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A New Adaptive Second Order Sliding Mode Control Design for Complex Interconnected Systems

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Abstract: In this paper, a new adaptive second order sliding mode control approach is extended to a class of complex interconnected systems with mismatched interconnections and unknown disturbances. The novel contribution of this paper is to remove two common limitations hindering the application of the sliding-mode control to complex interconnected systems: 1) the exogenous disturbances must be bounded by a known function of the outputs or by a known function of the state; 2) the control input is affected by chattering problems. First, based on a new adaptive law, a continuous decentralized adaptive sliding mode controller is designed to ensure the reachability of the system states without chattering problems by using output variables only. Second, existence conditions of linear sliding surfaces are derived to guarantee the asymptotic stability in terms of constrained linear matrix inequalities. Final, numerical examples are given to prove the effectiveness of the proposed method.

Keywords: Complex interconnected systems, output feedback controller, linear matrix inequalities, second order sliding mode control

1. INTRODUCTION

With the development of science and technology, there is an increasing need to control complex interconnected systems in changing environments where nonlinearity, mismatched uncertain, unmodeled dynamics, and exogenous disturbances are the main sources of complexity. An efficient control strategy for these systems is decentralized control. The advantage of decentralized control is to use local signals at the level of each subsystem in the controller implementation for large-scale interconnected systems, and this therefore overcomes the limitations of the traditional centralized control. Various decentralized controllers have been developed for large-scale interconnected systems in the presence of uncertainties (see, for example, Wu, 2002; Mahmoud, 2009; Ghosh et al., 2009; Zhang et al., 2012; Wu, 2012; Chai and Osman, 2015). However, most of them require the availability of the states of each subsystem, which cannot be guaranteed in practice because some state variables may be difficult/costly to measure and sometimes have no physical meaning, and thus cannot be measured at all. This motivated the authors of (Ye et al., 2005; Chen and Li, 2008; Xi and Ding, 2007; Mehraeen et al., 2011; Koo et al., 2014; Yan et al., 2004; Cheng and Chang, 2008; Kalsi et al., 2010; Yan et al., 2010) to develop decentralized output feedback controllers for large-scale systems. In (Ye et al., 2005), a decentralized adaptive control scheme is presented for large-scale systems with unknown control directions. The authors of (Chen and Li, 2008) developed a decentralized output-feedback neural network control scheme for a large-scale nonlinear system, where does not require any matching conditions on the parametric uncertainties. These approaches given in (Ye et al., 2005; Chen and Li, 2008) are achieved under assumption that large-scale nonlinear systems have special structure. In (Xi and Ding, 2007), a solution to the problem of global decentralized output regulation is proposed for a class of interconnected systems by using error information. In (Mehraeen et al., 2011), a new neural network -based nonlinear decentralized adaptive controller is developed for a class of large-scale nonlinear by using the dynamic surface control. In (Koo et al., 2014), a decentralized fuzzy control approach is proposed to stabilize a class of nonlinear large-scale systems by using an observer-based output-feedback scheme. The authors of (Yan et al., 2004) developed a new sliding mode control scheme for nonlinear large-scale system where the disturbances are the function of outputs. In order to handle the mismatched perturbations, the authors of (Cheng and Chang, 2008) have presented a new adaptive sliding surface in which all system states are assumed to be measurable. In (Kalsi et al., 2010), a decentralized dynamic output feedback based linear controller is proposed for a class of matched uncertain interconnected systems. In (Yan et al., 2010), a global decentralized static output feedback sliding mode control strategy is presented for interconnected time-delayed systems where the interconnection parts are functions of the system output. However, the approaches given in (Yan et al., 2004; Cheng and Chang, 2008; Kalsi et al., 2010; Yan et al., 2010) suffered the drawback of severe chattering in the control input. In addition, this chattering phenomenon is very harmful for actuators used in practical electromechanical systems.

Many authors have proposed the second order sliding mode control (SOSMC) scheme to avoid chattering problems such as (Chang, 2012; Li and Zheng, 2012; Mondal and Mahanta, 2013; Das and Mahanta, 2014). In (Chang, 2012), a new SOSMC law was developed for a class of mismatched uncertain systems with exogenous disturbance. This technique can guarantee that the system is stable and the chattering problem is removed. The study of (Li and Zheng, 2012) proposed a robust adaptive SOSMC scheme for a class of uncertain non-linear systems where the upper bounds of uncertainties are not required to be known in advance. Recently, an adaptive tuning law is proposed such that the mismatched perturbations are rejected during the sliding mode (Mondal and Mahanta, 2013). This approach can ensure asymptotical stability of the whole system and remove chattering in the control input. In (Das and Mahanta, 2014), based on the linear quadratic regulator method, an optimal second order sliding mode controller was proposed for a class of matched uncertain systems. However, these approaches given in (Chang, 2012; Li and Zheng, 2012; Mondal and Mahanta, 2013; Das and Mahanta, 2014) are achieved under assumption that the second derivative of all state variables must be existed, even though mathematical model of systems are the first order. This condition may be difficult to satisfy for many practical systems. In addition, these control method based on small scale systems therefore cannot be applied for complex interconnected systems. To the best of our knowledge, no SOSMC strategy has so far been developed for complex interconnected systems. The main contributions of this paper are as follows:

- A new continuous decentralized adaptive output feedback sliding mode control strategy is proposed to ensure the reachability of the system states without chattering problems by using output variables only.
- A new adaptive law is developed to solve the unknown exogenous disturbances in the complex interconnected systems.

2. SYSTEM DESCRIPTION AND PRELIMINARY RESULTS

Let the complex interconnected systems to be controlled be represented by the following form:

$$\dot{x}_i = (A_i + \Delta A_i)x_i + B_i(u_i + \xi_i(x_i,t)) + \sum_{\substack{j=1\\ j\neq i}}^{L}(H_{ij} + \Delta H_{ij})x_j \qquad (1)$$

$$y_i = C_i x_i$$

where $x_i \in R^{n_i}$ is the state vector, $u_i \in R^{m_i}$ is the control input, and $y_i \in R^{p_i}$ is the output for the i^{th} subsystem, respectively. A_i, B_i, C_i and H_{ij} are system matrices of the complex interconnected systems with appropriate dimensions. The vector $\xi_i(x_i,t)$ is a matched non-linearity in the i^{th} subsystem. The mismatched uncertainty of the complex interconnected systems ΔA_i and ΔH_{ij} are supposed to satisfy

$$\Delta A_i = D_i F_i(x_i,t)E_i, \qquad \Delta H_{ij} = M_{ij}F_{ij}(x_j,t)N_{ij}$$

where M_{ij}, N_{ij}, D_i, E_i are known constant real matrices with appropriate dimensions, and $F_i(x_i,t)$, $F_{ij}(x_j,t)$ are norm-bounded unknown matrix as $\|F_i(x_i,t)\| \le 1$ and $\|F_{ij}(x_j,t)\| \le 1$, respectively.

The following assumption is required for the complex interconnected systems (1):

Assumption 1: The matrices B_i and C_i satisfy the following relations:

$$\text{rank}(C_i) = p_i,\ \text{rank}(B_i) = m_i \text{ and } p_i \ge m_i. \qquad (2)$$

Under assumption 1, it follows from equations (11), (12) and (13) of paper (Yan et al., 2012) that there exists a coordinate transformation $z_i = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} = T_i x_i$ so that the complex interconnected system (1) can be rewritten as

$$\begin{bmatrix} \dot{z}_{i1} \\ \dot{z}_{i2} \end{bmatrix} = (\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} + \begin{bmatrix} D_{i1} \\ D_{i2} \end{bmatrix} F_i \begin{bmatrix} E_{i1} & E_{i2} \end{bmatrix}) \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix}(u_i + \xi_i) + \sum_{\substack{j=1\\ j\neq i}}^{L}(\begin{bmatrix} H_{ij1} & H_{ij2} \\ H_{ij3} & H_{ij4} \end{bmatrix} + \begin{bmatrix} M_{ij1} \\ M_{ij2} \end{bmatrix} F_{ij} \begin{bmatrix} N_{ij1} & N_{ij2} \end{bmatrix}) \begin{bmatrix} z_{j1} \\ z_{j2} \end{bmatrix}, \qquad (3)$$

$$y_i = \begin{bmatrix} 0 & C_{i2} \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} \qquad (4)$$

where $T_i A_i T_i^{-1} = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}$, $T_i H_{ij} T_j^{-1} = \begin{bmatrix} H_{ij1} & H_{ij2} \\ H_{ij3} & H_{ij4} \end{bmatrix}$, $T_i B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix}$, $T_i D_i F_i E_i T_i^{-1} = \begin{bmatrix} D_{i1} \\ D_{i2} \end{bmatrix} F_i \begin{bmatrix} E_{i1} & E_{i2} \end{bmatrix}$, $T_i M_{ij} F_{ij} N_{ij} T_j^{-1} = \begin{bmatrix} M_{ij1} \\ M_{ij2} \end{bmatrix} F_{ij} \begin{bmatrix} N_{ij1} & N_{ij2} \end{bmatrix}$, $C_i T_i^{-1} = \begin{bmatrix} 0 & C_{i2} \end{bmatrix}$.

Two matrices $B_{i2} \in R^{m_i \times m_i}$ and $C_{i2} \in R^{p_i \times p_i}$ are invertible.

3. MAIN RESULTS

In this section, a new adaptive second order sliding mode control strategy is developed to stabilize the complex interconnected systems (3)-(4). First the sliding surface $\sigma_i(x_i(t))$, which is linear with respect to the output variable y_i, is given by

$$\sigma_i(x_i(t)) = K_i C_{i2}^{-1} y_i = \begin{bmatrix} 0_{m_i \times (p_i - m_i)} & K_{i2} \end{bmatrix} C_{i2}^{-1} \begin{bmatrix} 0 & C_{i2} \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} = \begin{bmatrix} 0_{m_i \times (p_i - m_i)} & K_{i2} \end{bmatrix} \begin{bmatrix} \hat{N}_i & 0_{(p_i - m_i) \times m_i} \\ 0_{m_i \times (n_i - m_i)} & I_{m_i \times m_i} \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} = K_{i2} z_{i2} = 0 \qquad (5)$$

and the sliding manifold is defined as

$$s_i(t) = \dot{\sigma}_i(x_i(t)) + \mathrm{X}_i \sigma_i(x_i(t)) \qquad (6)$$

where $K_i = \begin{bmatrix} 0_{m_i \times (p_i - m_i)} & K_{i2} \end{bmatrix} \in R^{m_i \times p_i}$, $K_{i2} = \Xi_i P_i \Xi_i^T$, the matrix $\Xi_i \in R^{m_i \times (n_i - m_i)}$ is designated so that K_{i2} is invertible, $P_i \in R^{(n_i - m_i) \times (n_i - m_i)} > 0$ will be designed later, $\hat{N}_i = \begin{bmatrix} 0_{(p_i - m_i) \times (n_i - p_i)} & I_{(p_i - m_i) \times (p_i - m_i)} \end{bmatrix}$ and $\mathrm{X}_i \in R^{m_i \times m_i}$ is any diagonal matrix. With the definition of sliding surface and sliding manifold as above, the continuous decentralized adaptive output feedback second order sliding mode control scheme is given as below:

$$u_i(t) = u_i(0) - \int_0^t \{(K_{i2}B_{i2})^{-1}(\kappa_i\eta_i + \bar{\kappa}_i\|y_i\| + \hat{\kappa}_i\|\dot{y}_i\| + \zeta_i(t) + \alpha_i\|s_i\|)\mathrm{sign}(s_i)\}dt \tag{7}$$

where

$$\begin{aligned}\kappa_i &= \|K_{i2}\|\|A_{i3}\|(\|A_{i1}\| + \|D_{i1}\|\|E_{i1}\|) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{L}\|K_{j2}\|\|A_{j3}\|(\|H_{ji1}\| + \|M_{ji1}\|\|N_{ji1}\|) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{L}[\|K_{j2}\|\|H_{ji3}\|(\|A_{i1}\| + \|D_{i1}\|\|E_{i1}\|) \\ &+ \|K_{i2}\|\|H_{ij3}\| \times \sum_{\substack{i=1\\ j\neq i}}^{L}(\|H_{ji1}\| + \|M_{ji1}\|\|N_{ji1}\|)],\end{aligned} \tag{8}$$

$$\begin{aligned}\bar{\kappa}_i &= \|K_{i2}\|\|A_{i3}\|(\|A_{i2}\| + \|D_{i1}\|\|E_{i2}\|)\|K_{i2}^{-1}K_iC_{i2}^{-1}\| \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{L}\|K_{j2}\|\|A_{j3}\|(\|H_{ji2}\| + \|M_{ji1}\|\|N_{ji2}\|)\|K_{i2}^{-1}K_iC_{i2}^{-1}\| \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{L}[\|K_{j2}\|\|H_{ji3}\|(\|A_{i2}\| + \|D_{i1}\|\|E_{i2}\|) \\ &+ \|K_{i2}\|\|H_{ij3}\| \times \sum_{\substack{i=1\\ j\neq i}}^{L}(\|H_{ji2}\| + \|M_{ji1}\|\|N_{ji2}\|)]\|K_{i2}^{-1}K_iC_{i2}^{-1}\|,\end{aligned} \tag{9}$$

$$\begin{aligned}\hat{\kappa}_i &= \|\mathrm{X}_i\|\|K_iC_{i2}^{-1}\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\|K_{j2}\|\|H_{ji4}\|\|K_{i2}^{-1}K_iC_{i2}^{-1}\| \\ &+ \|K_{i2}\|\|A_{i4}\|\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\end{aligned} \tag{10}$$

and

$$\begin{aligned}\zeta_i(t) &= \hat{a}_i(t)\|K_{i2}\| + \hat{b}_i(t)\|K_{i2}\|\eta_i \\ &+ \hat{c}_i(t)\|K_{i2}\|\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\|y_i\|)\end{aligned} \tag{11}$$

and the adaptive laws are

$$\dot{\hat{a}}_i(t) = \hat{q}_i[-\hat{\upsilon}_i\hat{a}_i(t) + \|K_{i2}\|], \tag{12}$$

$$\dot{\hat{b}}_i(t) = \bar{q}_i[-\bar{\upsilon}_i\hat{b}_i(t) + \|K_{i2}\|\eta_i] \tag{13}$$

and

$$\dot{\hat{c}}_i(t) = \tilde{q}_i(-\tilde{\upsilon}_i\hat{c}_i(t) + \|K_{i2}\|\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\|y_i\|) \tag{14}$$

where the scalars $\alpha_i > 0$, $\eta_i > 0$, $\hat{q}_i > 0$, $\bar{q}_i > 0$, $\tilde{q}_i > 0$, $\hat{\upsilon}_i > 0$, $\bar{\upsilon}_i > 0$ and $\tilde{\upsilon}_i > 0$.

Then the following main results are derived.

Theorem 1: Consider the closed loop of the complex interconnected systems (3)-(4) with the continuous decentralized adaptive output feedback second order sliding mode controller (7). Then, every solution trajectory is directed towards the sliding manifold $s_i(t) = 0$ and once the trajectory hits the sliding manifold ($\sigma_i(x_i(t)) = 0$ and $\dot{\sigma}_i(x_i(t)) = 0$) it remains on the surface for all $\|z_{i1}(t)\| < \eta_i$. And the resulting ($n_i - m_i$) reduced-order dynamics of the closed loop system (3)-(4) restricted to the sliding manifold $s_i(t) = 0$ is asymptotically stable if there exists symmetric positive definite matrix $P_i \in R^{(n_i-m_i)\times(n_i-m_i)}$ such that

$$\begin{bmatrix} \Psi_i & P_iD_{i1} & E_{i1}^T & P_i \\ D_{i1}^TP_i & -\varphi_i I & 0 & 0 \\ E_{i1} & 0 & -\varphi_i^{-1}I & 0 \\ P_i & 0 & 0 & -\dfrac{\varepsilon_i}{L-1}I \end{bmatrix} < 0, \quad i = 1,2,..,L \tag{15}$$

where $\Psi_i = A_{i1}^TP_i + P_iA_{i1} + \sum_{\substack{j=1\\ j\neq i}}^{L}(\varepsilon_jH_{ji1}^TH_{ji1} + \bar{\varphi}_i^{-1}P_iM_{ij1}M_{ij1}^TP_i + \bar{\varphi}_jN_{ji1}^TN_{ji1})$, L is the number of subsystems and the scalars $\varphi_i > 0,\ \varepsilon_i > 0,\ \bar{\varphi}_i > 0,\ i = 1,2,...,L$.

Proof. There are two main parts involved in the proof of theorem 1. The first part is to prove that the the continuous decentralized adaptive output feedback second order sliding mode control law (7) drives the system trajectory onto the sliding manifold and maintain the trajectory on the sliding manifold for all subsequent time, i.e. the reachability condition is satisfied. The second part is to prove that the ($n_i - m_i$) reduced-order dynamics restricted to the sliding manifold $s_i(t) = 0$ is asymptotically stable. Before the proof of the first part, some statements should be noted in the following: Firstly, from equation (3), one can obtain

$$\begin{aligned}\dot{z}_{i1} &= (A_{i1} + D_{i1}F_iE_{i1})z_{i1} + (A_{i2} + D_{i1}F_iE_{i2})z_{i2} \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{L}[(H_{ij1} + M_{ij1}F_{ij}N_{ij1})z_{j1} + (H_{ij2} + M_{ij1}F_{ij}N_{ij2})z_{j2}],\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{z}_{i2} &= (A_{i3} + D_{i2}F_iE_{i1})z_{i1} + (A_{i4} + D_{i2}F_iE_{i2})z_{i2} \\ &+ B_{i2}(u_i + \xi_i) + \sum_{\substack{j=1\\ j\neq i}}^{L}[(H_{ij3} + M_{ij2}F_{ij}N_{ij1})z_{j1} \\ &+ (H_{ij4} + M_{ij2}F_{ij}N_{ij2})z_{j2}].\end{aligned} \tag{17}$$

Taking the derivative of $\sigma_i(x_i(t))$ and using equation (17) yields

$$\begin{aligned}\dot{\sigma}_i(x_i(t)) &= K_{i2}[A_{i3}z_{i1} + A_{i4}z_{i2} + B_{i2}u_i + \pi_i(t) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{L}(H_{ij3}z_{j1} + H_{ij4}z_{j2})]\end{aligned} \tag{18}$$

where

$$\begin{aligned}\pi_i(t) &= D_{i2}F_iE_{i1}z_{i1} + D_{i2}F_iE_{i2}z_{i2} + B_{i2}\xi_i + M_{ij2}F_{ij}N_{ij1}z_{j1} \\ &+ M_{ij2}F_{ij}N_{ij2}z_{j2}.\end{aligned} \tag{19}$$

The second time derivative of $\sigma_i(x_i(t))$ can be given as

$$\ddot{\sigma}_i(x_i(t)) = K_{i2}[A_{i3}\dot{z}_{i1} + A_{i4}\dot{z}_{i2} + B_{i2}\dot{u}_i + \dot{\pi}_i(t) + \sum_{\substack{j=1\\j\neq i}}^{L}(H_{ij3}\dot{z}_{j1} + H_{ij4}\dot{z}_{j2})]. \qquad (20)$$

According to equations (6) and (20), we achieve

$$\dot{s}_i(t) = K_{i2}[A_{i3}\dot{z}_{i1} + A_{i4}\dot{z}_{i2} + B_{i2}\dot{u}_i] + K_{i2}\dot{\pi}_i(t) + \sum_{\substack{j=1\\j\neq i}}^{L} K_{i2}[H_{ij3}\dot{z}_{j1} + H_{ij4}\dot{z}_{j2}] + \mathrm{X}_i\dot{\sigma}_i. \qquad (21)$$

Secondly, the following assumption is derived as:

Assumption 2. The disturbances $\dot{\pi}_i(t)$ in (21) are assumed to be bounded and satisfy the following condition:

$$\|\dot{\pi}_i(t)\| \leq a_i + b_i\|z_{i1}(t)\| + c_i\|z_{i2}(t)\| \qquad (22)$$

where a_i, b_i and c_i are unknown bounds. In most of systems, the bounds of the system uncertainties are commonly unknown in advance and therefore it is difficult to realize the error term $\dot{\pi}_i(t)$ in equation (21). So adaptive tuning laws given in equations (12), (13) and (14) are proposed to estimate $\dot{\pi}_i(t)$.

Thirdly, the following lemmas are recalled, which will be used in the proof of Theorem 1

Lemma 1 (Zhang and Xia, 2010): Let X, Y and F are real matrices of suitable dimension with $F^T F \leq I$ then, for any scalar $\varphi > 0$, the following matrix inequality holds:

$$XFY + Y^T F^T X^T \leq \varphi^{-1}XX^T + \varphi Y^T Y.$$

Lemma 2 (Horn and Johnson, 1985): Let X and Y are real matrices of suitable dimension then, for any scalar $\mu > 0$, the following matrix inequality holds:

$$X^T Y + Y^T X \leq \mu X^T X + \mu^{-1}Y^T Y.$$

Now, the first part of theorem 1 will be proved by defining a Lyapunov function as (Mondal and Mahanta, 2013; Xia et al., 2011):

$$V(t) = \sum_{i=1}^{L}(\|s_i(t)\| + \frac{0.5}{\hat{q}_i}\tilde{a}_i^2(t) + \frac{0.5}{\overline{q}_i}\tilde{b}_i^2(t) + \frac{0.5}{\tilde{q}_i}\tilde{c}_i^2(t)) \qquad (23)$$

where $\tilde{a}_i(t) = a_i - \hat{a}_i(t)$, $\tilde{b}_i(t) = b_i - \hat{b}_i(t)$ and $\tilde{c}_i(t) = c_i - \hat{c}_i(t)$ are the estimation errors of the adaptive gains. The time derivative of $V(t)$ is achieved as.

$$\dot{V}(t) = \sum_{i=1}^{L}(\frac{s_i^T}{\|s_i\|}\dot{s}_i(t) - \frac{1}{\hat{q}_i}\tilde{a}_i\dot{\hat{a}}_i - \frac{1}{\overline{q}_i}\tilde{b}_i\dot{\hat{b}}_i - \frac{1}{\tilde{q}_i}\tilde{c}_i\dot{\hat{c}}_i). \qquad (24)$$

Finally, we achieve

$$\dot{V}(t) < 0. \qquad (25)$$

The detailed proof of $\dot{V}(t) < 0$ can be found in Appendix A. The equation (25) indicates that the system states reach the sliding manifold and stay on it thereafter.

Following the proof of the first part, now it is necessary to prove that the ($n_i - m_i$) reduced-order dynamics restricted to the sliding manifold $s_i(t) = 0$ is asymptotically stable. The second-order sliding mode dynamics is determined from the basic equality condition $\sigma_i(x_i) = \dot{\sigma}_i(x_i) = 0$, whereas the proposed controller reaches the condition asymptotically. From equation (5), in the sliding mode $\sigma_i(x_i) = 0$ and $\dot{\sigma}_i(x_i) = 0$, we have $z_{i2} = 0$. Then, from the structure of system (3)-(4), the ($n_i - m_i$) reduced-order sliding mode dynamics of the system (3)-(4) related with the sliding surface (5) is designated by

$$\dot{z}_{i1} = (A_{i1} + D_{i1}F_i E_{i1})z_{i1} + \sum_{\substack{j=1\\j\neq i}}^{L}(H_{ij1} + M_{ij1}F_{ij}N_{ij1})z_{j1}. \qquad (26)$$

For the ($n_i - m_i$) reduced-order sliding mode dynamics (26), the Lyapunov function candidate is considered $V_0 = \sum_{i=1}^{L} z_{i1}^T P_i z_{i1}$ where $P_i > 0$ satisfies (15). Then, the time derivative of V_0 and using equation (26) is given by

$$\dot{V}_0 = \sum_{i=1}^{L} z_{i1}^T(A_{i1}^T P_i + P_i A_{i1} + P_i D_{i1}F_i E_{i1} + E_{i1}^T F_i^T D_{i1}^T P_i)z_{i1} + \sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}(z_{j1}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1}z_{j1} + z_{i1}^T P_i M_{ij1}F_{ij}N_{ij1}z_{j1} + z_{j1}^T N_{ij1}^T F_{ij}^T M_{ij1}^T P_i z_{i1}). \qquad (27)$$

Applying Lemma 1 and Lemma 2 to equation (27), it is easy to acquire that

$$\dot{V}_0 < 0. \qquad (28)$$

The detailed proof of $\dot{V}_0 < 0$ can be found in Appendix B. The inequality (28) shows that if LMI (15) is feasible, which further indicates that the ($n_i - m_i$) reduced-order sliding mode dynamics (26) is asymptotically stable. □

Design procedure: The proposed continuous decentralized adaptive output feedback second order sliding mode control strategy can be implemented as below.

Step 1: If assumption 1 is satisfied then calculate the coordinate transformation matrix T_i following the technique given in (Yan et al., 2012).

Step 2: Find a feasible solution of LMI (15) and calculate the sliding surface parameter $K_i \in R^{m_i \times p_i}$.

Step 3: Design the sliding surface $\sigma_i(x_i(t))$ and sliding manifold $s_i(t)$ according to equations (5) and (6).

Step 4: Design the continuous decentralized adaptive output feedback second order sliding mode controller $u_i(t)$ using equation (7).

Remark 1: The parameters α_i and η_i in the controller (7) play a significant role in determining the convergence rate of the sliding surface. It is evident that a higher value of α_i and η_i will force the system states to converge to the origin at a high speed. Since a large value of α_i and η_i will need a very high control input which is often not desirable in practice, these parameters cannot be chosen too large. Hence, a compromise has to be made between the response speed and the control input.

Remark 2: The scalars $\hat{\upsilon}_i > 0$, $\bar{\upsilon}_i > 0$ and $\tilde{\upsilon}_i > 0$ in the adaptive laws (12)-(14) determine the convergence rate of the estimated bounds $\hat{a}_i(t)$, $\hat{b}_i(t)$ and $\hat{c}_i(t)$. Practically, any scalars $\hat{\upsilon}_i > 0$, $\bar{\upsilon}_i > 0$ and $\tilde{\upsilon}_i > 0$ can be used to estimate the disturbance but a large value only is used for faster estimation of disturbance resulting in larger the band of the bounded region and vice versa.

Remark 3: It is obvious from (7) that $\dot{u}(t)$ is discontinuous but integration of $\dot{u}(t)$ yields a continuous control law $u(t)$. Therefore the undesired high frequency chattering of the control signal is removed.

Remark 4: The decentralized output feedback control strategy for the interconnected systems could be found in these papers (Yan et al., 2004; Cheng and Chang, 2008; Kalsi et al., 2010; Yan et al., 2010). However, these methods had the limitation of severe chattering in the control input. Therefore, the continuous decentralized output feedback control strategy equation (7) has been developed to avoid chattering problems in the control input.

Remark 5: These approaches given in (Chang, 2012; Li and Zheng, 2012; Mondal and Mahanta, 2013; Das and Mahanta, 2014) are achieved under assumption that the second derivative of all state variables $\ddot{x}_i$ must be existed, even though mathematical model of systems are the first order. Therefore, the control methods given in (Chang, 2012; Li and Zheng, 2012; Mondal and Mahanta, 2013; Das and Mahanta, 2014) could not be applied for the complex interconnected system (1).

4. NUMERICAL EXAMPLES

To evaluate the effectiveness of the present study, three examples are tested below.

Example 1 (Kalsi et al., 2010): The first subsystem parameters are as $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$,

$x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \in R^2$, $u_1 \in R^1$, $y_1 = [y_{11}] \in R^1$, $D_1 = \frac{1}{\sqrt{10}}[1 \quad 15]^T$, $E_1 = [1 \quad 1]$ and $F_1 = \sin(x_{12})$, $M_{12} = \frac{1}{\sqrt{10}}[1 \quad 8]^T$, $F_{12} = \cos(x_{22})$, $N_{12} = [1 \ 1 \ 1]$.

The second subsystem parameters are as $A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -40.8 & -41.5 & -9.35 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$,

$x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix} \in R^3$, $u_2 \in R^1$, $y_2 = [y_{21}] \in R^1$,

$D_2 = \frac{1}{\sqrt{15}}[0.9 \ \ 1.2 \ \ 200]^T$, $E_2 = [1 \quad 1 \quad 1]$, $F_2 = \sin(x_{21})$,

$N_{21} = [1 \quad 1]^T$, $M_{21} = \frac{1}{\sqrt{15}}[1.3 \quad 3 \quad 150]$ and $F_{21} = \cos(x_{11})$.

The disturbances $\pi_1(t)$ and $\pi_2(t)$ are assumed to satisfy

$$\pi_1(t) \le a_1 + b_1\|x_{11}\| + c_1\|x_{12}\| = 1 + 1\|x_{11}\| + 1\|x_{12}\| \tag{29}$$

and

$$\pi_2(t) \le a_2 + b_2\left\|\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}\right\| + c_2\|x_{23}\| = 1 + 1\left\|\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}\right\| + 1\|x_{23}\|. \tag{30}$$

Remark 6: Since $rank[B_1 \ \Delta A_1 \ \Delta H_{12}] = 2 > rank(B_1) = 1$, $rank[B_2 \ \Delta A_2 \ \Delta H_{21}] = 3 > rank(B_2) = 1$ and equations (29)-(30), it is to say that the interconnected system considered in this example consists of mismatched uncertainties and unknown disturbances.

It is easy to know that the assumption 1 is satisfied because $\mathrm{rank}(B_1) = \mathrm{rank}(B_2) = 1$ and $\mathrm{rank}(C_1) = \mathrm{rank}(C_2) = 1$. Therefore the coordinate transformation matrices of two subsystems are found to be $T_1 = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. From these transformation matrices, we achieve $B_{12} = B_{22} = C_{12} = C_{22} = 1$. By solving LMI (15), we have $P_1 = 6.0886$ and $P_2 = \begin{bmatrix} 30.6739 & -16.3025 \\ -16.3025 & 48.3512 \end{bmatrix} > 0$, the scalars $\varphi_1 = 60$, $\varphi_2 = 20$, $\bar{\varphi}_1 = 10$, $\bar{\varphi}_2 = 150$, $\varepsilon_1 = 30$ and $\varepsilon_2 = 100$. Since the LMI (15) is feasible and $m_i = p_i = 1$, $i = 1,2$ then the sliding matrices are

$$K_1 = K_{12} = \Xi_1^T P_1 \Xi_1 = 0.0061 \tag{31}$$

and

$$K_2 = K_{22} = \Xi_2^T P_2 \Xi_2 = 0.0017. \tag{32}$$

Based on the analysis in Remark 1 and Remark 2, the design parameters in the control scheme are chosen to be $\alpha_1 = 0.08$, $\alpha_2 = 0.07$, $\eta_1 = 0.001$, $\eta_2 = 0.002$, $\hat{q}_1 = 0.01$, $\hat{q}_2 = 0.02$, $\bar{q}_1 = 0.1$, $\bar{q}_2 = 0.1$, $\tilde{q}_1 = 0.1$, $\tilde{q}_2 = 0.1$, $\hat{\upsilon}_1 = 40$, $\hat{\upsilon}_1 = 40$, $\bar{\upsilon}_2 = 40$, $\bar{\upsilon}_1 = 40$ $\tilde{\upsilon}_1 = 40$ and $\tilde{\upsilon}_2 = 40$. From equation (6), the sliding manifold of two subsystems are $s_1 = 0.0061\dot{y}_1 + 0.0488y_1$ and $s_2 = 0.0016\dot{y}_2 + 0.0032y_2 = 0$. According to (7), the continuous decentralized adaptive output feedback second order sliding mode controller of the two subsystems are

$$u_1(t) = 0.001 - \int_0^t \{164.2[\zeta_1(t) + 0.08\|s_1\| + 0.074\|y_1\| + 0.054\|\dot{y}_1\|]\operatorname{sign}(s_1)\}dt, \tag{33}$$

and

$$u_2(t) = 0.001 - \int_0^t 588.2[\zeta_2(t) + 0.07\|s_2\| + 0.109\|y_2\| + 0.017\|\dot{y}_2\|]\operatorname{sign}(s_2)dt \tag{34}$$

where $\zeta_1(t) = 0.0061\hat{a}_1(t) + 0.0000034\hat{b}_1(t) + 0.0017\hat{c}_1\|y_1\|$, $\zeta_2(t) = 0.0016\hat{a}_2(t) + 0.0000034\hat{b}_2(t) + 0.0017\hat{c}_2\|y_2\|$ and the adaptive laws are designed as $\dot{\hat{a}}_1(t) = -0.4a_1(t) + 0.00006$, $\dot{\hat{b}}_1(t) = -0.8\hat{b}_1(t) + 0.000001$, $\dot{\hat{c}}_1(t) = -4\hat{c}_1(t) + 0.0006\|y_1\|$, $\dot{\hat{a}}_2(t) = -0.8a_2(t) + 0.00003$, $\dot{\hat{b}}_2(t) = -4\hat{b}_2(t) + 0.000001$ and $\dot{\hat{c}}_2(t) = -4\hat{c}_2(t) + 0.0003\|y_2\|$.

The initial conditions of two subsystems are chosen to be $x_1(0) = [5 \;\; 5]^T$ and $x_2(0) = [5 \;\; 5 \;\; 5]^T$. The system states, sliding manifold and the control input of the interconnected system by applying the proposed continuous decentralized adaptive output feedback sliding mode controller (7) are plotted in Fig. 1 to Fig. 6, respectively.

Remark 7: According to Fig. 1 and Fig. 6, the proposed controller (7) produces smaller input variation and smoother chattering free control input as compared to the one in (Kalsi et al., 2010).

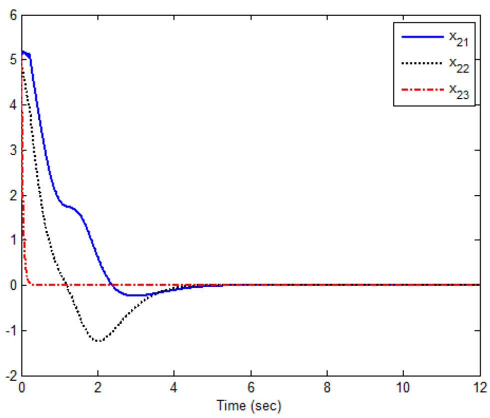

Fig. 1. Time responses of system states x_{21} (solid), x_{22} (dotted), x_{23} (dashed).

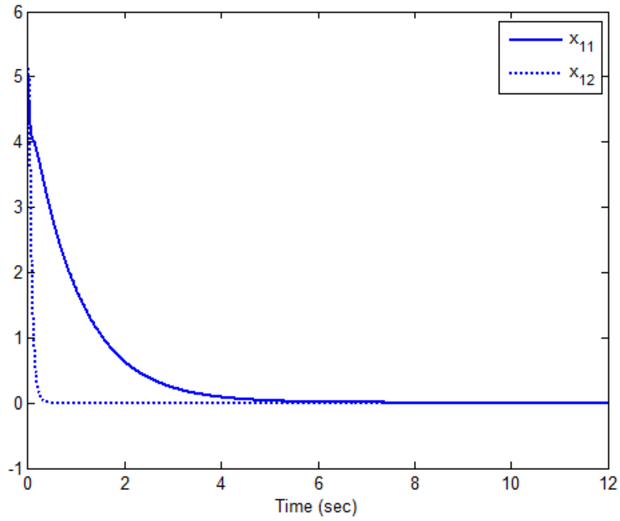

Fig. 2. Time responses of system states x_{11} and x_{12}.

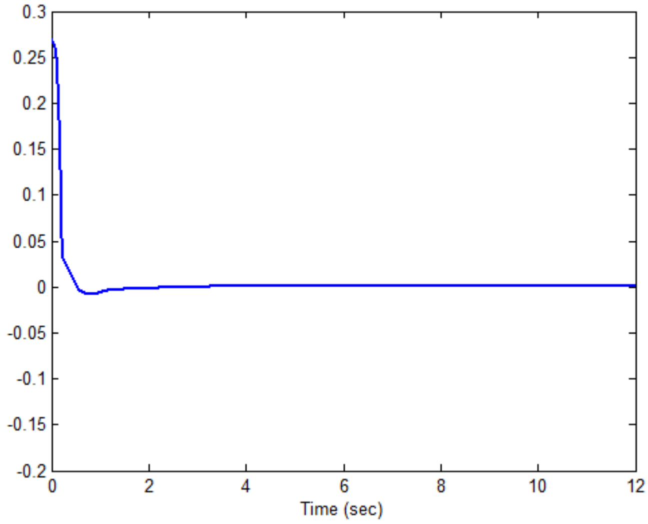

Fig. 3. Time responses of sliding manifold s_1.

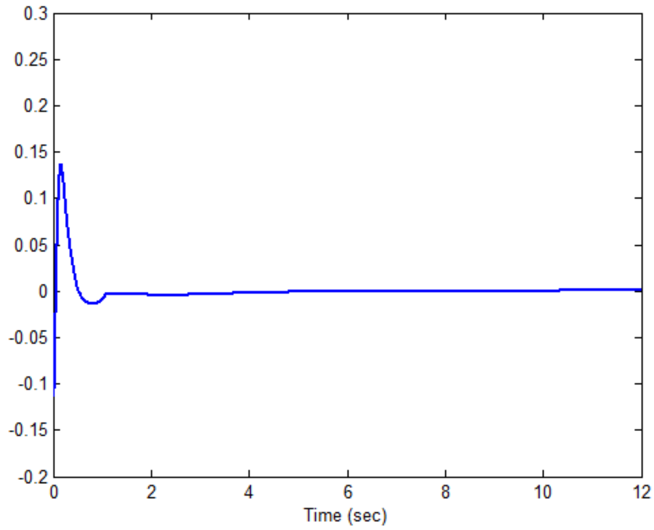

Fig. 4. Time responses of sliding manifold s_2.

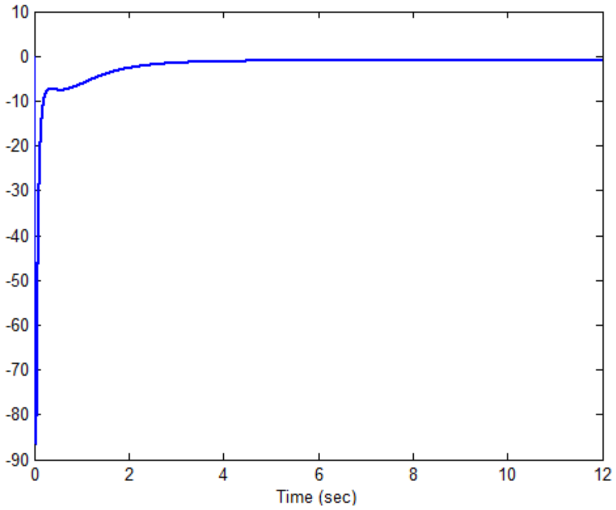

Fig. 5. Time responses of continuous control input u_1.

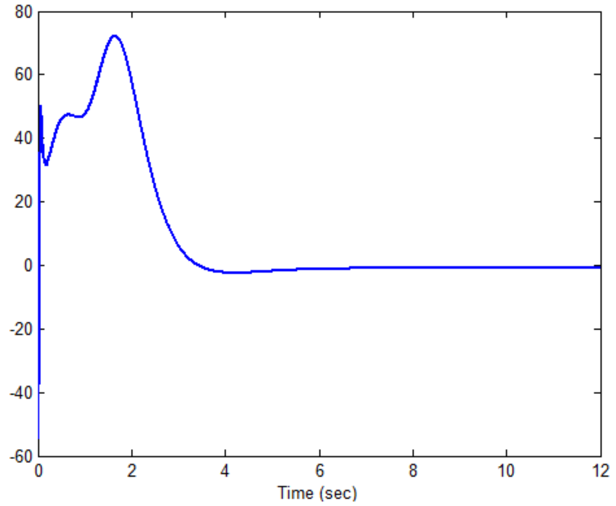

Fig. 6. Time responses of continuous control input u_2.

Remark 8: The control schemes given by (Kalsi et al., 2010) are based on dynamical output feedback which requires more hardware for implementation due to the associated closed-loop system possessing a dynamical order double that of the actual system. This limitation has been solved by applying the proposed continuous static output feedback controllers (33)-(34).

Example 2: The control systems of two inverted pendulums connected by a spring (Zhiming et al., 1996), and composed of two subsystems as follows:

$$\dot{x}_i = A_i x_i + B_i(u_i + \xi_i(x_i,t)) + \sum_{\substack{j=1 \\ j\neq i}}^{2} H_{ij} x_j,\ i=1,2;\ j=1,2 \tag{35}$$

$$y_i = C_i x_i$$

where $x_i = \begin{bmatrix} \theta_i \\ \dot{\theta}_i \end{bmatrix} \in R^2$, $u_i \in R^1$, $y_i = \begin{bmatrix} \dot{\theta}_i \end{bmatrix} \in R^1$, $A_i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,

$B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_i = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $H_{ij} = B_i \times \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$,

$\xi_i(x_i,t) = \Delta A_i x_i + \Delta B_i = -0.5\theta_i - 0.2$, $i=1,\ 2;\ j=1,\ 2;\ i\neq j$. Each pendulum might be positioned by a torque input u_i, $i=1,\ 2$ applied by a servomotor at its base.

The transformation matrices of two subsystems are $T_1 = T_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. The initial conditions are taken as $\theta_1(0) = 0.5$ and $\theta_2(0) = -0.4$, and the others initial values are chosen zeros. By applying the proposed continuous decentralized adaptive output feedback second order sliding mode control scheme (7) to control the large-scale system (35), the simulation results are given by Fig. 7 and Fig. 8, respectively, where Fig. 7 and Fig. 8 express the angular displacements of the pendulums from vertical and the trajectories of the control inputs.

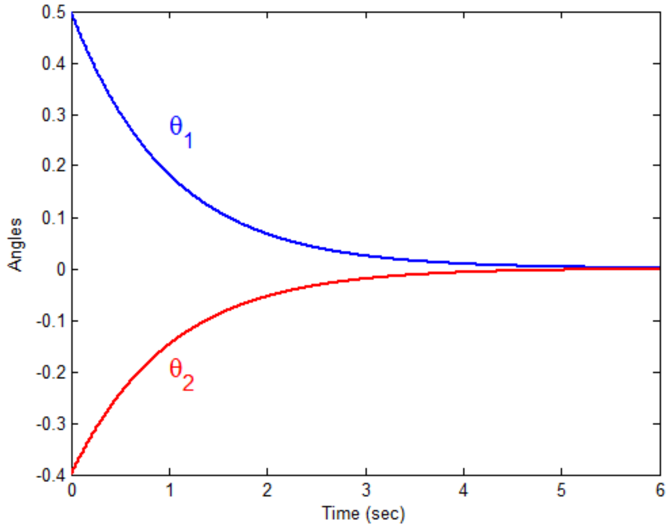

Fig. 7. System responses.

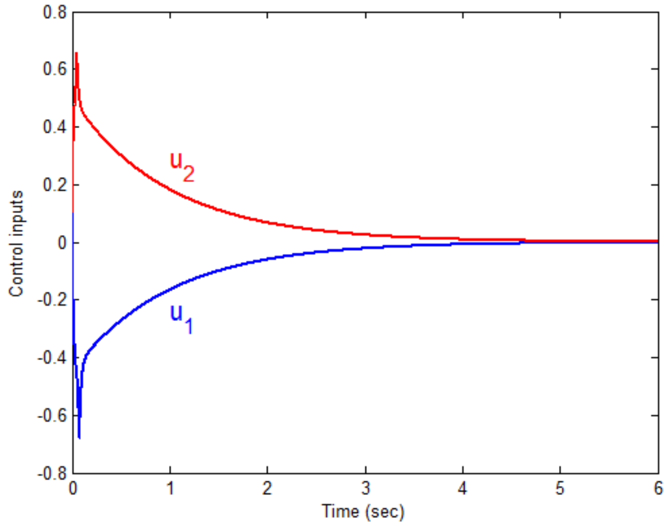

Fig. 8. Control inputs

Remark 9: From Fig. 7 and Fig. 8, it is marked that the proposed continuous decentralized adaptive output feedback second order sliding mode control scheme (7) yields faster settling time, smaller input variation, and lower control energy than those achieved by using the controller given in (Zhiming et al., 1996).

Remark 10: It would be declared that the existing decentralized control method in (Zhiming et al., 1996) can also be applied to the inverted pendulum system. However, the considered system (35) in this paper is not required to measure the angular displacements of the pendulums from vertical. Therefore, the aforementioned decentralized control approach cannot be applied to control the system (35).

Example 3: The design procedure presented is applied to design an adaptive robust decentralized load-frequency controller for a three-area interconnected power system, which are from paper (Lim, Wang, and Zhou, 1996). The state vector is

$$x_i(t) = \left[\Delta f_i(t)\ \Delta P_{gi}(t)\ \Delta X_{gi}(t)\ \Delta E_i(t)\ \Delta\delta_i(t)\right]^T \tag{36}$$

where $\Delta f_i(t)$, $\Delta P_{gi}(t)$, $\Delta X_{gi}(t)$, $\Delta E_i(t)$ and $\Delta\delta_i(t)$ are the changes of frequency, power output, governor valve position, integral control and rotor angle deviation, respectively. The disturbances $\dot{\pi}_1(t)$, $\dot{\pi}_2(t)$ and $\dot{\pi}_3(t)$ are assumed to satisfy $\dot{\pi}_1(t) \le a_1 = 1$, $\dot{\pi}_2(t) \le a_2 = 1$ and $\dot{\pi}_3(t) \le a_3 = 1$.

It is easy to know that the assumption 1 is satisfied. Therefore the coordinate transformation matrices of three subsystems are given by

$$T_1 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ -3.0867 & -3.0179 & -1 & -0.5759 & -0.1704 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ -9.1231 & -5.0849 & -1 & -3.1351 & -1.5942 \end{bmatrix} \quad \text{and}$$

$$T_3 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ -16.7645 & -8.4590 & -1 & -13.3088 & -2.5296 \end{bmatrix}.$$

Then the matrices $B_{12} = -13.021$,

$$C_{12} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.575 & 0.17 & -3.017 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad B_{22} = -14.468,$$

$$C_{22} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3.135 & 1.594 & -5.084 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad B_{32} = -14.881 \quad \text{and}$$

$$C_{32} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -13.3 & 2.529 & -8.459 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$ are non-singular.

Using the LMI-solver, it is found that the feasible solution of (15) is attained at

$$P_1 = \begin{bmatrix} 43.1482 & -25.8322 & 8.1622 & -20.3276 \\ -25.8322 & 15.7039 & -4.7802 & 12.1736 \\ 8.1622 & -4.7802 & 1.6900 & -3.8326 \\ -20.3276 & 12.1736 & -3.8326 & 9.6163 \end{bmatrix} > 0,$$

$$P_2 = \begin{bmatrix} 18.3889 & -8.2135 & 4.2891 & -8.2108 \\ -8.2135 & 3.8963 & -1.8270 & 3.6701 \\ 4.2891 & -1.8270 & 1.1467 & -1.8948 \\ -8.2108 & 3.6701 & -1.8948 & 3.7153 \end{bmatrix} > 0 \quad \text{and}$$

$$P_3 = \begin{bmatrix} 0.1237 & -0.0659 & 0.0609 & -0.0184 \\ -0.0659 & 0.3022 & 0.0758 & 0.0056 \\ 0.0609 & 0.0758 & 0.1484 & -0.0107 \\ -0.0184 & 0.0056 & -0.0107 & 0.0096 \end{bmatrix} > 0,$$ the scalars $\varphi_1 = 0.5$, $\varphi_2 = 0.5$, $\varphi_3 = 0.5$, $\bar{\varphi}_1 = 0.05$, $\bar{\varphi}_2 = 0.06$, $\bar{\varphi}_3 = 0.04$ $\varepsilon_1 = 10$, $\varepsilon_2 = 9$ and $\varepsilon_3 = 9.5$. The sliding surface matrices are

$K_1 = \begin{bmatrix} 0 & 0 & 0 & K_{12} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \Xi_1^T P_1 \Xi_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0.9 \end{bmatrix}$,

$K_2 = \begin{bmatrix} 0 & 0 & 0 & K_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \Xi_2^T P_2 \Xi_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0.9 \end{bmatrix}$ and

$K_3 = \begin{bmatrix} 0 & 0 & 0 & K_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \Xi_3^T P_3 \Xi_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0.9 \end{bmatrix}$.

From equation (6), the sliding manifold of the three subsystems are

$$\begin{aligned} s_1 &= [-0.1534\ \ -2.7161\ \ -0.9000\ \ -0.5183]\dot{y}_1 \\ &\quad + [-0.0218\ \ -0.3857\ \ -0.1278\ \ -0.0736]y_1 \end{aligned} \tag{37}$$

$$\begin{aligned} s_2(t) &= [-1.4348\ \ -4.5764\ \ -0.9\ \ -2.8216]\dot{y}_2 \\ &\quad + [-0.2052\ \ -0.6544\ \ -0.1287\ \ -0.4035]y_2 \end{aligned} \tag{38}$$

and

$$\begin{aligned} s_3(t) &= [-2.2766\ \ -7.6131\ \ -0.9\ \ -11.9779]\dot{y}_3 \\ &\quad + [-0.3301\ \ -1.1039\ \ -0.1305\ \ -1.7368]y_3. \end{aligned} \tag{39}$$

According to equation (7), the continuous decentralized adaptive output feedback second order sliding mode controller of the three subsystems are

$$\begin{aligned} u_1(t) &= \int_0^t [0.0853(0.9\hat{a}_1(t) + 868.3845\|s_1\| \\ &\quad + 311.697\|y_1\| + 7.8181\|\dot{y}_1\|)\mathrm{sign}(s_1)]dt, \end{aligned} \tag{40}$$

$$u_2(t) = \int_0^t [0.0768(0.9\hat{a}_2(t) + 957.7044\|s_2\| + 705.4611\|y_2\| + 9.7406\|\dot{y}_2\|)\mathrm{sign}(s_2)]dt \tag{41}$$

and

$$u_3(t) = \int_0^t [0.0747(0.9\hat{a}_3(t) + 2420.1\|s_3\| + 1096.4\|y_3\| + 150.3298\|\dot{y}_3\|)\mathrm{sign}(s_3)]dt \tag{42}$$

where the adaptive laws are considered as $\dot{\hat{a}}_1(t) = -a_1(t) + 0.9$, $\dot{\hat{a}}_2(t) = -2a_2(t) + 0.9$ and $\dot{\hat{a}}_3(t) = -2a_3(t) + 0.9$.

Case 1: For testing the robustness of the continuous decentralized adaptive output feedback sliding mode controller, the system with nominal parameters is used and load changes of 0.01 p.u. MW is applied to all areas. The simulation results of $\Delta f_i(t)$ and $\Delta X_{gi}(t)$ for the proposed controller are shown in Figs. 9-10. It is showed that the frequency deviations converge to zero in about 3.5 s. The increment in governor-valve position $\Delta X_{gi}(t)$ in each area is to compensate for the local load change of 0.01 p.u. MW.

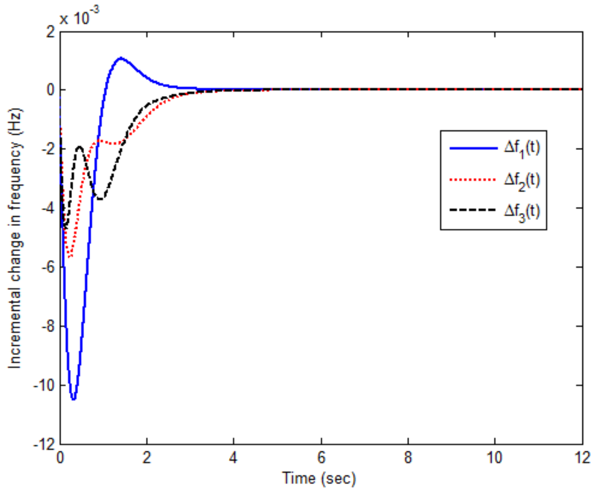

Fig. 9. The $\Delta f_i(t)$ for Case 1.

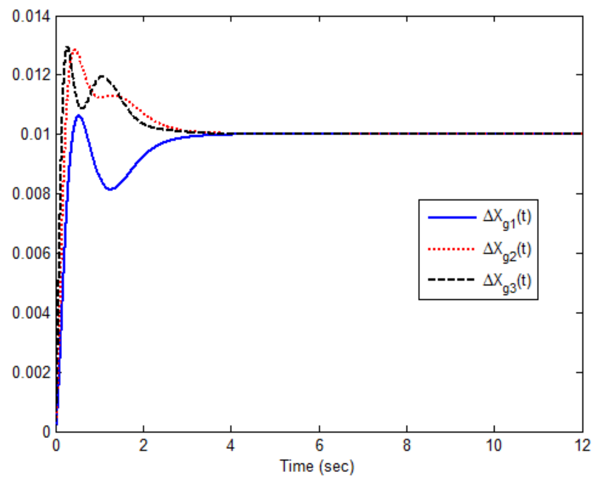

Fig. 10. The $\Delta X_{gi}(t)$ for Case 1.

Case 2: The unmatched uncertainties in the three areas are considered in this Case as:

$$D_i F_i E_i = 0.01 \times \begin{bmatrix} \cos(t) & 0 & 0 & 0 & 0 \\ 0 & \sin(t) & 0 & 0 & 0 \\ 0 & 0 & \cos(t) & 0 & 0 \\ 0 & 0 & 0 & \sin(t) & 0 \\ 0 & 0 & 0 & 0 & \cos(t) \end{bmatrix}, \; i = 1,\ 2,\ 3. \tag{43}$$

The system performances with the continuous decentralized adaptive output feedback second order sliding mode controller are tested under mismatched parameter uncertainty (43) and apply load changes of 0.02 p.u. MW to all areas. The simulation results of $\Delta f_i(t)$ and $\Delta X_{gi}(t)$ for the proposed robust controller are shown in Figs. 11-12. It is observed that the frequency deviations converge to zero in about 4 s. The increment in governor-valve position $\Delta X_{gi}(t)$ in each area is to compensate for the local load change of 0.02 p.u. MW.

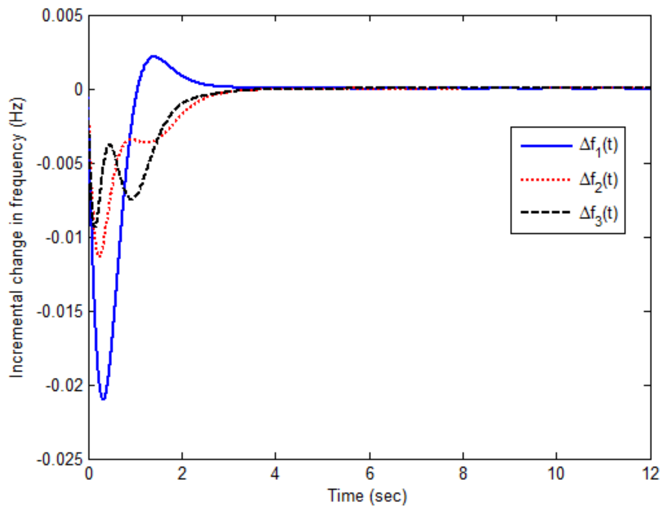

Fig. 11. The $\Delta f_i(t)$ for Case 2.

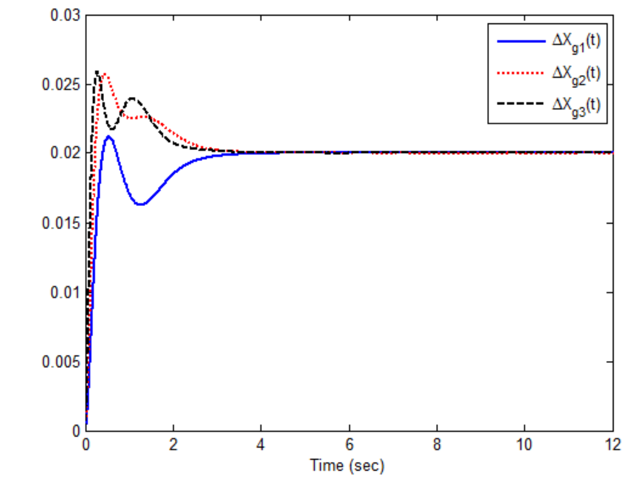

Fig. 12. The $\Delta X_{gi}(t)$ for Case 2.

Remark 11: In comparing the simulation results with the results given by (Lim, Wang, and Zhou, 1996), the proposed continuous decentralized adaptive output feedback sliding mode controller (40)-(42) can assure not only a fast response speed but also the smaller overshoot. Therefore, the designed controller is robust and effective to control the matching and unmatched parameter uncertainties of interconnected multi-area systems.

Remark 12: The method proposed by (Lim, Wang, and Zhou, 1996) can not be applied for a three-area interconnected power system presented if the state variable $\Delta f_i(t)$ is unmeasurable. This limitation has been removed by the proposed continuous decentralized adaptive output feedback second order sliding mode controller (40)-(42) because the proposed controller (40)-(42) only use four output variables ($\Delta P_{gi}(t)$, $\Delta X_{gi}(t)$, $\Delta E_i(t)$ and $\Delta\delta_i(t)$).

5. CONCLUSION

In this paper, a new second order sliding mode control law is presented for the purpose of stabilizing an mismatched uncertain complex interconnected systems with unknown disturbance. It has been shown that the sliding mode dynamics is asymptotically stable and the reachability of the system states is guaranteed. The main contribution of this paper is to introduce an adaptive second order sliding mode control approach to complex interconnected systems in order to control both the chattering and mismatched uncertainties without measurement all of state variables, which is impossible to achieve with the conventional sliding-mode control approach. Three examples showed the superiority of the proposed approach over the traditional sliding-mode control, particularly regarding the reduction of chattering levels on the control input.

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APPENDIXES

Appendix A: The proof of $\dot{V} < 0$

According to equation (24) and property $\|AB\| \le \|A\|\|B\|$, it generates

$$\begin{aligned}\dot{V}(t) \le{}& \sum_{i=1}^{L}\|K_{i2}\|[\|A_{i3}\|\|\dot{z}_{i1}\| + \|A_{i4}\|\|\dot{z}_{i2}\|] + \sum_{i=1}^{L}\frac{s_i^T}{\|s_i\|}K_{i2}B_{i2}\dot{u}_i \\ &+ \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\ne i}}^{L}\|K_{i2}\|[\|H_{ij3}\|\|\dot{z}_{j1}\| + \|H_{ij4}\|\|\dot{z}_{j2}\|] + \sum_{i=1}^{L}\|X_i\|\|\dot{\sigma}_i\| \qquad (44)\\ &+ \sum_{i=1}^{L}\|K_{i2}\|\|\dot{\pi}_i(t)\| - \sum_{i=1}^{L}\frac{1}{\hat{q}_i}\tilde{a}_i\dot{\hat{a}}_i - \sum_{i=1}^{L}\frac{1}{\bar{q}_i}\tilde{b}_i\dot{\hat{b}}_i - \sum_{i=1}^{L}\frac{1}{\tilde{q}_i}\tilde{c}_i\dot{\hat{c}}_i.\end{aligned}$$

Since $\|\dot{\pi}_i(t)\| \le a_i + b_i\|z_{i1}\| + c_i\|z_{i2}\|$, we get

$$\begin{aligned}\dot{V}(t) \le{}& \sum_{i=1}^{L}\|K_{i2}\|[\|A_{i3}\|\|\dot{z}_{i1}\| + \|A_{i4}\|\|\dot{z}_{i2}\|] + \sum_{i=1}^{L}\frac{s_i^T}{\|s_i\|}K_{i2}B_{i2}\dot{u}_i \\ &+ \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\ne i}}^{L}\|K_{j2}\|\|H_{ji4}\|\|\dot{z}_{i2}\| + \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\ne i}}^{L}\|K_{i2}\|\|H_{ij3}\|\|\dot{z}_{j1}\| \qquad (45)\\ &+ \sum_{i=1}^{L}\|X_i\|\|\dot{\sigma}_i\| + \sum_{i=1}^{L}\|K_{i2}\|(a_i + b_i\|z_{i1}\| + c_i\|z_{i2}\|) \\ &- \sum_{i=1}^{L}\frac{1}{\hat{q}_i}\tilde{a}_i\dot{\hat{a}}_i - \sum_{i=1}^{L}\frac{1}{\bar{q}_i}\tilde{b}_i\dot{\hat{b}}_i - \sum_{i=1}^{L}\frac{1}{\tilde{q}_i}\tilde{c}_i\dot{\hat{c}}_i.\end{aligned}$$

Since $z_{i2} = K_{i2}^{-1}K_iC_{i2}^{-1}y_i$ and equations (16) and (45), we achieve

$$\begin{aligned}\dot{V}(t) \le{}& \sum_{i=1}^{L}(\kappa_i\eta_i + \bar{\kappa}_i\|y_i\| + \hat{\kappa}_i\|\dot{y}_i\|) + \sum_{i=1}^{L}\frac{s_i^T}{\|s_i\|}K_{i2}B_{i2}\dot{u}_i \\ &+ \sum_{i=1}^{L}\|K_{i2}\|(a_i + b_i\eta_i + c_i\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\|y_i\|) \qquad (46)\\ &- \sum_{i=1}^{L}\frac{1}{\hat{q}_i}\tilde{a}_i\dot{\hat{a}}_i - \sum_{i=1}^{L}\frac{1}{\bar{q}_i}\tilde{b}_i\dot{\hat{b}}_i - \sum_{i=1}^{L}\frac{1}{\tilde{q}_i}\tilde{c}_i\dot{\hat{c}}_i\end{aligned}$$

where κ_i, $\bar{\kappa}_i$ and $\hat{\kappa}_i$ are given by equations (8), (9) and (10), respectively.

Substituting the controller (7) into (46), it is clear that

$$\begin{aligned}\dot{V}(t) \le{}& \sum_{i=1}^{L}\|K_{i2}\|(a_i + b_i\eta_i + c_i\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\|y_i\|) - \sum_{i=1}^{L}\zeta_i \\ &- \sum_{i=1}^{L}\alpha_i\|s_i\| - \sum_{i=1}^{L}\frac{1}{\hat{q}_i}\tilde{a}_i\dot{\hat{a}}_i - \sum_{i=1}^{L}\frac{1}{\bar{q}_i}\tilde{b}_i\dot{\hat{b}}_i - \sum_{i=1}^{L}\frac{1}{\tilde{q}_i}\tilde{c}_i\dot{\hat{c}}_i. \qquad (47)\end{aligned}$$

Considering equations (12), (13) and (14), the inequality (47) can be rewritten as

$$\begin{aligned}\dot{V}(t) \le{}& \sum_{i=1}^{L}\{\|K_{i2}\|(a_i + b_i\eta_i + c_i\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\|y_i\|) - \zeta_i - \alpha_i\|s_i\| \\ &+ \hat{\upsilon}_i[-(\hat{a}_i - \frac{a_i}{2})^2 + \frac{a_i^2}{4}] - \|K_{i2}\|a_i + \|K_{i2}\|\hat{a}_i \\ &+ \bar{\upsilon}_i[-(\hat{b}_i - \frac{b_i}{2})^2 + \frac{b_i^2}{4}] - b_i\|K_{i2}\|\eta_i + \hat{b}\|K_{i2}\|\eta_i \qquad (48)\\ &+ \tilde{\upsilon}_i[-(\hat{c}_i - \frac{c_i}{2})^2 + \frac{c_i^2}{4}] - (c_i + \hat{c}_i)\|K_{i2}\|\|K_{i2}^{-1}K_iC_{i2}^{-1}\|\|y_i\|\}.\end{aligned}$$

Then, equation (48) can be rewritten as

$$\begin{aligned}\dot{V}(t) \le{}& -\sum_{i=1}^{L}\alpha_i\|s_i(t)\| + \sum_{i=1}^{L}\hat{\upsilon}_i[-\hat{a}_i^2 + \hat{a}_ia_i] \\ &+ \sum_{i=1}^{L}\bar{\upsilon}_i[-\hat{b}_i^2 + \hat{b}_ib_i] + \sum_{i=1}^{L}\tilde{\upsilon}_i[-\hat{c}_i^2 + \hat{c}_ic_i] \qquad (49)\end{aligned}$$

Using equations (23) and (49) yield

$$\begin{aligned}\dot{V}(t) \le{}& -\bar{\alpha}V(t) + \sum_{i=1}^{L}[\bar{\alpha}\frac{0.5}{\hat{q}_i}a_i^2 + (\hat{\upsilon}_i - \frac{\bar{\alpha}}{\hat{q}_i})a_i\hat{a}_i \\ &+ (\bar{\alpha}\frac{0.5}{\hat{q}_i} - \hat{\upsilon}_i)\hat{a}_i^2] + \sum_{i=1}^{L}[\bar{\alpha}\frac{0.5}{\bar{q}_i}b_i^2 + (\bar{\upsilon}_i - \frac{\bar{\alpha}}{\bar{q}_i})b_i\hat{b}_i \\ &+ (\bar{\alpha}\frac{0.5}{\bar{q}_i} - \bar{\upsilon}_i)\hat{b}_i^2] + \sum_{i=1}^{L}[\bar{\alpha}\frac{0.5}{\tilde{q}_i}c_i^2 + (\tilde{\upsilon}_i - \frac{\bar{\alpha}}{\tilde{q}_i})c_i\hat{c}_i \qquad (50)\\ &+ (\bar{\alpha}\frac{0.5}{\tilde{q}_i} - \tilde{\upsilon}_i)\hat{c}_i^2]\end{aligned}$$

where $\bar{\alpha} = \min(\alpha_i)$, $i = 1, 2, ..., L$. By using Young's inequality $2ab \le a^2 + b^2$ and choosing $\hat{\upsilon}_i \ge \frac{\bar{\alpha}}{\hat{q}_i}, \bar{\upsilon}_i \ge \frac{\bar{\alpha}}{\bar{q}_i}$, $\tilde{\upsilon}_i \ge \frac{\bar{\alpha}}{\tilde{q}_i}$, it can be shown that

$$\dot{V}(t) \le -\bar{\alpha}V(t) - \sum_{i=1}^{L}(0.5\hat{\upsilon}_i\hat{a}_i^2 + 0.5\bar{\upsilon}_i\hat{b}_i^2 + 0.5\tilde{\upsilon}_i\hat{c}_i^2)$$
$$+\sum_{i=1}^{L}(0.5\hat{\upsilon}_i a_i^2 + 0.5\bar{\upsilon}_i b_i^2 + 0.5\tilde{\upsilon}_i c_i^2) \tag{51}$$
$$\le -\bar{\alpha}V(t) + \sum_{i=1}^{L}0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)$$

Equation (51) can be expressed as

$$0 \le V(t) \le e^{-\bar{\alpha}t}V(0)$$
$$+[\sum_{i=1}^{L}0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)]\int_0^t e^{-\bar{\alpha}(t-\tau)}d\tau$$
$$= e^{-\bar{\alpha}t}[V(0) - \sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}] \tag{52}$$
$$+\sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}$$

Taking the limit as t approaches infinity on both sides of equation (52), it can be seen

$$\lim_{t\to\infty}V(t) \le \lim_{t\to\infty}\{e^{-\bar{\alpha}t}[V(0) - \sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}]$$
$$+\sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}\} \tag{53}$$
$$= \sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}$$

In addition, using equations (23) and (52), it is easy to see that

$$\sum_{i=1}^{L}(\|s_i(t)\| + \frac{0.5}{\hat{q}_i}\tilde{a}_i^2(t) + \frac{0.5}{\bar{q}_i}\tilde{b}_i^2(t) + \frac{0.5}{\tilde{q}_i}\tilde{c}_i^2(t))$$
$$= V(t) \le e^{-\bar{\alpha}t}[V(0) - \sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}] \tag{54}$$
$$+\sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}$$

It follows that

$$\sum_{i=1}^{L}\|s_i\| \le e^{-\bar{\alpha}t}[\sum_{i=1}^{L}(\|s_i(0)\| + \frac{0.5}{\hat{q}_i}\tilde{a}_i^2(0) + \frac{0.5}{\bar{q}_i}\tilde{b}_i^2(0)$$
$$+\frac{0.5}{\tilde{q}_i}\tilde{c}_i^2(0) - \sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}] \tag{55}$$
$$+\sum_{i=1}^{L}\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}$$

Then, one gets

$$\|s_i\| \le e^{-\bar{\alpha}t}[\|s_i(0)\| + \frac{0.5}{\hat{q}_i}\tilde{a}_i^2(0) + \frac{0.5}{\bar{q}_i}\tilde{b}_i^2(0)$$
$$+\frac{0.5}{\tilde{q}_i}\tilde{c}_i^2(0) - \frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}] \tag{56}$$
$$+\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}$$

So that

$$\lim_{t\to\infty}\|s_i(t)\| \le \lim_{t\to\infty}\{e^{-\bar{\alpha}t}[\|s_i(0)\| + \frac{0.5}{\hat{q}_i}\tilde{a}_i^2(0) + \frac{0.5}{\bar{q}_i}\tilde{b}_i^2(0)$$
$$+\frac{0.5}{\tilde{q}_i}\tilde{c}_i^2(0) - \frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}] \tag{57}$$
$$+\frac{0.5(\hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2)}{\bar{\alpha}}\} = \frac{0.5\varpi_i}{\bar{\alpha}}$$

where $\varpi_i = \hat{\upsilon}_i a_i^2 + \bar{\upsilon}_i b_i^2 + \tilde{\upsilon}_i c_i^2$ and $\bar{\alpha} = \min(\alpha_i)$. From equations (53) and (57), the decrease of $V(t)$ eventually drives the trajectories of the closed-loop system into $\|s_i(t)\| \le \frac{0.5\varpi_i}{\bar{\alpha}}$ (Mondal and Mahanta, 2013; Krstic et al., 1995; Na et al., 2013). Therefore, the trajectories of the closed-loop system are bounded ultimately as

$$\lim_{t\to\infty}s_i(t) \in \left(\|s_i(t)\| \le \frac{0.5\varpi_i}{\bar{\alpha}}\right) \tag{58}$$

which is a small set containing the origin of the closed-loop system. In order to guarantee the bounded motion around the sliding surface, the positive parameter α_i is chosen large enough such that $\dot{V}(t) < 0$ when $V(t)$ is out of a certain bounded region which contains an equilibrium point (Mondal and Mahanta, 2013; Krstic et al., 1995; Na et al., 2013).

Appendix B: The proof of $\dot{V}_0 < 0$

Applying Lemma 1 to equation (27) yields

$$\dot{V}_0 \le \sum_{i=1}^{L}z_{i1}^T(A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1}P_i D_{i1}D_{i1}^T P_i + \varphi_i E_{i1}^T E_{i1})z_{i1}$$
$$+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\ne i}}^{L}(z_{j1}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1}z_{j1} + \bar{\varphi}_i z_{j1}^T N_{ij1}^T N_{ij1}z_{j1} \tag{59}$$
$$+\bar{\varphi}_i^{-1}z_{i1}^T P_i M_{ij1}M_{ij1}^T P_i z_{i1})$$

where the scalars $\varphi_i > 0$ and $\bar{\varphi}_i > 0$. By Lemma 2 and equation (59), it is obvious that

$$\dot{V}_0 \le \sum_{i=1}^{L}z_{i1}^T[A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1}P_i D_{i1}D_{i1}^T P_i + \frac{L-1}{\varepsilon_i}P_i P_i$$
$$+\varphi_i E_{i1}^T E_{i1} + \sum_{\substack{j=1\\j\ne i}}^{L}(\varepsilon_j H_{ji1}^T H_{ji1} + \bar{\varphi}_i^{-1}P_i M_{ij1}M_{ij1}^T P_i \tag{60}$$
$$+\bar{\varphi}_j N_{ji1}^T N_{ji1})]z_{i1}.$$

where the scale $\varepsilon_i > 0$. In addition, by the Schur complement of (Boyd et al., 1998), LMIs (15) is equivalent to the following inequality

$$A_{i1}^T P_i + P_i A_{i1} + \varphi_i E_{i1}^T E_{i1} + \frac{L-1}{\varepsilon_i}P_i P_i + \varphi_i^{-1}P_i D_{i1}D_{i1}^T P_i$$
$$+\sum_{\substack{j=1\\j\ne i}}^{L}(\varepsilon_j H_{ji1}^T H_{ji1} + \bar{\varphi}_i^{-1}P_i M_{ij1}M_{ij1}^T P_i + \bar{\varphi}_j N_{ji1}^T N_{ji1}) < 0. \tag{61}$$

According to equations (60) and (61), it is easy to get that

$$\dot{V}_0 < 0. \tag{62}$$