Comparative Study of Type-2 and Type-1 Fuzzy PI Controllers on Sensor Noise Suppression in a Control Loop

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Abstract: The main contribution of this work is to reveal that the Footprint of Uncertainty (FOU) in Type-2 (T2) Fuzzy Logic Sets (FLS) have a great impact on the system performance and the uncertainty caused by sensor noise can be effectively suppressed by properly setting the value of FOU. As a case study, in this paper, a novel technique for sensor noise suppression, i.e. minimizing the effects of measurement uncertainty, using T2 Proportional-Integral (PI) Fuzzy Logic Controller (FLC) has been investigated for a discrete nonlinear and open loop unstable system in the MATLAB environment. For T2 PI FLC, FOU was varied to find its optimum value to provide the best sensor noise suppression. The conducted investigations have shown that the FOU of 90% offered the best sensor noise suppression and in the considered case an improvement of 13-19% was recorded over its counterpart Type-1 (T1) PI FLC. T1 PI FLC was considered as a special case of T2 PI FLC with zero FOU and genetic algorithm was used to tune controller gains for minimum integral of absolute error in T1 PI i.e. T2 PI FLC with zero FOU. The simulation results clearly demonstrated that the T2 PI FLC is better able to handle the uncertainty due to sensor noise present in the control system in comparison to T1 PI FLC.

Keywords: Fuzzy control, Type-1 FLC, Type-2 FLC, Uncertainty, Sensor noise, Noise suppression.

1. INTRODUCTION

Fuzzy logic is being used in the control systems for past three decades and has it been successful to a good extent as it is able to control complex plants with uncertainties. Type-1 Fuzzy Logic Controller (T1 FLC) has been the point of focus for quite some time, but for the uncertain complex systems, Type-2 (T2) FLC is being explored. Castillo and Melin have presented an excellent review on the application of T2 FLC in intelligent control (Castillo and Melin, 2014). There have been two major approaches to implement T2 FLC – using T2 Fuzzy Logic Sets (FLS) mathematics or averaging two T1 FLS. Good research has been conducted in this area by large number of researchers such as (Castillo et al., 2012; Castillo and Melin, 2012, 2014; Mendel, 2000, 2001, 20013). Liang and Mendel presented the theory and design of interval T2 FLS with an efficient and simplified method to compute the input and antecedent operations for interval T2 FLS that is based on a general inference formula. They introduced the concept of upper and lower Membership Functions (MFs) and illustrated efficient inference methods for the case of Gaussian MF. They also proposed a method for designing an interval T2 FLSs in which they tuned its parameters. T2 FLS was utilized for time-series forecasting when a non-stationary time-series was corrupted by an additive noise. The T2 FLS demonstrated an improved performance over T1 FLS (Liang and Mendel, 2000). Further, Mendel has presented a T2 FLS and has described various set operations on T2 FLS, properties of membership grades of T2 FLS, T2 relations and their compositions, and defuzzification (Mendel, 2001). Karnik et al., introduced a T2 FLS, which could handle rule uncertainties. The implementation of T2 FLS involved the operations of fuzzification, inference, and output processing. The main focus was on output processing which consisted of type reduction and defuzzification. The used type-reduction methods were the extended versions of T1 defuzzification methods. Type reduction captured more information about rule uncertainties than the defuzzified value (a crisp number), however, it was computationally intensive, except for interval T2 FLS for which they also provided a simple type-reduction computation procedure. Finally, T2 FLS was applied to time-varying channel equalization and it was demonstrated that it yielded better performance than a T1 FLS and nearest neighbour classifier (Karnik et al., 1999). Du and Ying have described a method to derive the detailed mathematical structure for connecting the input-output of Mamdani interval T2 PI FLC. It may be noted that the proposed structure was not applied to evaluate the closed loop performance for any plant (Du and Ying, 2010). Zhou and Ying have derived analytical structures of a broad class of interval T2 Mamdani fuzzy controllers. Here also it may be noted that the developed control structures were not implemented in closed loop (Zhou and Ying, 2013). Computing the centroid and performing type-reduction for T2 FLSs and systems are the operations that are required to be performed iteratively using Karnik–Mendel (KM) algorithms. Good quality of research on centroid and type-reduction computations has been conducted on these iterative algorithms during the past decade. The tutorial presented by Mendel focused on the research that has been conducted to improve the KM algorithms and further he has used KM algorithms to solve the non-fuzzy logic system problem (Mendel, 2013). Cara *et al.* have performed a comparative study between optimized singleton T1, non-singleton T1, and singleton interval T2

FLSs under the presence of noise. For the optimization of singleton T1, non-singleton T1, and singleton interval T2 fuzzy systems for function approximation problems, a multi-objective evolutionary algorithm has also been presented. With the help of statistical analysis they showed that the T2 FLS was able to handle higher levels of noise than its non-singleton and singleton T1 counterparts (Cara et al., 2013). Sepulveda et al. have optimized interval T2 MFs using an average of two T1 systems and they also performed experiments where they optimized the standard deviation of Gaussian MFs for the FOU and noise values. They concluded that T1 FLC performed better than T2 FLC when no uncertainties were present and T2 performed better than T1 when uncertainty was present (Sepulveda et al., 2006).

In a recent work, Sepulveda et al. have designed a control system using T2 FLS for minimizing the effects of measurement uncertainty. They have implemented T2 FLS as described in (Mendel, 2001). All the controller gains were taken as unity. Based on the simulation study, it was concluded that best results are obtained using T2 FLS (Sepulveda et al., 2007). It may be noted that in this work the MFs parameters were optimized instead of the controller gains, as done in the present work. Galluzzo and Cosenza have studied T1 and T2 FLC for control of the biodegradation of mixed wastes in a continuous bioreactor. Presented simulation results showed that T2 FLC offered better performance as compared to the other controllers i.e. T1 FLC and a PI controller, in terms of robustness and response speed. It was proved that the use of T2 FLC can be a good choice for the control of nonlinear processes with bifurcations, in particular when uncertainties on some parameters of the controlled system are present (Galluzzo and Cosenza, 2009). Oh et al. optimized T1 and T2 cascaded FLC with the aid of Particle Swarm Optimization (PSO) for a ball and beam system and found T2 FLC to perform better than T1 FLC (Oh et al., 2011). It may be noted that lots of literature is available on the theoretical investigations on the various aspects of the T2 FLC and this is a unique contribution implementing the T2 FLC on a real world experiment. Cosenza and Galluzzo presented development and theoretical testing of a multivariable fuzzy control system that makes use of T2 FLS for the control of pH and temperature of a fed-batch reactor for penicillin production. They obtained best results with the T2 FLC particularly when uncertainties were present in the control system (Cosenza and Galluzzo, 2012).

More recently, there has been a trend in real time hardware implementation and evaluation of the optimization methods. Castillo and Melin presented a review of the methods used in the design of interval T2 FLCs. The fundamental focus of the work was based on the basic reasons for optimizing T2 FLC for different areas of applications. In this review, they considered the application of Genetic Algorithms (GA), PSO and Ant Colony Optimization (ACO) as three different paradigms that helped the design of optimal T2 fuzzy controllers. Alternative approaches were also suggested for the designing of T2 FLCs without optimization techniques. A comparison of the optimization methods was also presented for the case of designing T2 FLC (Castillo and Melin, 2012). Further, Castillo et al., described the application of ACO and PSO on the optimization of the MF parameters of T1 and T2 FLCs in order to find the optimal intelligent controller for an autonomous wheeled mobile robot. Obtained simulation results were statistically compared with the previous work and statistical analysis showed that ACO outperformed PSO and the GA, but PSO outperformed the GA (Castillo et al., 2012). Melin et al., addressed the tracking problem for the dynamic model of a unicycle mobile robot. A novel optimization method inspired by the chemical reactions was applied to solve this problem by integrating a kinematic and a torque controller based on fuzzy logic theory. Computer simulations were presented which confirmed that this optimization paradigm is able to outperform other optimization techniques applied to this particular robot application (Melin et al., 2013).

Sepúlveda et al., showed that interval T2 fuzzy inference systems can also be used in applications that require high speed processing. KM iterative method was adequately implemented using the appropriate combination of hardware and software components on Field Programming Gate Array (FPGA), to control a DC motor. Detailed timing aspects of the components of the T2 FLC were presented (Sepúlveda et al., 2012). Maldonado *et al.* proposed the optimization of the T2 MFs for the average approximation of an interval of T2 FLC using PSO and GA. The fuzzy controller was realized and its MFs were optimized using PSO and GA on a FPGA hardware platform. The controller was used to regulate the speed of a DC motor. Main contribution of this work was implementing the optimization techniques on FPGA and it was claimed that PSO is a superior optimization method (Maldonado et al., 2013).

T2 fuzzy logic has also been explored to solve real world applications in different areas, for example, in signal processing, Karnik and Mendel have used T2 FLS to predict Mackey-Glass chaotic time-series with uniform noise presence (Karnik and Mendel, 1999; Mendel, 2000); in medicine, Phong et al., have used T2 FLS to predict survival time of myeloma patients (Phong et al., 2009); in networking, Jammeh *et al.* have done congestion control for video streaming across IP networks (Jammeh et al., 2009); in data prediction, Kurniawan has used T2 FLS for electrical load time-series data forecasting (Kurniawan, 2010). Khanesar et al., proposed simpler T2 FLS with the novel T2 MFs having Ellipsoidal nature. The proposed T2 fuzzy neuro structure was tested on different input–output data sets, and it was shown that the T2 FLS with the proposed novel MF has better noise reduction property when compared to the T1 counterparts. Three theoretical case studies namely Prediction of Chaotic Mackey–Glass Time Series, Identification of a laboratory setup acting like hair dryer data and control of a non-BIBO nonlinear plant was used to prove the efficacy of the proposed approach (Khanesar et al., 2011).

Cazarez-Castro *et al.* proposed a systematic methodology to design T1 and T2 FLCs to solve the output regulation problem of a servomechanism with nonlinear backlash via Fuzzy Lyapunov Synthesis (Cazarez-Castro et al., 2012). Yang et al., designed a direct adaptive interval T2 fuzzy neural network (IT2-FNN) controller for the hypersonic flight control. Simulation results validated the effectiveness and robustness of the proposed controller under uncertainties (Yang et al., 2013). Recently, Aladi et al., presented the relationships between FOU values and uncertainty/noise levels applied to the Mackey-Glass chaotic time series prediction. They found that as uncertainty/noise increases, T2 FLSs with fuzzy sets having FOUs of increasing size become more and more viable (Aladi et al., 2014).

The survey conducted above clearly indicates that the T2 PI FLC using triangular MFs for both input as well as output has not been explored for sensor noise suppression. It can be clearly seen that most of the presented work made use of Gaussian MFs for input and singleton or Gaussian MFs for output. Furthermore, it may be noted that Gaussian MFs are fairly complex to implement and consumes more time and space on the computing platform and hence it is not as suitable as triangular MFs for real time implementation. The main motive for the proposed work is two-fold. Firstly, the use of triangular MFs in input as well as the output is explored. Secondly, the effectiveness of the FOU variations to achieve the desired sensor noise suppression in contrast to the optimization of the MFs itself as already done by several other researchers is demonstrated. Thus the proposed technique offers an effective alternative method of noise suppression.

Noise is an inherent part of every measurement and control system. It affects the decision-making process of a plant as it introduces uncertainty in the measured variable. In general, the user will tend to use a filter for noise suppression but FLC can perform both control as well as the filtering task simultaneously. Further, it is also not reasonable to use T1 FLC which uses nonflexible MF (i.e. T1 FLS) for uncertainties such as random sensor noise. Thus another type of fuzzy set is needed to handle these uncertainties i.e. T2 FLS. T2 FLS uses further fuzzified membership function values in contrast to T1 FLS. Usage of the T2 FLS for demonstrating the sensor noise suppression and its statistical analysis has been one of the key motivations for this work.

In this work, a comparative study of T1 PI and T2 PI FLC, for a given plant, subjected to sensor noise, has been performed. T2 PI FLC controller was designed and implemented in velocity form. As mentioned earlier, T1 PI FLC was implemented as a special case of T2 PI FLC i.e. FOU of T2 PI FLC was made zero. The resulting T1 FLC's controller gains were tuned using GA. These gains were also used for T2 PI FLC and FOU of both inputs and output was varied together to perform experiments on a non-linear plant under uncertainty. Uncertainty has been considered in the form of sensor noise which may arise due to vibration or aging of instrumentation elements, thermal noise, electromagnetic interference, improper soldering of wires, loosened wires, improper shielding of wires, grounding at different potentials, etc. This noise needs to be dealt carefully to control the system under consideration. For this experiment, this random behaviour of noise has been simulated in Simulink by varying the percentage of uniformly distributed random noise from 5% to 50% in the interval of 5% so as to imitate a real world control system. A series of simulations have been performed to test T1 and T2 PI FLC where the plant is subjected to varying percentage of uniformly distributed random noise. The experimental results and quantitative measures of errors have shown to support the above statement. For quantifying the errors, Integral of Absolute Error (IAE) has been used as the performance criteria.

The rest of paper is organized as follows. Section 2 presents a brief explanation of T2 and T1 PI FLC along with the respective implementations. Section 3 is devoted to simulations, tuning of controller gains and presenting the results of experiments. As mentioned earlier, the plant was tested under varying noise percentages. A performance comparison, between T1 and T2 PI FLC, has also been presented. Section 4 presents a comparative study between the developed controllers for random sensor noise suppression. Finally, in Section 5, the conclusions have been drawn.

2. FUZZY LOGIC CONTROLLER IMPLEMENTATION

FLC is composed of a knowledge base that comprises of the information given by the process operator in form of linguistic control rules; a fuzzification interface, for transforming crisp data into FLS; an inference system, that uses the FLS in conjunction with the knowledge base to make inferences by means of a reasoning method; and a defuzzification interface, which translates the fuzzy control action so obtained to a real control action using a defuzzification method. In this work T1 and T2 PI FLCs were implemented in velocity form. A typical structure of FLC control loop is shown in Fig. 1. Essentially it takes two inputs namely the error and the change in error and produces an incremental controller output. The incremental output needs an accumulation before its final implication on the plant. So, to implement the PI FLC one needs to fuzzify two input variables and an output variable. Furthermore, the fuzzified inputs are manipulated as per the designed set of rules and finally, output is defuzzified to obtain a crisp value. The brief mathematics given below shows the linkage of various variables starting from the conventional PI control.

The conventional PI controller, in time domain, can be described as in (1).

$$U_{PI}(t) = K_p e(t) + K_i \int e(t)\,dt\,, \qquad (1)$$

where, the error $e(t)$ is defined as $e(t)=r(t)-y(t)$; $r(t)$ is setpoint and $y(t)$ is plant output, $U_{PI}(t)$ is the controller output, and two constants K_p and K_i are proportional and integral gains, respectively.

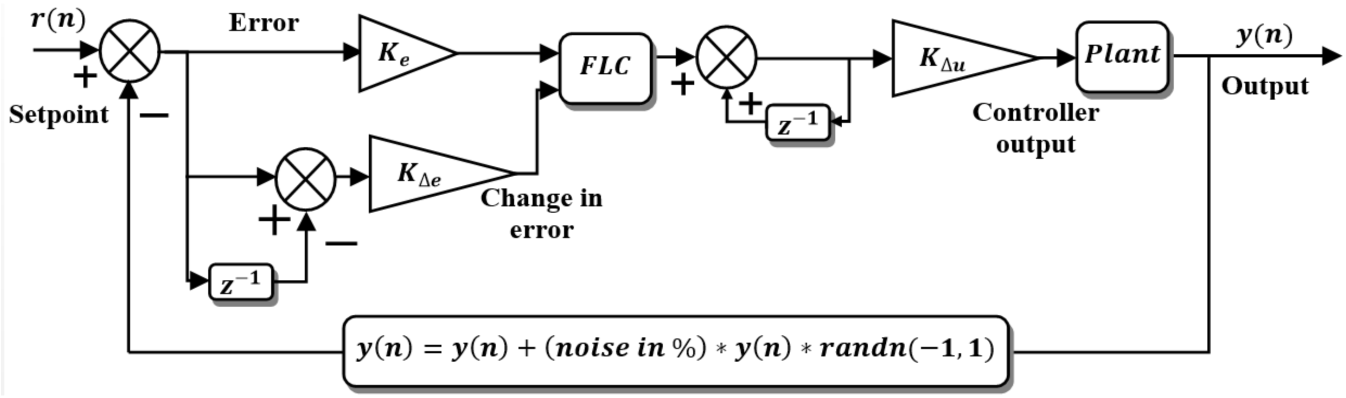


Fig. 1. Block diagram implementation of FLC.

Taking derivative of (1)

$$\frac{dU_{PI}(t)}{dt} = K_p \frac{de(t)}{dt} + K_i e(t), \tag{2}$$

In discrete form, (2) becomes

$$\frac{U_{PI}(n) - U_{PI}(n-1)}{T} = K_p \frac{e(n) - e(n-1)}{T} + K_i e(n), \tag{3}$$

where, T is the sampling time. Equation (3) can also be written as

$$\Delta U_{PI}(n) = K_e e(n) + K_{\Delta e} \Delta e(n), \tag{4}$$

where, $K_e = K_i T$ and $K_{\Delta e} = K_p$.

$\Delta U_{PI}(n) = U_{PI}(n) - U_{PI}(n-1)$ and

$\Delta e(n) = e(n) - e(n-1)$.

Further the control action can be written as

$$U_{PI}(n) = U_{PI}(n-1) + \Delta U_{PI}(n). \tag{5}$$

To increase the degree of freedom in FLC, an additional gain $K_{\Delta U}$ is introduced.

$$U_{PI}(n) = U_{PI}(n-1) + K_{\Delta U} \Delta U_{PI}(n) \tag{6}$$

Based on (4), FLC is designed and final control action is computed using (6). As seen in (4), FLC would require two inputs, error *e(n)* and change in error *Δe(n)* and would produce an incremental control action $\Delta U_{PI}(n)$. Accordingly, these three variables need to be fuzzified and processed. In this work, the structure of T1 is derived from T2 PI FLC by keeping FOU as zero. Hence, effectively only T2 needs to be designed and implemented.

2.1 T2 PI FLC Implementation

This sub-section describes the implementation of T2 PI FLC. T2 PI FLC, as shown in Fig. 2, includes a fuzzifier, a rule base, fuzzy inference engine, type-reducer and defuzzifier. Two inputs, error *e(n)* and change in error *Δe(n)*, are fed to the controller. Fuzzifier, the first block of T2 PI FLC, converts the crisp inputs into T2 fuzzified values. Fuzzy inference engine uses rule base and fuzzified values to produce fuzzy control actions. Type reducer converts the inferred T2 fuzzy values into T1 fuzzy values so that it can be processed by defuzzifier. Defuzzifier finally converts the fuzzified values into crisp output which can be fed into a plant, as fuzzified values cannot be used by plants (Liang and Mendel, 2000; Karnik et al., 1999; Sepulveda et al., 2006).

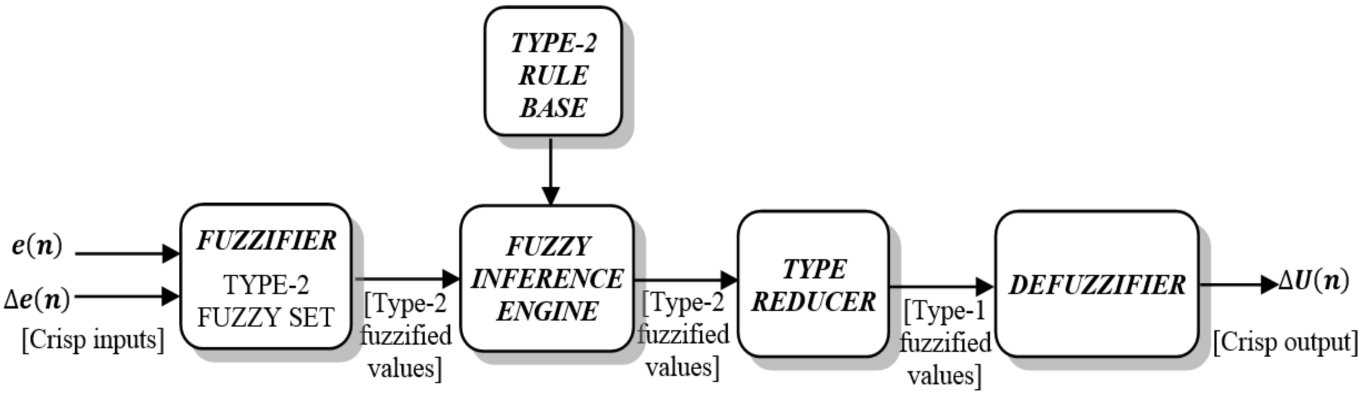


Fig. 2. Components of Type-2 FLC.

2.1.1 Fuzzifier

T2 PI FLC takes two inputs, error: $e(n)$ and change in error :$\Delta e(n)$. Equations (7) and (8) define and represent input variables in a discrete form:

$$e(n) = r(n) - y(n). \tag{7}$$

$$\Delta e(n) = e(n) - e(n-1). \tag{8}$$

where: *r(n)* is reference input and *y(n)* is plant output.

T2 MFs have FOU as shown in the Fig. 3. This additional degree of freedom is able to provide the robustness to the controller which helps in minimizing the effects of uncertainty such as sensor noise. The value of FOU can be varied within a certain range and correspondingly different variations of uncertainty of MFs of T2 PI FLC can be formulated. In this work, seven triangular MFs have been used for fuzzification of both the inputs, *e(n)* and *Δe(n)*, and the output $\Delta U_{PI}(n)$. Fig. 3 shows the input MFs where NB: negative big, NM: negative medium, NS: negative small, Z: zero, PS: positive small, PM: positive medium and PB: positive big. The Universe of Discourse (UOD) was set to ±10 units; total width of each triangular MF is kept as 5 units. Thus, FOU can be varied from 0 to 1.25 units. The value of 10 was arrived at after several experimentation and observing the large output value of the plant, as the plant is an open loop unstable system.

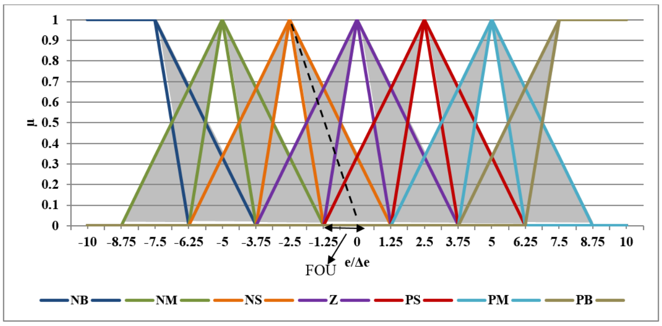


Fig. 3. Input MFs for T2 PI FLC.

2.1.2 Rule base

The structure of rules of T1 and T2 PI FLC is the same, but in the latter, the antecedents and the consequents will be represented by T2 FLS. The rule base was designed using process reaction curve and is as listed in Table 1.

2.1.3 Inference mechanism

In the T2 PI FLC, the inference engine combines rules and gives a mapping from input T2 FLS to output T2 FLS. Min-max t-norm was used for inferencing as defined below in (9) and (10) (Liang and Mendel, 2000; Karnik et al., 1999; Sepulveda et al., 2006).

$$\Delta U_l^i(x') = \min\left(\mu_{\widetilde{F}_1^l}(x_1'),\ldots,\mu_{\widetilde{F}_p^l}(x_q')\right). \tag{9}$$

$$\Delta U_r^i(x') = \min\left(\mu_{\widetilde{F}_1^r}(x_1'),\ldots,\mu_{\widetilde{F}_p^r}(x_q')\right). \tag{10}$$

where, $\mu_{\widetilde{F}_p^r}(x_q')$ =*Membership value of* p^{th} *right MF for* x_q'.

and $\mu_{\widetilde{F}_p^l}(x_q')$ =*Membership value of* p^{th} *left MF for* x_q'.

Table 1. Rule Base.

e(n) → Δe(n) ↓	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	Z
NM	NB	NB	NB	NM	NS	Z	PS
NS	NB	NB	NM	NS	Z	PS	PM
Z	NB	NM	NS	Z	PS	PM	PB
PS	NM	NS	Z	PS	PM	PB	PB
PM	NS	Z	PS	PM	PB	PB	PB
PB	Z	PS	PM	PB	PB	PB	PB

2.1.4 Output MFs

Output MFs are shown in Fig. 4. UOD was set to ±10 units, and width of each triangular MF is 5 units. Thus, FOU can be varied from 0 to 1.25 units. It can be observed that the output MFs are similar to input MFs, except that output MFs are terminated to 0 outside UOD whereas input MFs are 1 outside UOD.

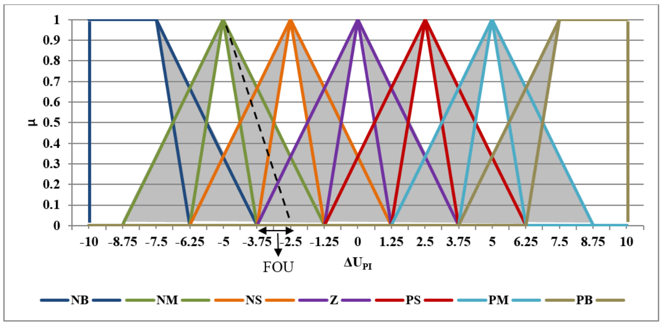


Fig. 4. Output MFs for T2 PI FLC

2.1.5 Type-reducer

The type reducer generates a T1 fuzzy set output which is then converted into a crisp output through the defuzzifier. Center of Sets (COS), U_{cos}, type defuzzifier has been used as in (Du and Ying, 2010).

$$\Delta U_{cos}(x) = [\Delta U_u, \Delta U_l]. \tag{11}$$

$$\Delta U_u = \frac{\sum_{i=1}^{M} f_u^i \Delta U_u^i}{\sum_{i=1}^{M} f_u^i}. \tag{12}$$

$$\Delta U_l = \frac{\sum_{i=1}^{M} f_l^i \Delta U_l^i}{\sum_{i=1}^{M} f_l^i}. \tag{13}$$

Where f_u^i = *area of* i^{th} *upper output membership funtion*

f_l^i = *area of* i^{th} *lower output membership funtion*

ΔU_u^i = *centroid of* i^{th} *upper output membership funtion*

$$\Delta U_l^i = centroid\ of\ i^{th}\ lower\ output\ membership\ funtion$$

2.1.6 Defuzzifier

From the type-reducer, an interval set ΔU_{cos} is obtained. The average defuzzifier (Du and Ying, 2010) was used to obtain the defuzzified output of T2 PI FLC as:

$$\Delta U_{PI}(x) = \frac{1}{N}\sum_{j=1}^{N} \Delta U_j(x). \tag{14}$$

$$Where\ N = Size\ of\ \Delta U_{cos}(x)\ set$$

2.2 T1 PI FLC

As mentioned earlier, the T1 PI FLC is implemented from T2 PI FLC keeping the FOU as zero. The effective structure of T1 PI FLC is as shown in Fig. 5.

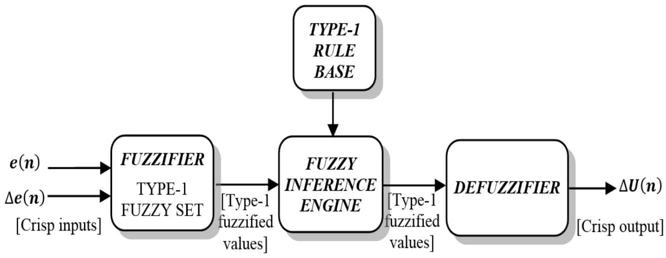


Fig. 5. Structure of T1 PI FLC.

The effective MFs of input for T1 are as shown in Fig. 6. These MFs correspond to T2 PI FLC for zero FOU.

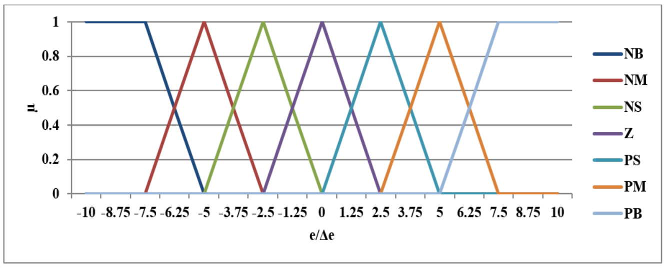


Fig. 6. Input MFs for T1 FLC

The rules of T1 PI FLC are same as T2 PI FLC, but the antecedents and the consequents will be represented by T1 FSs. Also, inference engine uses similar min-max t-norm. Output MFs are similar to input MFs, except that output MFs are terminated to zero outside UOD whereas input MFs are unity outside UOD. Fig. 7 shows the output MFs.

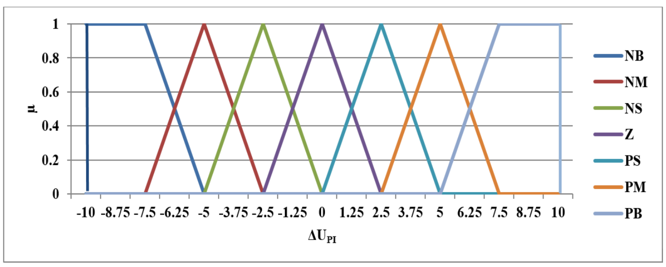
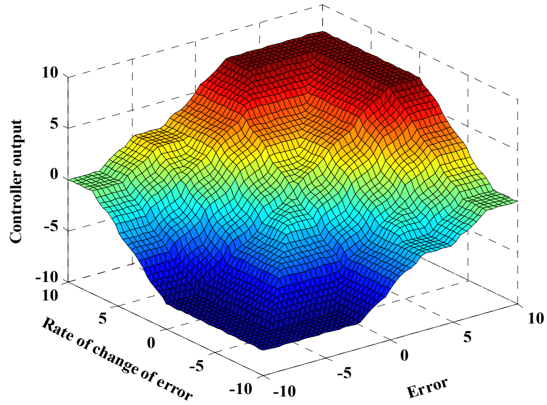
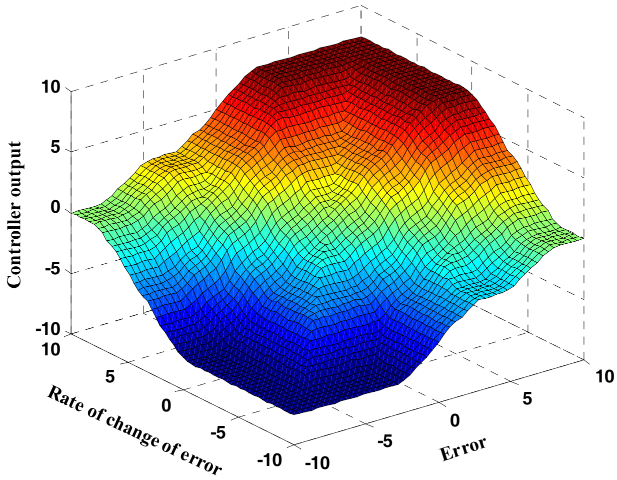


Fig. 7. Output MF for T1 PI FLC

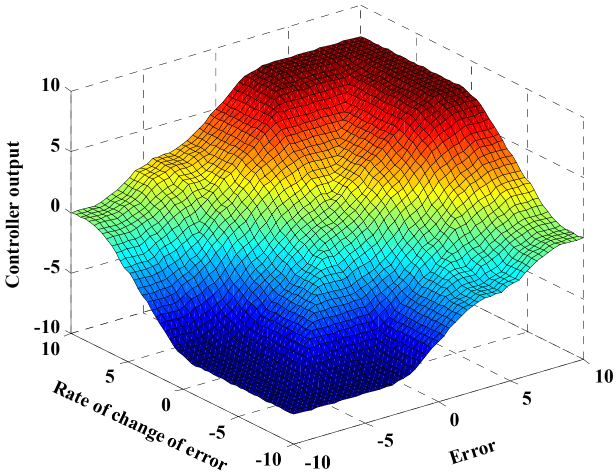
The type reducer of T2 PI FLC also acts as a defuzzifier for T1 PI FLC. Thus, effectively T1 PI FLC has a COS type defuzzifier. It needs a special mention here that the defuzzifier block of T2 PI FLC has no effect in T1 FLC and hence a single code works for both the controllers.



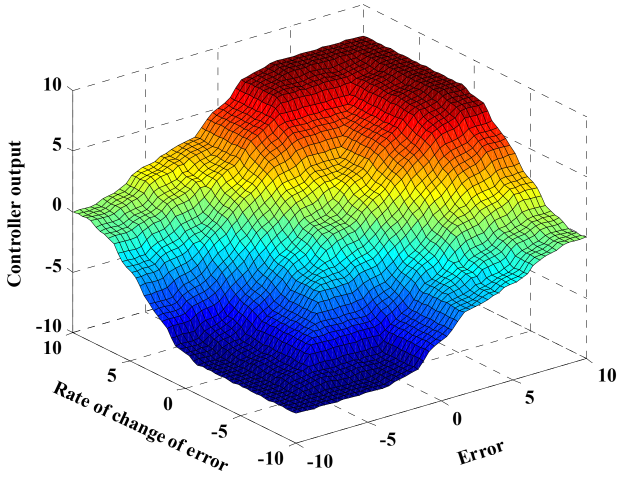
(a)



(b)



(c)



(d)

Fig. 8. Surface plot of FLC: (a) FOU = 0%, (b) FOU = 25% (c) FOU = 50% and (d) FOU = 75%.

The resulting control surface plots of T2 PI FLC, for varying FOU, are shown in Fig. 8 where Figs. 8(a), 8(b), 8(c) and 8(d) depict surface plots for FOU of 0%, 25%, 50% and 75%, respectively. From Fig. 8, corresponding control actions can be inferred for the entire UOD for the given FOU. Further, it can be clearly seen that as FOU is increased the plots become smoother i.e. discontinuities are reduced throughout the surface. This yields better and smooth control action as desired under an uncertain environment.

3. SIMULATION RESULTS

Simulations were performed using MATLAB and the code for T2 PI FLC was developed in Simulink. Rigorous simulations have been performed to study and compare the performance of T1 and T2 PI FLC for the following non-linear and unstable discrete system present (Sepulveda et al., 2006):

$$y(n) = 0.2 * y(n-3) * 0.07 y(n-2) + 0.9 * y(n-1) + 0.05 * u(n-1) + 0.5 * u(n-2) \tag{15}$$

A set of four simulation experiments viz. **A** to **D** have been carried out on this plant and are described in this section. In the experiment **A,** open loop behaviour of the plant has been presented. In experiment B, T1 PI FLC, i.e. T2 PI FLC with zero FOU, was tuned for setpoint tracking using GA and closed loop response was investigated in experiment **C**. Finally, the experiment **D** deals with T1 and T2 PI FLC random sensor noise suppression analysis where the plant is subjected to varying percentage of random sensor noise.

3.1 Experiment A: Plant open-loop characteristics

To study the open loop dynamic behaviour of plant, it is excited by a unit step input u(n) in open loop. As the Fig. 9 shows, plant's output grows exponentially to a large value, thus proving it as an unstable plant for a unit step input. Since the plant under study is discrete in nature, the sampling time was kept as unity.

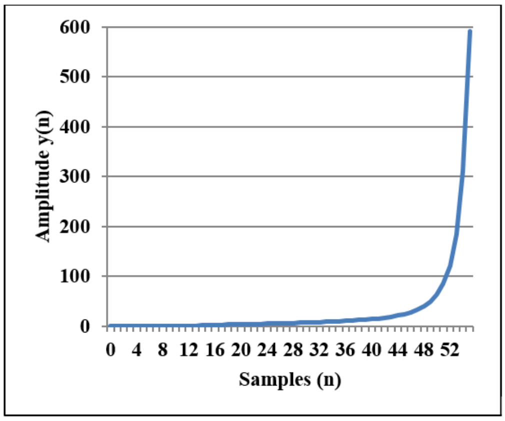


Fig. 9. Open loop response.

3.2 Experiment B: Controller tuning using GA

The standard GA toolkit of MATLAB was used to optimize the three gains, K_e (error gain), $K_{\Delta e}$ (change in error gain) and $K_{\Delta u}$ (controller's output gain) of T1 PI FLC, i.e. T2 PI FLC with zero FOU, for setpoint tracking and IAE was minimized for first 300 samples. The optimized IAE value was obtained as 3.24. Table 2 presents the various parameters of GA that have been used to optimize the controller gains. Fig. 10 depicts the generation versus IAE values during the optimization process. Table 3 presents the optimized values of the controller gains.

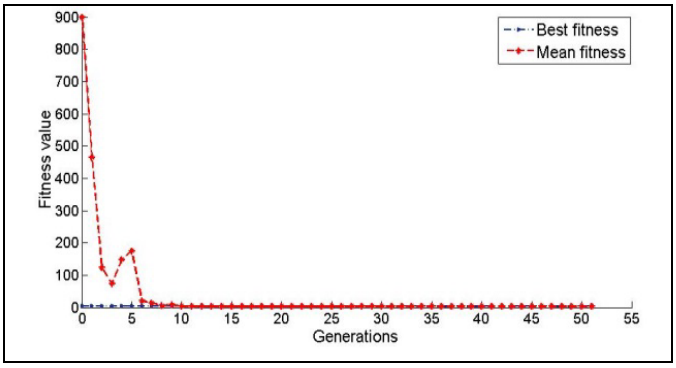


Fig. 10. Generation versus fitness values.

Table 2. Parameter settings of GA toolkit.

Parameter	Value
Fitness Function	IAE
Creation Function	Uniform
Crossover Fraction	0.8
Crossover Function	Scattered
Elite Count	2
Generations	100
Initial Population Size	40
Mutation Function	Uniform
Population Size	20
Scaling Function	Rank
Selection Function	Stochastic Uniform
Stall Generations Limit	50
Stall Time Limit	Infinite

Table 3. Optimized controller gains.

Controller gains	Optimized values
K_e	0.2316
$K_{\Delta e}$	1.0321
$K_{\Delta u}$	0.5781

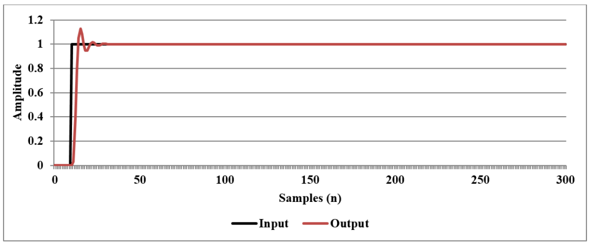


Fig. 11. Closed loop response.

3.3 Experiment C: Plant closed-loop characteristics

The plant was controlled using the developed FLCs with above gains as tuned by GA for zero FOU. Fig. 11 shows the unit step response. Effectively this response represents the performance of T1 PI FLC. An overshoot of around 10% can be clearly observed in the unit step response. It may be noted that this oscillatory behaviour is much less in magnitude as compared to the reported values in literature (Sepulveda et al., 2006).

3.4 Experiment D: Random sensor noise suppression

This section deals with the performance analysis of T1 and T2 PI FLC for suppression of the random sensor noise. The noise was injected to the system's output *y(n)* as defined in (16), using the Simulink's block 'Uniform Random Number Block', which generates uniformly distributed random numbers.

$$y(n) = y(n) + (noise\ in\ \%) * y(n) * randn(-1,1) \quad (16)$$

Noise (in percent of the unit setpoint) was varied from 5% to 50% in steps of 5% and the noise component was inducted to system from 100th sample to 300th sample, i.e. the system was initially allowed to settle. Accordingly, system performance parameter, IAE, was also computed from 100th sample to 300th sample. For studying the sensor noise suppression performance of T2 PI FLC, the FOU has been varied for a given noise percentage. As the injected noise is random in nature, for every value of noise percentage and FOU, the simulation was repeated 1000 times and mean and standard deviation of obtained IAE values were recorded and have been presented. Following subsection D-1 details the performance analysis of T1 PI FLC and subsection D-2 details the performance analysis of T2 PI FLC.

3.4.1 Experiment D-1: Noise suppression with T1 PI FLC

When the FOU is 0% (i.e. FOU = 0 units) the lower and upper MFs coincides to form a single MFs set thus reducing T2 PI FLC into T1 PI FLC. Keeping this value of FOU (i.e. FOU = 0 units) the noise percentage is varied from 5%-50% in the intervals of 5% for testing the performance of T1 PI FLC. Table 4 shows the performance of T1 PI FLC.

Table 4. Performance criterion values for different levels of random noise for T1 PI FLC.

Noise (%)	Mean of IAE	Standard Deviation of IAE
5	6.37	0.40
10	12.65	0.73
15	18.56	1.04
20	24.59	1.19
25	30.45	1.63
30	36.56	1.90
35	42.31	2.01
40	48.14	2.82
45	53.62	2.59
50	59.67	3.04

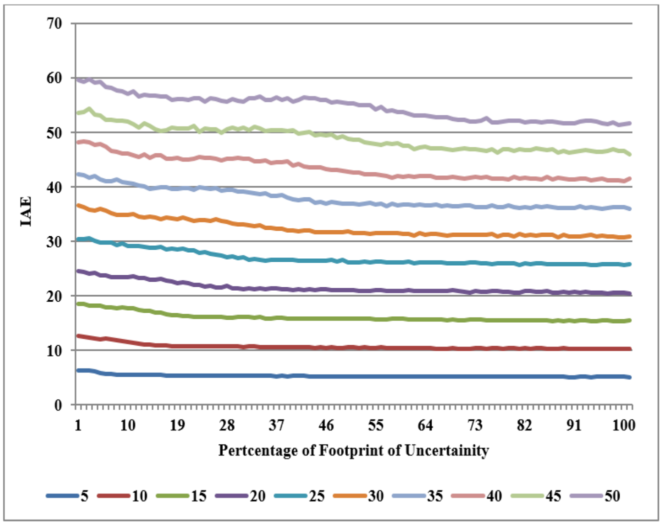


Fig. 12. Variation of IAE with FOU and noise percentage.

3.4.2 Experiment D-2: Noise suppression with T2 PI FLC

Noise was injected to study the performance of T2 PI FLC and was varied from 5% to 50% incrementing in the interval of 5%. Now, for a given noise FOU was gradually increased from 1% to 100% (i.e. from 0.0125 to 1.25 units) in the interval of 1% (i.e. 0.0125 units) thus converting the T1 PI FLC into T2 PI FLC of increasing uncertainties. FOU of both input MFs and output MFs were varied equally together, keeping them same for an iteration. This experiment was again performed 1000 times for each of the 100 FOU value at a given noise level. Mean and standard deviation of each set at a given noise percentage was calculated. Fig. 12 shows the variation of IAE with FOU and noise percentage.

Table 5. Variation of Mean IAE with FOU and noise.

Noise (%)	FOU (%)											Mean IAE for FOU 90%-100%
	90	91	92	93	94	95	96	97	98	99	100	
5	5.13	5.16	5.15	5.13	5.16	5.16	5.19	5.19	5.15	5.16	5.14	5.16
10	10.36	10.30	10.32	10.27	10.31	10.31	10.26	10.33	10.26	10.33	10.30	10.30
15	15.32	15.50	15.51	15.45	15.40	15.57	15.47	15.44	15.37	15.38	15.46	15.44
20	20.82	20.65	20.78	20.66	20.66	20.67	20.40	20.67	20.65	20.60	20.51	20.64
25	25.82	25.91	25.92	25.74	25.61	25.82	25.89	25.79	25.77	25.72	25.77	25.79
30	30.96	30.91	31.13	31.19	30.94	31.00	30.94	30.86	30.74	30.83	30.99	30.95
35	36.07	36.43	36.07	36.25	36.16	35.94	36.16	36.27	36.28	36.24	35.98	36.17
40	41.32	41.48	41.47	41.25	41.40	41.48	41.27	41.26	41.25	41.11	41.52	41.35
45	46.52	46.57	46.76	46.65	46.43	46.43	46.60	46.89	46.67	46.68	46.06	46.57
50	51.64	51.96	52.14	52.12	52.02	51.75	51.48	51.81	51.37	51.59	51.76	51.79

It can be clearly seen from Fig. 12 that as the FOU is increased, the IAE value decreases. This is because as the MFs are made more and more uncertain, T2 PI FLC becomes better in handling the random sensor noise. Furthermore, it can also be observed that for FOU >= 90% the IAE values are nearly constant and attains the minimum possible value of IAE. This can also be understood as that the IAE is nearly same for the FOU of 90% and above. So, the settings of recommended FOU for effective sensor noise suppression would be 90% and above for the given experimentation. On the other hand it can also be suggested that in no case FOU should be kept below 75% as the smaller values of FOU leaves behind the uncorrected IAE. Table 5 shows the mean of IAE for FOU of 90% to 100%, for each noise percent along with the mean of these 11 mean IAEs values. Table 6 shows the corresponding standard deviations of mean IAE values in Table 5. It can also be seen that standard deviations are 3%-5% of the mean value.

4. PERFORMANCE COMPARISON

This section presents a performance comparison of the proposed controller. From the experiments conducted above, a comparison between the performances of T2 PI FLC and T1 PI FLC has been presented. Table 7 shows the comparison between IAE of T1 PI FLC and T2 PI FLC for various levels of random noise with mean IAE for FOU of 90%-100%. It may be noted that these mean IAE values of T2 PI FLC are the mean of 11 sets of FOU wherein each of the 11 sets represents a mean value of 1000 iterations as mention earlier. Fig. 13 presents the tabulated results graphically. It can be observed that the T2 PI FLC outperforms T1 PI FLC for all investigated cases. The improvements of T2 PI FLC over T1 PI FLC range from 13-19%. The maximum improvement was observed at the lower values of the sensor noise in comparison with the higher noise levels. Overall the observed improvement is significant and proves the superiority of T2 PI FLC over T1 PI FLC.

Table 6. Variation of standard deviation of IAE with FOU and noise.

Noise (%)	FOU										
	90	91	92	93	94	95	96	97	98	99	100
5	0.17	0.25	0.23	0.21	0.21	0.22	0.23	0.22	0.22	0.25	0.21
10	0.46	0.40	0.41	0.42	0.43	0.46	0.46	0.41	0.38	0.44	0.38
15	0.70	0.64	0.65	0.69	0.60	0.65	0.62	0.63	0.63	0.60	0.64
20	0.86	0.99	0.85	0.90	0.87	0.80	0.78	0.93	0.87	0.87	0.95
25	1.03	1.13	0.88	1.09	1.22	1.15	1.22	1.03	0.99	1.10	1.09
30	1.26	1.36	1.34	1.42	1.15	1.14	1.25	1.15	1.28	1.36	1.46
35	1.52	1.81	1.39	1.53	1.60	1.33	1.48	1.39	1.47	1.66	1.52
40	1.73	1.67	1.55	1.88	1.61	1.75	1.87	1.89	1.81	1.66	1.41
45	2.17	1.95	2.20	2.00	2.05	1.63	1.94	1.79	1.84	2.05	2.06
50	2.21	2.06	2.25	1.92	2.18	1.82	2.23	1.94	2.34	2.26	2.11

Table 7. Performance comparison.

Noise (%)	IAE		Improvement (%)
	T1 PI FLC	T2 PI FLC	
5	6.37	5.16	19
10	12.65	10.30	19
15	18.56	15.44	17
20	24.59	20.64	16
25	30.45	25.79	15
30	36.56	30.95	15
35	42.31	36.17	15
40	48.14	41.35	14
45	53.62	46.57	13
50	59.67	51.79	13

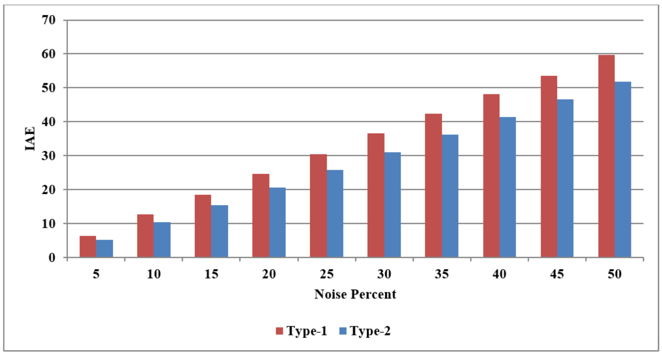
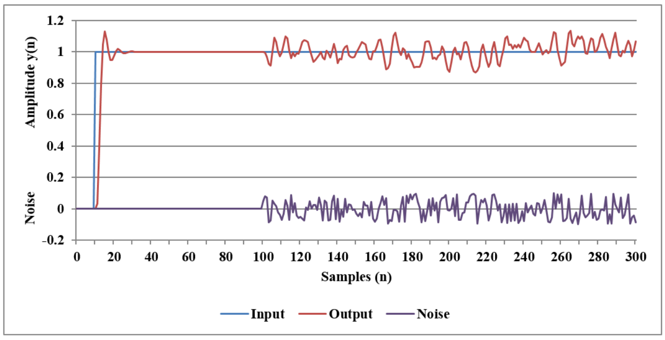
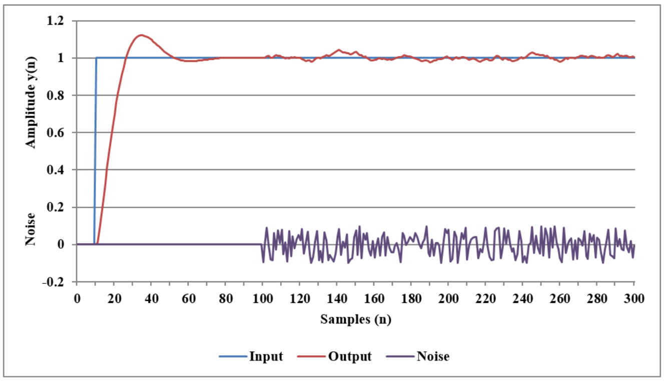


Fig. 13. Comparison of T1 PI FLC and T2 PI FLC.

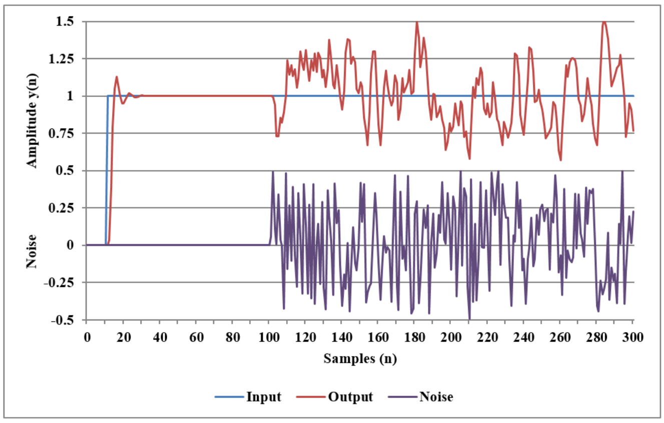
In order to illustrate the setpoint tracking and the sensor noise suppression along with the output variation, few select cases are presented in this section. Fig. 14 shows the noise suppression results for two particular cases i.e. for 10% and 50% noise. It can be clearly seen in Fig 14 (a) that when a noise of 10% is applied to T1 PI FLC, plant output varies by ±15% about setpoint. On the other hand, as seen in Fig. 14 (b) for T2 PI FLC, plant output varies just by ±4%. Furthermore, for the case of 50% noise it can be seen in Fig. 14 (c) that T1 PI FLC fails miserably and its plant output varies by nearly ±52% whereas, for T2 PI FLC it can be seen in Fig (d) that setpoint varies just by ±24%. Again, it can be clearly inferred that T2 PI FLC outperformed T1 PI FLC in handling sensor noise.



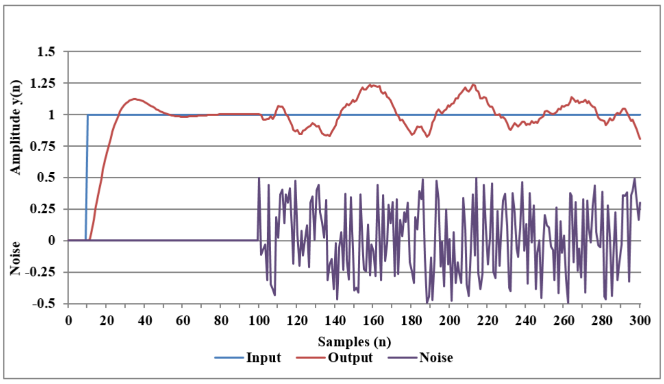
(a)



(b)



(c)



(d)

Fig. 14. Noise suppression performances: (a) T1 PI FLC for 10% noise (b) T2 PI FLC for 10% noise, FOU 90% (c) T1 PI FLC for 50% noise and (d) T2 PI FLC for 50% noise, FOU 90%.

5. CONCLUSION

In this paper, a novel technique for sensor noise suppression, i.e. minimizing the effects of measurement uncertainty, by T2 PI FLC has been investigated. A step by step procedure for designing of T1 PI FLC and T2 PI FLC was presented. T1 PI FLC was derived from the T2 PI FLC by keeping FOU as zero. The controllers were tuned using GA for minimum IAE, for a non-linear and unstable discrete time system. Large numbers of trials were conducted to evaluate the performance statistically. In the considered case, improvements of 13-19% have been successfully demonstrated for various noise levels. T2 PI FLC was found to outperform T1 PI FLC for sensor noise suppression in the presented study. Thus, T2 PI FLC could prove to be a better option than T1 PI FLC for systems with high uncertainty and where better controller performance are desirable. In summary, the main contribution of this work has been to show that the FOU of T2 FLS has a great impact on the system performance, and the uncertainty caused by sensor noise can easily be suppressed by FOU.

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