Hybrid Complex Projective Synchronization of Complex Chaotic Systems Using Active Control Technique with Nonlinearity in the Control Input

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Abstract: In this paper, an active control method based on Lyapunov function is used to study hybrid complex projective synchronization (HCPS). In the complex space, the response system is asymptotically synchronized up to the drive system by the state transformation by using a complex scale matrix. An extension of projective synchronization from the field of real numbers to the field of complex numbers has been done in this paper; i.e., the scaling factors are complex. The unpredictability of the scaling factors in the proposed synchronization scheme can additionally increase the security of communication. This synchronization method is studied between two non-identical complex chaotic nonlinear 3-dimensional systems. We take Lorenz system as the driving system and Chen system as the response system. In order to demonstrate the asymptotic convergence of the error states, dead-zone nonlinearity input is imposed to the control input. The closed loop stability conditions based on Lyapunov function are derived. Finally, numerical simulations are presented to verify the results of the proposed scheme.

Keywords: Complex chaotic system; Hybrid complex projective synchronization; Active control; Dead-zone nonlinearity input; Lyapunov function

1. INTRODUCTION

Chaos is a very complex nonlinear phenomenon that shows some specific features such as crucially dependence to initial conditions, broad Fourier transform spectra, strange attractors and fractal properties of the motion in phase space. A tiny change in the initial conditions and the system parameters leads to an enormous difference in the long-term behaviour of the system; this is the main special feature of chaotic systems (Setoudeh, 2014).

Complex systems appear in many important fields of physics and engineering, for example, in the secure communication, complex variables (doubling the number of variables) can carry more transmitted information and additionally increase security of information (Mahmoud et al., 2007; Mahmoud et al., 2008; Moghtadaei et al; 2012). Research property of the chaotic complex system is difficult in the field of complex numbers; however, by separating the imaginary part and real part of chaotic complex system, this system can be converted to corresponding real number chaotic system. Many physical phenomena could be described by chaotic or complex chaotic systems, for example, the detuned laser systems and the amplitudes of electromagnetic fields. Several complex chaotic nonlinear systems such as complex chaotic Lorenz system (Mahmoud et al., 2007a), complex chaotic Chen and Lu systems (Mahmoud et al., 2007b) and complex chaotic coupled system (Wu et al., 2012) have been proposed in literature. The numerical simulations of the chaotic systems are very sensitive with respect to the numerical integration method and the numerical computing errors (Yao, 2010).

In 1990, Pecora and Carroll (Pecora and Carroll, 1990) introduced a method to synchronize two identical chaotic systems and showed that it was possible for some other chaotic systems to be completely synchronized (Faieghi and Delavari, 2012a; Faieghi et al., 2012b; Shutang and Fangfang, 2014). In the synchronization of chaotic systems, the output of the response system tracks the output of the drive system. If the synchronization occurs, the synchronization errors will tend to zero asymptotically. Control and synchronization of nonlinear dynamical systems like chaotic systems have attracted increasing attention in different fields, such as secure communication, optimization of nonlinear system performance, ecological systems, modeling brain activity, system identification and pattern recognition (Li et al., 2011; Florin et al., 2011; Feki, 2003; Bai et al., 2005). Moreover, in (Liu et al., 2010) the global convergence and the superlinear convergence of the new modified BFGS method for unconstrained optimization or complexity systems is introduced.

A wide variety of impressive approaches have been suggested in the literature for the stabilization and synchronization of the nonlinear systems such as the linear and nonlinear feedback method (Huang, 2004), time delay feedback method (Park and Kwon, 2005), back-stepping design (Wu and Lü, 2003), sliding mode control method (Djari, 2014; Mohadeszadeh and Delavari, 2017a), fuzzy sliding mode control method (Mohadeszadeh and Delavari, 2017b; Faieghi et al., 2012) and active control method (Das et al., 2013).

Various kinds of synchronization have been proposed such as complete synchronization (Zhu et al., 2009), anti-synchronization (Srivastava et al., 2014), modified generalized synchronization (Wu et al., 2012), phase synchronization (Breve et al., 2009), lag synchronization (Luo and Wang, 2013), projective synchronization (Peng et al., 2008), modified function projective synchronization (Sun et al., 2012), hybrid projective synchronization (Hu et al., 2008) and HCPS (Wei et al., 2013).

The transformation matrix in this type of synchronization (HCPS) is a square matrix, and its elements are complex. The transformation matrix makes an appropriate waging on the states of drive system. The scaling factors in transformation matrix play an important role in such a cases like chaotic secure communication where the states of the drive system are manipulated with the scaling factors to transmit to the communication channel as well as to improve the security of the useful information signal.This type of synchronization is considered as a generalization of several kinds of synchronization that have been appeared in the recent literatures. Although for HCPS of complex chaotic systems, different components of the complex system synchronized to different scale complex numbers (Wei et al., 2013), but for the other mentioned synchronizations of complex chaotic systems like projective synchronization, the scaling factors can be complex. For example, $\omega = \mu\, e^{j\rho}\eta$, where ω and η denote the complex states of drive and response systems, respectively, $\mu > 0$ denotes the zoom rate and $\rho \in [0, 2\pi)$ denotes the rotate angle.

In the recent years, synchronization of dynamical systems subjected to nonlinearity control input has received a lot of attentions. In fact, the control inputs of practical systems are usually subjected to nonlinearity as a consequence of physical limitations. The presence of nonlinearity in control input was indicated to cause a serious degradation of the system performance and induce a decreasing rate of the system response (Yau and Yan, 2008a; Kebriaei and Yazdanpanah, 2010; Li et al., 2012; Márton, 2009; Roopaei et al., 2010). In (Roopaei et al., 2010), synchronization of gyroscope system using adaptive fuzzy sliding mode control technique is investigated when dead-zone nonlinearity is imposed to the control input. In (Márton, 2009), the backlash nonlinearity is designed to impose to the control input of the motion control systems in order to analyze the control performance.

Input nonlinearities are inevitable in real applications, it can severely degrade closed-loop performance due to integrator windup and other effects. There is a little information among all of the referenced papers. Here in this paper we study the effect of nonlinearity via active control method for complex chaotic systems which is not investigated until now. In order to show the stability of the closed loop system we proposed some new theoretical results. But here, in this paper, we study the HCPS between two non-identical complex chaotic systems using the active control method in the presence dead-zone nonlinearity.

In our contribution, we pursue four main research aims. First, the states of the error converge to the origin asymptotically and stability of the proposed method are analytically proved. Second, the proposed novel scheme is achieved for complex chaotic systems; to the best of our knowledge there has been very little information about this. Third, our proposed method can be applied for a wide class of complex chaotic systems, such as fractional-order counterpart. Fourth, the sufficient criterion for the error states to converge to the origin in accordance to the nonlinearity in the control input are designed using active control technique.

The rest of the paper is organized as follows: section 2 briefly describes the chaotic complex nonlinear systems. In section 3, a general scheme of HCPS is reviewed. The proposed scheme will be achieved between two non-identical complex chaotic Lorenz as drive system and Chen as response system, in section 4. Finally, a concluding remark is given in section 5.

2. COMPONENT SYSTEMS

To study the hybrid complex projective synchronization (HCPS), we take Lorenz system as the driving system and Chen system as the response system. Hence, consider the complex chaotic nonlinear Lorenz system as follows (Lorenz, 1963):

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) \\ \dot{y}_1 = a_2 x_1 - y_1 - x_1 z_1 \\ \dot{z}_1 = {}^1\!/_2\,(\overline{x_1} y_1 + x_1 \overline{y_1}) - a_3 z_1 \end{cases} \tag{1}$$

where $X = (x_1, y_1, z_1)^T$ is the complex state vector, $X = X^r + jX^i$, $x_1 = {x_1}^r + j{x_1}^i$, $y_1 = {y_1}^r + j{y_1}^i$, $z_1 = {z_1}^r$, $j = \sqrt{-1}$. Dots represent derivatives with respect to time, an overbar denotes complex conjugate variables, superscripts r and i stand for the real and imaginary parts of the complex state vector X. By separating real and imaginary parts of (1), a 5-dimensional continuous real autonomous system will obtain as follows:

$$\begin{cases} \dot{x}_1{}^r = a_1({y_1}^r - {x_1}^r) \\ \dot{x}_1{}^i = a_1({y_1}^i - {x_1}^i) \\ \dot{y}_1{}^r = a_2 {x_1}^r - {y_1}^r - {x_1}^r {z_1}^r \\ \dot{y}_1{}^i = a_2 {x_1}^i - {y_1}^i - {x_1}^i {z_1}^r \\ \dot{z}_1{}^r = ({x_1}^r {y_1}^r + {x_1}^i {y_1}^i) - a_3 {z_1}^r \end{cases} \tag{2}$$

where a_1, a_2 and a_3 are positive parameters. System (2) exhibits a chaotic behaviour, when $(a_1, a_2, a_3)^T = \big(10, 28, (8/3)\big)^T$. The chaotic attractors of (1) with initial conditions of $(1 + j2, 3 + j0.5, 4)^T$ are depicted in Figure 1.

Now, let us consider the complex chaotic Chen system as follows (Chen et al., 1999):

$$\begin{cases} \dot{x}_2 = b_1(y_2 - x_2) \\ \dot{y}_2 = (b_2 - b_1) x_2 - x_2 z_2 + b_2 y_2 \\ \dot{z}_2 = {}^1\!/_2\,(\overline{x_2} y_2 + x_2 \overline{y_2}) - b_3 z_2 \end{cases} \tag{3}$$

where $Y = (x_2, y_2, z_2)^T$ is the complex state vector, $Y = Y^r + jY^i$, $x_2 = {x_2}^r + j{x_2}^i$, $y_2 = {y_2}^r + j{y_2}^i$, $z_2 = {z_2}^r$. By separating real and imaginary parts of (3), a 5-dimensional continuous real autonomous system will obtain as follows:

$$\begin{cases} \dot{x}_2{}^r = b_1({y_2}^r - {x_2}^r) \\ \dot{x}_2{}^i = b_1({y_2}^i - {x_2}^i) \\ \dot{y}_2{}^r = (b_2 - b_1){x_2}^r - {x_2}^r {z_2}^r + b_2 {y_2}^r \\ \dot{y}_2{}^i = (b_2 - b_1){x_2}^i - {x_2}^i {z_2}^r + b_2 {y_2}^i \\ \dot{z}_2{}^r = ({x_2}^r {y_2}^r + {x_2}^i {y_2}^i) - b_3 {z_2}^r \end{cases} \tag{4}$$

where b_1, b_2 and b_3 are positive parameters.
When $(b_1, b_2, b_3)^T = (28, 22, 1)^T$, system (4) exhibits a chaotic behaviour. The chaotic attractors of (3) with initial conditions of $(4.7 + j6, 0.2 + j7.5, 1)^T$ are depicted in Figure 2.

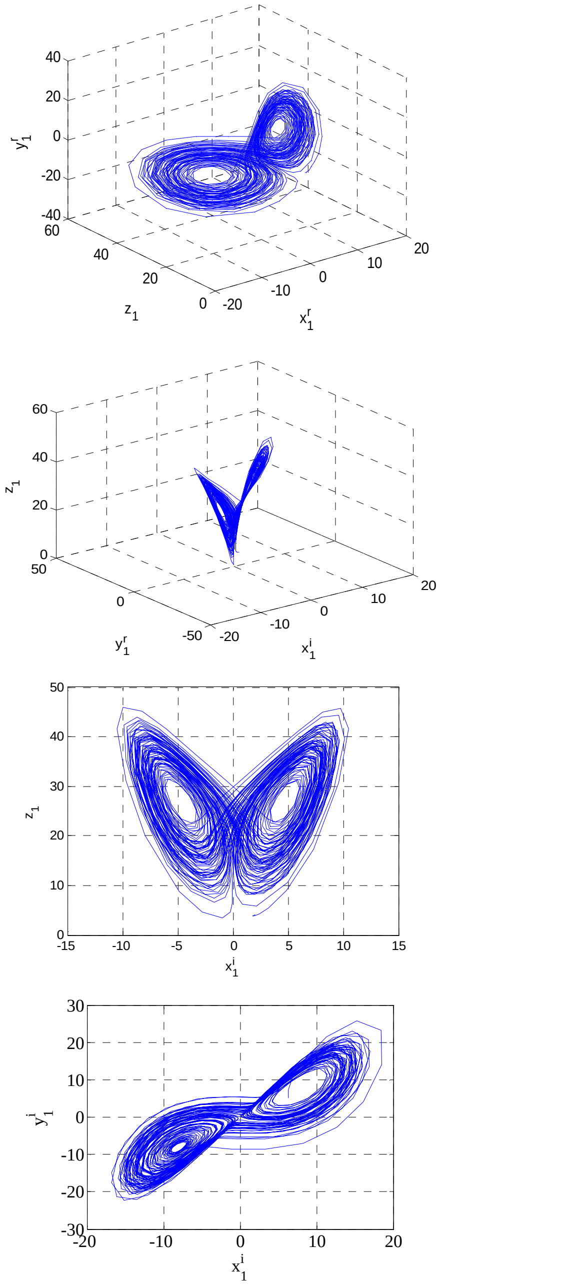

Fig. 1. 2D and 3D projections of chaotic attractors of complex chaotic Lorenz system.

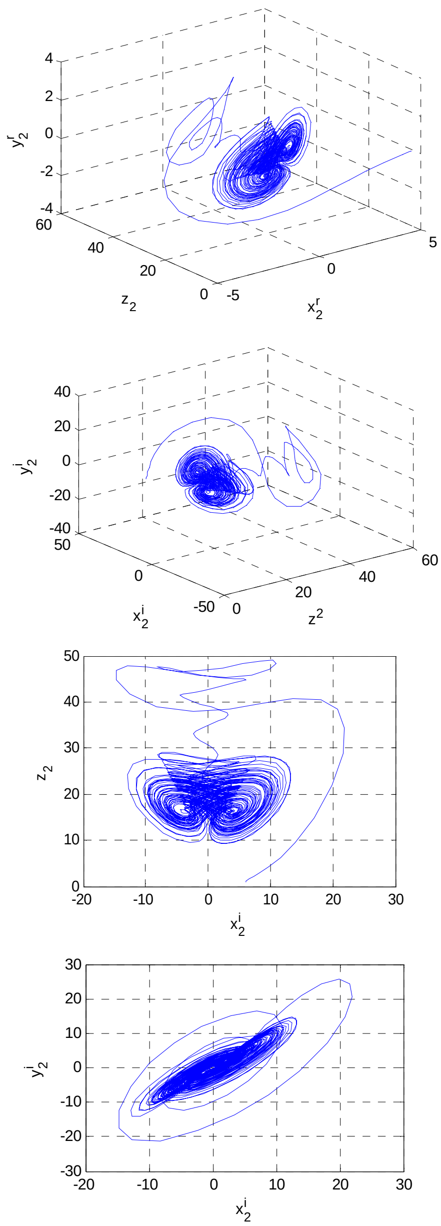

Fig. 2. 2D and 3D projections of chaotic attractors of complex chaotic Chen system.

3. A SCHEME TO ACHIEVE HCPS

3.1 HCPS of complex chaotic systems with active control

Consider two non-identical complex chaotic nonlinear systems. The drive system like system (1) is as follows:

$$\dot{X} = f(X(t)) \tag{5}$$

where $X \in C^n$ is a complex state vector, $f: C^n \to C^n$ is a vector of nonlinear complex functions. Now, consider the controlled response system like (3) in the following form:

$$\dot{Y} = g(Y(t)) + U \tag{6}$$

where $Y \in C^n$ is a complex state vector, $g: C^n \to C^n$ is a vector of nonlinear complex functions, $U \in C^n$ is a complex control function which will design in Section 3, $U = U^r + jU^i$, $U^r = (u_1{}^r, u_2{}^r, \dots, u_n{}^r)^T$, $U^i = (u_1{}^i, u_2{}^i, \dots, u_n{}^i)^T$.

In HCPS, the synchronization error between the drive system (5) and the controlled response system (6) is as follows:

$$E = Y - \Lambda X \tag{7}$$

where $E = E^r + jE^i$, $E^r = (e_1{}^r, e_2{}^r, \dots, e_n{}^r)^T$, $E^i = (e_1{}^i, e_2{}^i, \dots, e_n{}^i)^T$, and $\Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_n)$ is a complexscaling matrix.

Definition 1. For the drive system (5) and the controlled response system (6), it is said that HCPS can be achieved, if there exist a control function $U = U^r + jU^i$ such that:

$$lim_{t\to\infty}\|E\| = lim_{t\to\infty}\|Y - \Lambda X\| = 0 \tag{8}$$

In (8), $\|.\|$ is the Euclidean norm of a vector. Then by separating real and imaginary parts of (8), we have

$$\begin{cases} lim_{t\to\infty}\|E^r\| = lim_{t\to\infty}\|Y^r - \Lambda^r X^r + \Lambda^i X^i\| = 0 \\ lim_{t\to\infty}\|E^i\| = lim_{t\to\infty}\|Y^i - \Lambda^r X^i - \Lambda^i X^r\| = 0 \end{cases} \tag{9}$$

In this case, $\Lambda = \Lambda^r + j\Lambda^i = diag({\lambda_1}^r, {\lambda_2}^r, \dots, {\lambda_n}^r) + jdiag({\lambda_1}^i, {\lambda_2}^i, \dots, {\lambda_n}^i)$, ${\lambda_k}^r \in R$, ${\lambda_k}^i \in R$, $k = 1,2,\dots,n$.

3.2 HCPS of complex chaotic systems with active control input affected by dead-zone nonlinearity

Let us consider the following two drive and controlled response complex chaotic systems. In this case, the drive system and the response system denoted by vectors $X = (x_1, y_1, z_1)^T$ and $Y = (x_2, y_2, z_2)^T$, respectively. Also, the response system affected by dead-zone nonlinearity input.

Consider system (1) as drive system and the following system as non-autonomous response system

$$\begin{cases} \dot{x}_2 = b_1(y_2 - x_2) + \phi_1(u_1) \\ \dot{y}_2 = (b_2 - b_1)x_2 - x_2 z_2 + b_2 y_2 + \phi_2(u_2) \\ \dot{z}_2 = {}^1\!/_2\,(\overline{x_2} y_2 + x_2 \overline{y_2}) - b_3 z_2 + \phi_3(u_3) \end{cases} \tag{10}$$

The behaviour of the tracking system is investigated with dead-zone nonlinear functions $\phi_k(.)$, $k = 1,2,3$ which are imposed to the control inputs u_k, $k = 1,2,3$. These nonlinear control inputs which are applied to the chaotic complex system (10) are as follow:

$$\phi_1(u_o) = \phi_2(u_o) = \phi({u_o}^r) + j\phi({u_o}^i), \phi_3 = \phi({u_o}^r)$$

where

$$\phi(u_o) = \begin{cases} (u_o + b_-);\ u_o < -b_-\ , \\ 0; \quad -b_- \le u_o \le b_+, \\ (u_o - b_+);\ u_o > b_+ \end{cases} \tag{11}$$

where b_+ and b_- are positive constants. The dead-zone nonlinearity input function is depicted in Figure 3.

According to the section 3, we have the error variables as follows:

$$\begin{cases} e_1 = x_2 - \lambda_1 x_1 \\ e_2 = y_2 - \lambda_2 y_1 \\ e_3 = z_2 - \lambda_3 z_1 \end{cases} \tag{12}$$

where $\lambda_1 = {\lambda_1}^r + j{\lambda_1}^i$, $\lambda_2 = {\lambda_2}^r + j{\lambda_2}^i$ and $\lambda_3 = {\lambda_3}^r$; ${\lambda_k}^r (k = 1,2,3) \in R, {\lambda_k}^i (k = 1,2) \in R$.

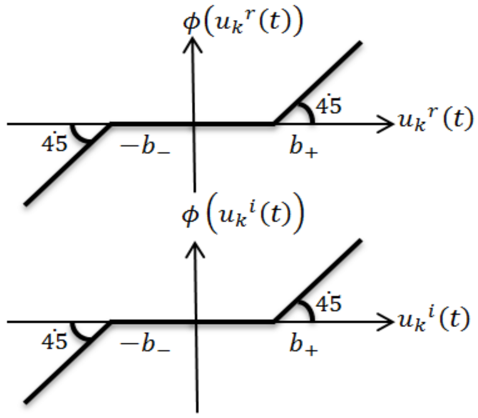

Fig. 3. A dead-zone nonlinearity input function $\phi(u_k(t)) = \phi({u_k}^r(t)) + j\,\phi({u_k}^i(t))$.

Therefore, the tracking errors can be rewritten in the following form:

$$\begin{cases} {e_1}^r = {x_2}^r - {\lambda_1}^r {x_1}^r + {\lambda_1}^i {x_1}^i, \\ {e_1}^i = {x_2}^i - {\lambda_1}^r {x_1}^i - {\lambda_1}^i {x_1}^r, \\ {e_2}^r = {y_2}^r - {\lambda_2}^r {y_1}^r + {\lambda_2}^i {y_1}^i, \\ {e_2}^i = {y_2}^i - {\lambda_2}^r {y_1}^i - {\lambda_2}^i {y_1}^r, \\ {e_3}^r = {z_2}^r - {\lambda_3}^r {z_1}^r \end{cases} \tag{13}$$

The goal of this paper is that for any given chaotic complex system, such as (1) and (10), an active nonlinear controller is designed in spite of the dead-zone nonlinear inputs $\phi_k(u_k)$, $k = 1,2,3$, such that the asymptotic stability of the resulting tracking errors (12) can be achieved in the sense of (8).

The design procedure of the active nonlinear control input has two main steps. The first part is to eliminate the nonlinearity and the second step is to make the error states asymptotically stable. In fact, one of the shortcomings of active control is that an accurate knowledge of mathematical model of the system is needed, but in practical application there are always unknown factors which affect the control systems. The block diagram for the considered synchronization system is depicted in Figure 4.

Composite internal variables of the controller can be defined as follows:

$$\begin{cases} \boldsymbol{\mu_1}^r = b_1({e_2}^r - {e_1}^r) + b_1{\lambda_2}^r{y_1}^r - b_1{\lambda_2}^i{y_1}^i \\ \quad -b_1{\lambda_1}^r{x_1}^r + b_1{\lambda_1}^i{x_1}^i - a_1{\lambda_1}^r{y_1}^r \\ \quad +a_1{\lambda_1}^r{x_1}^r + a_1{\lambda_1}^i{y_1}^i - a_1{\lambda_1}^i{x_1}^i \\ \boldsymbol{\mu_1}^i = b_1({e_2}^i - {e_1}^i) + b_1{\lambda_2}^r{y_1}^i + b_1{\lambda_2}^i{y_1}^r \\ \quad -b_1{\lambda_1}^r{x_1}^i - b_1{\lambda_1}^i{x_1}^r - a_1{\lambda_1}^r{y_1}^i \\ \quad +a_1{\lambda_1}^r{x_1}^i - a_1{\lambda_1}^i{y_1}^r + a_1{\lambda_1}^i{x_1}^r, \\ \boldsymbol{\mu_2}^r = (b_2 - b_1){e_1}^r + b_2{e_2}^r + (b_2 - b_1){\lambda_1}^r{x_1}^r \\ \quad -(b_2 - b_1){\lambda_1}^i{x_1}^i - {e_1}^r{e_3}^r - {e_1}^r{\lambda_3}^r{z_1}^r \\ \quad -{\lambda_1}^r{x_1}^r{e_3}^r - {\lambda_1}^r{x_1}^r{\lambda_3}^r{z_1}^r + {\lambda_1}^i{x_1}^i{e_3}^r \\ \quad +{\lambda_1}^i{x_1}^i{\lambda_3}^r{z_1}^r + b_2{\lambda_2}^r{y_1}^r - b_2{\lambda_2}^i{y_1}^i \\ \quad -{\lambda_2}^r a_2{x_1}^r + {\lambda_2}^r{y_1}^r + {\lambda_2}^r{x_1}^r{z_1}^r + {\lambda_2}^i a_2{x_1}^i \\ \quad -{\lambda_2}^i{y_1}^i - {\lambda_2}^i{x_1}^i{z_1}^r, \\ \boldsymbol{\mu_2}^i = (b_2 - b_1){e_1}^i + b_2{e_2}^i + (b_2 - b_1){\lambda_1}^r{x_1}^i \\ \quad +(b_2 - b_1){\lambda_1}^i{x_1}^r - {e_1}^i{e_3}^r - {e_1}^i{\lambda_3}^r{z_1}^r \\ \quad -{\lambda_1}^r{x_1}^i{e_3}^r - {\lambda_1}^r{x_1}^i{\lambda_3}^r{z_1}^r - {\lambda_1}^i{x_1}^r{e_3}^r \\ \quad -{\lambda_1}^i{x_1}^r{\lambda_3}^r{z_1}^r + b_2{\lambda_2}^r{y_1}^i + b_2{\lambda_2}^i{y_1}^r \\ \quad -{\lambda_2}^r a_2{x_1}^i + {\lambda_2}^r{y_1}^i + {\lambda_2}^r{x_1}^i{z_1}^r - {\lambda_2}^i a_2{x_1}^r \\ \quad +{\lambda_2}^i{y_1}^r + {\lambda_2}^i{x_1}^r{z_1}^r, \\ \boldsymbol{\mu_3}^r = -b_3{e_3}^r + {e_1}^r{e_2}^r + {e_1}^r{\lambda_2}^r{y_1}^r \\ \quad -{e_1}^r{\lambda_2}^i{y_1}^i + {\lambda_1}^r{x_1}^r{e_2}^r + {\lambda_1}^r{x_1}^r{\lambda_2}^r{y_1}^r \\ \quad -{\lambda_1}^r{x_1}^r{\lambda_2}^i{y_1}^i - {\lambda_1}^i{x_1}^i{e_2}^r - {\lambda_1}^i{x_1}^i{\lambda_2}^r{y_1}^r \\ \quad +{\lambda_1}^i{x_1}^i{\lambda_2}^i{y_1}^i + {e_1}^i{e_2}^i + {e_1}^i{\lambda_2}^r{y_1}^i \\ \quad +{e_1}^i{\lambda_2}^i{y_1}^r + {\lambda_1}^r{x_1}^i{e_2}^i + {\lambda_1}^r{x_1}^i{\lambda_2}^r{y_1}^i \\ \quad +{\lambda_1}^r{x_1}^i{\lambda_2}^i{y_1}^r + {\lambda_1}^i{x_1}^r{e_2}^i + {\lambda_1}^i{x_1}^r{\lambda_2}^r{y_1}^i \\ \quad +{\lambda_1}^i{x_1}^r{\lambda_2}^i{y_1}^r - b_3{\lambda_3}^r{z_1}^r - {\lambda_3}^r{x_1}^r{y_1}^r \\ \quad -{\lambda_3}^r{x_1}^i{y_1}^i + a_3{\lambda_3}^r{z_1}^r \end{cases} \tag{14}$$

The next step is to design an active nonlinear controller, when the control input function contains dead-zone nonlinearity. The adaptive control laws $u_k = u_k{}^r + ju_k{}^i$, $k = 1,2,3$ are as follow:

$$\begin{cases} u_k{}^r = \begin{cases} -\beta|\mu_k{}^r| - b_-; & e_k{}^r > 0 \\ 0; & e_k{}^r = 0\ , \quad k = 1,2,3 \\ \beta|\mu_k{}^r| + b_+; & e_k{}^r < 0 \end{cases} \\ u_k{}^i = \begin{cases} -\beta|\mu_k{}^i| - b_-; & e_k{}^i > 0 \\ 0; & e_k{}^i = 0\ , \quad k = 1,2 \\ \beta|\mu_k{}^i| + b_+; & e_k{}^i < 0 \end{cases} \end{cases} \tag{15}$$

where parameter $\beta > 0$.

From (11) and (15), we have

$$\begin{cases} \phi(u_k{}^r) = \begin{cases} -\beta|\mu_k{}^r|; & e_k{}^r > 0 \\ 0; & e_k{}^r = 0\ , \quad k = 1,2,3 \\ \beta|\mu_k{}^r|; & e_k{}^r < 0 \end{cases} \\ \phi(u_k{}^i) = \begin{cases} -\beta|\mu_k{}^i|; & e_k{}^i > 0 \\ 0; & e_k{}^i = 0\ , \quad k = 1,2 \\ \beta|\mu_k{}^i|; & e_k{}^i < 0 \end{cases} \end{cases} \tag{16}$$

where $|.|$ denotes the absolute value.

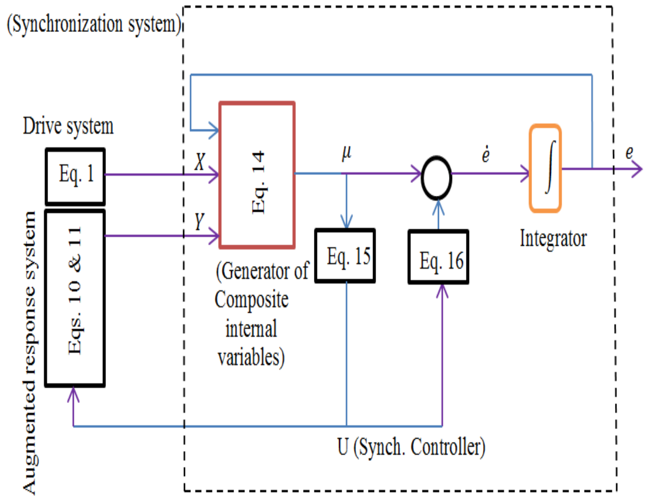

Fig. 4. The block diagram for the designed synchronization system.

From (12) and (14), one can derive the error dynamics as

$$\begin{cases} \dot{e}_1{}^r = \mu_1{}^r + \phi_1(u_1{}^r) \\ \dot{e}_1{}^i = \mu_1{}^i + \phi_1(u_1{}^i) \\ \dot{e}_2{}^r = \mu_2{}^r + \phi_2(u_2{}^r) \\ \dot{e}_2{}^i = \mu_2{}^i + \phi_2(u_2{}^i) \\ \dot{e}_3{}^r = \mu_3{}^r + \phi_3(u_3{}^r) \end{cases} \tag{17}$$

By taking into account (16), then (17) becomes as

$$\begin{cases} \dot{e}_k{}^r = \begin{cases} \mu_k{}^r - \beta|\mu_k{}^r|; & e_k{}^r > 0 \\ \mu_k{}^r; & e_k{}^r = 0\ , \quad k = 1,2,3 \\ \mu_k{}^r + \beta|\mu_k{}^r|; & e_k{}^r < 0 \end{cases} \\ \dot{e}_k{}^i = \begin{cases} \mu_k{}^i - \beta|\mu_k{}^i|; & e_k{}^i > 0 \\ \mu_k{}^i; & e_k{}^i = 0\ , \quad k = 1,2 \\ \mu_k{}^i + \beta|\mu_k{}^i|; & e_k{}^i < 0 \end{cases} \end{cases} \tag{18}$$

If $\beta > 1$, $x - \beta|x| \le 0$ and if $x + \beta|x| \ge 0$, $x \in R$; Consequently, we have

$$\begin{cases} e_k{}^r \dot{e}_k{}^r \le 0, k = 1,2,3 \\ e_k{}^i \dot{e}_k{}^i \le 0, k = 1,2 \end{cases} \tag{19}$$

Then, we choose a Lyapunov function candidate according to error states (12) as follows:

$$V(t) = {}^1/_2 \left(\sum_{k=1}^{3}(e_k{}^r)^2 + \sum_{k=1}^{2}(e_k{}^i)^2\right) \tag{20}$$

Taking the time derivative from (20), yields

$$\dot{V}(t) = \sum_{k=1}^{3} e_k{}^r \dot{e}_k{}^r + \sum_{k=1}^{2} e_k{}^i \dot{e}_k{}^i \tag{21}$$

Thus from (19) and (21), the stability of system is guaranteed.

4. NUMERICAL EXAMPLE

In this section, to verify the effectiveness and feasibility of the proposed synchronization scheme, the HCPS between two non-identical complex chaotic Lorenz system as drive system and complex chaotic Chen system as response system is accomplished, in which control input is subjected to dead-zone nonlinearity. The control inputs become active at $t = 5\ s$. For the case of HCPS, fourth-order Runge-Kutta method with initial conditions $\left(x_1{}^r(0) + jx_1{}^i(0), y_1{}^r(0) +\right.$ jy1i0, z1r0T=1+j2, 3+j0.5, 4T, x2r0+jx2i0, y2r0+jy2i0, z2r0T=4.7+j6, 0.2+j7.5, 1T and time step size of 0.001 are used.

The scaling matrix is $\Lambda = diag\{1 + j0.5, 1 - j,\ 1.5\}$ and $b_- = b_+ = 1$.

Figure 5 displays the synchronized states of the complex chaotic Lorenz and Chen systems, where the control inputs are subjected to dead-zone nonlinearity. It is clear that the synchronization errors converge to zero quickly, which implies that the chaos synchronization between the complex chaotic Lorenz and Chen systems is realized, as shown in Figure 6.

Moreover, using the fourth-order Runge-Kutta method with the time step size of 0.1 the simulation is done and the simulation is depicted in Figure 7. Also, the simulation result, when the ODE 113 (Adams) method as the integral method is used with the time step size of 0.001 is depicted in Figure 8. The time history of control inputs affected by dead zone nonlinearity input is depicted in Figure 9.

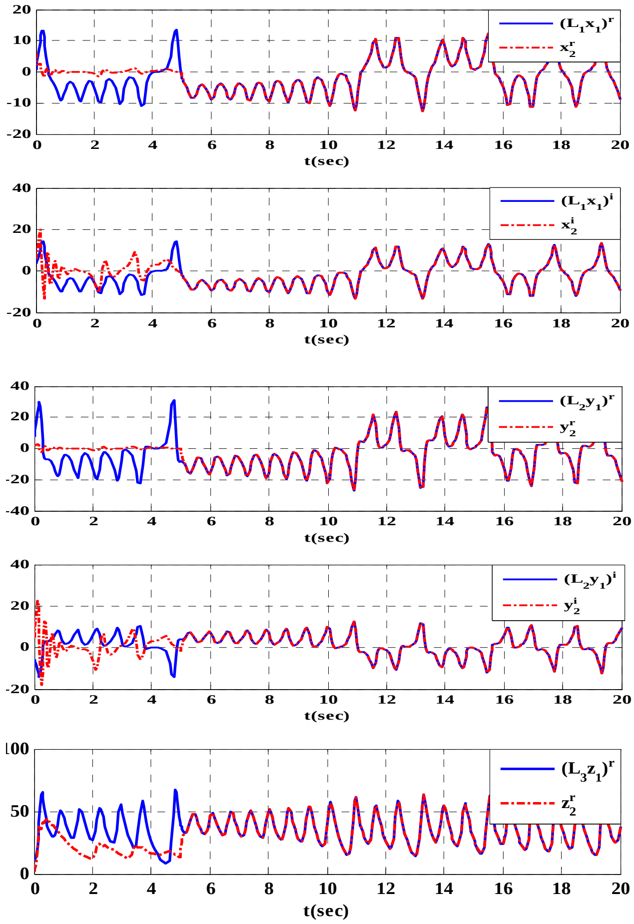

Fig. 5. State trajectories of complex chaotic Lorenz and Chen systems.

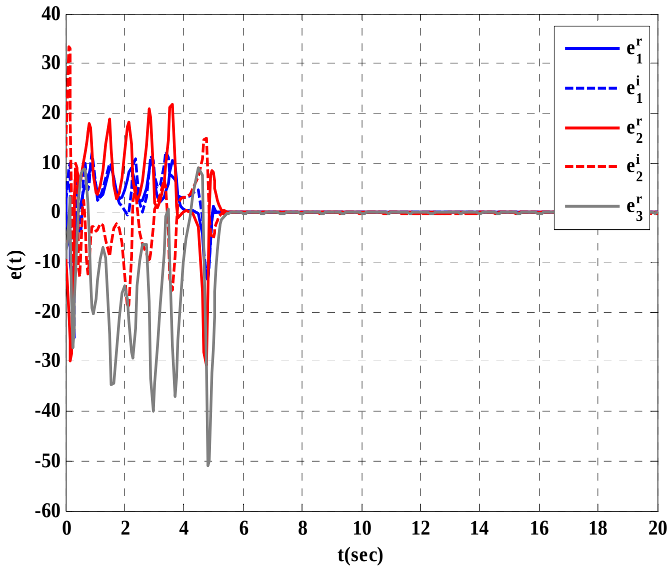

Fig. 6. Synchronization errors between the Lorenz and Chen systems.

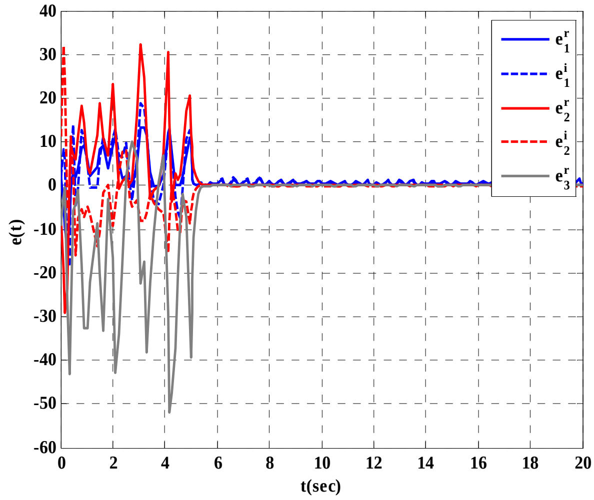

Fig. 7. Synchronization errors between the Lorenz and Chen systems.

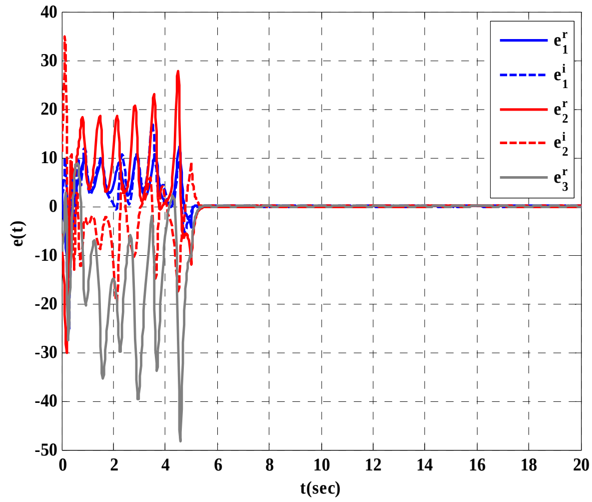

Fig. 8. Synchronization errors between the Lorenz and Chen systems.

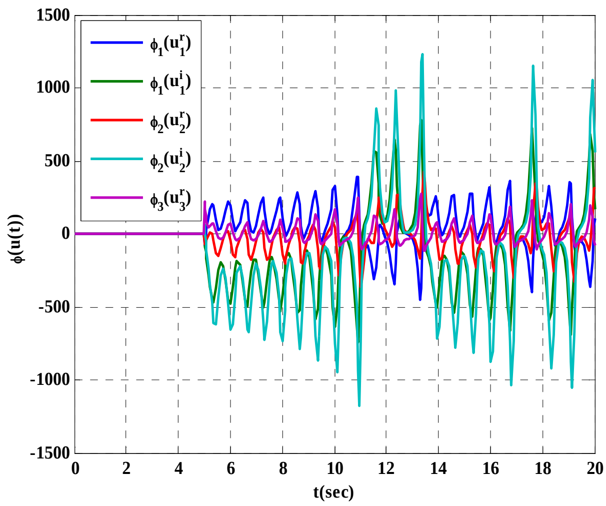

Fig. 9. The time history of control inputs affected by dead zone nonlinearity input

Figure 7 shows that to obtain a better evaluation of the proposed synchronization method, the size of the integration time step used in the simulations should be chosen very small. So in accordance of Figure 6, it is better to decrease the time step size to have the minimum error in the synchronization. According to Figures 6 and 8, it is obvious that for the different integral methods in the numerical simulation, the synchronization errors not changed, significantly. In Figure 9, the smoothness of the control input is obvious.

5. CONCLUSIONS

In this paper, an active nonlinear controller subject to input nonlinearity has been addressed. In the many applications, an active control technique is not robust against nonlinearity; but under some conditions, this control scheme can eliminate some perturbations like nonlinearities. Because the complex chaotic systems are not widely considered in the previous literatures, we investigate the effect of nonlinearity for these systems. Techniques for achieving hybrid complex projective synchronization between two different complex chaotic systems demonstrated. At First step, an active control scheme is suggested to tackle the existence of the general nonlinearities in control inputs. Based on the Lyapunov stability theorem, sufficient conditions to guarantee stable synchronization are given and the components of the error states tend to zero as time becomes large. Moreover, the complex behavior and high dependence of the proposed chaotic system to the initial condition in this paper is illustrated.

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