Fault Detection and Isolation in Stochastic Nonlinear Systems using Unscented Particle Filter based Likelihood Ratio Approach

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Abstract: A novel unscented particle filtering based log-likelihood ratio (LLR) approach for Fault detection and isolation (FDI) in nonlinear stochastic systems is proposed. It is well-known that the particle filter (PF) is used for the state estimation of nonlinear and non-Gaussian system but the key step in the filter design is the selection of a suitable proposal density to represent the true posterior density. The traditional PF when used for the FDI problem does not always guarantee that all particles lie in the likelihood region as the proposal density for this filter is independent of the measurement data. The new approach utilizing PF with unscented Kalman filter (UKF) proposal to solve the FDI problem assures that the estimated states (particles) lie within the high likelihood region because the proposal density in the unscented particle filter (UPF) is dependent on the current measurement. The detection and isolation of faults are carried out through maximum likelihood estimation and hypothesis testing method. The efficacy of the proposed method is demonstrated through an implementation on two highly nonlinear systems- a chemical reactor system and a three phase induction machine. The simulation results obtained from this method are compared with that of FDI technique using the generic particle filtering algorithm as state estimator.

Keywords: Fault detection and isolation, proposal density, unscented particle filter, log-likelihood ratio, nonlinear stochastic system.

1. INTRODUCTION

Modern engineering systems greatly focus on increasing safety and reliability. So the degree of automation used on these systems has become more complex. Sometimes, this increased level of complexity causes such systems susceptible to unexpected faults. As a result, it has motivated the researchers to propose various methods for solving the fault detection problem (Isermann, 1984; Baseville, 1997; Zarei and Shokri, 2014; Van Eykeren and Chu, 2014; Flett and Bone, 2016; Huang et al., 2016). Fault detection and isolation (FDI) is an increasingly important issue in designing systems with safety and reliability. The FDI methods provide an alarm tool which can detect if a fault is present in the monitored system and also determine the type, location and time of fault. Thus, this detection enables one to take appropriate corrective action before catastrophic failures. It is also noted that if the abnormal process behaviour and faults are detected and isolated as quickly as possible; the industries can largely maintain high safety and reliability standards (Baseville, 1997).

The FDI approaches are classified into two major categories such as model-based and non-model-based approaches. It is a well-known fact that the best analytical model normally represents complete concise knowledge of the system. The model-based approaches use an explicit mathematical model of the system to be monitored. Hence, the model-based methods tend to be more powerful and provide a better performance if the process is well modelled (Chen and Patton, 1999). Model-based FDI is performed by two steps: residual generation that reflects the fault on the system; then residual evaluation for decision making based on these residuals (Kadirkamanathan et al., 2002). The residual computed is used to decide when and where a fault has occurred during the abnormal behaviour of the process. The residual is usually generated either using an observer for deterministic systems or a filter for stochastic systems which leads to the observer-based and innovation–based FDI approaches respectively. Initially, FDI schemes was applied to the stochastic linear systems in which the noises being Gaussian. Under such cases, the Kalman filter (KF) is normally employed for state estimation and measurement prediction (Frank and Ding, 1997). The predicted measurement is then compared with the actual measurement (sensor data) in order to calculate the residual, based on which some statistical hypothesis tests are carried out for fault detection. Fault isolation is achieved by using the multiple model and log-likelihood ratio (LLR) test methods (Willsky and Jones, 1976).

Extension of the KF based FDI approach for the nonlinear stochastic systems have also been considered in the literature by replacing the KF by the extended Kalman filter (EKF) (Foo et al., 2013). The EKF based approach approximates the nonlinear system by employing the linearization technique through Taylor series method but such approximations can sometimes result in divergence of the filter. Therefore, this approach cannot be always guaranteed to work well for all fault detection problems as it can lead to high rate of false alarms. The unscented Kalman filter (UKF) based FDI approach overcomes the limitations faced in the above approach by using an unscented transformation (UT) technique to pick a minimal set of sample points (sigma points) around the mean in order to generate more accurate estimated state and predicted measurement (Mirzaee and Salahshoor, 2012). Unlike EKF, the UKF eliminates the need to perform any analytical calculations by using the concept of sample statistics (Julier and Uhlmann, 2004). But the FDI approaches making use of EKF or UKF can be applied only to nonlinear systems which assume the noises to be Gaussian.

The Particle filter (PF) is considered as a more powerful state estimation technique than Kalman-based estimators because of its asymptotic optimality and ability to tackle non-Gaussian systems (Li and Kadirkamanathan, 2001). In recent years, the PF based FDI approach for nonlinear and non-Gaussian systems have attracted much attention in the literature (Orchard and Vachtsevanos, 2009; Marseguerra and Zio, 2009; Alrowaie et al., 2012; Tadic and Durovic, 2014; Bozhao et al., 2014). The transition prior is usually chosen as the proposal density in the design of PF. As the selected proposal density is independent of current measurement, the state space is explored without any knowledge of observations (Jayaprasanth and Jovitha, 2014). Hence, such PFs when used for FDI can sometimes become inefficient and sensitive to outliers, thereby causing the filter to diverge. So under such conditions, the performance of FDI approach based on PF can be greatly affected.

In this paper, the PF with UKF proposal which incorporates the most recent measurement known as unscented particle filter (UPF) is employed to develop a new method for solving the FDI problem in general stochastic nonlinear and non-Gaussian systems. The potential advantage of using UPF for fault diagnosis is that it involves UKF to generate proposal density which moves the particles towards the region of high likelihood. The validity of the proposed approach is illustrated through simulations on a chemical reactor system and three phase induction machine.

2. PROBLEM STATEMENT

The model-based FDI approach based on a multi-model method is developed in this work for stochastic nonlinear systems. It is assumed that in this method if N possible known faults occurs in the system then $N+1$ models are taken in to consideration as

$$M = M_0 + \{M_s\}_{s=1}^{N} \tag{1}$$

where M indicates the total model and M_0 corresponds to the normal or fault-free system model and M_s, for $s = 1, 2, ..., N$ represents the s^{th} faulty model.

It is considered throughout this paper that the normal system behaviour and all the possible faults of the system can be expressed by the following discrete stochastic nonlinear state space models indexed by $M = M_0, M_1, ..., M_N$.

$$x_k^{(M)} = f^{(M)}(x_{k-1}^{(M)}, u_{k-1}^{(M)}, v_{k-1}^{(M)}) \tag{2}$$

$$y_k^{(M)} = g^{(M)}(x_k^{(M)}, n_k^{(M)}) \tag{3}$$

where $f^{(M)}$ and $g^{(M)}$ are the nonlinear state transition function and the nonlinear measurement function for the respective models and both are assumed to be known and bounded. x_k is the system state vector at the sample instant k, u_k is the system input and y_k is the measurement vector. The system noise v_k accounts for unknown disturbances in the system and the measurement noise n_k accounts for sensor inaccuracies. The noises v_k and n_k are two sequences of independent and identically distributed random variables with zero mean and its respective known covariance matrices are denoted as $Q = \mathrm{E}(v_k v_k^{\mathrm{T}})$ and $R = \mathrm{E}(n_k n_k^{\mathrm{T}})$, where E(.) denotes the mathematical expectation operator.

The two main steps considered to provide solution to the FDI problem are as follows:

- *Fault detection:* This step decides a model shift or detects a change from the normal model M_0 to one of the faulty models $\{M_s\}_{s=1}^{N}$ and also estimates the time t_f at which the change has occurred.
- *Fault isolation:* This step determines which of the faulty model among $\{M_s\}_{s=1}^{N}$, the considered system has moved to.

The model given by (2) and (3) is generic enough to define system faults as well as sensor faults. It must be noted that the calibration faults are considered as a key source of errors (Balaban et al., 2009; Sharma et al., 2010). The calibration error in the sensor can occur either due to the bias fault or scaling fault. This paper specifically emphasizes on detection and isolation of component fault in the system and bias fault in the sensor which are most frequently occurring in practice.

One of the greatest difficulty in FDI approach for the nonlinear system described by (2) and (3) is the presence of unknown and unmeasured state variables. Generally, the solution to the above problem is provided by the design of state estimators or filters. Hence, the model-based FDI approach proposed in this work employs a bank of UPFs where each UPF corresponds to a known possible fault and runs in parallel with a UPF that corresponds to the normal model.

3. BAYESIAN STATE ESTIMATION

The dynamic state estimation problem is usually solved by using recursive Bayesian state estimation methods which involves the construction of the probability density function (pdf) of the current state x_k, given the measurements up to time k, i.e., $y_{1:k}$. So the key in calculating the conditional pdf $p(x_k \mid y_{1:k})$ is Bayes theorem and the estimation of this pdf normally involves two steps- prediction and update. This section focuses on the state estimation (filtering) algorithms that form the basis for the development of a new FDI approach.

3.1 Particle Filter

Sequential Monte Carlo (SMC) methods are a set of simulation based methods which provide a convenient and attractive approach for computing the conditional pdf $p(x_k \mid y_{1:k})$. The PFs are based on SMC method which can deal with both the nonlinear and non-Gaussian state estimate estimation problem (Arulampalam et al., 2002). The basic idea of PF is to approximate $p(x_k \mid y_{1:k})$ using a set of particles (random samples) $\{x_k^i\}_{i=1}^{N_p}$ with associated weights $\{w_k^i\}_{i=1}^{N_p}$ as

$$p(x_k \mid y_{1:k}) \approx \sum_{i=1}^{N_p} w_k^i \delta(x_k - x_k^i) \tag{4}$$

where δ is the Dirac delta function. As the number of particles, N_p increases, the PF tends to provide a good approximation to $p(x_k \mid y_{1:k})$. Furthermore, it has been a very important alternative to Kalman-based filters because of its generality and robustness. The potential advantage of the PF is that the complete probability distribution information of the state is obtained, and not just the expectation of the state estimate.

The concept of importance sampling is used to obtain the particles and its associated weights. The particles x_k^i are generated from a known density called importance density or proposal density $\pi(x_k \mid y_{1:k})$ which is easy to sample from, and the corresponding weights of the particles are defined as

$$w_k^i \alpha \frac{p(x_k^i \mid y_{1:k})}{\pi(x_k^i \mid y_{1:k})} \tag{5}$$

If the density function $\pi(x_k \mid y_{1:k})$ is only dependent on the current measurement y_k and the past state x_{k-1}, then the importance weights can be updated as

$$w_k^i \alpha w_{k-1}^i \frac{p(y_k \mid x_k^i)\, p(x_k^i \mid x_{k-1}^i)}{\pi(x_k^i \mid x_{k-1}^i, y_k)} \tag{6}$$

The PF normally uses the transition prior density $p(x_k^i \mid x_{k-1}^i)$ as the proposal density from which the particles are drawn as

$$x_k^i \sim p(x_k^i \mid x_{k-1}^i) \tag{7}$$

However, as this proposal density is independent of current measurement y_k, the states are estimated without the knowledge of measurements (Ristic et al., 2004).

The assumption in (7) is used to obtain a simple form of importance weights from (6) as

$$w_k^i = p(y_k \mid x_k^i) \tag{8}$$

and then the normalized weights are calculated as

$$w_k^i = \frac{p(y_k \mid x_k^i)}{\sum_{i=1}^{N_p} p(y_k \mid x_k^i)} \tag{9}$$

After a few iterations, the weights of most particles become insignificant due to the modelling errors and noise which makes the particles drift away from the true state. As a result, the problem of degeneracy occurs which cannot be avoided because the variance of the importance weights increases over time (Doucet et al., 2000). Therefore, this problem is solved by a resampling technique which removes the particles with small normalized importance weights and concentrates upon particles with large weights. Thus, a new particle set is generated by sampling with replacement from the original set $\{x_k^i\}_{i=1}^{N_p}$ with probability $\mathrm{pr}(x_k^j = x_k^i) = w_k^i$ and j corresponds to the particle index after resampling.

3.2 Unscented Particle Filter

The proposal density $\pi(x_k \mid x_{k-1}, y_k)$ in the particle filtering algorithm can very well approximate the true posterior density by incorporating the most recent measurement y_k through a bank of UKFs. Such PF using UKF to proposal density is referred as UPF (Van der Merwe et al., 2000). The idea is to use for each particle (index *i*), a separate UKF to generate and propagate a Gaussian proposal distribution; that is,

$$\pi(x_k^i \mid x_{k-1}^i, y_k) = \mathcal{N}(x_k^i; \hat{x}_k^i, P_k^i) \tag{10}$$

where $\hat{x}_k^i$ and P_k^i are estimates of the mean and covariance of a particle (index *i*) computed by UKF at time k using measurement y_k. In summary, the unscented particle filtering algorithm for the current time step k is as follows (Jayaprasanth and Kanthalakshmi, 2016):

a) For $i = 1 : N_p$
- Run UKF
  $[\hat{x}_k^i, P_k^i] = \mathrm{UKF}[x_{k-1}^i, P_{k-1}^i, y_k]$
- Draw a particle from the proposal density as $x_k^i \sim \mathcal{N}(x_k^i; \hat{x}_k^i, P_k^i)$

End

b) For $i = 1 : N_p$
- Calculate importance weight for each particle as $w_k^i = \dfrac{p(y_k \mid x_k^i)\, p(x_k^i \mid x_{k-1}^i)}{\pi(x_k^i \mid x_{k-1}^i, y_k)}$
- Normalize: $w_k^i = \dfrac{w_k^i}{\sum_{i=1}^{N_p} w_k^i}$

End

c) Resample to get an updated particle set $\{x_k^j, i^j\}_{j=1}^{N_p}$, where *j* refers to the particle index after resampling. Now the updated relationship is represented as $\text{parent}(j) = i$.

d) For $j = 1 : N_p$

- Assign Covariance: $P_k^j = P_k^{i^j}$

End

The above steps form a single iteration and are recursively applied at each instant k.

4. FAULT DETECTION AND ISOLATION ALGORITHM

The model-based FDI methods are by nature the most powerful fault diagnosis methods. The UPF based LLR approach for detection and isolation of fault is discussed in the following sections.

4.1 FDI based LLR Test

The more versatile approach in fault diagnosis problem is modelling faults as changes in system parameters θ which are reflected by changes in the (2) and/or (3). One of the popular ways to monitor changes in θ is to monitor the likelihood function of the measurements, $p(y_k \mid \theta)$ under the hypothesis testing methods (Alrowaie et al., 2012). The hypothesis testing method involves a null hypothesis and an alternative hypothesis which is defined as follows:

Null Hypothesis $H_0 : \theta = \theta_0$

Alternative Hypothesis $H_1 : \theta \neq \theta_0$

Suppose if $\theta = \theta_0$ then it is assumed that the null hypothesis is true but instead if $\theta \neq \theta_0$ then it is considered that the alternative hypothesis is deemed to be true.

In this paper, the normal model and the faulty model is referred as null hypothesis and alternative hypothesis respectively. Hence, if a faulty condition is occurred in the actual system then it is assumed that alternative hypothesis is true. These hypotheses are tested by performing the well-known LLR test. Neyman-Pearson lemma states that this LLR test is the most powerful test for testing hypotheses. This test is conducted in order to monitor the logarithm of the ratio between the likelihood function of the measurements from the faulty model and from the normal model. The LLR test statistic which is used in this paper is defined as

$$\Lambda^k = \log\left[\frac{\sup_\theta \{p(y_k \mid \theta)\}}{p(y_k \mid \theta_0)}\right] \tag{11}$$

where sup is the supremum function.

The evaluation of the LLR test statistic can be successful only if the estimation of the likelihood function is maximum which is commonly known as maximum likelihood estimation (MLE). But the estimation of likelihood function is not straight-forward due to the hidden states in the system model. So these states of the system are estimated using the Bayesian filters and then from the estimated states, the likelihood function of the measurements is evaluated. Therefore, the selection of an optimal filter plays a vital role in maximizing the likelihood estimate. Hence, it must be noted that for the system subjected to fault at the unknown fault onset time t_f, the approach of modelling faults as changes in system parameters θ is replaced by its MLE, $p(y_k \mid x_k)$.

Consider if there are N faulty models then the problem of fault isolation is to identify the most likely alternative hypothesis among the following N hypotheses:

Alternative Hypothesis $H_1 : \theta = \theta_1$

$\vdots$

Alternative Hypothesis $H_N : \theta = \theta_N$

As a result, N LLR test statistics are obtained and the hypothesis corresponding to the largest value of test statistic Λ^k is accepted. The larger value of Λ^k indicates that its corresponding faulty model has a maximum probability to provide good representation of the system fault. It is so because the estimate of the likelihood function of this faulty model is of higher value compared to that of other faulty models.

4.2 UPF based LLR Approach to FDI

Li and Kadirkamanathan combined the LLR test with PF and developed the particle filtering based LLR method for detection and isolation of faults (Li and Kadirkamanathan, 2001). In the PF based FDI approach, the PF assumes the transition prior as proposal density which does not take the current measurement in to account. The performance of this filter may diverge because of such weaker assumption. In this paper, the LLR test is combined with the UPF for FDI in stochastic nonlinear systems in order to overcome the difficulties faced in the above method. The UPF based FDI approach presented here uses the UKF as the proposal density in the UPF algorithm which is dependent on the current measurement.

Consider the stochastic nonlinear system governed by one of $N+1$ models given by (2) and (3). In order to identify the faulty model that best describes the fault in the system, UPF is implemented for each of these faulty models M_s, for $s = 1, 2, ..., N$ and all these filters are made to run in parallel with an UPF implemented for the normal model M_0 and the corresponding LLR test statistic is evaluated.

The primary step in the UPF based LLR approach to FDI is to compute the likelihood function for each hypothesized model by using UPF, and then activating in parallel N LLR tests for all the faulty models. The LLR test statistic is calculated as

$$\Lambda^k(M_s) = \log\left[\frac{p(y_k \mid x_k^{(M_s)})}{p(y_k \mid x_k^{(M_0)})}\right] \tag{12}$$

where $M_s = M_1, M_2, ..., M_N$ represents the N faulty models and Λ^k is calculated for each of the faulty model. $p(y_k \mid x_k^{(M_s)})$ and $p(y_k \mid x_k^{(M_0)})$ are the likelihood functions of the measurement y_k for the faulty model and normal model respectively. These likelihood functions are estimated at each instant k from the hidden state vector x_k which are recursively estimated using the UPF. This proposed approach guarantees that the UPF propagates the particles towards the likelihood function $p(y_k \mid x_k^{(M_s)})$ and ensures maximum likelihood for one of the faulty model which signifies the system fault.

In this approach, N_p particles are generated from the UPF at each instant k for each of the faulty model as $\{x_k^{(M_s)i} : i = 1, ..., N_p\}$ and the normal model as $\{x_k^{(M_0)i} : i = 1, ..., N_p\}$. The likelihood function $p(y_k \mid x_k^{(M_s)})$ is evaluated for the faulty model by taking average of likelihood obtained for each of the particle (index *i*) as

$$p(y_k \mid x_k^{(M_s)}) = \frac{1}{N_p}\sum_{i=1}^{N_p} p(y_k \mid x_k^{(M_s)i}) \tag{13}$$

The likelihood function for the normal model is also evaluated in the same manner as

$$p(y_k \mid x_k^{(M_0)}) = \frac{1}{N_p}\sum_{i=1}^{N_p} p(y_k \mid x_k^{(M_0)i}) \tag{14}$$

For both the above cases, the likelihood is computed for each of the particle (index *i*) using the formula,

$$p(y_k \mid x_k^i) = \frac{1}{\sqrt{(2\pi)^m\left[\det(R)\right]}}\exp\left\{-\frac{1}{2}r_k^{(i)\mathrm{T}}R^{-1}r_k^{(i)}\right\} \tag{15}$$

where $r_k^{(i)}$ and is the prediction error based on the i^{th} particle which corresponds to the difference between the actual measurement and predicted measurement of the system with m-dimensional state vector and R is the measurement noise covariance. The critical term $r_k^{(i)\mathrm{T}}R^{-1}r_k^{(i)}$ in (15) is the square of the prediction error normalized by its covariance based on the i^{th} particle.

The decision function h_k for fault detection based on LLR between the two hypotheses, H_0 and H_1 is given by

$$h_k = \max_{1 \le i \le k} \Lambda_i^k \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \tau \tag{16}$$

where Λ_i^k corresponds to individual LLR value evaluated for the period from i to k. $\tau > 0$ is a threshold which is chosen in such a way to provide a reasonable tradeoff between false and missing alarms. Equation (16) suggests to accept H_1 whenever $h_k > \tau$ or else accept H_0.

Likewise, the decision function d_k involving double maximization for both detection and isolation of the fault from the N possible faulty models is given by

$$d_k = \max_{1 \le i \le k}\ \max_{M_1 \le M_s \le M_N} \Lambda_i^k(M_s) > \tau \tag{17}$$

The solution of (17) indicates which of the N faulty models has the maximum value of LLR and that corresponding model can be considered to provide a good representation of the fault in the system. The unknown fault onset time t_f in the true system is calculated as

$$\hat{t}_f = \min\{k : d_k > \tau\} \tag{18}$$

where $\hat{t}_f$ is the estimated fault time which gives the minimum value of k among all $d_k > \tau$.

Generally, if the states of a system are independent to each other, then the number of required particles, N_p increases exponentially with the state dimension. The UPF algorithm has the promising capability to provide more accurate estimated states with very minimum number of particles compared to PF because it is more stable and allows one to control the rate at which the tails of the proposal density go to zero. So the PF based FDI approach usually requires large number of particles, whereas, the proposal density used in the UPF in the proposed approach largely contributes for reduction in the required number of particles to achieve improved FDI performance. In practice, the number of particles required in the filter design is usually decided empirically by conducting some initial experiments on the considered system (Chen et al., 2005). The computation time of the UPF based FDI algorithm is comparatively higher than the PF based FDI algorithm but the time required to run an algorithm is not an issue with the advent of fast processors in the market. This paper assumes that the faults defined by the models $\{M_s\}_{s=1}^N$ are not dependent on each other and also do not occur simultaneously as the likelihood of the simultaneous faults are generally not very high.

5. RESULTS AND DISCUSSIONS

This section presents the efficacy of the proposed UPF based LLR method for FDI by conducting simulation studies on two highly nonlinear stochastic systems- a chemical reactor and a three phase induction machine.

5.1 Application to a Chemical Reactor System

5.1.1 System Description

The chemical reactor system considered in this section is taken from (Prakash and Senthil, 2008). A chemical reactor that is mostly commonly used in the process industry is continuous stirred tank reactor (CSTR). It is used primarily for liquid phase reactions. The CSTR considered here is a constant volume reactor in which an irreversible and exothermic reaction A$\rightarrow$ B takes place. It is normally operated under steady state conditions. Due to the occurrence of non-isothermal nature of the reaction, a cooling jacket is used to remove heat from the reactor. A mathematical model of the reactor derived from its mass-balance relationship takes the following form:

$$\frac{dC_A}{dt} = \frac{q}{V}\left(C_{Af} - C_A\right) - K_0 C_A e^{\left(\frac{-E}{RT}\right)} \tag{19}$$

$$\frac{dT}{dt} = \frac{q}{V}\left(T_f - T\right) - \frac{\left(-\Delta H\right) K_0 C_A}{\rho C_p} e^{\left(\frac{-E}{RT}\right)} + \frac{\rho_c C_{pc}}{\rho C_p V} q_c \left\{1 - e^{\left(\frac{-\mathrm{hA}}{q_c \rho C_p}\right)}\right\}\left(T_c - T\right) \tag{20}$$

Table 1. Steady-state values for CSTR.

Process variable	Steady state operating condition
Product concentration (C_A)	0.0989 mol/L
Reactor temperature (T)	438.7763 K
Coolant flow rate (q_c)	103 L/min
Process flow rate (q)	100 L/min
Feed concentration (C_{Af})	1 mol/L
Feed temperature (T_f)	350 K
Inlet coolant temperature (T_c)	350 K
CSTR volume (V)	100 L
Heat transfer term (hA)	7 x 10^5 cal/(min K)
Reaction rate constant (K_0)	7.2 x 10^{10} min^{-1}
Activation energy term (E/R)	1 x 10^4 K
Heat of reaction $(-\Delta H)$	-2 x 10^5 cal/mol
Liquid density (ρ, ρ_c)	1000 g/L
Specific heats (C_p, C_{pc})	1 cal/(g K)

The state vector x and the measurement vector y for this system is $[C_A \quad T]^{\mathrm{T}}$ and $[T]$ respectively. The system parameters and its steady state operating data are given in Table 1. The steady state operating condition of the states are chosen as initial states of the system. The system and measurement noise covariance matrices are assumed as

$$Q = \begin{bmatrix} (0.00098)^2 & 0 \\ 0 & (0.438)^2 \end{bmatrix} \quad \text{and} \quad R = \left[(0.438)^2\right]$$

5.1.2 FDI Algorithm Performance on CSTR System

The simulation study on the CSTR system is carried out by considering two kinds of fault- the component fault $(M = M_1)$ and sensor fault $(M = M_2)$. The UPF-based LLR approach for FDI developed in this paper is used to detect and isolate these two faults and in order to highlight its effectiveness, it is compared with the PF based LLR approach. Furthermore, a performance comparison of both the approaches is carried out under the same conditions.

Simulations have been carried out with a sampling time of 0.083 min as the CSTR is a slow dynamical process. The analysis of this system under fault is carried out by considering the number of particles, $N_p = 30$ for UPF based approach and $N_p = 200$ for PF based approach. The false alarms can be normally minimized by selecting higher value of threshold for decision making and the missing alarms can be minimized by choosing its lower value. Hence, through simulation studies, the decision making threshold for this faulty system is chosen as $\tau = 20$ because the false alarms and missing alarms are greatly reduced under this value. The performance of the proposed approach on the CSTR system is analyzed both under normal conditions (error-free faulty model) and in the presence of modelling errors as follows:

Case 1: Normal conditions

The component fault is considered in this system by assuming a leakage fault in the cooling jacket of reactor which therefore results in the reduction of coolant flow rate. Hence, the component fault is modelled by the reduction in the nominal coolant flow rate by 3%. This fault is simulated to occur in the actual system at instant $k = 45$ at which the system model is shifted from $M = M_0$ (normal model) to $M = M_1$ (component fault). The sensor fault is modelled by a jump from the nominal value provided by the temperature sensor to 2% faulty biased sensor value in order to test the sensitivity of the proposed approach over the existing approach. The sensor fault is simulated in this case to occur at instant $k = 60$ at which the system model is shifted from $M = M_0$ to $M = M_2$ (sensor fault).

It is observed from Figs. 1 and 2, the LLR computed by the proposed UPF based approach suddenly raises to high value, $\Lambda = 134$ at the time of detection of component fault compared to that of $\Lambda = 54$ in the PF based approach. Figs. 1 and 2 also indicate that there is no sensor fault as its LLR value lies around zero. Hence, the component fault can be detected and isolated by proper evaluation of LLR test statistic.

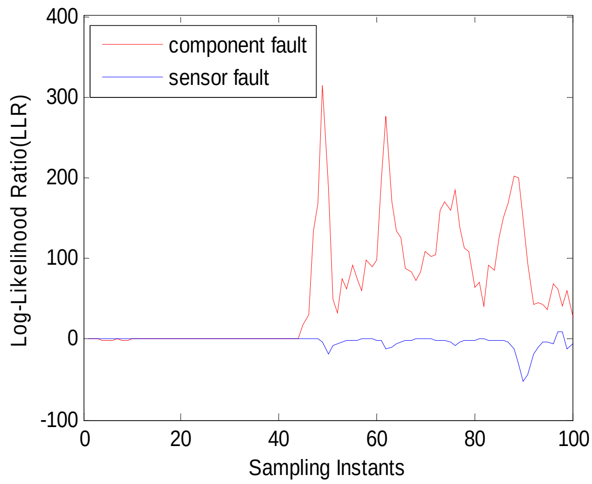

Fig. 1. Detection and isolation of component fault in CSTR using UPF based LLR algorithm.

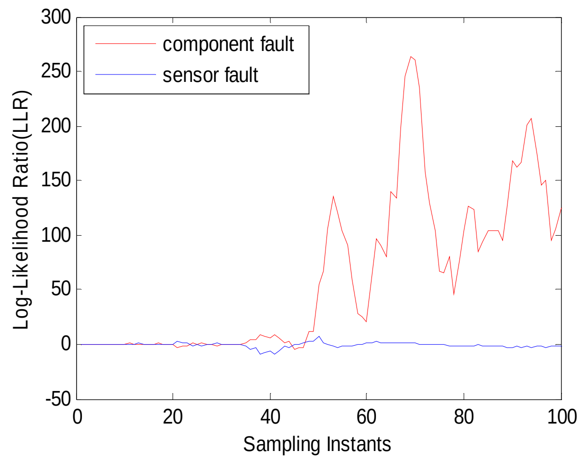

Fig. 2. Detection and isolation of component fault in CSTR using PF based LLR algorithm.

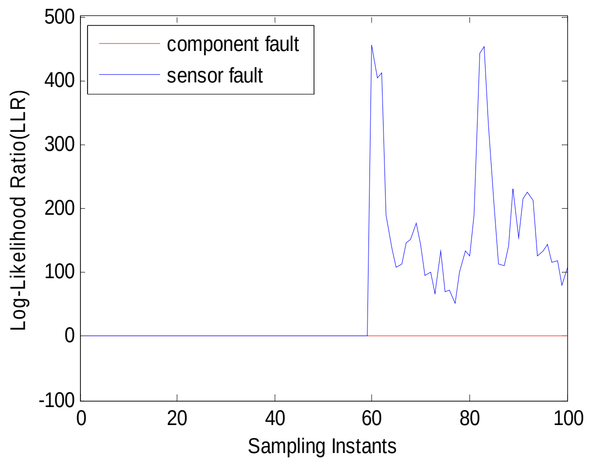

Fig. 3. Detection and isolation of sensor fault in CSTR using UPF based LLR algorithm.

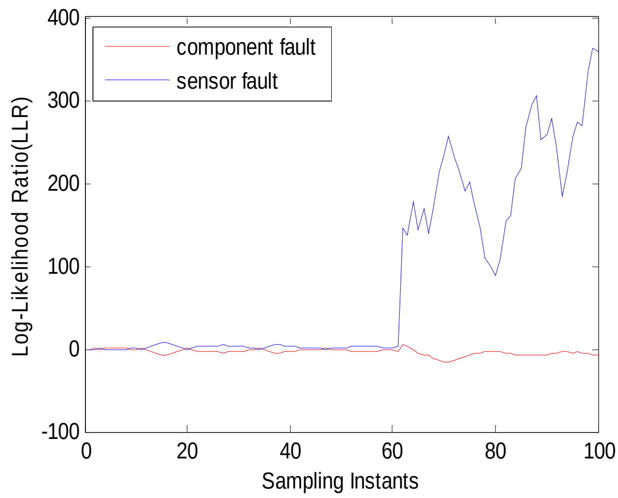

Fig. 4. Detection and isolation of sensor fault in CSTR using PF based LLR algorithm.

Similarly, Figs. 3 and 4 show that at the time of detection of sensor fault, the proposed approach has a LLR test statistic of large peak, $\Lambda = 455$ compared to that of $\Lambda = 146$ in the PF based approach. It is inferred that the UPF based approach provides sharp increase in LLR at the time of detection of such abrupt faults, whereas, in the PF based approach, the LLR response to abrupt faults is not so rapid. Also the large LLR obtained from the proposed algorithm under the occurrence of faults guarantees a very minimum chance of missing alarm.

Table 2. FDI algorithm performance on CSTR under normal conditions

Type of Fault	UPF based FDI		PF based FDI	
	Fault detection instant	LLR test statistic	Fault detection instant	LLR test statistic
Component fault	47	134	51	54
Sensor fault	60	455	62	146

The performance comparison of both the FDI approaches under different faults in CSTR is shown in Table 2. The true component fault onset instant is $t_f = 45$. The component fault is detected at $\hat{t}_f = 47$ in the UPF based approach and $\hat{t}_f = 51$ in the PF based approach among which the new method gives a relatively closer estimate of fault onset instant. The true sensor fault onset instant is $t_f = 60$. The sensor fault is also detected at $\hat{t}_f = 60$ in the proposed approach but it is detected only at $\hat{t}_f = 62$ in the PF based approach.

Table 3. Comparison of fault detection delay under normal conditions for CSTR.

Type of Fault	Average fault detection delay	
	UPF based FDI	PF based FDI
Component fault	1.2	4
Sensor fault	0	2.2

Further, 100 Monte Carlo simulations are carried out to assess the performance of the proposed approach. Table 3 shows the average fault detection delay for both the approaches for 100 Monte Carlo runs, clearly indicating the superior performance of the UPF based FDI approach over PF based approach.

Case 2: Presence of modelling errors

The problem that arises in the generic PF under the presence of modelling error is that its likelihood function becomes very minimum because of the transition prior as proposal which can move the weights of particles towards zero. As a result, these particles tend to lie either in the tail of the likelihood or far away from the likelihood region. Thus, the PF exhibits less robustness to the plant-model mismatch. The robustness issues with the PF are also reported in (Shenoy et al., 2011). Such problems can also cause the PF based FDI approach to perform poorly in the presence of modelling errors. The UPF generally has the capability to move the particles towards the regions of high likelihood. This potential advantage causes the stability of the UPF to be better than the PF even under plant-model mismatch. Hence, the UPF based FDI approach can also prove to be more robust than the PF based approach in the presence of modelling error.

This case illustrates the performance of both the FDI approaches in the presence of modelling error. In this case also, the component fault is simulated to occur in the actual system causing reduction in the coolant flow rate by 3% and the sensor fault is simulated to occur with 2% biased sensor. But the component fault is modelled by considering the reduction in the coolant flow rate by 6%, thereby introducing model error. Similarly, the sensor fault is modelled by considering 4% biased sensor. As described in the previous case, the component fault and sensor fault are made to occur at instant $k = 45$ and $k = 60$ respectively.

It is noticed from Figs. 5 and 6, the component fault is detected in the proposed UPF based approach at $\hat{t}_f = 48$ at which large LLR test statistic $\Lambda = 81$ is obtained compared to that of $\Lambda = 28$ obtained in the PF based approach at $\hat{t}_f = 56$. Figs. 7 and 8 shows that the sensor fault is detected in the proposed approach at $\hat{t}_f = 60$ at which $\Lambda = 186$ is obtained compared to that of $\Lambda = 75$ obtained in the PF based approach at $\hat{t}_f = 66$. It is observed from Figs. 5 to 8 that the proposed approach in the presence of modelling errors assures very less missed alarm rate compared to that of PF based approach. Thus, the new UPF based FDI approach exhibits more robustness than the existing PF based approach.

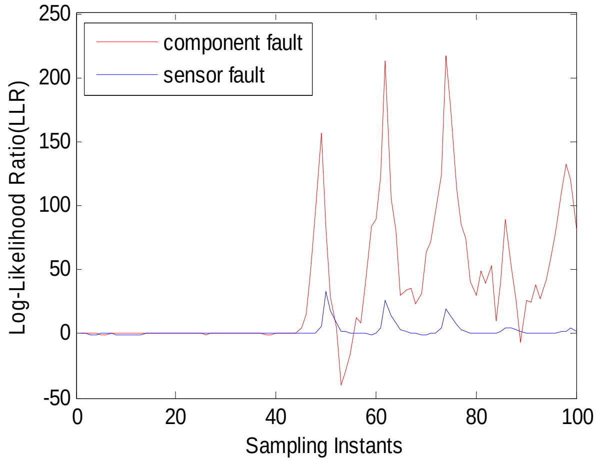

Fig. 5. Detection and isolation of component fault in CSTR using UPF based LLR algorithm in the presence of modelling error.

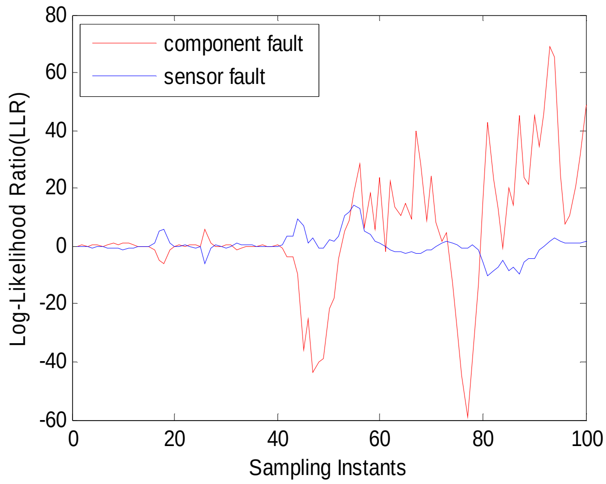

Fig. 6. Detection and isolation of component fault in CSTR using PF based LLR algorithm in the presence of modelling error.

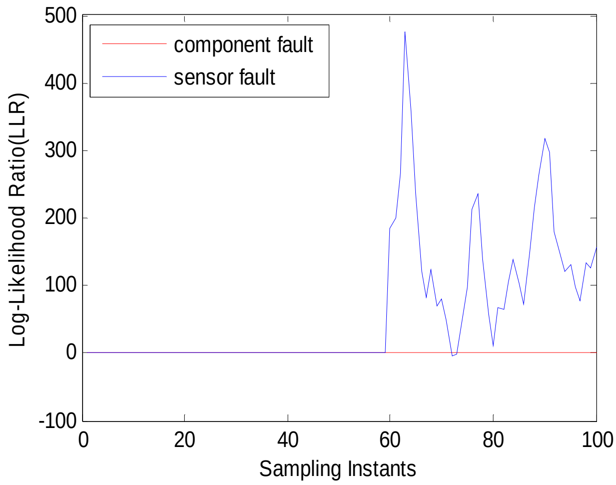

Fig. 7. Detection and isolation of sensor fault in CSTR using UPF based LLR algorithm in the presence of modelling error.

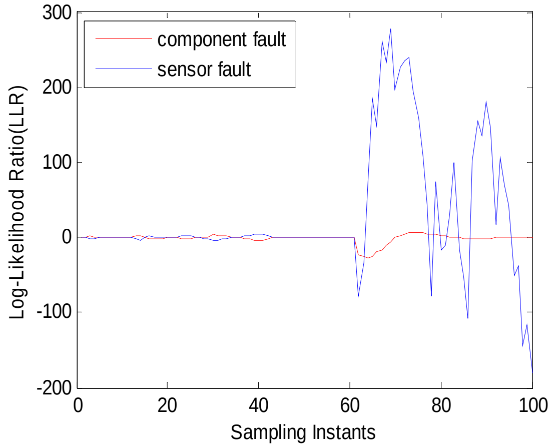

Fig. 8. Detection and isolation of sensor fault in CSTR using PF based LLR algorithm in the presence of modelling error.

Table 4. Comparison of fault detection delay under modelling errors.

Type of Fault	Average fault detection delay	
	UPF based FDI	PF based FDI
Component Fault	3.4	9.7
Sensor Fault	0.6	7.1

The average fault detection delay calculated for both the approaches is shown in Table 4 which indicates that the FDI performance using UPF based LLR approach can outperform the PF based LLR approach in the presence of modelling errors.

5.2 Application to a Three Phase Induction Machine

5.2.1 System Description

The highly nonlinear three phase induction machine considered in this section is taken from (Kandepu et al., 2008). The three phase induction machines are widely used in industrial drives because they are rugged, reliable and economical. Nowadays, induction machines are very widely used in variable-frequency drive (VFD) applications. The state space model for a symmetrical three phase induction machine is,

$$\begin{aligned}
\dot{x}_1 &= m_1 x_1 + z_1 x_2 + m_2 x_3 + z_2, \\
\dot{x}_2 &= -z_1 x_1 + m_1 x_2 + m_2 x_4, \\
\dot{x}_3 &= m_3 x_1 + m_4 x_3 + (z_1 - x_5) x_4, \\
\dot{x}_4 &= m_3 x_2 - (z_1 - x_5) x_3 + m_4 x_4, \\
\dot{x}_5 &= m_5 (x_1 x_4 - x_2 x_3) + m_6 z_3
\end{aligned} \tag{21}$$

where the state variables x_1 and x_2 are the stator flux components, states x_3 and x_4 are the components of rotor flux and x_5 is angular velocity of the rotor. All the state variables are normalized. The system inputs, the frequency and the amplitude of stator voltage are indicated by z_1 and z_2 respectively, and the load torque is indicated by z_3 . The parameters m_1 to m_6 used in the system state equation are the parameters depending on the considered induction machine. The output (measurement) equations with parameters m_7 and m_8 are given as

$$\begin{aligned}
y_1 &= m_7 x_1 + m_8 x_3, \\
y_2 &= m_7 x_2 + m_8 x_4
\end{aligned} \tag{22}$$

where the outputs y_1 and y_2 represents the normalized stator currents. For simulation, the system inputs and the model parameters are obtained from (Kandepu et al., 2008).

5.2.2 FDI Algorithm Performance on Induction Machine

The experimental study is carried out for the induction machine by considering sensor bias fault in the measurement. In this section, the sensor fault in the outputs y_1 and y_2 are considered as sensor-1 fault $(M = M_1)$ and sensor-2 fault $(M = M_2)$ respectively. The sensor-1 and sensor-2 faults are modelled as a jump in the nominal value of sensor to 5% faulty biased sensor value. The sensor-1 fault is simulated to occur in the true system at an earlier instant $k = 5$ at which the system model is shifted from $M = M_0$ to $M = M_1$. Similarly, the sensor-2 fault is assumed to occur at instant $k = 10$ at which the system model is shifted from $M = M_0$ to $M = M_2$.

The initial condition of the normalized states of this system are considered as

$$x = [0.2 \quad -0.6 \quad -0.4 \quad 0.1 \quad 0.3]^{\mathrm{T}}$$

The system and measurement noise covariance matrices are chosen as $Q = 10^{-4} I_5$ and $R = 10^{-2} I_2$ respectively and I_n indicates the identity matrix of order $n \times n$. Simulations have been carried out with a sampling interval of 0.1sec. In this case, the particle count N_p is considered as 20 and 100 for the UPF based approach and PF based approach respectively. The decision making threshold for this system under fault is chosen as $\tau = 2$ which provides a reasonable tradeoff between false and missing alarms.

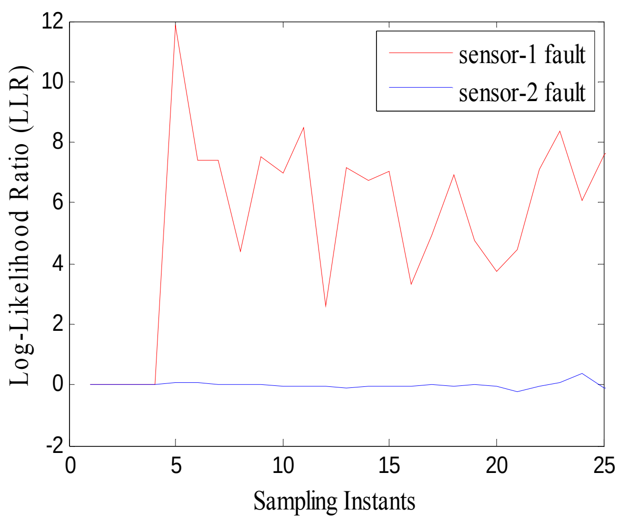

Fig. 9. Detection and isolation of sensor-1 fault in three phase induction machine using UPF based LLR algorithm.

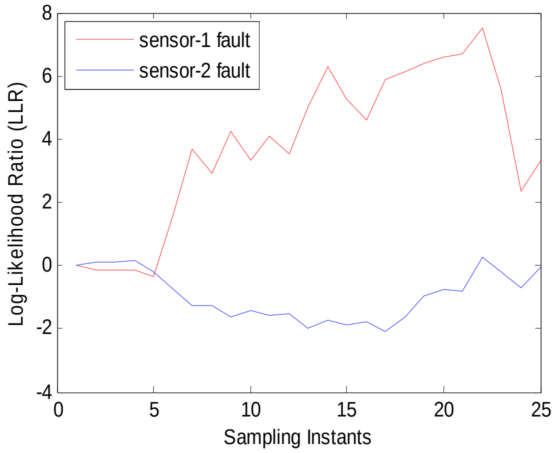

Fig. 10. Detection and isolation of sensor-1 fault in three phase induction machine using PF based LLR algorithm.

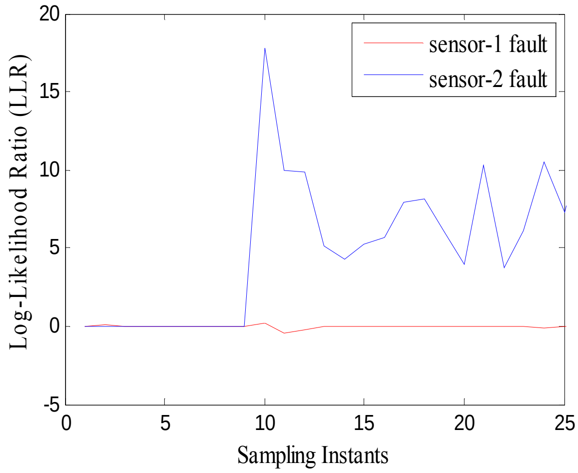

Fig. 11. Detection and isolation of sensor-2 fault in three phase induction machine using UPF based LLR algorithm.

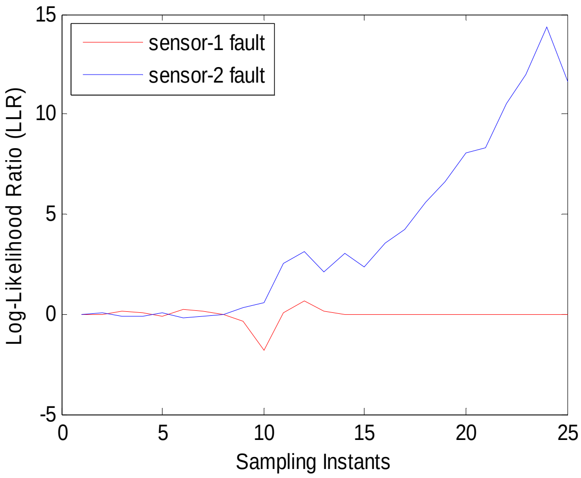

Fig. 12. Detection and isolation of sensor-2 fault in three phase induction machine using PF based LLR algorithm.

The true sensor-1 fault onset instant is $t_f = 5$. It is noticed from Figs. 9 and 10, the sensor-1 fault is detected in the proposed UPF based approach at $\hat{t}_f = 5$ at which larger LLR test statistic, $\Lambda = 12$ is obtained compared to that of $\Lambda = 3.6$ obtained in the PF based approach at $\hat{t}_f = 7$. The true sensor-2 fault onset instant is $t_f = 10$. Figs. 11 and 12 shows that the sensor-2 fault is detected in the proposed approach at $\hat{t}_f = 10$ at which $\Lambda = 18$ is attained compared to that of $\Lambda = 2.5$ obtained in the PF based approach at $\hat{t}_f = 11$.

Table 5. FDI algorithm performance on Induction machine.

Type of Fault	UPF based FDI		PF based FDI	
	Fault detection instant	LLR test statistic	Fault detection instant	LLR test statistic
Sensor-1 fault	5	12	7	3.6
Sensor-2 fault	10	18	11	2.5

Table 6. Comparison of fault detection delay for Induction machine.

Type of Fault	Average fault detection delay	
	UPF based FDI	PF based FDI
Sensor-1 fault	0	2.3
Sensor-2 fault	0.3	1.6

The performance comparison of both the FDI approaches under sensor faults in three-phase induction machine is shown in Table 5. Hence, it is clear from Table 5 that the proposed method provides relatively larger LLR value during the occurrence of fault and minimum fault detection instant than the PF based approach. Table 6 shows the average fault detection delay for both the approaches for 100 Monte Carlo simulations for the considered induction machine.

6. CONCLUSIONS

A new approach for solving the FDI problem in stochastic nonlinear systems has been developed by combining the UPF algorithm with the LLR test in the multi-model environment. This new UPF based FDI approach detects changes in the system behaviour and isolates a corresponding fault using a bank of UPFs running in parallel. Its effectiveness has been demonstrated through exhaustive simulation studies on a highly nonlinear chemical reactor system and a three phase induction machine. The Simulation results indicate clearly that the FDI performance of the proposed approach outperforms PF based approach in terms of efficacy by giving relatively minimum fault detection delay and large LLR test statistic during the occurrence of fault in the system. This approach is designed for detection of abrupt faults and is unable to handle incipient faults. It requires sufficient knowledge of the possible system faults. The UPF based approach for detection and isolation of faults provides higher degree of robustness than the PF based approach even in the presence of modelling errors.

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