first-order irreversible reaction takes place (Henson and Seborg, 1990; Doyle et al., 1998). The relevant mass balance is:

$$\frac{dC_A}{dt} = -K\, C_A + (C_{Ai} - C_A)\, u \tag{14}$$

where C_A is the concentration of the reactant and u is the dilution rate. The rate constant K is $0.028\ min^{-1}$ and the initial concentration C_{Ai} is $1.0\ mol/L$.

First, the system is excited by an adequate signal $u(k)$ in the range $[0;1]$ to collect the measurement $C_A(k+1)$ at different instants. The input vectors $[C_A(k+1),u(k)]$ are subjected to fuzzy classification as discussed earlier. The total length of the sequence is 1000. By linearizing the system around the cluster's center, the dynamics of the system can be modelled by a first order transfer function. Then, the gap metric is performed on the obtained models to compute the validity criterion. The minimization of the proposed criterion D_C allows to find the optimal partition corresponding to the number of models.

For this example, Table 2 illustrates the results obtained for different numbers of models with different validity criteria. All the criteria, including the proposed criterion, indicate that $C=2$ is the optimal model number.

Table 2. Values of different criteria for the first example.

Number of models	2	3	4	5	6
D_c	0.038	0.2464	0.2988	0.3639	0.3525
PC	**0.9**	0.87	0.67	0.89	0.65
PE	**0.18**	0.19	0.2	0.27	0.28
XB	**0.5**	2	4.92	7.8	3.5

Using the proposed criteria, D_C, the local liner models obtained by the optimal decomposition are:

$$P_1 = \frac{7.35}{16s+1} \tag{15}$$

$$P_2 = \frac{0.144}{2.25s+1} \tag{16}$$

Two local linear H_∞ controllers are designed to control the presented system where the corresponding transfer functions are given by the following expressions:

$$K_1 = \frac{615.6s^2 + 315.7s + 17.32}{s^3 + 312.9s^2 + 62.64s + 31.27} \tag{17}$$

$$K_2 = \frac{1031s^2 + 530.9s + 32.85}{s^3 + 135.8s^2 + 27.23s + 13.56} \tag{18}$$

The global multimodel H_∞ controller based on the two local controllers is combined according to (12).

Figure 3 shows the responses using our proposed scheme and a neural network controller (NNC) where the inverse model of the process was used as a non-adaptive controller, with respective control structure (Fig. 2) (Andrasik, 2004). As is seen in Fig. 3, it can easily be observed that the dynamic response of the NNC is sluggish and present static errors. An excellent tracking performance is achieved via the proposed method where the two local linear models are used for controller design. The accurate tracking behaviour approves the adopted method to get a reduced model bank to design multilinear controller.

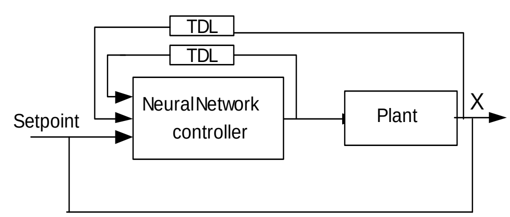

Fig. 2. Block diagram of direct inverse control.

In our proposed method, the knowledge of the extent of influence of each controller provides a know-how of the dominant dynamics in that operating space. The weighting functions used to combine the local H_∞ controllers works on the assumption that when a particular gap is small, the corresponding weight is high (Fig. 4). In addition, it is indicated that the corresponding local controller dominated the control action. For instance, it can be concluded that controller K_1 is most active and that controller K_2 is the least active when the process operates at a set point of 0.94.

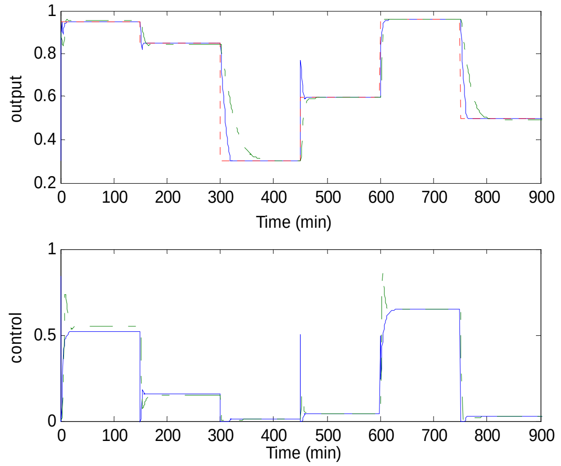

Fig. 3. Closed-loop responses and control inputs moves ((dotted) setpoint; (solid) with the proposed method; (dash-dotted) with NNC).

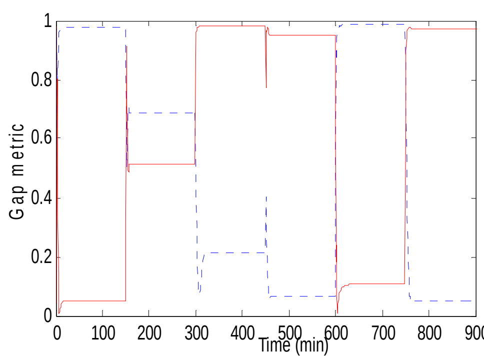

Fig. 4. Calculated gap metrics ((dotted) δ_1; (solid) δ_2)

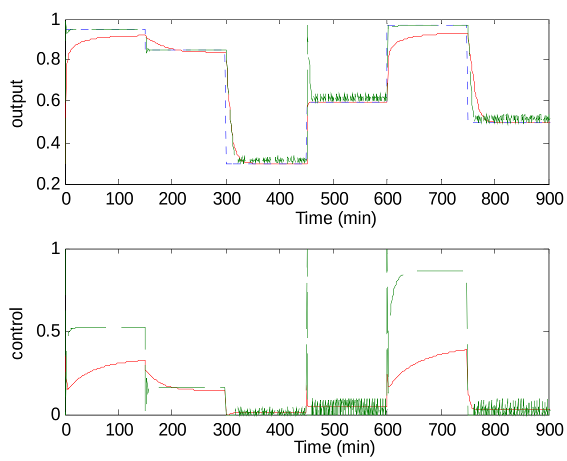

Fig. 5. Closed-loop responses and control inputs moves of CSTR using one model ((dotted) setpoint; (solid) with K_1; (dashed) with K_2).

Fig. 5 shows the performances of the two local controllers in the entire operating range. The output under controller K_1 (solid line) cannot track the reference at high concentration values and at small values the output under controller K_2 (dashed line) oscillates fiercely. Neither local controller is able to perform the tracking task in the entire operating space.

4.2 Example 2: Nonisothermal CSTR

A benchmark CSTR where an irreversible first-order reaction takes place is considered. The mathematical model was taken from the paper (Uppal *et al.*,1974), where x_1, x_2, and u are the dimensionless reagent conversion, the temperature (output), and the coolant temperature (input), respectively. The system under study is described by the following set of nonlinear differential equations.

$$\begin{cases} \dot{x}_1 = -x_1 + D_a\left(1-x_1\right)\exp\left(\dfrac{x_2}{1+x_2/\gamma}\right) \\ \dot{x}_2 = -x_2 + BD_a\left(1-x_1\right)\exp\left(\dfrac{x_2}{1+x_2/\gamma}\right) + \beta(u-x_2) \\ y = x_2 \end{cases} \tag{19}$$

The nominal values for the constants in (19) are D_a=*0.72*, *B=8*, γ=*20* and β=*0.3*.

The system is excited in its full operating range by a control input *u(k)* of random amplitude in the range *[-2;2]*. Then, the measurements $x_1(k+1)$ and $x_2(k+1)$ are collected at different instants. The total length of the sequence is 2000. The input vectors $[x_1(k+1),x_2(k+1),u(k)]$ are, then, presented to the FCM algorithm. By linearizing the system around the cluster's center, the dynamic data in each cluster can be modeled by second-order transfer function.

Table 3 shows results of the validity indexes. Of the indexes considered, only the *PC* criterion indicates that *C=3* may be as the optimal partition. The other indexes consider that *C= 2* as the optimal number of models.

Table 3. Values of different criteria for the second example.

Number of models	2	3	4	5	6
D_c	**0.001**	0.2904	0.3082	0.299	0.31
PC	0.64	**0. 74**	0.71	0. 62	0.65
PE	**0.2**	0.23	0.29	0.25	0.28
XB	**7.9**	18.2	8.5	15.8	13.5

Based on the proposed criterion, the following models are obtained from linearization around the cluster's center.

$$P_1 = \frac{0.3s+0.3229}{s^2+1.18s+0.8711} \tag{20}$$

$$P_2 = \frac{0.3s+0.538}{s^2+0.1082s-0.4376} \tag{21}$$

The H_∞ controllers designed based on corresponding local models are respectively given by:

$$K_1 = \frac{13.67s^2 + 24.77s + 11.97}{s^3 + 6.53s^2 + 5.688s + 0.05622} \tag{22}$$

$$K_2 = \frac{239.5s^2 + 205.9s + 24.38}{s^3 + 71.45s^2 + 98.38s + 0.9766} \tag{23}$$

Simulation results of the NNC and of the proposed controller are shown in Fig. 6. It is obvious, from simulation results, that a NNC is unable to satisfactorily control the process because of imperfect inverse mapping. In fact, the dynamic responses are far from satisfactory, as indicated by the large static errors. Based on the two linear local models, obtained by the proposed method, it is seen that all the output quickly and accurately follows the reference signal over the operating space.

The metric gaps, in Fig. 7, between the dynamic local model at the current state and the two obtained local models defined in the model bank are used to calculate weighting functions in order to combine the local H_∞ controllers into global controller. It is worth noting that the closed-loop transient behavior is more than likely influenced by the controller synthesized around the unstable region.

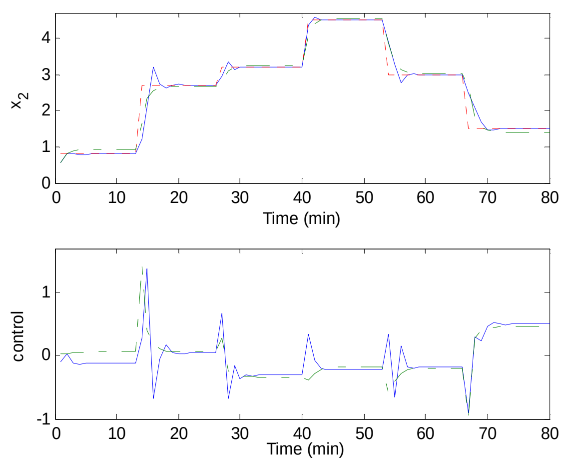

Fig. 6. Closed-loop responses and control inputs moves ((dotted) setpoint; (solid) with the proposed method; (dash-dotted) with NNC).

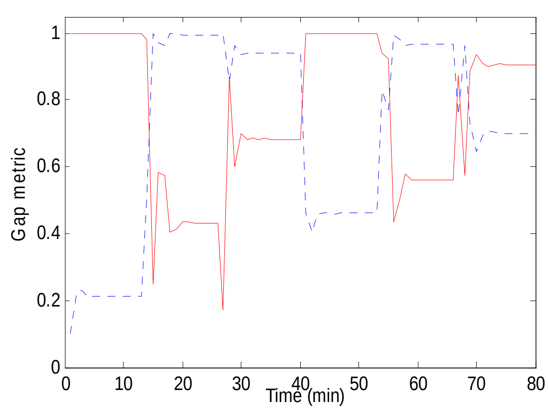

Fig. 7. Calculated gap metrics ((dotted) δ_1; (solid) δ_2).

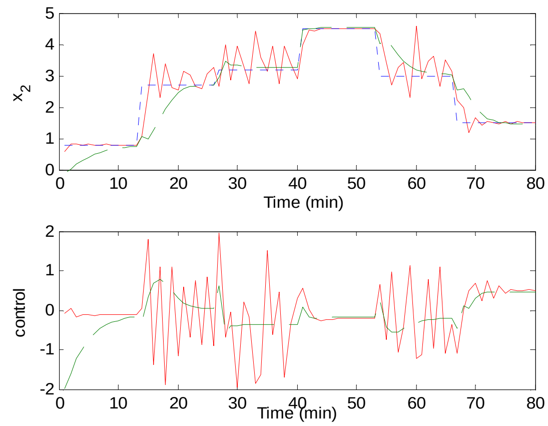

Fig. 8. Closed-loop responses and control inputs moves of CSTR using one controller ((dotted) setpoint; (solid) with K_1; (dashed) with K_2).

If only one model is used to control the system (Fig. 8), we also get an expected result. In fact, the results indicate when only the controller K_1 is used some degradation in performance is observed especially around unstable regions whereas when only the controller K_2 is used the setpoint is not well attained. In this case, the degradation due to a large plant-model mismatch confirms the proposed decomposition method aiming to get a smaller and effective linear model bank for multimodel controller design.

4.3 Example 3: a multivariable process

The proposed method can be applied to both SISO systems and MIMO systems. Consider the following square process two-inputs–two-outputs (TITO) (Koivo, 2002):

$$\begin{cases} \dot{x}_1 = x_2 - u_1 \\ \dot{x}_2 = (4.75 - 4.5x_1)u_2 - 0.7(x_2 - u_1) - 0.25x_1 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases} \tag{24}$$

where u_1, u_2 are the inputs, y_1, y_2 are the outputs, and x_1, x_2 are the state variables. The ranges of state variables and inputs are $y_i \in [0,1], u_i \in [0,1]$.

In order to excite a rich variety of dynamical modes in the plant, the system identification data were generated using uniformly random inputs signal that varies between 0 and 1. Then, the collected outputs are used to construct the vector to be clustered to know $[x_1(t), x_2(t), u_1(t-1), u_2(t-1)]$.

By linearizing the system around the cluster's center, a local model is fitted to each cluster. On the obtained models, the gap metric was performed to compute the proposed criterion of validity.

For the data set from the TITO system, D_C and PE indicate that $C=3$ is an optimal model number. However, the PC and XB index give us, respectively, that $C=4$ and $C=2$ are a good model number estimate.

Table 4. Values of different criteria for the third example.

Number of models	2	3	4	5	6
D_c	0.46	**0.332**	0.349	0.37	0.36
PC	0.62	0. 66	**0.88**	0. 68	0.65
PE	0.11	**0.09**	0.22	0.25	0.29
XB	**12.4**	27.1	20.9	14.5	16.5

Based on the proposed criterion, a decentralized control structure will be used. By linearizing the system around the cluster's center, the obtained local models are:

$$P_1 = \begin{pmatrix} \dfrac{-0.3s+4.045}{s^2+0.7s+4.045} & 0 \\ 0 & \dfrac{0.462s+0.462}{s^2+0.7s+4.045} \end{pmatrix} \quad (25)$$

$$P_2 = \begin{pmatrix} \dfrac{-0.3s+0.98}{s^2+0.7s+0.98} & 0 \\ 0 & \dfrac{1.04s+1.04}{s^2+0.7s+0.98} \end{pmatrix} \quad (26)$$

$$P_3 = \begin{pmatrix} \dfrac{-0.3s+1.46}{s^2+0.7s+1.46} & 0 \\ 0 & \dfrac{3.7s+3.7}{s^2+0.7s+1.46} \end{pmatrix} \quad (27)$$

Based on the obtained models of the studied system, the transfer functions for the H_∞ controllers are:

$$K_1 = \begin{pmatrix} \dfrac{1105.5\ s^2+73.86\ s+426.8}{s^3+112.9\ s^2+419.8\ s+4.187} & 0 \\ 0 & \dfrac{5.195\ s^2+3.636\ s+21.01}{s^3+1.508\ s^2+2.217\ s+0.0220} \end{pmatrix} \quad (28)$$

$$K_2 = \begin{pmatrix} \dfrac{136.7\ s^2+95.68\ s+134.9}{s^3+88\ s^2+219.7\ s+2.188} & 0 \\ 0 & \dfrac{135.4\ s^2+94.76\ s+133.6}{s^3+186.7\ s^2+116.9\ s+1.15} \end{pmatrix} \quad (29)$$

$$K_3 = \begin{pmatrix} \dfrac{251\ s^2+175.7\ s+366.5}{s^3+187.4\ s^2+508.9\ s+5.07} & 0 \\ 0 & \dfrac{134.4\ s^2+94.09\ s+196.2}{s^3+528.7\ s^2+485.9\ s+4.806} \end{pmatrix} \quad (30)$$

The global multimodel H_∞ controller based on the three local controllers is combined by their weighting functions according to (12).

The response of the global controller is evaluated for successive set-point changes in the whole operating space. The closed-loop profile for the complete model bank (P_1 ;P_2 ;P_3) shows satisfactory performance, as depicted in Fig.9. The selection of the linear model bank is approved by the fast and efficient tracking behavior.

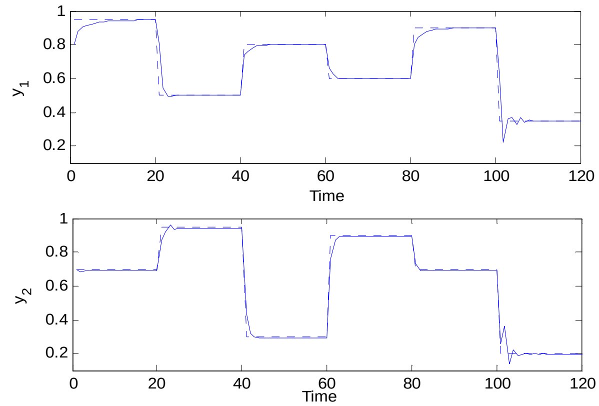

Fig. 9. Closed-loop response of TITO system using three controllers.

Figure 10 shows the metric gaps, between the linearized model of the system at the current state and the three local models defined in the model bank, used to calculate the global controller.

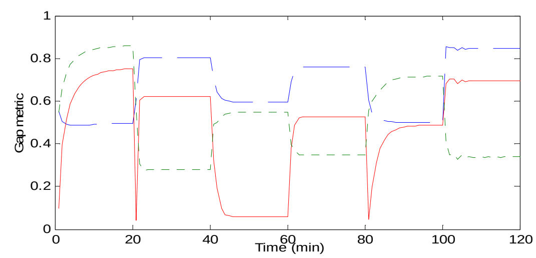

Fig. 10. Calculated gap metrics ((dotted) δ_1; (solid) δ_2; (dashed) δ_3)

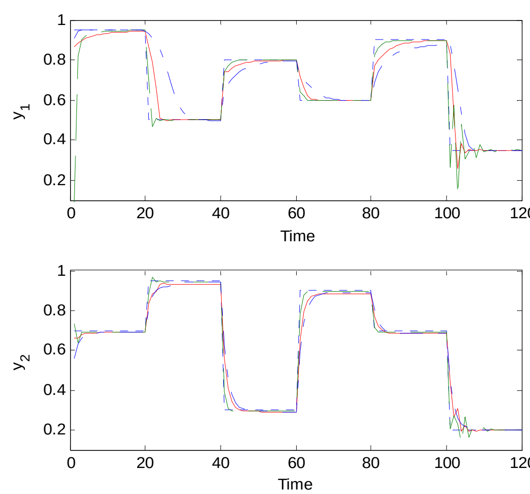

Fig. 11. Closed-loop response of TITO system using two controllers ((dotted) setpoint; (solid) with K_1 *and* K_2; (dashed) with K_2 *and* K_3; *(dash-dotted)* with K_1 *and* K_3).

When only two models are used the results indicate that performances in tracking are not satisfactory. If only the two linear models (P_1 and P_3) are used for controller design the dynamic responses are sluggish. The second combination, using models P_2 and P_3, exhibits some degradation in performance as indicated by the static errors. And for the last combination (P_1 and P_2) the performance is quite bad in the last period: The outputs are oscillating.

Closed-loop simulations confirm that the proposed method is a useful and effective tool for choosing linear model bank for multivariable systems.

5. CONCLUSION

This study presents a new validity crietrion able to decompose the operating space and to determine the minimum linear model bank for a nonlinear system. Based on the gap metric, the proposed criterion quantifies the degree of similarity between local models for decomposition. The optimal decomposition is obtained by minimizing the proposed criterion with respect to the number of models. Multimodel H_∞ control algorithm is validated for set point tracking of two benchmark nonlinear chemical processes and a multivariable process. The examples we have chosen show the ability of our criterion to determine the optimal linear model bank for nonlinear systems.

It should be noted that the proposed weighting functions used to combine local controllers into a global one is based on the process model. In case of an unavailability of the process model, a dynamic model can be identified by a linearization procedure. To accommodate this situation, we may extend the proposed method in our future.

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