Bidirectional Platoon Control of Arduino Controlled Cars with Actuator Saturation and Time-varying Delay

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Abstract: The concept of the bidirectional platoon control system is developed, which enjoys significant advantages over present day adaptive cruise control systems in terms of string stability, tracking safety and fuel economy. A novel bidirectional platoon control system model is established, in which the effect of engine time uncertainty, time-varying actuator delay (including fuelling delay and braking delay) and actuator saturation is involved. Based on the new model, a H_∞ controller is presented that can robustly stabilize the vehicular platoon system only use information from their immediate neighbors. The theoretical results show that the proposed system can achieve the objective of a smaller inter-vehicle spacing and bidirectional string stable. The effectiveness and advantage of the presented methodology are demonstrated by both numerical simulations and experiments with laboratory scale Arduino controlled cars.

Keywords: Bidirectional platoon control, actuator delay, actuator saturation, bidirectional string stable, engine time uncertainty.

1. INTRODUCTION

Over the past decade, a considerable attention has been paid to the research theme of automated vehicles in intelligent vehicle highway systems (Li, 2011). There are so many advantages of moving vehicle based on the notion of platoons, such as driving safety and comfort, reducing fuel consumption and air pollution, and improving the throughput in the highway (Tiberiu, 2005). Due to this, a lot of research works on platoon control have been extensively studied in (Jovanovic, 2005; Seiler, 2004).

The control architectures of platoon investigated in the literature can be classified into three broad categories: *predecessor-following, predecessor and leader following* and *bidirectional control*. The architecture is called *predecessor following* if the control action on a particular vehicle depends on the information with the predecessor, i.e., the vehicle in front of it. This scheme is decentralized, since the control action on each following vehicle is computed based upon measurements obtained by on-board sensors. It was shown that this architecture suffers from a drawback known as string instability (Seiler, 2004). That is, the response of a disturbance on an individual vehicle will be amplified along the string of vehicles. Constant time-gap spacing strategy was introduced by (Vahidi, 2003) to overcome this difficulty, and in which the inter-vehicle distances are dependent on vehicle velocities. However, this only helps when the control bandwidths are allowed to diverge as the number of vehicles grows (Middleton, 2010). Alternatively, in (Xiao, 2011) shown that string stability can be achieved if a *predecessor and leader following* structure is adopted, where the control action on a particular vehicle is based on the distance between the preceding vehicle as well as the velocity and acceleration of the lead vehicle. This scheme is centralized, since the lead vehicle has to broadcast its information to all following vehicles. The use of the network to provide the following vehicles in the platoon with the lead vehicle information immediately cause some questions on the effect of disruptions of the wireless communication. Under this framework, these works presents in (Hedrick, 2001; Liu, 2001) studied the effects of communication delays on string stability; longitudinal platoon control and state estimation via communication channels with packed-dropout are addressed in (Guo, 2011); a decentralized communication and control strategy is presented in (Guo, 2014) for automated driving assistance to a platoon of vehicles in heavy traffic and scarce visibility.

Another control architecture investigated in the literature, and on which we focus in this research is decentralized *bidirectional control*. This control scheme is advantageous because, apart from its simplicity in achieving string stability, it does not require wireless communication. The control action on an individual vehicle depends on the information of its own velocity and the spacing errors between it and its predecessor and its follower vehicles, which can be obtained by on-board sensors alone. Still, the bidirectional platoon control suffers from the high sensitivity to the length of the vehicular platoon and lower performance (Hao, 2013). In (Jovanovic, 2005), the authors investigate optimal control strategies for a bidirectional platoon with an increasing number of vehicles and show that some related LQR problems are ill-posed. In order to enhance the coherence of the bidirectional platoon, an optimal controller was designed in (Liu, 2012), which integrated the previous results**.**

It is worth noting that most existing results on bidirectional platoon control are limited in at least the following three aspects. Firstly, ignoring the saturations in the engine, this can deteriorate the control performance or even render the platoon system unstable. In (Ibtissem, 2011), an error governor scheme was discussed for dealing with saturation, which can eliminate the windup phenomenon and guarantee stability. Nevertheless, this control method suggested is not applicable to the bidirectional platoon system. Secondly, without considering the uncertain factors such as the inaccuracies of model parameters and the errors of sensors and actuators, degrade the track performance and safety during the driving process. In recent years, many results have been reported to deal with the uncertainties in order to guarantee the stability of the platoon (Swaroop, 2001). The combined actuating delay is the third aspect that may add to the limitations since the delay effect may accumulate as it propagates both directions in the platoon. An upper bound of the actuator delay was derived in (Huang, 1998) under which the so-called slinky-effect can be avoided, and in (Yanakiev, 2001) for a control method dealing with large actuator delays. However, these results are based on a single direction vehicular platoon and hence are not adequate for achieving more stringent performance requirement for bidirectional platoons control. To the authors' knowledge, strategies systematically taking into account the desired system performance, uncertainties, saturations and actuator delay have not yet been reported**.**

The aim of this paper is to set up a bidirectional platoon control framework that takes full consideration of the uncertainty in the engine time, the actuator saturation and actuator delay. In the suggested framework, each vehicle can only detect the distance between it and the adjoined vehicle with an on-board distance sensor. Then the time-varying fuelling and braking delay is taken into account. In addition, full consideration is given to the uncertainty in engine time and actuator saturation in our framework, which further highlights the completeness of the result. As will be shown later in both numerical simulations and experiments with Arduino cars, the presented method can serve as an effective algorithm for practical use**.**

2. PROBLEM FORMULATION

Consider a bidirectional platoon control system composed by *n* vehicles running in a horizontal environment. All vehicles in the platoon can measure the relative distance and velocity with respect to their nearest neighbours by on-board sensors. In what follows, the vehicle dynamics, actuator lumped delay, actuator saturation, and the engine time uncertainty will be formulated in detail.

Denote by z_i and v_i the *i*th (*i*=1,…,*n*) vehicle's position and velocity, and *i*=0 represents the lead vehicles with $z_0 = 0$. Based on the constant time-gap spacing strategy (Vahidi, 2003), the spacing error for the *i*th vehicle can be written as:

$$\delta_i = z_{i-1} - z_i - L_i - hv_i \tag{1}$$

where h is the time gap, L_i is the length of the vehicle. Then the dynamics of the *i*th following vehicle can be modelled by the following nonlinear differential equations:

$$\dot{\delta}_i = v_{i-1} - v_i - h\dot{v}_i\,,\ \Delta\dot{v}_i = a_{i-1} - a_i\,,\ \dot{a}_i = f_i(v_i, a_i) + g_i(v_i)c_i \tag{2}$$

where c_i is the control input of the *i*th vehicle's engine/brake, with $c_i \geq 0$ and $c_i < 0$ representing the throttle input and the brake input, respectively, $f_i(v_i, a_i)$ and $g_i(v_i)$ are given by:

$$f_i(v_i, a_i) = -\left(\dot{v}_i + \sigma A_i c_{di} v_i^2 / 2m_i + d_{mi}/m_i\right)/\varsigma_i - \sigma A_i c_{di} v_i a_i / m_i\ ,$$

$$g_i(v_i) = 1/\varsigma_i m_i\ ,$$

where σ is the specific mass of the air, m_i is the vehicle mass, A_i is the cross-sectional area, $\sigma A_i c_{di} / 2m_i$ is the air resistance, c_{di} is the drag coefficient, d_{mi} is the mechanical drag, ς_i is the engine time constant.

The following control law was adopted:

$$c_i = u_i m_i + \sigma A_i c_{di} v_i^2 / 2 + d_{mi} + \varsigma_i \sigma A_i c_{di} v_i a_i\ , \tag{3}$$

where u_i is the additional input signal to be designed so that the closed-loop system can satisfy certain performance criteria. Obviously, this control law achieves feedback linearization, since, after introducing (3), the third equation in (2) becomes:

$$\dot{a}_i(t) = -a_i(t)/\varsigma_i + u_i(t)/\varsigma_i\ . \tag{4}$$

A more realistic dynamic model should consider the changes of the engine time, actuator lumped delay (including fuelling delay and braking delay) and the actuator saturation nonlinearities in the vehicle *i*. Taking these properties into account (4) can be rewritten as,

$$\dot{a}_i(t) = -(1/\varsigma_i + 1/\Delta\varsigma_i)a_i(t) + (1/\varsigma_i + 1/\Delta\varsigma_i)u_{\mathrm{sat}_i}(t - \tau_i(t)) \tag{5}$$

where $|\Delta\varsigma_i| = f_i(t)$, with $f_i(t)$ being a Lebesgue-measurable continuous function satisfying $f_i^2(t) \geq D_i$, $D_i > 0$. The actuator delay $\tau_i(t)$ is time-varying continuous function, and satisfies,

$$\tau_{i1} \leq \tau_i(t) \leq \tau_{i2}\,,\ 0 \leq \dot{\tau}_i(t) \leq \mu_i \tag{6}$$

where τ_{i1} and τ_{i2} represent the lower and upper bounds of the lumped delay of the *i*th vehicle, respectively, and μ_i is the delay variation rate bound. Take sat_i to be the saturation level of the *i*th vehicle's actuator, and describe as

$$u_{\mathrm{sat}_i}(t) = [u^1_{\mathrm{sat}_i}(t), u^2_{\mathrm{sat}_i}(t) \ldots u^q_{\mathrm{sat}_i}(t)]^T\ ,$$

$$u_{\mathrm{sat}_i}^{j}(t) \overset{\Delta}{=} \begin{cases} sat_i^{j\max} & if\ u_i \geq sat_i^{j\max} \\ u_i(t) & if\ -sat_i^{j\max} \leq u_i \leq sat_i^{j\max} \\ -sat_i^{j\max} & if\ u_i \leq -sat_i^{j\max} \end{cases}, \tag{7}$$

Here, the controller $u_i(t)$ assumes the following form:

$$u_i(t) = k_{pf}\delta_i + k_{pb}\delta_{i+1} + k_{vf}\Delta v_i + k_{vb}\Delta v_{i+1} \tag{8}$$

where k_{pf} , k_{pb} , k_{vf} and k_{vb} are the controller gains to be determined. Note that the control law for the *i*th vehicle is solely based on the relative position errors and the relative velocity errors between nearest neighbors.

By combining the dynamics of the vehicle (1), (5) and (8) and setting $a_{i-1}(t) = d_i(t)$ as a measurable disturbance from the preceding vehicle, the following state space equation for the bidirectional platoon system can be derived,

$$\dot{x}_i(t) = (A_i + \Delta A_i)x_i(t) + (B_i + \Delta B_i)u_{sat_i}(t - \tau_i(t)) + B_{di}d_i(t),$$

$$y_i(t) = C_i[x_i, x_{i+1}]^T, \tag{9}$$

where $x_i(t) = [\delta_i \quad \Delta v_i \quad a_i]^T$ ($a_0 = 0$ in x_0) is the state of the system, $y_i(t) = [\delta_i, \Delta v_i, \delta_{i+1}, \Delta v_{i+1}]^T$ is the measurement output, and

$$A_i = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ 0 & 0 & -1/\varsigma_i \end{bmatrix},\ B_i = \begin{bmatrix} 0 \\ 0 \\ 1/\varsigma_i \end{bmatrix},\ B_{di} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \Delta B_i = \begin{bmatrix} 0 \\ 0 \\ 1/\Delta\varsigma_i \end{bmatrix},$$

$$\Delta A_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1/\Delta\varsigma_i \end{bmatrix},\ C_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Using $\Delta\varsigma_i$ in (5) this leads to $[\Delta A_i \quad \Delta B_i] = L_{i1}F_i(t)[E_{i1} \quad E_{i2}]$, where $L_{i1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}^T \Big/ \sqrt{D_i}$, $E_{i1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$, $E_{i2} = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$ ($i \geq 2$), for i=1, $E_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $F_i(t) = [\sqrt{D_i}/f_i(t)]I$, which clearly satisfies $F_i^T(t)F_i(t) \leq I$.

Before designing the controller for each following vehicle, the following control objectives should be considered.

(***a***). Asymptotic stability: The state of each vehicle in the bidirectional platoon control system can be asymptotically stabilized to the origin, i.e., spacing error and velocity error approach to zero when all vehicles running with a constant velocity.

(***b***). Bidirectional string stability: If the $i_{dis}th$ ($i_{dis} \in [1, n]$) vehicle suffers from a sudden disturbance, the oscillations are not amplifying downstream or upstream the vehicular string with the vehicle index, namely, $|G(jw)| \leq 1$ for any w, where $G(s) = \begin{cases} a_i(s)/a_{i-1}(s), & for\ i \in [i_{dis}, 1] \\ a_i(s)/a_{i+1}(s), & for\ i \in [n, i_{dis}) \end{cases}$ with $a_i(s)$, $a_{i-1}(s)$ and $a_{i+1}(s)$ denotes the Laplace transforms of the acceleration $a_i(t)$, $a_{i-1}(t)$ and $a_{i+1}(t)$, respectively.

Note that the asymptotic stability and bidirectional string stability discussed in here is different from the existing stability issues in (Xiao, 2011), which caused by the broadcast measurement is replaced by the bidirectional measurement.

(***c***). Fuel economy: For the purpose of fuel consumption, in (Li, 2011), the authors provide criteria as, the absolute acceleration of all the following vehicles should be constrained and minimized, namely, minimizing $a_i(t)$ in the controller design process with $|a_i(t)| < a_{i\max}$, thus, a new measurement output is defined as $y_{i1}(t) = a_i(t)$.

Furthermore, the H_∞ norm is employed to measure the performance, whose value actually gives an upper bound of the root mean square gain. Hence, our goal is to minimize the H_∞ norm $\|T_{y_{i1}(t)d_i(t)}\|_\infty < \gamma_i$, where $T_{y_{i1}(t)d_i(t)}$ denotes the closed loop transfer function from $d_i(t)$ to the output $y_{i1}(t)$.

(***d***). Tracking safety: The designed controller should be capable to prevent the following vehicles from colliding with the preceding vehicle, the requirement is $y_{i2}(t) = |\delta_i(t)| \leq \delta_{id\min}$, where $\delta_{id\min}$ is the minimal safety distance under all operational changes from vehicle in the platoon.

Then, the platoon system can be described by the following state-space equations:

$$\dot{x}_i(t) = \overline{A}_i x_i(t) + \overline{B}_i u_{sat_i}(t - \tau_i(t)) + B_{di}d_i(t) \tag{10}$$

$$y_{i1} = C_{i1}x_i(t) \tag{11}$$

$$y_{i2} = C_{i2}x_i(t) \tag{12}$$

where $\overline{A}_i = A_i + \Delta A_i$, $\overline{B}_i = B_i + \Delta B_i$, $C_{i1} = [0 \quad 0 \quad 1]^T$, $C_{i2} = [1 \quad 0 \quad 0]^T$.

In this research, our goal is to find an output feedback controller for each vehicle in the platoon system

$$u_i(t) = K_i y_i(t), \tag{13}$$

where $K_i = [k_{pf} \quad k_{pb} \quad k_{vf} \quad k_{vb}]$ is the controller gain, such that the requirements in ***(a)-(d)*** can be satisfied.

3. H∞ CONTROLLER DESIGN

In this subsection, a sufficient condition is given for the bidirectional platoon system to ensure that all the vehicles in the string are robust asymptotically stable with the effect of uncertainties, time-varying delay and actuator saturation effects.

To begin with, for output feedback gain matrix K_i, we define

$$L(K_i)\overset{\Delta}{=}\{y_i \in R^n : |k_i^j y_i(t)| \le sat_i^{j\max},\ j=1,2,...q\},$$

where k_i^j is the jth row of K_i. Then $L(K_i)$ is the region in the output state space where the control input is linear in y_i.

Next, as shown in [Hu, 2002], we utilize the technique of auxiliary feedback matrices here to reduce the conservatism of dealing with the actuator saturation. Namely, for two matrices K_i, $H_i \in R^{q\times n}$ and a vector $V_i \in R^q$, a matrix set is introduced as

$$W_i(V_i,K_i,H_i)\overset{\Delta}{=}\left\{W_i \in R^{q\times n} : W_i=\begin{bmatrix} v_i^1 k_i^1+(1-v_i^1)h_i^1 \\ \vdots \\ v_i^q k_i^q+(1-v_i^q)h_i^q \end{bmatrix}\right\},$$

where v_i^j =0 or 1, define $\psi(V_i)\overset{\Delta}{=}\{V_i \in R^n : v_i^j=0\ or\ 1\}$ and the auxiliary matrix H_i satisfies $|h_i^j y_i(t)| \le sat_i^{j\max}$, $j=1,2,...q$. And a subset of the set $L(K_i)$ will be found and chosen to be an ellipsoid of the form

$$\xi(P_i,1)\overset{\Delta}{=}\{y_i \in R^n : y_i^T P_i y_i \le 1\}$$

where $P_i>0$ will be determined. Combine $\xi(P_i,1)$ with $\begin{bmatrix} sat_i^{j\max} & h_i^j \\ * & sat_i^{j\max}P_i \end{bmatrix}$, $j=1,2,...q$, which means that if $y_i^T P_i y_i \le 1$, we have $2|h_i^j y_i(t)| \le sat_i^{j\max}(1+y_i^T P_i y_i) \le 2sat_i^{j\max}$. So we can ensure that $\xi(P_i,1)\subset L(H_i)$.

Remark 1. There are 2^q elements in $\psi(V_i)$. V_i is used to choose from the rows of K_i and H_i to form a new matrix $W_i(V_i,K_i,H_i)$. If v_i^j =0, then the jth row of $W_i(V_i,K_i,H_i)$ is h_i^j, and if v_i^j =1, then the jth row of $W_i(V_i,K_i,H_i)$ is k_i^j. For example, assume q=2, then $\{W_i(V_i,K_i,H_i): V_i \in \psi(V_i)\overset{\Delta}{=}\left\{H_i, \begin{bmatrix} k_i^1 \\ h_i^2 \end{bmatrix}, \begin{bmatrix} h_i^1 \\ k_i^2 \end{bmatrix}, K_i\right\}$.

Based on the above ideas, the following theorem gives the existence conditions of a desired output feedback controller for system (10).

Theorem 1: The bidirectional platoon control system in (10) under the controller in (13) with actuator saturation and time-varying delay is asymptotically stable and satisfies $\|T_{y_i d_i}\|_\infty < \gamma_i$ for all $d_i(t)$ under zero initial condition if there exist matrices $P_i>0$, $T_i>0$, $Q_{ij}>0$, j=1, 2, 3, $Z_{im}>0$, m=1, 2, N_{il}, S_{il}, M_{il}, l=1, 2,…,5, $W_i(V_i,K_i,H_i)$, $W_i(S_i,K_i,H_i)$ and K_i satisfying,

$$\begin{bmatrix} \hat{\Pi}_i & \tau_{i2} & \tau_{i12}S_i & \tau_{i12}M_i \\ * & -\tau_{i2}Z_1 & \mathbf{0} & \mathbf{0} \\ * & * & -\tau_{i12}(Z_{i1}+Z_{i2}) & \mathbf{0} \\ * & * & * & -\tau_{i12}Z_{i2} \end{bmatrix}<0, \tag{14}$$

$$\begin{bmatrix} -I & \sqrt{\vartheta_i}C_{i2} \\ * & -\delta_{id\min}^2 P_i \end{bmatrix}<0 \tag{15}$$

where $\hat{\Pi}_i=\begin{bmatrix} \Pi_{i11}+C_{i1}^T C_{i1} & \Pi_{i12} & \Pi_{i13} & \Pi_{i14} & \Pi_{i15} & \Pi_{i16} \\ * & \Pi_{i22} & \Pi_{i23} & \Pi_{i24} & \Pi_{i25} & \mathbf{0} \\ * & * & \Pi_{i33} & \Pi_{i34} & \Pi_{i35} & \mathbf{0} \\ * & * & * & \Pi_{i44} & \Pi_{i45} & \mathbf{0} \\ * & * & * & * & \Pi_{i55} & \Pi_{i56} \\ * & * & * & * & * & -\gamma_i^2 \end{bmatrix}$,

$$N_i^T=[N_{i1}^T \quad N_{i2}^T \quad N_{i3}^T \quad N_{i4}^T \quad N_{i5}^T \quad 0],$$

$$S_i^T=[S_{i1}^T \quad S_{i2}^T \quad S_{i3}^T \quad S_{i4}^T \quad S_{i5}^T \quad 0],$$

$$M_i^T=[M_{i1}^T \quad M_{i2}^T \quad M_{i3}^T \quad M_{i4}^T \quad M_{i5}^T \quad \mathbf{0}], \tag{16}$$

$$\Pi_{i11}=\sum_{j=1}^{3}Q_{ij}+N_{i1}+N_{i1}^T+T_i\overline{A}_i+\overline{A}_i^T T_i,$$

$$\Pi_{i12}=N_{i2}^T-N_{i1}+S_{i1}-M_{i1}+T_i\overline{B}_i W_i(V_i,K_i,H_i),$$

$$\Pi_{i13}=M_{i1}+N_{i3}^T,\ \Pi_{i14}=-S_{i1}+N_{i4}^T,\ \Pi_{i15}=N_{i5}^T-T_i+P_i+\overline{A}_i^T T_i,$$

$$\Pi_{i16}=T_i B_{id},\ \Pi_{i22}=(\mu_i-1)Q_{i3}+S_{i2}+S_{i2}^T-N_{i2}-N_{i2}^T-M_{i2}-M_{i2}^T,$$

$$\Pi_{i23}=M_{i2}-N_{i3}^T+S_{i3}^T-M_{i3}^T,\ \Pi_{i24}=-S_{i2}-N_{i4}^T+S_{i4}^T-M_{i4}^T,$$

$$\Pi_{i25}=S_{i5}^T-N_{i5}^T-M_{i5}^T+W_I^T(S_i,K_i,H_i)\overline{B}_i^T T_i,$$

$$\Pi_{i33}=-Q_{i1}+M_{i3}+M_{i3}^T,$$

$$\Pi_{i34}=-S_{i3}+M_{i4}^T,\ \Pi_{i35}=M_{i5}^T,\ \Pi_{i44}=-Q_{i2}-S_{i4}-S_{i4}^T,$$

$$\Pi_{i45}=-S_{i5}^T,\ \Pi_{i55}=\tau_{i2}Z_{i1}+\tau_{i12}Z_{i2}-2T_i,\ \Pi_{i56}=T_i B_{id},$$

$$\tau_{i12}=\tau_{i2}-\tau_{i1}.$$

Proof. See Appendix.

Remark 2. Theorem 1 supplies a sufficient condition for the bidirectional platoon to be robust asymptotically stable with the uncertainties, time-varying delay and actuator saturation effects, implying that the control objective ***(a)***, ***(c)*** and ***(d)*** can be achieved. A stabilizing controller design method will be given in the following.

Theorem 2: Suppose ϑ_i, τ_{i1}, τ_{i2} and μ_i are prescribed positive scalars. Consider the bidirectional platoon system in

(10), if there exist matrices $\overline{P}_i > 0$, $\overline{T}_i > 0$, $\overline{W}_i$, $\overline{Q}_{ij} > 0$, j=1, 2, 3, $\overline{Z}_{im} > 0$, m=1, 2, $\overline{N}_{il}$, $\overline{S}_{il}$, $\overline{M}_{il}$, l=1, 2,…,5, and $\varepsilon_i > 0$ satisfying,

$$\begin{bmatrix} \Lambda_{i11} & \Lambda_{i12} \\ \Lambda_{i12}^T & \Lambda_{i22} \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} -I & \sqrt{\vartheta_i} C_{i2} \overline{T}_i \\ * & -\delta_{id\min}^2 \overline{P}_i \end{bmatrix} < 0, \tag{18}$$

where

$$\Lambda_{i12} = \begin{bmatrix} \tau_{i2}\overline{N}_{i1} & \tau_{i12}\overline{S}_{i1} & \tau_{i2}\overline{M}_{i1} & \overline{T}_i C_{i1}^T & \overline{T}_i E_{i1}^T & \overline{T}_i E_{i1}^T \\ \tau_{i2}\overline{N}_{i2} & \tau_{i12}\overline{S}_{i1} & \tau_{i2}\overline{M}_{i1} & \mathbf{0} & E_{i2}^T & E_{i2}^T \\ \tau_{i2}\overline{N}_{i3} & \tau_{i12}\overline{S}_{i1} & \tau_{i2}\overline{M}_{i1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \tau_{i2}\overline{N}_{i4} & \tau_{i12}\overline{S}_{i1} & \tau_{i2}\overline{M}_{i1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \tau_{i2}\overline{N}_{i5} & \tau_{i12}\overline{S}_{i1} & \tau_{i2}\overline{M}_{i1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Lambda_{i22} = diag\{-\tau_{i2} Z_{i1}, -\tau_{i12}(Z_{i1} + Z_{i2}), -\tau_{i12} Z_{i2}, -I, -\varepsilon_i I\},$$

$$\Pi_{i11} = \sum_{j=1}^{3} Q_{ij} + N_{i1} + N_{i1}^T + T_i \overline{A}_i + \overline{A}_i^T T_i,$$

$$\Pi_{i12} = N_{i2}^T - N_{i1} + S_{i1} - M_{i1} + B_i \overline{W}_i,$$

$$\overline{\Pi}_{i13} = \overline{M}_{i1} + \overline{N}_{i3}^T,\ \overline{\Pi}_{i14} = -\overline{S}_{i1} + \overline{N}_{i4}^T,\ \overline{\Pi}_{i15} = \overline{N}_{i5}^T - \overline{T}_i + \overline{P}_i + \overline{T}_i A_i^T,$$

$$\overline{\Pi}_{i16} = B_{id},\ \overline{\Pi}_{i22} = (\mu_i - 1)\overline{Q}_{i3} + \overline{S}_{i2} + \overline{S}_{i2}^T - \overline{N}_{i2} - \overline{N}_{i2}^T - \overline{M}_{i2} - \overline{M}_{i2}^T,$$

$$\overline{\Pi}_{i23} = \overline{M}_{i2} - \overline{N}_{i3}^T + \overline{S}_{i3}^T - \overline{M}_{i3}^T,\ \overline{\Pi}_{i24} = -\overline{S}_{i2} - \overline{N}_{i4}^T + \overline{S}_{i4}^T - \overline{M}_{i4}^T,$$

$$\overline{\Pi}_{i25} = S_{i5}^T - N_{i5}^T - M_{i5}^T + \overline{W}_i B_i^T T_i,\ \overline{\Pi}_{i33} = -\overline{Q}_{i1} + \overline{M}_{i3} + \overline{M}_{i3}^T,$$

$$\overline{\Pi}_{i34} = -\overline{S}_{i3} + \overline{M}_{i4}^T,\ \overline{\Pi}_{i35} = \overline{M}_{i5}^T,\ \overline{\Pi}_{i44} = -\overline{Q}_{i2} - \overline{S}_{i4} - \overline{S}_{i4}^T,$$

$$\overline{\Pi}_{i45} = -\overline{S}_{i5}^T,\ \overline{\Pi}_{i55} = \tau_{i2}\overline{Z}_{i1} + \tau_{i12}\overline{Z}_{i2} - 2\overline{T}_i,\ \overline{\Pi}_{i56} = B_{id},$$

then a stabilizing controller in (13) exists, and according to (11) and (12), the controller gain can be given as

$$K_i = \overline{W_i T_i^{-1}} D_i, \tag{19}$$

where $C_i D_i = I$.

Proof: by Shur complement, (17) is equivalent to

$$\Xi_{i1} + \varepsilon_i \Xi_{i3} \Xi_{i3}^T + \varepsilon_i^{-1} \Xi_{i2}^T \Xi_{i2} < 0, \tag{20}$$

where

$$\Xi_{i2} = \begin{bmatrix} E_{i1}\overline{T}_i & E_{i2}\overline{K}_i\overline{C}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ E_{i1}\overline{T}_i & E_{i2}\overline{K}_i\overline{C}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Xi_{i3}^T = \begin{bmatrix} L_{i1}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & L_{i1}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

By invoking Lemma 1 in (Guo, 2014), (20) holds if

$$\Xi_{i1} + \Xi_{i3}\overline{\Gamma}_i(t)\Xi_{i2} + \Xi_{i2}^T\overline{\Gamma}_i^T(t)\Xi_{i3}^T < 0, \tag{21}$$

where $\overline{\Gamma}_i(t) = diag\{F_i(T_i), F_i(T_i)\}$.

From the norm bounded parameter uncertainty defined in (8) note that (21) is equivalent to (14) by defining $\overline{T}_i = T_i^{-1}$, $\overline{N}_{il} = T_i^{-1} N_{il} T_i^{-1}$, $\overline{S}_{il} = T_i^{-1} S_{il} T_i^{-1}$, $P_{il} = T_i^{-1} P_{il} T_i^{-1}$, $\overline{M}_{il} = T_i^{-1} M_{il} T_i^{-1}$, $\overline{Z}_{il} = T_i^{-1} Z_{il} T_i^{-1}$, $\overline{Q}_{il} = T_i^{-1} Q_{il} T_i^{-1}$, $\overline{W}_i = K_i C_i T_i^{-1}$, $J_i = diag\{T_i^{-1}, T_i^{-1}, T_i^{-1}, T_i^{-1}, T_i^{-1}, I, T_i^{-1}, T_i^{-1}, T_i^{-1}\}$, and performing a congruence transformation to (14) with J_i^{-1}. Similarly, it also follows that (18) is equivalent to (15). Hence, the bidirectional platoon system is asymptotically stable with a H_∞ disturbance attenuation level of γ_i if (17) and (18) hold. This completes the proof.

Remark 3. Theorem 2 shows that the conditions are LMIs not only over the matrix variables, but also over the objective scalar γ_i is given, which implies that γ_i can be included as an optimization variable to obtain a lower bound of the guaranteed H_∞ performance. That is, the controller design problem has been transformed into a set of LMI conditions. Based on these conditions, the robust multi-objective (***a***, ***c*** and ***d***) controller design can be accomplished by solving the convex optimization problem as: min γ_i subject to (17) and (18).

4. BIDIRECTIONAL STRING STABILITY AND CONTROL ALGORITHM

In the above section, considerations have been focused primarily on robust asymptotical stability of all the vehicles in the bidirectional platoon system. This section is concerned with the issue of bidirectional string stability, which is associated with objectives ***(b)*** given in section II. The analysis and results are based on the output feedback controller (13) obtained above.

Suppose each following vehicles in the bidirectional platoon system is under control of (13). Substituting (13) and (8) into (5) yields

$$\begin{aligned} \dot{a}_i(t) &= -a_i(t)/\varsigma_i + [k_{pb}\delta_i(t-\tau_i) + k_{pf}\delta_{i+1}(t-\tau_i) \\ &\quad + k_{vb}\Delta v_i(t-\tau_i) + k_{vf}\Delta v_{i+1}(t-\tau_i)]/\varsigma_i \end{aligned}. \tag{22}$$

Taking Laplace transformation to (22), and assuming that $a_i(0) = 0$, then (22) can be changed to

$$\begin{aligned} &(\varsigma_i s + 1) a_i(s) \\ &= [k_{pb}\delta_i(s) + k_{pf}\delta_{i+1}(s) + k_{vb}\Delta v_i(s) + k_{vf}\Delta v_{i+1}(s)] e^{-\tau_i s} \end{aligned}. \tag{23}$$

By using (1) and (2) yield

$\delta_i(s) = (a_{i-1}(s) - a_i(s))/s^2 - ha_i(s)/s$,

$$\Delta v_i(s) = (a_{i-1}(s) - a_i(s))/s . \tag{24}$$

For the platoon to achieve bidirectional string stability, assume that the $i_{dis}th$ vehicle suffers from a sudden disturbance (such as a wind gust or a slope), corresponding to this situation, substituting (24) into (23), and collecting similar terms together yields,

$$G(s) = \begin{cases} a_i(s)/a_{i-1}(s), & for\ i \in [i_{dis}, 1] \\ a_i(s)/a_{i+1}(s), & for\ i \in [n, i_{dis}) \end{cases}$$

$$= \begin{cases} \dfrac{(k_{pb} + k_{vb}s)e^{-\tau_i s}}{\{[(k_{pb}h + k_{vb} - k_{vf})s + k_{pb} - k_{pf}]e^{-\tau_i s} + s^2 + \varsigma_i s^3\}}, & for\ i \in [i_{dis}, 1] \\ \dfrac{[-(k_{vf} + k_{pf}h)s - k_{pf}]e^{-\tau_i s}}{\{[(k_{pb}h + k_{vb} - k_{vf})s + k_{pb} - k_{pf}]e^{-\tau_i s} + s^2 + \varsigma_i s^3\}}, & for\ i \in [n, i_{dis}) \end{cases} \tag{25}$$

Based on the transfer function (25), the following result on bidirectional string stability is derived.

Theorem 3. The platoon system (10) with a disturbance on the $i_{dis}th$ vehicle is bidirectional string stable if the following conditions are satisfied:

$$hk_{pb} - k_{vf} = 0 \tag{26}$$

For $i \in [i_{dis}, 1]$:

$$k_{pf} \geq 2k_{pb} , \tag{27}$$

$$1 + 2\varsigma_i(k_{pb} - k_{pf})\tau_i - 2k_{vb}\tau_i - \tau_i^2(k_{pf} - k_{pb}) - 2\varsigma_i k_{vb} \geq 0 \tag{28}$$

For $i \in [n, i_{dis})$:

$$k_{pb} \geq 2k_{pf} , \tag{29}$$

$$1 - 2\varsigma_i k_{vb} - 2\varsigma_i(k_{pb} - k_{pf})\tau_i - 2k_{vb}\tau_i \geq 0 , \tag{30}$$

$$k_{vb}^2 - (k_{vf} + k_{pf}h)^2 \geq 2(k_{pb} - k_{pf}) , \tag{31}$$

Proof. First, according to objective (b), $|G(jw)|$ can be written as the following two forms,

1. *Downstream* ($i \in [i_{dis}, 1]$), $|G(jw)| = |a_i(jw)/a_{i-1}(jw)| = \sqrt{\alpha_1/(\alpha_1 + \beta_1)}$

$\alpha_1 = k_{pb}^2 + k_{vb}^2 w^2$,

$$\begin{aligned} \beta_1 &= \varsigma_i^2 w^6 + w^4 + [(hk_{pb} - k_{vf} + k_{vb})^2 - k_{vb}^2]w^2 + k_{pf}^2 \\ &- 2k_{pb}k_{pf} - 2w^2(k_{pb} - k_{pf})\cos(\tau_i w) + 2w^3\varsigma_i(k_{pb} - k_{pf}) \\ &\cdot \sin(\tau_i w) - 2w^3(hk_{pb} - k_{vf} + k_{vb})\sin(\tau_i w) \\ &- 2w^4\varsigma_i(hk_{pb} - k_{vf} + k_{vb})\cos(\tau_i w) \end{aligned} \tag{32}$$

2. *Upstream* ($i \in [n, i_{dis})$), $G(jw) = |a_i(jw)/a_{i+1}(jw)| = \sqrt{\alpha_2/(\alpha_2 + \beta_2)}$

$\alpha_2 = (k_{vf} + k_{pf}h)^2 w^2 + k_{pf}^2$,

$$\begin{aligned} \beta_2 &= \varsigma_i^2 w^6 + w^4 + [2(hk_{pb} - k_{vf} + k_{vb})^2 - (k_{vf} + k_{pf}h)^2]w^2 \\ &+ 2w^3\varsigma_i(k_{pb} - k_{pf})\sin(\tau_i w) \\ &- 2w^3(hk_{pb} - k_{vf} + k_{vb})\sin(\tau_i w) + k_{pb}^2 - 2k_{pb}k_{pf} \\ &- 2w^2(k_{pb} - k_{pf})\cos(\tau_i w) \\ &- 2w^4\varsigma_i(hk_{pb} - k_{vf} + k_{vb})\cos(\tau_i w) \end{aligned} \tag{33}$$

Due to $\alpha_1 > 0$ and $\alpha_2 > 0$, $|G(jw)| \leq 1$ holds true, i.e., the platoon is bidirectional string stable, if $\beta_1 \geq 0$ and $\beta_2 \geq 0$. From (26), (27), (29) and the fact that $\cos(\tau_i w) \leq 1$, $\cos(\tau_i w) \geq 1 - w^2\tau^2/2$, $\sin(\tau_i w) \leq \tau_i w$ and $\sin(\tau_i w) \geq -\tau_i w$, we have for $w > 0$ that,

For *downstream*,

$$\begin{cases} 2w^2(k_{pf} - k_{pb})\cos(\tau_i w) \geq 2w^2(k_{pf} - k_{pb})(1 - w^2\tau^2/2) \\ -2w^4\varsigma_i(hk_{pb} - k_{vf} + k_{vb})\cos(\tau_i w) \geq -2w^4\varsigma_i k_{vb} \\ 2w^3\varsigma_i(k_{pb} - k_{pf})\sin(\tau_i w) \geq 2w^3\varsigma_i(k_{pb} - k_{pf})\tau_i w \\ -2w^3(hk_{pb} - k_{vf} + k_{vb})\sin(\tau_i w) \geq -2w^3 k_{vb}\tau_i w \end{cases} \tag{34}$$

For *upstream*,

$$\begin{cases} -2w^2(k_{pb} - k_{pf})\cos(\tau_i w) \geq -2w^2(k_{pb} - k_{pf}) \\ -2w^4\varsigma_i k_{vb}\cos(\tau_i w) \geq -2w^4\varsigma_i k_{vb} \\ 2w^3\varsigma_i(k_{pb} - k_{pf})\sin(\tau_i w) \geq -2w^3\varsigma_i(k_{pb} - k_{pf})\tau_i w \\ -2w^3(hk_{pb} - k_{vf} + k_{vb})\sin(\tau_i w) \geq -2w^4 k_{vb}\tau_i \end{cases} \tag{35}$$

Substituting the (34) and (35) into (32) and (33), respectively, and reorganized as

Downstream, $\beta_1 \geq \varsigma_i^2 w^6 + [1 + 2\varsigma_i(k_{pb} - k_{pf})\tau_i - 2k_{vb}\tau_i$

$- \tau_i^2(k_{pf} - k_{pb}) - 2\varsigma_i k_{vb}]w^4 + 2(k_{pf} - k_{pb})w^2 + k_{pf}^2 - 2k_{pb}k_{pf}$

Upstream, $\beta_2 \geq \varsigma_i^2 w^6 + [1 - 2\varsigma_i k_{vb} - 2\varsigma_i(k_{pb} - k_{pf})\tau_i - 2k_{vb}\tau_i]w^4$

$-2\varsigma_i(k_{pb} - k_{pf})\tau_i - 2k_{vb}\tau_i]w^4$

$+[k_{vb}^2 - (k_{vf} + k_{pf}h)^2 - 2(k_{pb} - k_{pf})]w^2$

Thus, if the conditions (28) (29) and (31) hold, then $\beta_1 \geq 0$ and $\beta_2 \geq 0$. This implies that $|G(jw)| \leq 1$ for any $w > 0$. This completes the proof.

Remark 4. From Theorem 3 (28) and (30), the upper bound for the actuator delay can be derived as

$$\tau_{i2} \leq \min\{(0.5 - \varsigma_i k_{vb})/(\varsigma_i k_1 + k_{vb}), \\ [k_{vb} - \varsigma_i k_1 \pm \sqrt{(k_{vb} - \varsigma_i k_1)^2 - k_1(1 - 2\varsigma_i k_{vb})}]/k_1\} ,$$

where $k_1 = k_{pb} - k_{pf}$.

Finally, based on the above discussions and all the results established heretofore, the following control algorithm is given.

Algorithm. Our robust controller design algorithm is given as follows:

1). Design the feedback-linearization controller in (3), which is a routine procedure.

2). Calculate the robust H_∞ controller gain, according to Theorem 2 using standard LMI mathematic tool.

3). Constrain the obtained H_∞ controller gain K_i with the conditions given in Theorem 3. If this is feasible, then the resulted controller gain can be used for bidirectional string stabilizing control. Otherwise, reset matrices $P_i > 0$, $T_i > 0$, $Q_{ij} > 0$, $Z_{im} > 0$, N_{il} , S_{il} , M_{il} and other related parameters and return to step 2).

5. SIMULATION AND EXPERIMENTS

5.1 Numerical simulations

The goal of the following simulations is to evaluate the performance of the proposed controller. These simulations are carried out with the virtual environment established by System Build software package in MATLAB. For comparison of performance, an optimal localized control algorithm **(OLC)** (Liu, 2012) is also designed based on the same control plant (10)-(12). Like the fuel economy consideration in proposed controller, its control law is also constrained by the fuel consumption criterion.

The following parameters are used in the simulations: length of vehicle $L_i = 4m$, engineer time constant $\varsigma_i = 0.25$, the time gap $h = 1$ and the noisy measurements of δ_i and $\dot{\delta}_i$ is assumed to be white and zero mean with standard deviations 0.1 m and 0.01m/s, respectively. The other parameters used in the simulations are the same as (Guo, 2014), namely, specific mass of the air $\sigma = 1.2\, kg/m^3$, cross-sectional area of vehicle $A_i = 2.2m^2$, drag coefficient $c_{di} = 0.35$, vehicle mass $m_i = 1464kg$, mechanical drag $d_{mi} = 5N$, saturation level $sat_i = 3.5m/s^2$.

The following parameters were used in the controller design: the delay variation rate $\mu = 1.2$ and the engine time uncertainties are expressed as $|\Delta\varsigma_i| = f_i(t) = D_i/\sin(t)$, $D_i = 4$.

By using Theorem 2 and Theorem 3, the controller gains can be obtained as,

For Downstream: $K_i = [4.2 \quad 8.8 \quad 1.7 \quad 4.2]$ and for *Upstream:* $K_i = [0.5 \quad 3.1 \quad 1.6 \quad 3.1]$.

From Remark 4 the bidirectional string stability can be achieved when $\tau_{i2} \le 0.095s$ for the output feedback controller and set the actuator delay as $\tau = 0.08s$.

In order to demonstrate the special performance of the proposed controller algorithm, the simulation is tested under three kinds of traffic scenario: Fictitious lead vehicle rapid accelerating, fictitious lead vehicle emergency braking and the fifth vehicle suffered a sudden disturbance. The proposed controller will be regarded as successful if its application can improve the string stability, the fuel consumption and tracking capability.

A. Fictitious lead vehicle rapid acceleration

In this scenario, it is assumed that all vehicles in the platoon run at the same initial speed 10m/s. At 5s, the fictitious lead vehicle accelerates at $3m/s^2$ from 10*m/s* to 25*m/s*. All the following vehicles are controlled to follow it by using the proposed controller and OLC algorithms, respectively. The results are shown in Figs. 1 and Figs.2.

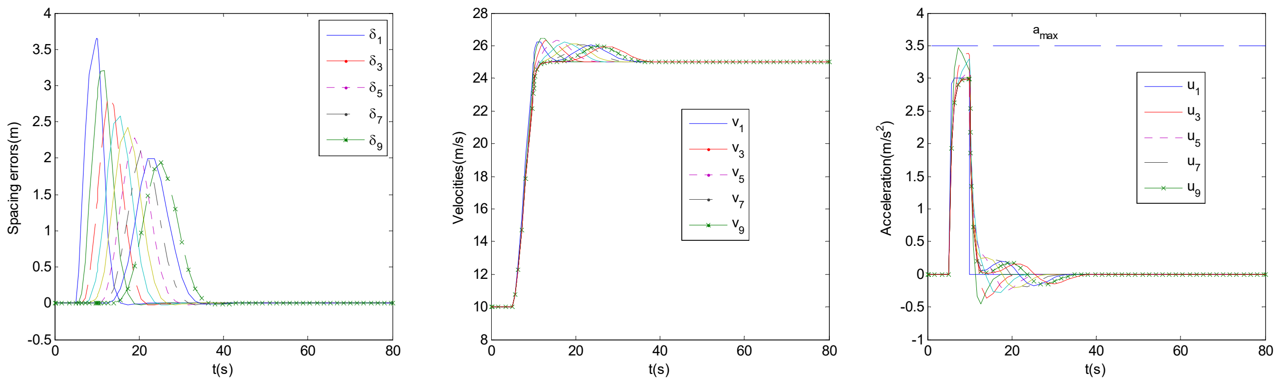

Fig. 1. (a) Spacing errors; (b) Velocities; (c) Acceleration.

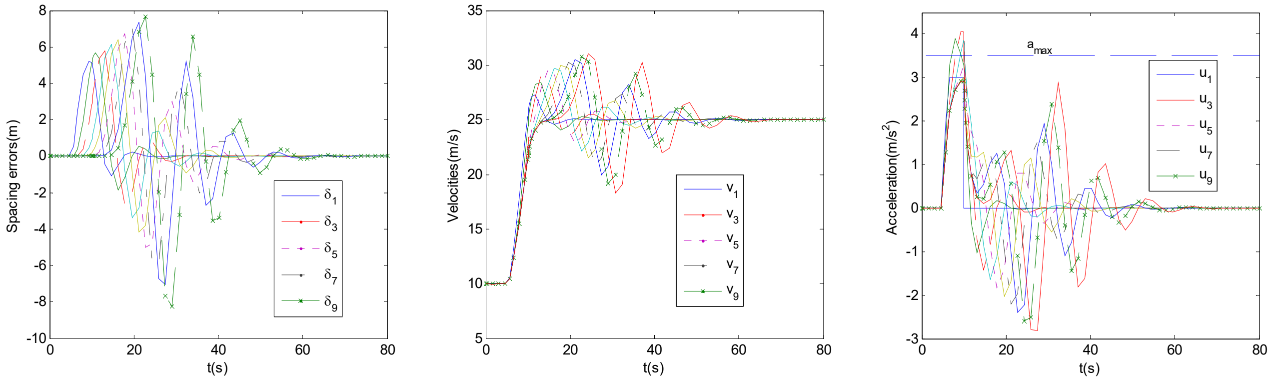

Fig. 2. (a) Spacing errors; (b) Velocities; (c) Acceleration.

It is found from Fig. 1 (c) that the maximum accelerations for all following vehicles in the bidirectional platoon control system under the proposed controller is 3.4 m/s^2 , satisfying the fuel economy. The control input satisfies $u_i < sat_i$, hence the control input windup is avoided. From Figs. 1 (b), it can be seen that the whole platoon can achieve tracking control accurately. The maximum spacing errors is 3.6m, and the string stability can be achieved as shown in Fig. 1 (a). In this same case, when the method suggested in (Liu, 2012) is used, the system is string unstable (see Fig.2). The maximum spacing errors and acceleration are 7.8m and 4 m/s^2 , respectively, which are much higher than in our case in Fig.1 (a) and (c). As shown from Fig. 1 (b) and Fig. 2 (b), the maximum of velocities under the proposed controller is smaller than that under OLC. So, both better fuel economy and better tracking capability can be foreseen in our research.

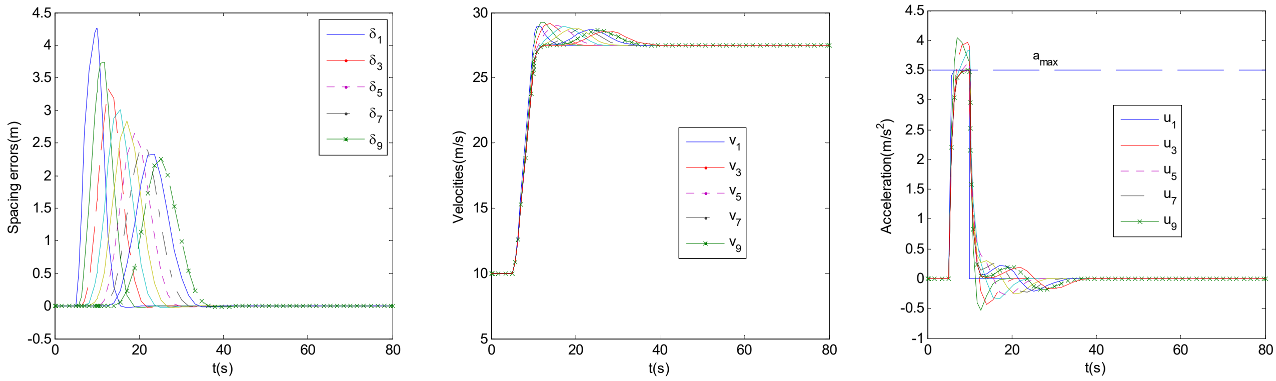

Fig. 3. (a) Spacing errors; (b) Velocities; (c) Acceleration.

When the fictitious lead vehicle accelerates at 3.5 m/s^2 from $0m/s$ to $17.5m/s$ (the input has reached the limits). From Figs. 3 (b), it can be seen that the whole platoon can achieve tracking control accurately as well. The maximum spacing errors and acceleration are $4.2m$ and 4 m/s^2 , respectively.

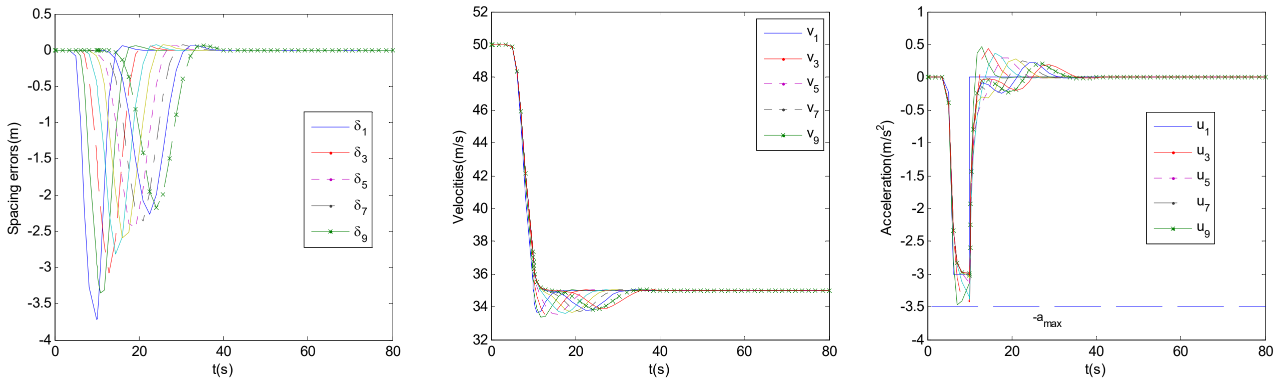

Fig. 4. (a) Spacing errors; (b) Velocities; (c) Acceleration.

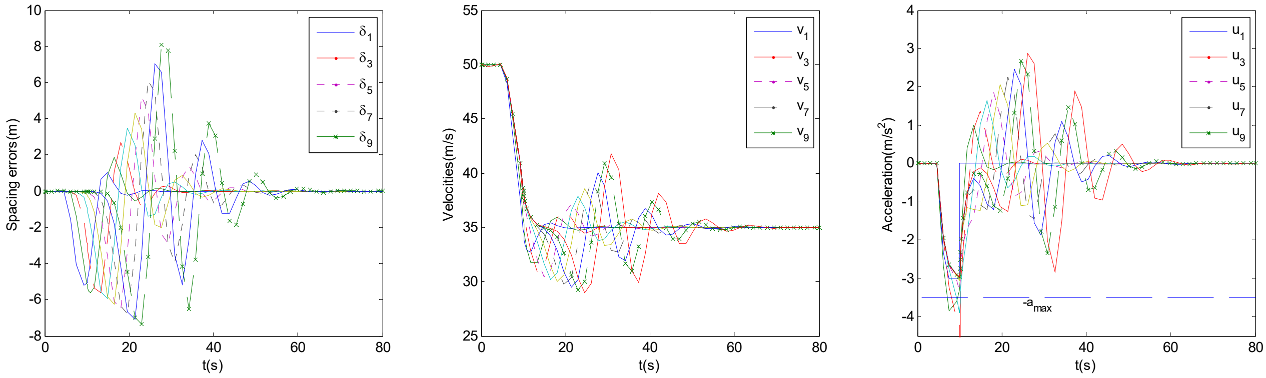

Fig. 5 (a) Spacing errors; (b) Velocities; (c) Acceleration.

B. Fictitious lead vehicle emergency braking

In preceding car's emergency braking scenario, the preceding car decelerates at the acceleration of -3 m/s^2 from 50*m/s* to 35*m/s*, and the minimal safety distance $\delta_{id\,\min} = 4\ m$. All the following vehicles are controlled to follow the fictitious lead by the proposed controller and OLC algorithms, respectively. The results are shown in Figs. 4 and 5. From Fig 4 (a), it is found that the whole platoon can hold string stability. The acceleration is lower than the minimum acceleration and the longitudinal ride comfort can be satisfied. In contrast, as shown in Fig. 5 (a), the maximum spacing error is -5.9*m*, which mean a rear end collision has happened, and the platoon is string instability. The last vehicle in the platoon cannot achieve the cruise velocity, and the maximum acceleration is -4 m/s^2 , as shown in Fig. 5 (b) and (c).

5.2 Experiments

The simulations in the previous sections indicate that the proposed controller is simple in structure and the parameters are easy to tune so that it can be quickly calibrated for a certain vehicle and can meet real-time requirements. In this section, experimental studies are carried out to demonstrate some properties of the proposed controller. The experiments are based on five radio controlled Arduino cars, shown in Fig. 6. (a) and (b), which are driven by two rear wheels and steered by a serve on the front wheel. The infrared sensor is used to measure the front distance and rear distance between the adjacent vehicles, respectively. The actuator lumped delay is 0.04*s*, which is identified using operational data.

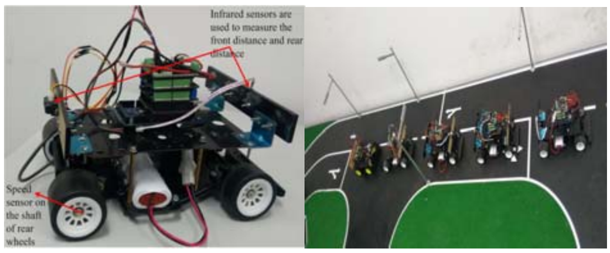

Fig. 6. (a) Arduino car; (b) Arduino platoon system.

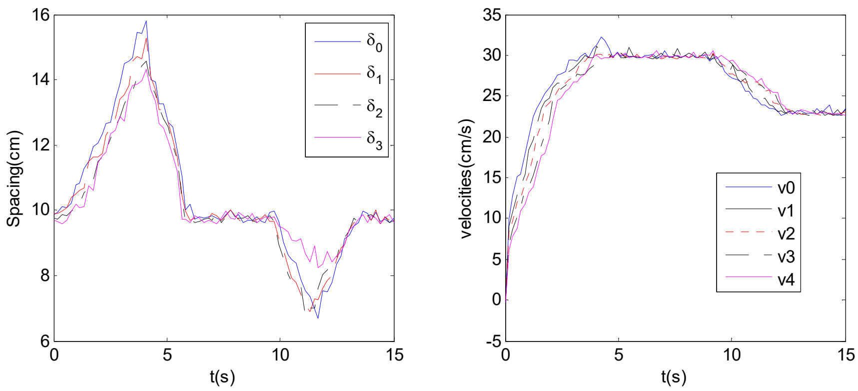

Fig. 7. (a) Spacings; (b) Velocities.

The most important objective of the experiment is to show the effectiveness of the presented method in dealing with a sudden disturbance on the following vehicle. In this case, the desired spacing is set to be 10cm, and a sudden drop of speed from 30cm/s to 22.5cm/s is applied to the third car in the interval [7s, 8s], and actuated by a command from the computer via a wireless communication. When the proposed controller is applied, the spacing error appears to be significantly larger than in the simulation. This is partly caused by measurement noise, the level of which is indicated by the constant velocity sections of the test scenario. Another cause is that the bidirectional platoon system behaviour is sensitive to variations of the neighbouring vehicle's input. Although all test cars are of the same type, these "heterogeneous" occur, among others, because the batteries do not have the same state of charge. As shown in Fig. 7. The second and the fourth car becomes biased during the sudden braking (the maximum spacing error is 6.1cm), while the entire platoon still remains string stable and the objectives in Subsection 2.3 are achieved.

6. CONCLUSIONS

In this paper, a bidirectional platoon control scheme has been developed with the effects of uncertainty, saturations and time delay. A robust controller was designed which can guarantee the individual stability as well as string stability without using the wireless communication. The results were further tested via experiments conducted with the laboratory scale Arduino cars.

There are several interesting questions that worth this research. Such as, if the closed platoon system performance can be improved by using more than two vehicles' information, and derive the fundamental limitations in this framework. In this scheme, the inter-vehicular communication will be required, and the wireless communication among the vehicles share should be considered, but may still be advantageous compared to the predecessor and leader following scheme. These issues raise various open problems that are worth investigating in future research work.

APPENDIX

Here, we present the proof of Theorem 1

Define the following Lyapunov-Krasovskii functional for system (9) as

$$V_i(t) = x_i^T(t)P_i x_i(t) + \int_{t-\tau_{i1}}^{t} x_i^T(s)Q_{i1}x_i(s)ds$$
$$+\int_{t-\tau_{i2}}^{t} x_i^T(s)Q_{i2}x_i(s)ds + \int_{t-\tau_i(t)}^{t} x_i^T(s)Q_{i3}x_i(s)ds \quad \text{(A1)}$$
$$+\int_{-\tau_{i2}}^{0}\int_{t+\theta}^{t} \dot{x}_i^T(s)Z_{i1}\dot{x}_i(s)dsd\theta + \int_{-\tau_{i2}}^{-\tau_{i1}}\int_{t+\theta}^{t} \dot{x}_i^T(s)Z_{i2}\dot{x}_i(s)dsd\theta$$

where $P_i > 0$, $Q_{ij} > 0$, j=1, 2, 3, $Z_{im} > 0$, m=1, 2, are matrices to be determined.

Then, the time derivative of $V_i(t)$ along the trajectory of the platoon system in (9) is given by:

$$\dot{V}_i(t) = \dot{x}_i^T(t)P_i x_i(t) + x_i^T(t)P_i\dot{x}_i(t) + x_i^T(t)Q_{i1}x_i(t)$$
$$- x_i^T(t-\tau_{i1})Q_{i1}x_i(t-\tau_{i1}) - x_i^T(t-\tau_{i2})Q_{i2}x(t-\tau_{i2})$$
$$+ x_i^T(t)Q_{i3}x_i(t) - \int_{t-\tau_{i2}}^{t-\tau_{i1}} \dot{x}_i^T(s)Z_{i2}\dot{x}_i(s)ds + d_{i2}\dot{x}_i^T(t)Z_{i1}\dot{x}_i(t)$$
$$-(1-\dot{\tau}_i(t))x_i^T(t-\tau_i(t))Q_{i3}x_i(t-\tau_i(t))$$
$$+ x_i^T(t)Q_{i2}x_i(t) - \int_{t-\tau_{i2}}^{t} \dot{x}_i^T(s)y_{i1}\dot{x}_i(s)ds + d_{i2}\dot{x}_i^T(t)Z_{i2}\dot{x}_i(t) \quad \text{(A2)}$$

Then, for any appropriately dimensioned matrices $T_i > 0$ and N_{il}, S_{il}, M_{il}, l=1, 2,…,5 yields

$$2\Omega_{i1}\left[x_i(t) - x_i(t-\tau_i(t)) - \int_{t-\tau_i(t)}^{t} \dot{x}_i(s)ds \right] = 0,$$

$$2\Omega_{i2}\left[x_i(t-\tau_i(t)) - x_i(t-\tau_{i2}) - \int_{t-\tau_{i2}}^{t-\tau_i(t)} \dot{x}_i(s)ds \right] = 0,$$

$$2\Omega_{i3}\left[x_i(t-\tau_{i1}) - x_i(t-\tau_i(t)) - \int_{t-\tau_i(t)}^{t-\tau_{i1}} \dot{x}_i(s)ds \right] = 0, \quad \text{(A3)}$$

$$2\left[x_i^T(t)T_i + \dot{x}_i^T(t)T_i\right]\left[-\dot{x}_i(t) + \overline{A}_i x_i(t) + \overline{B}_i u_{\text{sat}_i}(t-\tau_i(t))\right.$$
$$\left.+ B_{id}d_i(t)\right] = 0 \quad \text{(A4)}$$

where

$$\Omega_{i1} = x_i^T(t)N_{i1} + x_i^T(t-\tau_i(t))N_{i2} + x_i^T(t-\tau_{i1})N_{i3}$$
$$+ x_i^T(t-\tau_{i2})N_{i4} + \dot{x}_i^T(t)N_{i5},$$

$$\Omega_{i2} = x_i^T(t)S_{i1} + x_i^T(t-\tau_i(t))S_{i2} + x_i^T(t-\tau_{i1})S_{i3}$$
$$+ x_i^T(t-\tau_{i2})S_{i4} + \dot{x}_i^T(t)S_{i5}$$

$$\Omega_{i3} = x_i^T(t)M_{i1} + x_i^T(t-\tau_i(t))M_{i2} + x_i^T(t-\tau_{i1})M_{i3}$$
$$+ x_i^T(t-\tau_{i2})M_{i4} + \dot{x}_i^T(t)M_{i5}.$$

Noticing that the following equations hold

$$2x_i^T(t)T_i\overline{B}_i u_{\text{sat}_i}(t-\tau_i(t)) = 2\sum_{j=1}^{q} x_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}^j(t-\tau_i(t))$$

$$2\dot{x}_i^T(t)T_i\overline{B}_i u_{\text{sat}_i}(t-\tau_i(t)) = 2\sum_{j=1}^{q} \dot{x}_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}^j(t-\tau_i(t)),$$

where $u_{\text{sat}_i}^j(t-\tau_i(t)) = k_i^j c_i^j x_i(t-\tau_i(t))$.

Then, according to (7), for each term $2x_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}^j(t-\tau_i(t))$,

a. if $x_i^T(t)T_i\overline{b}_i^{\,j} \geq 0$ and $k_i^j c_i^j x_i(t-\tau_i(t)) \leq -sat_i^{j\max}$, then for $-sat_i^{j\max} \leq h_i^j c_i^j x_i(t-\tau_i(t))$ we have

$$2x_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}(t-\tau_i(t)) = -2x_i^T(t)T_i\overline{b}_i^{\,j}sat_i^{j\max}$$
$$\leq 2x_i^T(t)T_i\overline{b}_i^{\,j}h_i^j c_i^j x_i(t-\tau_i(t)).$$

b. if $x_i^T(t)T_i\overline{b}_i^{\,j} \geq 0$ and $k_i^j c_i^j x_i(t-\tau_i(t)) \geq -sat_i^{j\max}$, we have

$$2x_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}^j(t-\tau_i(t)) \leq 2x_i^T(t)T_i\overline{b}_i^{\,j}k_i^j c_i^j x_i(t-\tau_i(t)).$$

c. if $x_i^T(t)T_i\overline{b}_i^{\,j} \geq 0$ and $k_i^j c_i^j x_i(t-\tau_i(t)) \geq -sat_i^{j\max}$, then for $sat_i^{j\max} \geq h_i^j c_i^j x_i(t-\tau_i(t))$ we have

$$2x_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}(t-\tau_i(t)) = 2x_i^T(t)T_i\overline{b}_i^{\,j}sat_i^{j\max}$$
$$\leq 2x_i^T(t)T_i\overline{b}_i^{\,j}h_i^j c_i^j x_i(t-\tau_i(t)).$$

d. if $x_i^T(t)T_i\overline{b}_i^{\,j} \leq 0$ and $k_i^j c_i^j x_i(t-\tau_i(t)) \leq sat_i^{j\max}$, we have

$$2x_i^T(t)T_i\overline{b}_i^{\,j}u_{\text{sat}_i}^j(t-\tau_i(t)) \leq 2x_i^T(t)T_i\overline{b}_i^{\,j}k_i^j c_i^j x_i(t-\tau_i(t)).$$

By combining all the above four cases, we have

$$2x_i^T(t)T_i\bar{b}_i^j u_{\text{sat}_i}^j(t-\tau_i(t)) \le \max\{2x_i^T(t)T_i\bar{b}_i^j k_i^j c_i^j x_i(t-\tau_i(t)), 2x_i^T(t)T_i\bar{b}_i^j h_i^j c_i^j x_i(t-\tau_i(t))\}$$

for any $x_i \in \xi(P_i,1)$ and each $j \in [1,q]$.

Now if $2x_i^T(t)T_i\bar{b}_i^j u_{\text{sat}_i}^j(t-\tau_i(t)) < 2x_i^T(t)T_i\bar{b}_i^j h_i^j c_i^j x_i(t-\tau_i(t))$, we set $v_i^j=1$, otherwise we set $v_i^j=0$. Then it is clear that $2x_i^T(t)T_i\bar{B}_i u_{\text{sat}_i}(t-\tau_i(t)) \le 2x_i^T(t)T_i\bar{B}_i W_i(V_i,K_i,H_i)x_i(t-\tau_i(t))$, where $v_i \in \psi_i(v_i)$. Similarly, it also follows that $2\dot{x}_i^T(t)T_i\bar{B}_i u_{\text{sat}_i}(t-\tau_i(t)) \le 2\dot{x}_i^T(t)T_i\bar{B}_i W_i(S_i,K_i,H_i)x_i(t-\tau_i(t))$ where $S_i \in \psi_i(S_i)$.

Hence, we can see from (A4) that for every $x_i \in \xi(P_i,1)$ it holds that

$$\begin{aligned}
0 &= 2\left[x_i^T(t)T_i + \dot{x}_i^T(t)T_i\right]\left[-\dot{x}_i(t) + \bar{A}_i x_i(t) + \bar{B}_i u_{\text{sat}\,i}(t-\tau_i(t)) + B_{id}d_i(t)\right] \\
&\le 2x_i^T(t)T_i\left[-\dot{x}_i(t) + \bar{A}_i x_i(t) + \bar{B}_i W_i(V_i,K_i,H_i)x_i(t-\tau_i(t)) + B_{id}d_i(t)\right] \\
&\quad + 2\dot{x}_i^T(t)T_i\left[-\dot{x}_i(t) + \bar{A}_i x_i(t) + \bar{B}_i W_i(S_i,K_i,H_i)x_i(t-\tau_i(t)) + B_{id}d_i(t)\right]
\end{aligned} \tag{A5}$$

After adding (A3) and (A5) to (A2) and some algebraic manipulations it yields

$$\begin{aligned}
\dot{V}_i(t) &= \Phi_i^T(t)[\Pi_i + d_{i2}N_iZ_{i1}^{-1}N_i^T + d_{i12}S_i(Z_{i1}+Z_{i2})^{-1}S_i^T + d_{i2}M_iZ_{i2}^{-1}M_i^T]\Phi_i(t) \\
&- \int_{t-\tau i(t)}^{t}[\Phi_i^T(t)N_i + \dot{x}_i(s)Z_{i1}]Z_{i1}^{-1}[\Phi_i^T(t)N_i + \dot{x}_i(s)]^T ds \\
&- \int_{t-\tau_{i2}}^{t-\tau_i(t)}[\Phi_i^T(t)S_i + \dot{x}_i^T(s)(Z_{i1}+Z_{i2})](Z_{i1}+Z_{i2})^{-1} \cdot [\Phi_i^T(t)S_i + \dot{x}_i^T(s)(Z_{i1}+Z_{i2})]^T ds \\
&- \int_{t-\tau_i(t)}^{t-\tau i_1}[\Phi_i^T(t)M_i + \dot{x}_i^T(s)Z_i]Z_i^{-1}[\Phi_i^T(t)M_i + \dot{x}_i^T(s)Z_{i1}]^T ds
\end{aligned}$$

$$\le \Phi_i^T(t)[\Pi_i + \tau_{i2}N_iZ_{i1}^{-1}N_i^T + \tau_{i12}S_i(Z_{i1}+Z_{i2})^{-1}S_i^T + \tau_{i2}M_iZ_{i2}^{-1}M_i^T]\Phi_i(t) \tag{A6}$$

where

$$\Phi_i(t) = \left[x_i(t) \quad x_i(t-\tau_i(t)) \quad x_i(t-\tau_{i1}) \quad x_i(t-\tau_{i2}) \quad \dot{x}_i(t) \quad d_i(t)\right]^T.$$

By Shur complement, $\Pi_i + \tau_{i2}N_iZ_{i1}^{-1}N_i^T + \tau_{i12}S_i(Z_{i1}+Z_{i2})^{-1}S_i^T + \tau_{i2}M_iZ_{i2}^{-1}M_i^T < 0$ is equivalent to

$$\begin{bmatrix} \Pi_i & \tau_{i2}N_i & \tau_{i12}S_i & \tau_{i2}M_i \\ * & -\tau_{i2}Z_{i1} & \mathbf{0} & \mathbf{0} \\ * & * & -\tau_{i12}(Z_{i1}+Z_{i2}) & \mathbf{0} \\ * & * & * & -\tau_{i2}Z_{i2} \end{bmatrix} < 0 \tag{A7}$$

Next, the asymptotic stability of the system in (10) is established with $d_i(t)=0$, that is $\dot{x}_i(t) = \bar{A}_i x(t) + \bar{B}_i u_{sat_i}(t-\tau_i(t))$, then $\dot{V}(t)$ can be reduced to $\dot{V}_i(t) \le \tilde{\Phi}_i^T(t)\Pi_i\tilde{\Phi}_i(t)$,

where
$\tilde{\Phi}_i(t) = \left[x_i(t) \quad x_i(t-\tau_i(t)) \quad x_i(t-\tau_{i1}) \quad x_i(t-\tau_{i2}) \quad \dot{x}_i(t)\right]^T$,

and $\tilde{\Pi}_i = \begin{bmatrix} \Pi_{ir} & \tau_{i2}N_{ir} & \tau_{i12}S_{ir} & \tau_{i2}M_{ir} \\ * & -\tau_{i2}Z_{i1} & 0 & 0 \\ * & * & -\tau_{i12}(Z_{i1}+Z_{i2}) & 0 \\ * & * & * & -\tau_{i2}Z_{i2} \end{bmatrix} < 0$,

where

$N_{ir} = \left[N_{i1} \quad N_{i2} \quad N_{i3} \quad N_{i4} \quad N_{i5}\right]^T$, $S_{ir} = \left[S_{i1} \quad S_{i2} \quad S_{i3} \quad S_{i4} \quad S_{i5}\right]^T$,

$M_{ir} = \left[M_{i1} \quad M_{i2} \quad M_{i3} \quad M_{i4} \quad M_{i5}\right]^T$.

Equation (14) implies that $\Pi_{ir} < 0$, which further leads to $\dot{V}_i(t) < 0$. Therefore, the system (10) with the uncertainties, time-varying delay and actuator saturation is robust asymptotically stable.

Now, the H_∞ performance of the system under zero initial conditions should be established. Considering the following index:

$$J_i = \int_0^\infty [y_{i1}^T(t)y_{i1}(t) - \gamma_i^2 d_i^T(t)d_i(t)]dt \tag{A8}$$

This implies

$$J_i \le \int_0^\infty [y_{i1}^T(t)y_{i1}(t) - \gamma_i^2 d_i^T(t)d_i(t) + \dot{V}_i(t)]dt \tag{A9}$$

for all nonzero $d_i(t)$.

Via algebraic manipulations and Schur complement, yields

$$y_{i1}^T(t)y_{i1}(t) - \gamma_i^2 d_i^T(t)d_i(t) + \dot{V}_i(t) \le \tilde{\Phi}_i^T(t)\Pi_{is}\tilde{\Phi}_i(t) \tag{A10}$$

Then, using $\Pi_{is} < 0$, yields $y_{i1}^T(t)y_{i1}(t) - \gamma_i^2 d_i^T(t)d_i(t) + \dot{V}_i(t) < 0$, which implies $J_i < 0$. Hence $\|y_{i1}\|_2 < \gamma_i\|d_i(t)\|_2$ is guaranteed for any nonzero $d_i(t) \in L_2[0,\infty]$, and the H_∞ performance is established.

Finally, the safe distance constraint needs to be guaranteed. From above it is ensured that $y_{i1}^T(t)y_{i1}(t) - \gamma_i^2 d_i^T(t)d_i(t) < 0$, and by integrating both sides of which yields

$V_i(t) - V_i(0) < \gamma_i^2 \int_0^t d_i^T(t)d_i(t)dt < \gamma_i^2\|d_i\|_2^2$, then $x_i^T(t)P_ix_i(t) < \gamma_i^2 d_{i\max} + V_i(0) = \vartheta_i$. Moreover, it is also true that

$$\max\left|y_{i2}(t)\right|^2 = \max\left|x_i^T(t)C_{i2}^T C_{i2} x_i(t)\right|^2$$
$$= \max\left|x_i^T(t)P_i^{1/2}P_i^{-1/2}C_{i2}^T C_{i2} P_i^{1/2}P_i^{-1/2}x_i(t)\right|^2 ,$$
$$< \vartheta_i \cdot \lambda_{i\max}(P_i^{-1/2}C_{i2}^T C_{i2} P_i^{-1/2})$$

where $\lambda_{i\max}(.)$ represents the maximal eigenvalue of a matrix. From above, it is easy to see that the safe distance constraint is guaranteed if $\vartheta_i \cdot \lambda_{i\max}(P_i^{-1/2}C_{i2}^T C_{i2} P_i^{-1/2}) < \delta_{id\min}^2 I$, which is equivalent to (15), according to Schur complement. This completes the proof.

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