# Passive Fault Tolerant Control of Induction Motors Using Nonlinear Block Control

Hossein Tohidi<sup>\*</sup>. Koksal Erenturk<sup>\*\*</sup>, Sajjad Shoja-Majidabad<sup>\*\*\*</sup>

\* Electrical Engineering Department, Malekan Branch, Islamic Azad University, Malekan, Iran,
 \*\* Department of Electrical and Electronic Engineering, Ataturk University, 25240, Erzurum, Turkey,
 \*\*\* Electrical Engineering Department, University of Bonab, Bonab, Iran,
 (e-mail: hosseintohidi@ymail.com, keren@atauni.edu.tr, shoja.sajjad@gmail.com)

Abstract: This paper studies passive fault tolerant control (PFTC) of three phase induction motors (IMs). First, a nonlinear block control (NBC) transformation is applied to handle nonlinearities of motor faulty model. Afterwards, a fault tolerant speed controller is developed based on sliding mode control (SMC) technique with linear sliding surface. However, the utilized NBC transformation needs multiple derivatives calculations which make it computationally burden. In addition, this NBC transformation has unmatched fault rejection problem which SMC controller is not able to remove the fault. To avoid the mentioned drawbacks, a novel and simple NBC transformation is proposed for IM. Also, a new nonlinear sliding surface is presented in order to enhance the SMC controller performance. Comparative simulations demonstrated performance of the proposed controllers for matched and unmatched faults and load torque uncertainties.

*Keywords:* Induction motor, Fault tolerant control, Nonlinear block control, Sliding mode control, Matched and unmatched faults.

# 1. INTRODUCTION

When a fault occurs in a system components including sensors, actuators, and plant, it can cause performance degradation and even instability of the system. Therefore, there is a crucial need to design fault tolerant controllers (FTCs) to compensate faults effects and guarantee system reliability and stability with an acceptable performance. Generally speaking, the FTCs can be classified into two types (Kanev, 2006; Zhang and Jiang, 2008; Campos-Delgado et al., 2008): active fault tolerant control (AFTC) and passive fault tolerant control (PFTC). The AFTC technique usually needs a fault detection and isolation (FDI) scheme that has the task to detect and localize faults that eventually occur in the system. The PFTC is based on robust control techniques which makes system insensitive to certain faults that taken into account in the design step. This technique requires no redesign and online detection of faults, and therefore is computationally more interesting.

Three phase induction motors (IMs) are very important in industry because of their reliability, high performance and low costs (Gaied, 2014b). However, due to mechanical, environmental, thermal and electrical stresses, IMs are subjected to multiple faults. Stator and rotor failures including eccentricity and broken bars are the most famous asymmetric faults. The eccentricity faults can be categorized into two types: static eccentricity and dynamic eccentricity. The static and dynamic eccentricities can cause stator and rotor asymmetries, respectively (Vas, 1994; Bonivento et al., 2004). From AFTC point of view, different FDI techniques have been proposed for dynamical systems such as IMs for designing reliable controllers. In (Cho et al., 1992; Bachir et al., 2006; Karami et al., 2010), the authors have detected the broken bar fault by model-based parameter estimation techniques. Multiple signal processing techniques like fast Fourier transform (FFT) are studied in (Hedayati Kia et al., 2004; Vaimann and Kallaste, 2011). In (Filippetti et al., 2000; Benbouzid et al., 2007; Zidani et al., 2008), artificial intelligence techniques have discussed for fault detection. Also, some other sliding mode (Yan and Edwards, 2007), adaptive (Najafabadi et al., 2011), and sensorless methods (Holtz, 2006; Aurora and A. Ferrara, 2007; Ghanes and Zheng, 2009a; Ghanes et al., 2010b; Gaeid et al., 2012a) are proposed in the recent years.

From PFTC point of view, in recent years, many methods have been employed and combined with each other's for providing reliable controllers for linear and nonlinear systems. A reliable  $H_{\infty}$  controller has been proposed in (Shi et al., 2014). In (Jin et al., 2013), a non-fragile  $H_{\infty}$ compensation filter has been designed in the framework of linear matrix inequalities (LMIs) technique. An output feedback fuzzy controller has been developed in (Wang et al, 2013) for T–S fuzzy systems with sensor faults based on LMI. Passivity technique is discussed in (Benosman and Lum, 2010a, 2010b) with respect to loss of actuator effectiveness. Moreover, SMC technique have received great attentions for spacecraft and quadrotor vehicles control due to its robust behaviour against model uncertainties (Li et al., 2013; Mirshams et al., 2014; Merheb et al., 2015). In recent two decades, some studies have been developed for IMs control with PFTC (Bonivento *et al.*, 2004, Fekih, 2007, Djeghali *et al.*, 2011, 2013), and the most interesting approach is the SMC. In (Fekih, 2007), the author have proposed an approach which detects the occurrence of fault and switch itself between a nominal operation controller and sliding mode faulty controller. In (Djeghali *et al.*, 2011, 2013), a second-order sliding mode controller (SOSMC) is designed based of backstepping technique. The SOSMC is utilized to avoid the chattering phenomenon, also the backstepping technique is applied to bring the IM model nonlinearities into the control inputs direction. But both of the mentioned literatures only have studied the rotor broken bar fault and there is no attention on other faults like the stator eccentricity.

In this paper, two passive fault tolerant sliding mode controllers are developed for IM in order to attenuate the effects of the stator and rotor faults (broken bar plus eccentricity) and the load torque variations. The first controller is developed based on NBC technique which is reported in (Loukianov, 2002a; Loukianov et al., 2002b). This NBC technique is a powerful tool to handle nonlinearities of IM model. But it is computationally burden and it is not able to remove the complete effects of nonvanishing unmatched faults/uncertainties. To solve the mentioned problem an improved NBC technique is proposed, which is the first novelty of this study. On the other hand, since the SMC with linear sliding surface presents slow transient responses and considerable tracking errors (Shoja Majidabad and Toosian Shandiz, 2012), then a novel nonlinear sliding surface is suggested that shows fast transient response with small tracking errors. This technique is known as nonlinear sliding mode control (NSMC).

This paper is organized as follows: In Section 2, the IM healthy and faulty dynamical models are presented. Two NBC transformation techniques are implemented for IM faulty model in Section 3. In Section 4, two robust fault tolerant sliding mode controllers are developed based on the NBC transformed models. In Section 5, the merits of the designed controllers are verified by the simulations on IM subjected to the matched/unmatched faults and load torque disturbances. Finally, conclusions are given in Section 6.

## 2. DYNAMIC MODEL OF IM AND VECTROR CONTROL

In this section, healthy and faulty models of IM are expressed. In addition, the model simplification based on field-oriented control (FOC) technique is explained.

### 2.1. Healthy Model

Under balance operation, the state space model of IM is presented in the synchronously rotating reference frame (d-q) by the following equations (Fekih, 2007; Djeghali et al., 2013; Krause et al., 2013):

$$v_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_s \psi_{qs}$$

$$v_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_s \psi_{ds}$$

$$v_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} - (\omega_s - \omega_r)\psi_{qr}$$

$$v_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} + (\omega_s - \omega_r)\psi_{dr}$$

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr}$$
  

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr}$$
  

$$\psi_{dr} = L_r i_{dr} + L_m i_{ds}$$
  

$$\psi_{qr} = L_r i_{qr} + L_m i_{qs}$$

$$\begin{aligned} \frac{d\omega_m}{dt} &= \frac{1}{J} \left( T_e - F \omega_m - T_L \right), \qquad \omega_r = p \, \omega_m \\ T_e &= p \, \frac{L_m}{L_r} \left( \psi_{dr} i_{qs} - \psi_{qr} i_{ds} \right) \end{aligned} \tag{1}$$

where  $i_{ds}$ ,  $i_{qs}$  are the components of the stator current,  $\psi_{dr}$ ,  $\psi_{qr}$  are the components of the rotor flux,  $v_{ds}$ ,  $v_{qs}$  are the stator voltage components,  $v_{dr}$ ,  $v_{qr}$  are the rotor voltage components,  $\omega_s$  is the stator pulsation,  $\omega_r$  is the electrical angular speed,  $\omega_m$  is the mechanical speed,  $R_s$  and  $R_r$  are stator and rotor resistances,  $L_s$  and  $L_r$  are stator and rotor inductances and  $L_m$  is the mutual inductance, p is the number of pole pairs,  $T_e$  is the electromagnetic torque,  $T_L$  is an unknown load torque, J is the moment of inertia coefficient, and F is the friction coefficient.

Based on the FOC strategy, the coupling between the rotor flux vector  $[\psi_{dr}, \psi_{qr}]^T$  and the electromagnetic torque  $T_e$  should be reduced. This coupling is reducible by selecting the rotor flux orientation as follows (Fekih, 2007; Djeghali et al., 2013):

$$\psi_{dr} = \psi_r \quad , \psi_{ar} = 0 \tag{2}$$

From the above assumption, the IM model will be simplified as:

$$\frac{d\omega_m}{dt} = \frac{pL_m}{L_r J} i_{qs} \psi_{dr} - \frac{F}{J} - \frac{T_L}{J}$$

$$\frac{d\psi_{dr}}{dt} = \frac{L_m}{\tau_r} i_{ds} - \frac{1}{\tau_r} \psi_{dr}$$

$$\frac{di_{ds}}{dt} = -ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r} \psi_{dr} + \frac{v_{ds}}{\sigma L_s}$$

$$\frac{di_{qs}}{dt} = -ai_{qs} - \omega_s i_{ds} - \frac{L_m P p}{\sigma L_s L_r} \omega_m \psi_{dr} + \frac{v_{qs}}{\sigma L_s}$$
(3)

With

$$\omega_s = p\omega_m + \frac{L_m}{\tau_r \psi_{dr}} i_{qs} \tag{4}$$

where  $\tau_r = \frac{L_r}{R_r}$  is the rotor time constant,  $\sigma = 1 - \frac{L_m^2}{L_s L_r}$  is the

coefficient of dispersion, and  $a = \left(\frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma \tau_r}\right)$  is a constant.

# 2.2. Faulty Model

In this study, two classes of faults in the IM model are considered (Vas, 1994; Bonivento et al., 2004):

1- Rotor asymmetries, mainly due to broken rotor bars or dynamic eccentricity. Broken rotor bars lead to the rotor resistance variation. This variation can be modeled by substituting  $R_r + \Delta R_r$  instead of  $R_r$  in the IM dynamic model (3).

#### 2- Stator asymmetries, mainly due to static eccentricity.

From [6], it turns out that in the presence of static and dynamic eccentricities, faults generates asymmetries in the induction motor, yielding some slot harmonics in the stator winding. In the (d-q) reference frame, it is possible to model this effect by adding a sinusoidal corruption term to the stator currents values. Specifically, letting  $i_d^{uf}(t)$  and  $i_q^{uf}(t)$  denote the values of the stator current in the absence of faults and  $i_d^f(t)$  and  $i_q^f(t)$  are corresponding values in the presence of faults, the current can be expressed in the form

$$i_{sd}^{f}(t) = i_{sd}^{uf} + A\sin(\omega_{c}(t) + \theta_{s}(t) + \phi),$$
  

$$i_{sq}^{f}(t) = i_{sq}^{uf} + A\cos(\omega_{c}(t) + \theta_{s}(t) + \phi)$$
(5)

Where the amplitude A and the phases  $\phi$  are unknown, they depend on the stator or rotor faults entity.

Supposing that faults only occur when induction motor is working in a steady operation, then  $\omega_c(t) + \theta_s(t)$  the faults concerning static eccentricity are concerned,

$$\omega_c(t) + \theta_s(t) = 2\pi f t + (p\omega_{m,ref} + \omega_{sl})t + \theta_{s0}$$
(6)

The faults concerning dynamic eccentricity are concerned,

$$\omega_c(t) + \theta_s(t) = 2(\pi f \pm 2k\omega_{sl})t + (p\omega_{m,ref} + \omega_{sl})t + \theta_{s0}$$
(7)

where  $\omega_{sl} = \omega_s - p\omega_{m,ref}$  is the slip angular frequency,  $\omega_{m,ref}$  is the rotor speed reference, f is the supply frequency,  $\theta_{s0}$  is the unknown position of the reference frame once the fault occurs, k = 1,...,N is the finite integer.

From the above mentioned faults, the IM faulty model can be described as

$$\frac{d\omega_m}{dt} = \frac{pL_m}{L_r J} i_{qs} \psi_{dr} - \frac{F}{J} + h_1(t)$$

$$\frac{d\psi_{dr}}{dt} = \frac{L_m}{\tau_r} i_{ds} - \frac{1}{\tau_r} \psi_{dr} + h_2(t)$$

$$\frac{di_{ds}}{dt} = -ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r} \psi_{dr} + \frac{v_{ds}}{\sigma L_s} + h_3(t)$$

$$\frac{di_{qs}}{dt} = -ai_{qs} - \omega_s i_{ds} - \frac{L_m P p}{\sigma L_s L_r} \omega_m \psi_{dr} + \frac{v_{qs}}{\sigma L_s} + h_4(t)$$
(8)

Where  $h_1(t) = -\frac{T_L}{J} + h_1(\Delta i_{sdq})$ ,  $h_2(t) = h_2(\Delta i_{sdq}, \Delta R_r)$ ,  $h_3(t) = h_3(\Delta i_{sdq}, \Delta R_r)$  and  $h_4(t) = h_4(\Delta i_{sdq}, \Delta R_r)$  represent the fault and uncertainty terms due to rotor resistance variation, harmonics in the stator winding  $(\Delta i_{sdq} = i_{sdq}^f - i_{sdq}^{uf})$ and load torque variation. It is worthy to notify that the load torque  $T_L$  is added with the fault terms due to its typically uncertain nature.

The control objective is to design sliding mode controllers for governing the rotor mechanical speed  $\omega_m$  and the rotor flux  $\psi_r = [\psi_{dr}, \psi_{qr}]^T$  in the presence of the faults and uncertainties. By taking a glance in the IM faulty model (8), it is evident that the model is not in the companion form. Therefore, SMC technique is not applicable directly. To solve this problem, the NBC transformation technique will be developed in the next section.

#### 3. NBC TRANSFORMATION

In this section, two types of NBC transformation techniques are expressed. The first one is the conventional NBC technique which is presented in (Loukianov *et al.*, 2002; Loukianov, 2002). But because of non-vanishing uncertainty problem and high computational characteristic, we proposed the second and new method. More details are presented in below:

To make the IM faulty notations (8) less complex, consider the following new variables:

$$x_{11} = \omega_m, \quad x_{12} = \psi_{dr}, \quad x_{21} = i_{ds}, \quad x_{22} = i_{qs}$$
 (9)

Hence, the motor dynamics can be written as

$$\frac{d}{dt}x_{11} = \frac{pL_m}{L_rJ}x_{12}x_{22} - \frac{F}{J}x_{11} + h_1(t)$$

$$\frac{d}{dt}x_{12} = \frac{L_m}{\tau_r}x_{21} - \frac{1}{\tau_r}x_{12} + h_2(t)$$

$$\frac{d}{dt}x_{21} = -ax_{21} + \omega_s x_{22} + \frac{L_m}{\sigma L_s L_r \tau_r}x_{12} + \frac{1}{\sigma L_s}v_{ds} + h_3(t)$$

$$\frac{d}{dt}x_{22} = -ax_{22} - \omega_s x_{21} - \frac{L_m Pp}{\sigma L_s L_r}x_{11}x_{12} + \frac{1}{\sigma L_s}v_{qs} + h_4(t)$$
(10)

The system model (10), can be represented in the following NBC-form as

$$\dot{X}_1 = f_1(X_1) + B_1(X_1)X_2 + H_1(t)$$

$$\dot{X}_2 = f_2(X_1, X_2) + B_2u(t) + H_2(t)$$
(11)

where

$$\begin{aligned} X_{1} &= \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}, \qquad f_{1}(X_{1}) = \begin{bmatrix} -\frac{F}{J} x_{11} \\ -\frac{1}{\tau_{r}} x_{12} \end{bmatrix} \\ X_{2} &= \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}, \qquad f_{2}(X_{1}, X_{2}) = \begin{bmatrix} -ax_{21} + \omega_{s}x_{22} + \frac{L_{m}}{\sigma L_{s}L_{r}\tau_{r}} x_{12} \\ -ax_{22} - \omega_{s}x_{21} - \frac{L_{m}p}{\sigma L_{s}L_{r}} x_{11}x_{12} \end{bmatrix} \\ B_{1}(X_{1}) &= \begin{bmatrix} 0 & \frac{pL_{m}}{L_{r}} x_{12} \\ \frac{L_{m}}{\tau_{r}} & 0 \end{bmatrix}, \qquad B_{2} = \begin{bmatrix} \frac{1}{\sigma L_{s}} & 0 \\ 0 & \frac{1}{\sigma L_{s}} \end{bmatrix} \\ u(t) &= \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix}, \qquad H_{1}(t) = \begin{bmatrix} h_{1}(t) \\ h_{2}(t) \end{bmatrix}, \qquad H_{2} = \begin{bmatrix} h_{3}(t) \\ h_{4}(t) \end{bmatrix} \end{aligned}$$

The NBC-form (11) is composed of two blocks, where  $X_2$  is the fictitious control for the first block, and  $H_1(t)$  and  $H_2(t)$ are the unmatched and matched fault vectors, respectively. **Assumption 1:** Assume that the fault vectors be bounded as

$$\left\|H_1(t)\right\| < H_{1\max} \tag{12}$$

$$\left\|H_2(t)\right\| < H_{2\max} \tag{13}$$

where  $H_{1\text{max}}$  and  $H_{2\text{max}}$  are known positive constants.

3.1. Conventional NBC

Now, let us to define the following tracking errors

$$Z_{1} = \begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} = X_{1,ref} - X_{1} = \begin{bmatrix} x_{11,ref} - x_{11} \\ x_{12,ref} - x_{12} \end{bmatrix} = \begin{bmatrix} \omega_{m,ref} - \omega_{m} \\ \psi_{dr,ref} - \psi_{dr} \end{bmatrix}$$
(14)

where  $X_{1,ref}$  is the desired value of  $X_1$ .

The time derivative of (14) is

$$\dot{Z}_1 = \dot{X}_{1,ref} - \dot{X}_1$$
 (15)

Substituting the first block of (11) in (15), yields

$$\dot{Z}_1 = -f_1(X_1) + \dot{X}_{1,ref} - B_1(X_1)X_2 - H_1(t)$$
(16)

Let the fictitious control input  $X_2$  for (16) be chosen as

$$X_2 = B_1^{-1}(X_1)[-f_1(X_1) + \dot{X}_{1,ref} + K_1Z_1 - Z_2]$$
(17)

where 
$$Z_2$$
 is a new variable, and  $K_1 = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{12} \end{bmatrix}$ ,  $k_{11} > 0$ ,  $k_{12} > 0$  is the control gain matrix.

Inserting the virtual control law (17) in (16), results in the first transformation block in the new coordinates

$$\dot{Z}_1 = -K_1 Z_1 + Z_2 - H_1(t) \tag{18}$$

From (17),  $Z_2$  can be derived as follows

$$Z_2 = f_1(X_1) - \dot{X}_{1,ref} - K_1 Z_1 + B_1(X_1) X_2$$
(19)

The time derivative of (19) is

$$\dot{Z}_2 = \frac{d}{dt} (f_1(X_1)) - \ddot{X}_{1,ref} - K_1 \dot{Z}_1 + \frac{d}{dt} (B_1(X_1)) X_2 + B_1(X_1) \dot{X}_2$$
(20)

The above equation can be represent as

$$\dot{Z}_2 = F(Z_1, Z_2, X_1, X_{1, ref}, X_2) + B(X_1)u(t) + H(t)$$
 (21)

with the following details

$$F(Z_1, Z_2, X_1, X_{1,ref}, X_2) = \begin{bmatrix} -\frac{F}{J} & 0\\ 0 & -\frac{1}{\tau_r} \end{bmatrix} (K_1 Z_1 - Z_2 + \dot{X}_{1,ref})$$
$$- \ddot{X}_{1,ref} - K_1 (-K_1 Z_1 + Z_2)$$
$$+ \begin{bmatrix} 0 & \frac{pL_m}{L_r J} (k_{12} z_{12} - z_{22} + \dot{x}_{12,ref})\\ 0 & 0 \end{bmatrix} X_2$$
$$+ B_1(X_1) f_2(X_1, X_2)$$

$$B(X_1) = B_1(X_1)B_2$$

$$H(t) = \begin{pmatrix} K_1 + \begin{bmatrix} -\frac{F}{J} & 0\\ 0 & -\frac{1}{\tau_r} \end{bmatrix} \end{pmatrix} H_1(t) + B_1(X_1)H_2(t) + \begin{bmatrix} \frac{pL_m}{L_rJ} \\ 0 \end{bmatrix} h_{12}$$

From (18) and (21), the IM first transformed model can be expressed in the new coordinates  $(Z_1, Z_2)$  as follows

$$\dot{Z}_1 = -K_1 Z_1 + Z_2 - H_1(t)$$
  
$$\dot{Z}_2 = F(Z_1, Z_2, X_1, X_{1, ref}, X_2) + B(Z_1, X_{1, ref})u(t) + H(t)$$
(22)

**Remark 1:** The above transformed model contains three main drawbacks:

1- Deriving the second block dynamic is troublesome due to the high derivational computation (20). Also, for high dimensional systems this problem will be a real challenge, and needs to be solved.

2- In designing controllers, low tracking error  $(Z_1 \rightarrow 0)$  is

hard to achieve due to the non-vanishing uncertainties like  $T_L$  in the unmatched vector  $H_1(t)$ . To avoid this problem, the load torque  $T_L$  should be known by measuring or approximating it.

3- There is no reference value for  $X_2$ . In the other word, governing the virtual state  $X_2$  is not considered so important, while this inconsideration cases some problems in practical applications.

The next transformation technique is able to improve the mentioned problems.

#### 3.2. Improved NBC

Consider the following tracking errors

$$Z_{1} = X_{1,ref} - X_{1} = \begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} = \begin{bmatrix} x_{11,ref} - x_{11} \\ x_{12,ref} - x_{12} \end{bmatrix} = \begin{bmatrix} \omega_{m,ref} - \omega_{m} \\ \psi_{dr,ref} - \psi_{dr} \end{bmatrix}$$

$$Z_{2} = X_{2,ref} - X_{2} = \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix} = \begin{bmatrix} x_{21,ref} - x_{21} \\ x_{22,ref} - x_{22} \end{bmatrix} = \begin{bmatrix} i_{ds,ref} - i_{ds} \\ i_{qs,ref} - i_{qs} \end{bmatrix}$$
(23)

Taking time derivative of  $Z_1$  and substituting the first block of (11) in it, yields

$$\dot{Z}_{1} = \dot{X}_{1,ref} - \dot{X}_{1}$$

$$= \dot{X}_{1,ref} - f_{1}(X_{1}) - B_{1}(X_{1})(X_{2,ref} - Z_{2}) - H_{1}(t) \qquad (24)$$

$$= -f_{1}(X_{1}) + \dot{X}_{1,ref} - B_{1}(X_{1})X_{2,ref} + B_{1}(X_{1})Z_{2} - H_{1}(t)$$

In (24), we replaced  $X_2$  by  $X_{2,ref} - Z_2$ . Hence,  $X_{2,ref}$  will be the fictitious control input instead of  $X_2$ . This replacement is the main reason for the derivational calculation reduction.

Now, let propose the fictitious control input  $X_{2,ref}$  for (24) as

$$X_{2,ref} = B_1^{-1}(X_1) \Big( -f_1(X_1) + \dot{X}_{1,ref} + K_1 Z_1 + K_{sw1} \tanh(Z_1/\rho_1) \Big)$$
(25)

where  $tanh(\cdot)$  is the hyperbolic tangent function and is the approximation of  $sgn(\cdot)$ ,  $\rho_1$  is an enough small positive  $\lceil k_{mull} = 0 \rceil$ 

constant, and  $K_{sw1} = \begin{bmatrix} k_{sw11} & 0 \\ 0 & k_{sw12} \end{bmatrix}$  is the sliding gain and

should be selected in such a way that  $\left\| K_{swl} \right\| > H_{1\max}$ .

By substituting the virtual control law (25) in (24), we have

$$\dot{Z}_1 = -K_1 Z_1 + B_1(X_1) Z_2 - K_{sw1} \tanh(Z_1 / \rho_1) - H_1(t)$$
(26)

Moreover, calculating time derivative of  $Z_2$  and inserting the second block of (11) into it, yields

$$\dot{Z}_2 = \dot{X}_{2,ref} - \dot{X}_2 = -f_2(X_1, X_2) + \dot{X}_{2,ref} - B_2 u(t) - H_2(t) \quad (27)$$

From (26) and (27), the IM second transformation model can be expressed in the new coordinates  $(Z_1, Z_2)$  as follows

$$\dot{Z}_{1} = -K_{1}Z_{1} + B_{1}(X_{1})Z_{2} - K_{swl} \tanh(Z_{1} / \rho_{1}) - H_{1}(t)$$
  
$$\dot{Z}_{2} = -f_{2}(X_{1}, X_{2}) + \dot{X}_{2,ref} - B_{2}u(t) - H_{2}(t)$$
(28)

It is worthy to notify that compared to the conventional NBC: 1- The second block dynamic (27) is determined easily with no more derivational calculations.

2- The non-vanishing unmatched fault and uncertainty effects

can be attenuated by the robust term  $K_{swl} \tanh(Z_1 / \rho_1)$ .

3- Converging  $Z_2 \rightarrow 0$ , will result  $X_2 \rightarrow X_{2,ref}$ .

## 4. PASSIVE FAULT TOLERANT SLIDING MODE CONTROL

In this section, two passive fault tolerant sliding mode controllers are designed for the conventional and the improved NBC transformed models.

#### 4.1. SMC with the conventional NBC

Let us define the following sliding manifold

$$s(t) = Z_2(t) = 0 \tag{29}$$

By taking time derivative from (29) and using (22), we can get

$$\dot{s}(t) = F(Z_1, Z_2, X_1, X_{1, ref}, X_2) + B(X_1)u(t) + H(t)$$
(30)

**Theorem 1:** Consider the IM first transformed NBC model (22) with the linear sliding surface (29) and Assumption 1, then the following control law

$$u(t) = B^{-1}(X_1) \left[ -F(Z_1, Z_2, X_1, X_{1, ref}, X_2) - K_2 Z_2 - K_{sw2} \operatorname{sgn}(s(t)) \right]$$
(31)

guarantees zero convergence of  $Z_2$ , and boundedness of  $Z_1$ around zero. Hence, the IM speed and flux tracking errors convergence around the origin will be assured. Where  $K_2 = \begin{bmatrix} k_{21} & 0 \\ 0 & k_{22} \end{bmatrix}$ ,  $k_{21} > 0$ ,  $k_{22} > 0$ ,  $K_{sw2} = \begin{bmatrix} k_{sw21} & 0 \\ 0 & k_{sw22} \end{bmatrix}$ , and  $K_{sw} > H_{max} > ||H(t)||$ .

**Stability proof:** In order to assure the closed-loop system stability, consider the following Lyapunov candidate

$$V = \frac{1}{2} s_2^T s_2$$
 (32)

Taking time derivative from (32) and using (30) and (31), results in

$$\dot{V} = s_2^T \dot{s}_2 = Z_2^T \dot{Z}_2$$

$$= Z_2^T (F(Z_1, Z_2, X_1, X_{1, ref}, X_2) + B(X_1)u(t) + H(t))$$

$$= Z_2^T (-K_2 Z_2 - K_{sw2} \operatorname{sgn}(Z_2) + H(t))$$

$$= -K_2 Z_2^T Z_2 + Z_2^T (-K_{sw2} \operatorname{sgn}(Z_2) + H(t))$$
(33)

Selecting  $K_{sw} > H_{max}$ , yields

$$\dot{V} = -K_2 Z_2^T Z_2 < 0 \tag{34}$$

which fulfills  $Z_2 \rightarrow 0$ . Then, the first transformation model (22) can be reduced as

$$\dot{Z}_1 = -K_1 Z_1 + 0 - H_1(t) \tag{35}$$

From (35), it is obvious that  $Z_1$  will converge to zero for vanishing uncertainties, while  $H_1(t)$  have the non-vanishing term  $T_L$  in  $h_1(t)$ . Therefore, the flux convergence is possible  $(\psi_{dr} \rightarrow \psi_{dr,ref})$ , but the mechanical speed will have deviations from the reference speed  $(\omega_{m,ref} - \omega_m \rightarrow \varepsilon \neq 0)$ .

It is worthy to note that choosing large values for  $K_1$ , can reduce the speed tracking error. But it is not able to remove the error permanently.

#### 4.2. NSMC with the improved NBC

In this part, the NSMC is applied for the second transformed NBC model (28). In the other word, a nonlinear sliding surface is proposed to improve the conventional SMC slow responses.

Consider the following nonlinear sliding manifold

$$s(t) = Z_2(t) + c \int_0^t Z_2^{\mu}(\tau) d\tau$$
(36)

where  $s(t) = [s_1(t), s_2(t)]^T$ ,  $0 < \mu = \frac{p}{q} < 1$ , and p, q are

rational odd numbers, also c is a positive constant.

Taking time derivative from (36), yields

$$\dot{s}(t) = Z_2 + cZ_2^{\mu} \tag{37}$$

Inserting (28) in (37), results in

$$\dot{s}(t) = -f_2(X_1, X_2) + \dot{X}_{2,ref} + cZ_2^{\mu} - B_2 u(t) - H_2(t)$$
(38)

**Theorem 2:** Consider the IM second transformed NBC model (28) with the nonlinear sliding surface (36) and Assumption 1, then the following control law

$$u(t) = B_2^{-1} \Big( -f_2(X_1, X_2) + \dot{X}_{2,ref} + cZ_2^{\mu} + K_2 s(t) + K_{sw2} \operatorname{sgn}(s(t)) \Big)$$
(39)

guarantees  $Z_1$  and  $Z_2$  zero convergence. Hence, the IM speed and flux tracking errors convergence is assured. Where

$$K_2 = \begin{bmatrix} k_{21} & 0 \\ 0 & k_{22} \end{bmatrix}$$
, and  $||K_{sw2}|| > H_{2\max}$ .

Stability analysis: Let us define the following Lyapunov function

$$V(t) = 0.5s^{T}(t)s(t)$$
(40)

Taking time derivative from (40) and using (38) and (39), we can get

$$V(t) = s^{T}(t)\dot{s}(t)$$
  

$$\dot{V}(t) = s^{T}(t)\left(-f_{2}(X_{1}, X_{2}) + \dot{X}_{2,ref} + cZ_{2}^{\mu} - B_{2}u(t) - H_{2}(t)\right)$$
  

$$\dot{V}(t) = s^{T}(t)\left(-K_{2}s(t) - K_{sw2}\operatorname{sgn}(s(t)) - H_{2}(t)\right)$$
  

$$\dot{V}(t) = -s^{T}(t)K_{2}s(t) + s^{T}(t)\left(-K_{sw2}\operatorname{sgn}(s(t)) - H_{2}(t)\right)$$
  
(41)

Selecting  $||K_{sw2}|| > H_{2\max}$ , yields

$$\dot{V}(t) = -s^{T}(t)K_{2}s(t) < 0$$
(42)

which guarantees the second block state vector zero convergence  $(Z_2 \rightarrow 0)$ . Then, the equation (28) can be rewritten as

$$\dot{Z}_1 = -K_1 Z_1 + 0 - K_{sw1} \tanh(Z_1 / \rho_1) - H_1(t)$$
(43)

To analyze (43) stability, consider the following Lyapunov function

$$V_1(t) = 0.5 Z_1^T Z_1 \tag{44}$$

Calculating the time derivative of (44) and substituting (43) in it, results in

$$\dot{V}_{1}(t) = Z_{1}^{T} \dot{Z}_{1} = Z_{1}^{T} \left( -K_{1} Z_{1} - K_{sw1} \tanh(Z_{1} / \rho_{1}) - H_{1}(t) \right)$$

$$= -Z_{1}^{T} K_{1} Z_{1} + Z_{1}^{T} \left( -K_{sw1} \tanh(Z_{1} / \rho_{1}) - H_{1}(t) \right)$$
(45)

By selecting the sliding gain  $\left\|K_{swl}\right\| > H_{1\max}$ , we have

$$\dot{V}_1(t) = -Z_1^T K_1 Z_1 < 0 \tag{46}$$

which assures  $Z_1 \rightarrow 0$ . The inequality (46), is the main advantage of the second transformation form (28) and the control law (39) against the first transformation form (22) with the control law (31).

## 5. SIMULATION RESULTS

To testify the performance of the sliding mode block controllers, series of MATLAB/SIMULINK simulation tests for a 4 kW cage rotor induction machine with 50 Hz and 220 V power supply is carried out. The nominal electrical and mechanical parameters of the IM are shown in Table 1. Also, the induction motor control block diagram based on FOC technique is illustrated in Fig. 1. This figure contains rotor flux estimation block which is applied to avoid the use of flux sensor. The flux estimation block dynamic is selected as:

$$\hat{\psi}_{dr}(s) = \frac{L_m}{1 + \tau_r s} i_{ds}(s)$$

Comparative simulations of the discussed PFTC controllers (31) and (39) (SMC with the conventional NBC vs. NSMC with the improved NBC) are presented in Figs. 2-7. The IM speed response, electromagnetic torque, rotor flux, stator currents, speed and flux tracking errors and new transformation variables are showed in these figures.

Description	Parameter	Value	Units
stator resistance	R <sub>s</sub>	1.2	Ω
rotor resistance	R <sub>r</sub>	1.8	Ω
stator inductance	L <sub>s</sub>	0.1554	Н
rotor resistance	L <sub>r</sub>	0.1566	Н
mutual inductance	L <sub>m</sub>	0.15	Н
rotor inertia	J	0.024	Kg.m <sup>2</sup>
friction coefficient	F	0.011	N.m.s/rad
number of pole	р	2	-
pairs			
rated speed	$\omega_{mN}$	1440	rpm

**Table. 1** Nominal parameters of the IM adopted for thesimulation (Benbouzid et al., 2008).

The simulation test involves the following operating sequences:

1- To test the robustness of the proposed controllers with respect to rotor faults, we have considered the responses of the IM in the presence of rotor resistance variation  $+100\% R_r$ ,

this resistance variation is taken into account at t = 2s.

2- The motor shaft is subject to a step load torque variation of 4Nm at t=3s which is considered unknown.

3- To investigate the proposed controller's performance under stator faults, simulations are performed in the presence of static eccentricity. To simulate stator fault, the stator currents have been destroyed by assuming that A=0.5,  $\phi=0$ and  $\omega_c(t) + \theta_s(t)$  is computed using (6). The time of stator fault occupancy is considered at t=9s.



Fig. 1. Block diagram of the designed fault-tolerant controllers for IM.





b) NSMC with the improved NBC.

Fig. 2. IM speed response.



a) SMC with the conventional NBC.



Fig. 3. Electromagnetic torque.



b) NSMC with the improved NBC.

Fig. 6. Speed and flux tracking errors.







b) NSMC with the improved NBC.

Fig 7. New coordinate variables.

From Figs. 2-7, in is evident that the NSMC with the improved NBC technique has low tracking error and low variation against different faults in comparison with the SMC with the conventional NBC approach. In addition, the unmatched fault effects are removed effectively. Also, NSMC with the improved NBC method chattering is less than the SMC with the conventional NBC method.

# 6. CONCLUSIONS

In this study, passive fault tolerant sliding mode control of three phase induction motors is investigated. First, the SMC with the conventional NBC technique is designed. But the conventional NBC technique has some flaws like unmatched fault/uncertainty rejection problem. In addition, the SMC controller shows a slow response with considerable tracking error. Then in the second step, to solve the mentioned problems, the NSMC with the improved NBC technique has been proposed. Comparative simulation results revealed the effectiveness of the proposed method for multiple rotor and stator faults.

## REFERENCES

- Aurora. C and Ferrara. A (2007) A sliding mode observer for sensorless induction motor speed regulation. *International Journal of Systems Science*, 38: 913-29.
- Bachir. S, Tnani. S, Trigeassou. J and Champenois. G (2006) Diagnosis by parameter estimation of stator and rotor faults occurring in induction machines. *IEEE Trans. on Industrial Electronics*, 53: 963-73.
- Benbouzid. M.E.H, Diallo. D and Zeraoulia. M (2007) Advanced fault-tolerant control of induction-motor drives for EV/HEV traction applications: from conventional to modern and intelligent control techniques. *IEEE Trans. Vehicular Technology*, 56: 519-28.
- Benosman. M, Lum. KY (2010) Application of passivity and cascade structure to robust control against loss of actuator effectiveness. *International Journal of Robust and Nonlinear Control*, 20: 673–93.
- Benosman. M, Lum. KY (2010) Passive actuators faulttolerant control for affine nonlinear systems. *IEEE Trans. Control Systems Technology*, 18:152–63.
- Bonivento. C, Isidori. A, Marconi. L and Paoli. A (2004) Implicit fault-tolerant control: application to induction motors. *Automatica*, 40: 355-71.
- Campos-Delgado. D. U, Espinoza-Trejo. D. R and Palacios. E (2008) Fault tolerant control in variable speed drives: a survey. *IET Electric Power Applications*, 2: 121-34.
- Cho. K. R, Lang. J. H and Umans. S. D (1992) Detection of broken rotor bars in induction motors using state and parameter estimation. *IEEE Trans. on Industry Applications*, 28: 702-9.
- Djeghali. N, Ghanes. M, Djennoune. S and Barbot. J (2011) Backstepping fault tolerant control based on second order sliding mode observer: application to induction motors. Proc. of the 50th IEEE Conference on Decision and Control and European Control Conference, 4598-603.
- Djeghali. N, Ghanes. M, Djennoune. S and Barbot. J (2013) Sensorless fault tolerant control for induction motors. *International Journal of Control, Automation, and Systems*, 11: 563-76.
- Fekih. A (2007) Effective fault tolerant control design for nonlinear systems: application to a class of motor control system. *IET Control Theory and Applications*, 2: 762-72.
- Filippetti. F, Franceschini. G, Tassoni. C and Vas. P (2000) Recent developments of induction motor drives fault diagnosis using AI techniques. *IEEE Trans. on Industrial Electronics*, 47: 994-1004.
- Gaeid. K. S, Ping. H. W, Khalid. M and Masaoud. A (2012a) Sensor and sensorless fault tolerant control for induction motors using wavelet index sensors. *Sensors*, 12: 4031-50.
- Gaied. K. S, (2014b) Wavelet-based prognosis for faulttolerant control of induction motor with stator and speed sensor faults. *Transactions of the Institute of Measurement and Control.* DOI: 0142331214533121.

- Ghanes. M and Zheng. G (2009a) On sensorless induction motor drives: sliding mode observer and output feedback controller. *IEEE Trans. on Industrial Electronics*, 56: 3404-13.
- Ghanes. M, Barbot. J. P, Leon. J. D and Glumineau. A (2010b) A robust sensorless output feedback controller of the induction drives: new design and experimental validation. *International Journal of Control*, 83: 484-97.
- Hedayati Kia. S, Henao. H and Capolino. G. A (2007) A high-resolution frequency estimation method for threephase induction machine fault detection. *IEEE Trans. on Industrial Electronics*, 54: 2305-14.
- Holtz. J (2006) Sensorless control of induction machines with or without signal injection. *IEEE Trans. on Industrial Electronics*, 53: 7-30.
- Jin X. Z, Yang G. H, Ye D. (2013) Insensitive reliable H filtering against sensor failures. *Information Science*, 224:188–99.
- Kanev. S (2006) Robust fault-tolerant control. Ph.D. Thesis, *University of Twente*, Netherlands.
- Karami. F, Poshtan. J and Poshtan. M (2010) Detection of broken rotor bars in induction motors using nonlinear Kalman filters. *ISA Transactilions*, 49: 189-95.
- Krause. P. C, Wasynczuk. O, Sudhoff. S and Pekarek. S (2013) Analysis of electric machinery and drive systems. 3nd ed., *New York: Wiley-IEEE*.
- Li. T, Zhang. Y and Gordon. B. W (2013) Passive and active nonlinear fault-tolerant control of a quadrotor unmanned aerial vehicle based on the sliding mode control technique. *Journal of Systems and Control Engineering*, 227: 12-23.
- Loukianov. A. G (2002a) Robust block decomposition sliding mode control design. *Mathematical Problems In Engineering*, 8: 349-65.
- Loukianov. A. G, Toledo. B. C and Dodds. S (2002b) Robust stabilization of a class of uncertain system via block decomposition and VSC. *International Journal of Robust and Nonlinear Control*, 12: 1317-38.
- Merheb. A. R, Noura. H and Bateman. F (2015) Design of passive fault-tolerant controllers of a quadrotor based on sliding mode theory. *International Journal of Applied Mathematics and Computer Science*, 25:561-76.

- Mirshams. M, Khosrojerdi. M and Hasani. M (2014) Passive fault-tolerant sliding mode attitude control for flexible spacecraft with faulty thrusters. *Journal of Aerospace Engineering*, Doi: 10.1177/0954410013517671.
- Najafabadi. T.A, Salmasi. F.R and Jabehdar-Maralani. P (2011) Detection and isolation of speed-, DC-link voltage-, and current-sensor faults based on an adaptive observer in induction-motor drives. *IEEE Trans. Industrial Electronics*, 58: 1662-72.
- Shi. Y. T, Kou. Q, Sun. D. H, Li. Z. X, Qiao. S. J and Hou. Y. J (2014)  $H_{\infty}$  Fault tolerant control of WECS based on the PWA model. *Energies*, 7: 1750-69.
- Shoja Majidabad. S and Toosian Shandiz H (2012) Discretetime based sliding-mode control of robot manipulators. *International Journal of Intelligent Computing and Cybernetics*, 5: 340-58.
- Vaimann. T and Kallaste. A (2011) Detection of broken rotor bars in three-phase squirrel-cage induction motor using fast Fourier transform. Proc. of the 10th International Symposium, *Topical Problems in the Field of Electrical* and Power Engineering, 52-6.
- Vas. P (1994) Parameter estimation, condition monitoring and diagnosis of electrical machines. Oxford, UK: Oxford Science Publications.
- Wang C, Dong J, Yang G and Kang H. (2013) Fuzzy fault tolerant control for nonlinear systems with sensor faults. *Proceedings of the 32nd Chinese Control Conference* (CCC), 6214–9.
- Zhang. Y and Jiang. J (2008) Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control*, 32: 229–52.
- Zidani. F, Diallo. D, Benbouzid. M and Said. R (2008) A fuzzy-based approach for the diagnosis of fault modes in a voltage-fed PWM inverter induction motor drive. *IEEE Trans. Industrial Electronics*, 55: 586-93.
- Yan. X. G and Edwards. C (2007) Sensor fault detection and isolation for nonlinear systems based on a sliding mode observer. *International Journal of Adaptive Control and Signal Processing*, 21: 657-73.