A novel adaptive approach for synchronization of uncertain chaotic systems using fuzzy PI controller and active control method

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Abstract:
This paper proposes a new adaptive synchronization scheme in order to avoid many defects which may occur in the estimation of unknown parameters during synchronization of uncertain chaotic systems. To carry out the synchronization of chaotic systems and to estimate the unknown parameters, a controller is designed into two parts: a fuzzy PI controller used as an eliminator of perturbations (EOP), and a main controller based on the use of active control method. This approach ensures a good estimation of the parameters without taking linear independence (LI) condition into account, which is considered as an important requirement for estimation of the unknown parameters.

Keywords: Chaotic systems, Synchronization, Adaptive control, Active control, Fuzzy PI controller.

1. INTRODUCTION

Since the discovery of chaos theory by Edward Lorenz (Lorenz 1963), many problems have been appeared in physical theory in many fields. One of the bright problems was emerged by the pioneer work of Pecora et al. in 1990 (Pecora & Carroll 1990). They proved that the synchronization of many chaotic systems was possible by using only one signal to synchronize them. This discovery attracted the attention of many researchers in many fields such as secure communication (Li, Liao & Wong 2004), chemical reaction (Srivastava, Srivastava & Chattopadhyay 2013) , modeling some complex systems like brain activity (Ivancevic & Ivancevic 2007). This diffusion makes synchronization and control of chaotic systems are an interesting field to study.

In the synchronization of chaotic systems, one of the systems is considered as a master or drive system and the other is a slave or response system. The main idea of chaos synchronization is to create a synchronization signal generator using an efficient control approach. The role of the synchronization signal is to make the slave system outputs track the master system outputs when there are uncertainties on the parameters of both systems.

First, synchronization of chaotic systems was investigated in the ideal case where there are no uncertainties on the parameter. In this case, the majority of control approaches were applied successfully. However, In the presence of uncertainties on the parameters, the efficiency of many conventional control approaches is severely reduced to ensure the stability and the synchronization. Hence, different methods such as backstepping control (Zhang, Li, Zhang & Yu 2004), H_∞ control (Suykens, Curran, Vandewalle & Chua 1997), sliding mode control method (Zhang, Ma & Liu 2004), adaptive control method (Chua, Yang, Zhong & Wu 1996), sampled-data feedback method (Zhao & Lu 2008), fuzzy logic control (Yau & Shieh 2008) and many others, have been suggested for synchronization of uncertain chaotic systems.

Among all these methods, adaptive synchronization has taken a big area in the literature. Many adaptive approaches have been applied successfully, but some problems have appeared in many of them. In (Sun, Zhu, Si, Ge & Zhang 2013), the adaptive synchronization defects are categorized into five cases: infeasible parameter updating laws (Sudheer & Sabir 2009), the neglect of the linear independence condition (Wang & Sun 2011), wrongly designed controller functions (Xiang-Jun Wu 2011), adaptive synchronization or parametric synchronization? (Yang 2011), the impracticability of a pragmatic adaptive synchronization scheme (M. M. El-Dessoky 2012). In order to correct the defects in these articles, they presented modified schemes and emphasized the linear independence condition(LI), which was considered as an essential factor to ensure a good estimation of unknown parameters.

Since the emergence of fuzzy set theory by (Zadeh 1965), many fields have taken advantage of this theory in order to resolve outstanding problems such as image processing (Van De Ville, Nachtegael, Van der Weken, Kerre, Philips & Lemahieu 2003),biology analysis (Woolf & Wang 2000), control robots (Das & Kar 2006).

Fuzzy logic control (FLC) is another strategy that has been widely used for controlling of nonlinear systems (KOTHANDARAMAN, Satyanarayana & Ponnusamy 2015) (Rebai, Guesmi & Hemici 2015) (Kumar 2015); furthermore, it has shown a good performance towards problems that concern uncertainties on the parameters and modeling error. In many cases, the fusion of FLC with a formal controller generates a new effective controller. For instance, the fusion of FLC with conventional PI controller generates powerful non-conventional controller called fuzzy PI controller (Tang, Man, Chen & Kwong 2001a), which is more efficient and suitable for controlling nonlinear systems.

On the other hand, FLC has been used with classical control approaches as an assistant in order to avoid many problems(Driss & Mansouri 2015a). For example, the use of FLC with sliding-mode control (FSMC) helps to avoid chattering problem caused by the discontinuation of the sign function in the reaching control law in (SMC) (Roopaei, Jahromi, Ranjbar-Sahraei & Lin 2011) (Yau & Chen 2006) (Bouarroudj 2015).

In this paper, a novel adaptive approach for synchronization of uncertain chaotic systems is presented in order to avoid many defects, which have appeared in the estimation of unknown parameters due to the negligence of the LI condition in many articles such as (Al-sawalha & Noorani 2010),(Miao, Tang, Lu & Fang 2009),(Mossa Al-sawalha & Noorani 2012),(Li, Leung, Han, Liu & Chu 2011). To achieve the synchronization, the controller is divided into two parts. The task of the first part is to eliminate the perturbation terms due to uncertainties on the parameters, and it is named the eliminator of perturbation (EOP). The second part represents the main controller, and its task is to synchronize the two chaotic systems. For the design of the EOP, a fuzzy PI controller is used, whereas the main controller is based on the use of active control method.

The other parts of this paper are arranged as follows: Section 2 presents the synchronization problem. The design of the EOP using a fuzzy PI controller is developed in Section 3, while Section 4 is consecrated to the design of the main controller using active control method. The effectiveness of the proposed approach is illustrated by different results presented in Section 5, and the conclusion is given in Section 6.

2. SYNCHRONIZATION PROBLEM AND SYSTEM DESCRIPTION

Consider two chaotic systems that must be synchronized. As a master, The following system is defined:

$$\begin{cases} \dot{x}_1 = & f_1(x,t) \\ \dot{x}_2 = & f_2(x,t), \\ \quad\vdots & \quad\vdots \\ \dot{x}_n = & f_n(x,t) \end{cases} \tag{1}$$

$$x = [x_1, x_2, ..., x_n] \in \Re^n,$$

and as slave

$$\begin{cases} \dot{y}_1 = & h_1(y,t) + g_1(y,t) + u_1(y,x,t) \\ \dot{y}_2 = & h_2(y,t) + g_2(y,t) + u_2(y,x,t), \\ \quad\vdots & \quad\vdots \\ \dot{y}_n = & h_n(y,t) + g_n(y,t) + u_n(y,x,t) \end{cases} \tag{2}$$

$$y = [y_1, y_2, ..., y_n] \in \Re^n,$$

where $x(t), y(t) \in \Re^n$ represent the state vectors of the master and the slave system respectively.

$f, h : \Re^n \to \Re^n$ are continuous nonlinear vector functions, $g(t,y) : \Re^n \to \Re^n$ are the perturbation terms resulting from uncertainties on the parameters; they are supposed to be bounded. $u_1(t,x,y), u_2(t,x,y), ...u_n(t,x,y)$ are the control inputs to be designed.

Chaotic systems represent the intersection between deterministic systems and randomness. The deference between chaotic systems and normal nonlinear systems is the butterfly effect or sensitivity to initial conditions, which makes chaotic systems exhibit random behavior. Because of this feature, synchronization of chaotic systems has been considered as an interesting topic to study.

The aim of the synchronization is to design a feedback controller such that the error between the two systems converges to zero

$$\lim_{t\to\infty} \|y(t) - x(t)\| = 0, \tag{3}$$

for all $x(0) \in \Re^n$ and $y(0) \in \Re^n$.

The synchronization errors are defined by

$$e_m = y_m - x_m, m = 1, 2, ..., n, \tag{4}$$

and their dynamics by

$$\begin{cases} \dot{e}_1 = & h_1(y,t) - f_1(x,t) + g_1(y,t) + u_1(y,x,t) \\ \dot{e}_2 = & h_2(y,t) - f_2(x,t) + g_2(y,t) + u_2(y,x,t). \\ \quad\vdots & \quad\vdots \\ \dot{e}_n = & h_n(y,t) - f_n(x,t) + g_n(y,t) + u_n(y,x,t) \end{cases} \tag{5}$$

In this paper, the uncertainties on the parameters are considered as terms added to the slave system. These terms affect the behavior of the system, and the main controller cannot handle any new dynamic out of its environment. However, if another system operates to eliminate these terms, the design of the main controller becomes very simple. The main aim to eliminate the perturbation terms is to avoid the problem of linear dependence of the function terms.

Therefore, the controller is designed into two parts. The first part is the eliminator of perturbation (EOP), and its task is to eliminate the perturbation terms due to uncertainties on the parameters, and the second one serves to synchronize the two chaotic systems.

3. DESIGN OF THE ELIMINATOR OF PERTURBATION (EOP)

Fuzzy logic control has a special structure that helps to design an efficient controller for nonlinear systems even though the models are not accurate or not available, and the parameters are uncertain. It is easy to ensure the stability by understanding the behavior of the controlled system and by adjusting the rules instead of using the nonlinear terms of the models which trigger some problems such as uncertainties on the parameters(Chen & Ying 1993)(Misir, Malki & Chen 1996)(Tang, Man, Chen & Kwong 2001b). FLC has been used for synchronization of uncertain chaotic systems and has shown a good performance than many classical control approaches (Precup, Tomescu & Dragos 2014)(Ginarsa, Soeprijanto & Purnomo 2013)(Sadeghi & Menhaj 2012)(Kuo, Pai & Yau 2009)(Driss & Mansouri 2015b). Thus, fuzzy PI controller is used to deal with uncertainties as an eliminator of perturbations. Fuzzy PI controller is derived from the convention analog PI controller which is given in the frequency s-domain as follows:

$$u_{PIm}(s) = (K_{pm}^c + \frac{K_{im}^c}{s})E_m(s), \tag{6}$$

where K_{pm}^c and K_{im}^c, and $E_m(s)$ are the proportional gain , integral gain and the tracking error signal, respectively.

To get the digital version, the bilinear transform is applied $s = (2/T)(z-1)/(z+1)$, where $T > 0$, is the sampling time, which leads to the following form:

$$u_{PIm}(z) = (K_{pm}^c - \frac{K_{im}^c T}{2} + \frac{K_{im}^c T}{1-z^{-1}})E_m(z), \tag{7}$$

letting

$$K_{pm} = K_{pm}^c - \frac{K_{im}^c T}{2} \quad \text{and} \quad K_{im} = K_{im}^c T,$$

and using the inverse z-transform, we get the digital form of the controller

$$\begin{aligned} u_{PIm}(kT) - u_{PIm}(kT-T) = K_{pm}[e_m(kT) - \\ e_m(kT-T) + K_{im}e_m(kT)]. \end{aligned} \tag{8}$$

Dividing (8) by T, we obtain

$$\Delta u_{PIm}(kT) = K_{pm}e_{vm}(kT) + K_{im}e_{pm}(kT), \tag{9}$$

where

$$\Delta u_{PIm}(kT) = \frac{u_{PIm}(kT) - u_{PIm}(kT-T)}{T}, \tag{10}$$

$$e_{vm}(kT) = \frac{e_m(kT) - e_m(kT-T)}{T}, \tag{11}$$

$$e_{pm}(kT) = e_m(kT), \tag{12}$$

$\Delta u_{PIm}(kT)$ is the incremental control output of the PI controller, $e_{pm}(kT)$ the error between the master and the salve system, and $e_{vm}(kT)$ is the error rate. Equation (10) can be written as the folloing form:

$$u_{PIm}(kT) = u_{PIm}(kT-T) + T\Delta u_{PIm}(kT). \tag{13}$$

To get the fuzzy PI controller, the increment control input $T\Delta u_{PIm}(kT)$ will be replaced by a fuzzy control action $K_{uPIm}\Delta u_{PIm}(kT)$ such that

$$u_{PIm}(kT) = u_{PIm}(kT-T) + K_{uPIm}\Delta u_{PIm}(kT), \tag{14}$$

where K_{uPIm} is a fuzzy control gain.

The inputs of the Fuzzy PI controller are the error between the two systems, $e_{pm}(kT)$, and the error rate, $e_{vm}(kT)$. On the other hand, the fuzzy PI controller has only one output, the incremental control output $\Delta u_{PIm}(kT)$, which is used as input to the slave system. The design of the fuzzy PI controller needs three parts: fuzzification, control rule base, and defuzzification. Fig.1 gives the membership functions for the fuzzification of the inputs, whereas Fig.2 gives the membership functions of the output.

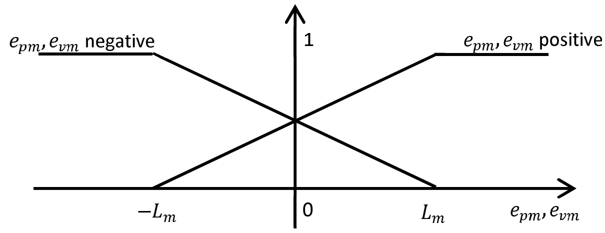

Fig. 1. The input membership functions.

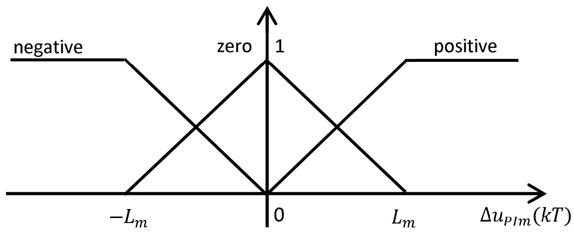

Fig. 2. The output membership functions.

The design of fuzzy control rules is very important part in order to ensure the stability of the controlled system. The choice of the rules gives the relation between the inputs and the output which represents the role of the controller. The fuzzy PI controller serves as the eliminator of perturbation terms that come from the uncertainties on the parameters. More precisely, the output of the controller must eliminate the perturbation terms according to the inputs which represent the error and the change of the error rate between the two chaotic systems, but how the fuzzy PI controller can act only to the variation of the error comes from the effect of uncertainties on the parameters. In order to solve this problem, another controller must be designed. The role of the new controller is to ensure the synchronization when the parameters used in the design of it are the same as the parameters of the master and the slave systems without uncertainties. The ability of the second control is restricted to ensure the synchronization when there are no uncertainties. In presence of uncertainties, the parameters will be unknown and because the main controller is designed using known parameters without adaptation, the synchronization is destroyed, which leads to the emergence of errors come from the effect of uncertainties on the parameters.

Using the membership functions, the following control rules are assigned for the fuzzy PI controller:

(R1) IF $e_{pm} = e_{pm}.n$ AND $e_{vm} = e_{vm}.n$ THEN PI-output=$o.p$,
(R2) IF $e_{pm} = e_{pm}.n$ AND $e_{vm} = e_{vm}.p$ THEN PI-output=$o.z$,
(R3) IF $e_{pm} = e_{pm}.p$ AND $e_{vm} = e_{vm}.n$ THEN PI-output=$o.z$,
(R4) IF $e_{pm} = e_{pm}.p$ AND $e_{vm} = e_{vm}.p$ THEN PI-output=$o.n$,

where $e_{pm}.n$ means error negative, $o.n$ means output negative and PI-output is $\Delta u_{PIm}(kT)$. The role of the fuzzy PI controller is to make the error converge to zero. Thus, if e_{pm} is negative($e_{pm}.n$) means that the error is below zero, and if the error rate is negative($e_{pm}.n$) implies the controller at the previous step is driving the system output downward, the control output, $\Delta u_{PIm}(kT)$, must be set to be positive, for example. Similarly, for rule 4 (R4), if the error is positive, and the error rate is positive implies that the error is above zero, and the previous control action is driving the system output upward. Therefore, the control output must set to be negative. For the rules 2 (R2) and 3 (R3), the control output is set to be zero because the error rate is negative when the error is positive, which drives the system output upward for R2, and vice versa for R3.

Center of mass formula is used in order to defuzzify the incremental control of the fuzzy control law

$$\Delta u_{PIm} = \frac{\sum \text{MVI} \times \text{MVO}}{\sum \text{membership value of input}}, \tag{15}$$

where MVI is membership value of input and MVO is output corresponding to the membership value of input. Twenty adjacent input-combination (IC) regions are used to decompose the value-ranges of the two inputs, as shown in Fig.3.

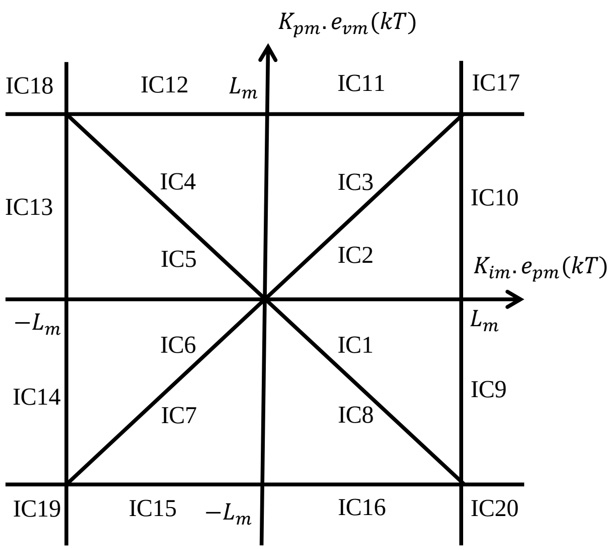

Fig. 3. Regions of the fuzzy controller values.

The membership functions of the error signal, which given by e_{pm} in Fig.1, are put over the horizontal $K_{im}.e_{pm}(kT)$-axis, whereas the membership functions of the rate of change of the erro signal, which given by e_{vm} in Fig.1, are put over the horizontal $K_{pm}.e_{vm}(kT)$-axis, which form the third axis. The intersection between the inputs selects which IC region is concerned for the defuzzification. For instance, if these three conditions are held,$0 < K_{im}.e_{pm}(kT) < L_m$, $-L_m < K_{pm}.e_{vm}(kT) < 0$ and $K_{pm}.e_{vm}(kT) + K_{im}.e_{pm}(kT) > 0$ implies that the IC1 region is selected. By using the rules and Zadeh's logical "AND" , the following input-output membership functions are selected:

$$(R1)\begin{cases}\text{The input membership value is } e_{pm}.n\\ \text{The output membership value is } o.p\end{cases}$$

$$(R2)\begin{cases}\text{The input membership value is } e_{pm}.n\\ \text{The output membership value is } o.z\end{cases}$$

$$(R3)\begin{cases}\text{The input membership value is } e_{vm}.n\\ \text{The output membership value is } o.z\end{cases}$$

$$(R4)\begin{cases}\text{The input membership value is } e_{vm}.p\\ \text{The output membership value is } o.n\end{cases}$$

It is easy to verify that the above combinations are true for the region IC2, too. Using formula (15) , the defuzzification formula of the regions IC1 and IC2 is given as follows:

$$\Delta u_{PIm} = \frac{Q}{e_{pm}.n + e_{pm}.n + e_{vm}.n + e_{vm}.p}, \tag{16}$$

where

$Q = e_{pm}.n \times o.p + e_{pm}.n \times o.z + e_{vm}.n \times o.z + e_{vm}.p \times o.n.$

By using the following formulas that describe the geometry of the input membership functions:

$$e_{pm}.p = \frac{K_{im}e_{pm}(kT) + L_m}{2L_m}, e_{pm}.n = \frac{-K_{im}e_{pm}(kT) + L_m}{2L_m},$$

$$e_{vm}.p = \frac{K_{pm}e_{vm}(kT) + L_m}{2L_m}, e_{vm}.n = \frac{-K_{pm}e_{vm}(kT) + L_m}{2L_m},$$

and by applying $o.p = L_m$, $o.n = -L_m$ and $o.z = 0$, the output in the regions IC1 and IC2 is given as follows:

$$\Delta u_{PIm}(kT) = \frac{-L_m[K_{im}.e_{pm}(kT) + K_{pm}.e_{vm}(kT)]}{2(2L_m - K_{im}.e_{pm}(kT))}. \tag{17}$$

Similarly, the output in the regions IC5 and IC6 is given as follows:

$$\Delta u_{PIm}(kT) = \frac{-L_m[K_{im}.e_{pm}(kT) + K_{pm}.e_{vm}(kT)]}{2(2L_m + K_{im}.e_{pm}(kT))}. \tag{18}$$

The difference between the IC1,IC2 and IC5,IC6 is the sign of $K_{im}.e_{pm}(kT)$, which is positive in IC1, IC2 and negative in IC5,IC6. Therefore, the four regions can be combined using the following equation:

$$\Delta u_{PIm}(kT) = \frac{-L_m[K_{im}.e_{pm}(kT) + K_{pm}.e_{vm}(kT)]}{2(2L_m - K_{im}.\left|e_{pm}(kT)\right|)}. \quad (19)$$

In the same way, the output $\Delta u_{PIm}(kT)$ in the 20 regions is given as follows:

in the regions IC 1,2,5,6

$$\Delta u_{PIm}(kT) = \frac{-L_m[K_{im}.e_{pm}(kT) + K_{pm}.e_{vm}(kT)]}{2(2L_m - K_{im}.\left|e_{pm}(kT)\right|)}, \quad (20)$$

in the regions IC 3,4,7,8

$$\Delta u_{PIm}(kT) = \frac{-L_m[K_{im}.e_{pm}(kT) + K_{pm}.e_{vm}(kT)]}{2(2L_m - K_{pm}.\left|e_{vm}(kT)\right|)}, \quad (21)$$

in the regions IC 9,10

$$\Delta u_{PIm}(kT) = -1/2[K_{pm}e_{vm}(kT) + L_m], \quad (22)$$

in the regions IC 11,12

$$\Delta u_{PIm}(kT) = -1/2[K_{im}e_{pm}(kT) + L_m], \quad (23)$$

in the regions IC 13,14

$$\Delta u_{PIm}(kT) = -1/2[K_{pm}e_{vm}(kT) - L_m], \quad (24)$$

in the regions IC 15,16

$$\Delta u_{PIm}(kT) = -1/2[K_{im}e_{pm}(kT) - L_m], \quad (25)$$

in the regions IC 18,20

$$\Delta u_{PIm}(kT) = 0, \quad (26)$$

in the region IC17

$$\Delta u_{PIm}(kT) = -L_m, \quad (27)$$

in the region IC19

$$\Delta u_{PIm}(kT) = L_m. \quad (28)$$

From the rules, the EOP output acts against the error variation. The appearance of errors between the master and the slave systems indicates that the main controller loses the synchronization due to the existence of the perturbation terms. The EOP reacts by adding new terms ($\varphi_m = u_{PIm}$) to (5) so as to eliminate them. Thus, (5) becomes

$$\begin{cases} \dot{e}_1 &= h_1(y,t) - f_1(x,t) + g_1(y,t) + U_1(t) \\ \dot{e}_2 &= h_2(y,t) - f_2(x,t) + g_2(y,t) + U_2(t), \\ \vdots & \qquad \vdots \\ \dot{e}_n &= h_n(y,t) - f_n(x,t) + g_n(y,t) + U_3(t) \end{cases} \quad (29)$$

with

$$\begin{cases} U_1(t) &= u_1(y,x,t) + \varphi_1(t) \\ U_2(t) &= u_2(y,x,t) + \varphi_2(t). \\ \vdots & \qquad \vdots \\ U_n(t) &= u_n(y,x,t) + \varphi_n(t) \end{cases} \quad (30)$$

If EOP eliminates the perturbation terms during the synchronization process, the following condition is held:

$$g_m(t) + \varphi_m(t) = 0, \quad (31)$$

and (29) becomes

$$\begin{cases} \dot{e}_1 &= h_1(y,t) - f_1(x,t) + u_1(y,x,t) \\ \dot{e}_2 &= h_2(y,t) - f_2(x,t) + u_2(y,x,t). \\ \vdots & \qquad \vdots \\ \dot{e}_n &= h_n(y,t) - f_n(x,t) + u_n(y,x,t) \end{cases} \quad (32)$$

This set of equations represents a simple system without perturbations, and it is easy to design a controller using Lypunov theory.

The rules are important to reach the stability; however, they are not enough to ensure asymptotic stability. Therefore, the following theorem (Chen & Ying 1993) (Misir *et al.* 1996) , which is based on bounded-input bounded-output Stability (BIBO) and the small gain theorem, is used to find the gain K_{im}, K_{pm}, K_{uPIm} and L_m.

Theorem 1. A sufficient condition for the nonlinear fuzzy PI control system to be stable is that the given nonlinear process has a bounded norm (gain) $\|N\| < \infty$ and the parameters of the fuzzy PI controller satisfy

$$\frac{K_{uPIm}(\gamma K_{pm} + K_{im})L_m}{T(2L_m - K_{Mm})} \|N\| < 1, \quad (33)$$

where

$$\gamma = max\{1, T\}$$

and

$$K_{Mm} = max\{K_{pm}M_{pm}, K_{im}M_{vm}\},$$

with

$$M_{pm} = \sup_{k\geq 0} \left|e_{pm}(kT)\right|,$$

$$M_{vm} = \sup_{k\geq 0} \left|e_{vm}(kT)\right| \leq \frac{2}{T} M_{pm}.$$

The gain of MIMO nonlinear system is the largest gain value across all input/output channels

$$\|N\| = max\{\|N_1\|, \|N_2\|, .., \|N_n\|\}.$$

To calculate the gain of one of the input/output channels, the following formula is used:

$$\|N_m\| = \sup_{v_1\neq v_2, k\geq 0} \frac{|N_m(v_1(kT)) - N_m(v_2(kT))|}{|v_1(kT) - v_2(kT)|}, \quad (34)$$

where v_1 and v_2 are two bounded signals. The slave system is used to calculate the gain $\|N_m\|$

$$\begin{cases} \dot{y}_1 = & h_1(y,t) + g_1(y,t) + v_j \\ \dot{y}_2 = & h_2(y,t) + g_2(y,t) + v_j, \\ \vdots & \qquad \vdots \\ \dot{y}_n = & h_n(y,t) + g_n(y,t) + v_j \end{cases} \quad (35)$$

where $j = 1, 2$.

4. DESIGN OF THE MAIN CONTROLLER USING ACTIVE CONTROL METHOD

For the design of the main controller, active control method is chosen (Vincent 2005) (Yassen 2005) (Bhalekar & Daftardar-Gejji 2010). To illustrate the effectiveness of the proposed approach, it is applied for the synchronization of two uncertain Lorenz systems.

The Lorenz master system is

$$\begin{cases} \dot{x}_1 &= a_1(x_2 - x_1) \\ \dot{x}_2 &= (-x_1x_3 + c_1x_1 - x_2), \\ \dot{x}_3 &= x_1x_2 - b_1x_3 \end{cases} \tag{36}$$

where x_1, x_2, x_3 are the state variables, a_1, c_1 and b_1 are positive uncertain parameters. The Lorenz chaotic trajectories are shown in Fig.5.

The Lorenz slave system is

$$\begin{cases} \dot{y}_1 &= a_2(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (-y_1y_3 + c_2y_1 - y_2) + u_2, \\ \dot{y}_3 &= y_1y_2 - b_2y_3 + u_3 \end{cases} \tag{37}$$

where y_1, y_2, y_3 are the state variables, a_2, c_2 and b_2 are positive uncertain parameters, and u_1, u_2, u_3 are the controllers to be designed.

The dynamics of the error system is defined as

$$\begin{cases} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2, \\ e_3 &= y_3 - x_3 \end{cases} \tag{38}$$

while the errors of the parameters of the systems are given by

$$\begin{cases} e_a &= a_2 - a_1 \\ e_c &= c_2 - c_1. \\ e_b &= b_2 - b_1 \end{cases} \tag{39}$$

After a simple calculation, the error states are given by

$$\begin{cases} \dot{e}_1 &= g_1(y) + a_1(e_2 - e_1) + u_1 \\ \dot{e}_2 &= g_2(y) - y_2 - y_1y_3 + c_1e_1 + x_2 + x_1x_3 + u_2, \\ \dot{e}_3 &= g_3(y) - b_1e_3 + y_1y_2 - x_1x_2 + u_3 \end{cases} \tag{40}$$

where

$$\begin{cases} g_1(y) &= e_a(y_2 - y_1) \\ g_2(y) &= e_cy_1, \\ g_3(y) &= -e_by_3 \end{cases} \tag{41}$$

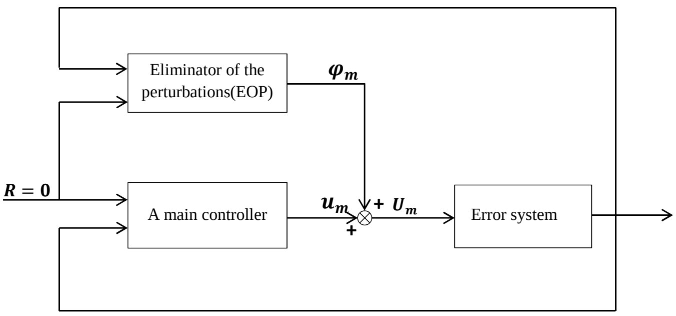

Fig. 4. Block diagram of the EOP and the main controller, where $R(t) = 0$ is the reference of the error system.

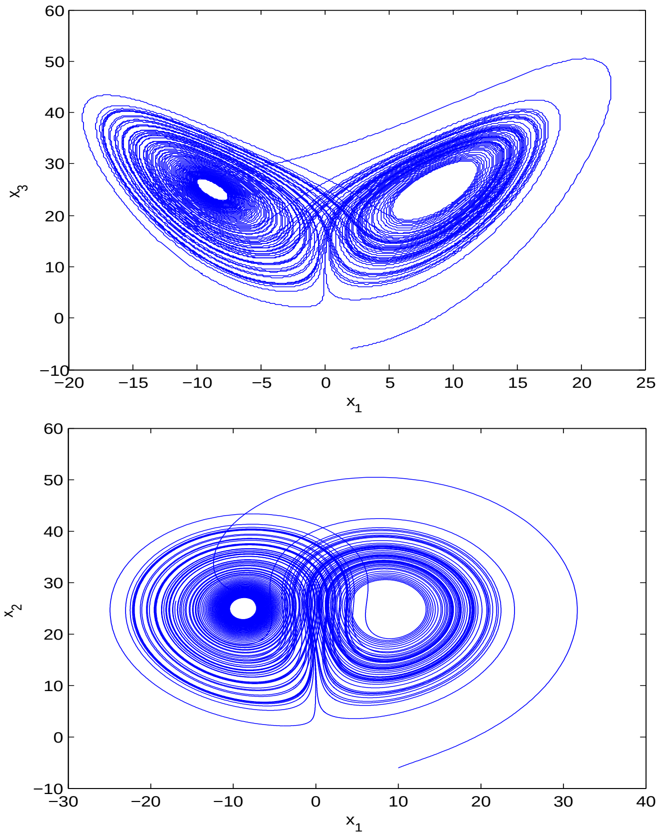

Fig. 5. The chaotic trajectories of Lorenz system.

represents the perturbation terms . All the perturbation terms can be neglected due to the actions of the EOP, and (40) becomes

$$\begin{cases} \dot{e}_1 &= a_1(e_2 - e_1) + u_1 \\ \dot{e}_2 &= -y_2 - y_1y_3 + c_1e_1 + x_2 + x_1x_3 + u_2. \\ \dot{e}_3 &= -b_1e_3 + y_1y_2 - x_1x_2 + u_3 \end{cases} \tag{42}$$

Hence, the active controllers are defined as follows:

$$\begin{cases} u_1 &= -e_1 - a_1(e_2 - e_1) \\ u_2 &= -e_2 + y_2 + y_1y_3 - c_1e_1 - x_2 - x_1x_3. \\ u_3 &= -e_3 - y_1y_2 + x_1x_2 + b_1e_3 \end{cases} \tag{43}$$

The parameters used in the design of the active controllers are fixed to be $a_1 = a_2 = 10, b_1 = b_2 = 8/3$ and $c_1 = c_2 = 28$, which restrict the controller to reach the synchronization when the parameters of the two systems are the same as the fixed parameters of the active controllers. The substitution of (43) into (42) gives a linear system of the form

$$\begin{cases} \dot{e}_1 &= -e_1 \\ \dot{e}_2 &= -e_2. \\ \dot{e}_3 &= -e_3 \end{cases} \tag{44}$$

Choose a Lyapunov function of the form

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2). \tag{45}$$

The derivative of V is

$$\dot{V}(e) = -e_1^2 - e_2^2 - e_3^2, \tag{46}$$

this implies that the controlled system is globally asymptotically stable.

The estimation of the parameters a_2, b_2 and c_2 is concluded for (41) and (31) as follows:

$$\begin{cases} \widehat{a}_2(t) = \dfrac{-\varphi_1(t)}{y_2(t)-y_1(t)} + a_1 & y_2 - y_1 \neq 0 \\ \widehat{c}_2(t) = \dfrac{-\varphi_2(t)}{y_1(t)} + c_1 & y_1 \neq 0. \\ \widehat{b}_2(t) = \dfrac{\varphi_3(t)}{y_3(t)} + b_1 & y_3 \neq 0 \end{cases} \tag{47}$$

Fig.6 shows the detailed control scheme used for the synchronization of uncertain chaotic systems.

5. NUMERICAL RESULTS

The described approach is applied for the synchronization of two uncertain Lorenz systems. There are two objectives to reach: the first is to make the error dynamics asymptotically stable, and the second is to estimate the unknown parameters of the systems.

For the numerical simulation, the fourth order Runge-Kutta method with $T = 0.001$ is used. The parameters of the master and the slave systems without perturbations are chosen as follows:

$$a_1 = 10, c_1 = 28, b_1 = 8/3,$$
$$a_2 = 10, c_2 = 8/3, b_2 = 8/3.$$

The parameters of the systems are randomly perturbed using a generator of perturbation (GP) based on MATLAB function "rand".

The initial conditions are taken as:

$$x_1(0) = -5, x_2(0) = 5, x_3(0) = 0,$$
$$y_1(0) = -6, y_2(0) = -8, y_3(0) = 12.$$

The parameters ,K_{im}, K_{pm}, K_{uPIm} and L_m, can be defined using theorem 1 and (34) which is used to determine the gain $\|N_m\|$. However, the gain $\|N_m\|$ depends on the uncertainties which are unknown. In order to solve this problem, worst-case gains must be defined, which represents the maximum gains over-all possible values of the parterres of the slave system. Tables 1,2 and 3 give the gain $\|N_m\|$ variations as a function of the parameters a_2, b_2, c_2.

The gain of the controlled system (slave system) is the maximum of the worst-case gains

$$\|N\| = max\,\{4.6192, 5.6824, 4.5644\} = 5.6824.$$

Thus , the parameters of the controllers are given as follows:

$L_1 = L_2 = L_3 = 1$,
$K_{p1} = 0.9, K_{i1} = 2, K_{uPI1} = 2.43$,
$K_{p2} = 1.4, K_{i2} = 0.2, K_{uPI2} = 1.3$,
$K_{p3} = 1, K_{i3} = 1.2, K_{uPI3} = 2$.

In order to illustrate the performances of the EOP and the main controller, the range of time is divided into five parts. Each part corresponds to a state; this allows to figure out acts of the EOP. The different considered situations are reported in Table 4.

Fig.7 and Fig.8 show the behavior of the main controller and the EOP concerning the synchronization of the two systems. The following behaviors are observed for the different cases reported in Table 4:

1. For $0 < t < 5$, the controllers are turned off, so the trajectories of the two systems are uncorrelated.
2. For $5 < t < 15$, there are no perturbation terms due to uncertainties on the parameters, the active controller is turned on, and the two systems are perfectly synchronized.
3. For $15 < t < 25$, the generator of perturbation is turned on, this factor destroys the synchronization between the two systems. The active controller cannot handle the new situation.
4. For $25 < t < 35$, the EOP is turned on, the error dynamics converge to zero, and the active controller works well as there is no perturbation.
5. For $35 < t < 50$,the EOP stops exactly when the perturbation on the parameters is turned off.

Table 4. The table of the states.

Time(sec)	Active controller	GP	EOP
0 to 5	Off	Off	Off
5 to 15	On	Off	Off
15 to 25	On	On	Off
25 to35	On	On	On
35 to 50	On	Off	On

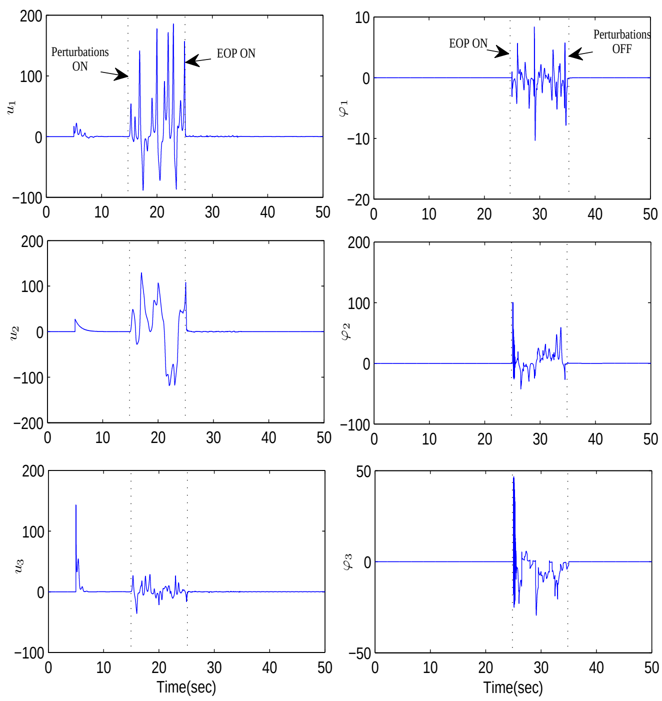

Fig. 8. The actions of EOP and the main controller.

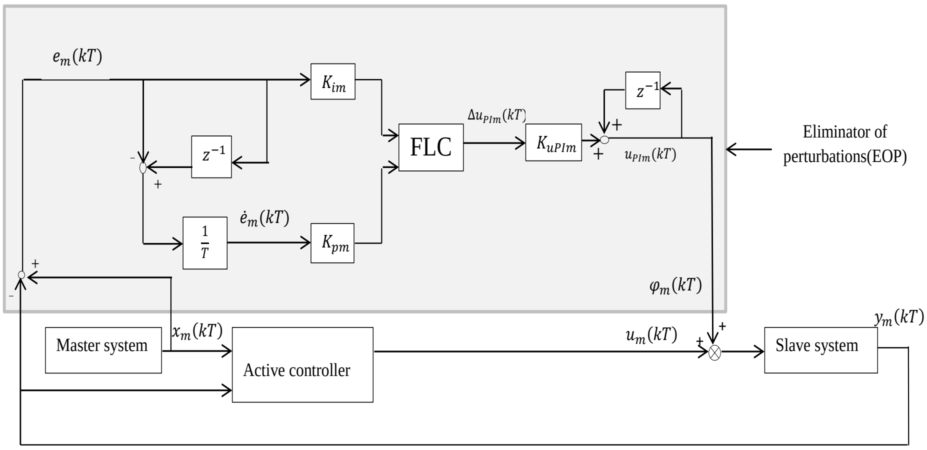

Fig. 6. The proposed control scheme.

Table 1.

a_2	4	5	6	7	8	9	10	11	12	13	14	15	16
$\|N_1\|$	0.5728	0.8932	3.9990	3.9441	4.0783	3.9083	3.4187	3.1995	2.2931	4.3068	4.6192	4.0652	4.5133

Table 2.

c_2	22	23	24	25	26	27	28	29	30	31	32	33	34
$\|N_2\|$	0.8964	3.1250	4.4389	5.2312	3.8311	3.6413	5.3288	4.2208	4.8563	5.0255	5.0460	5.6824	5.6548

Table 3.

b_2	8/3-1.2	8/3-1	8/3-0.8	8/3-0.6	8/3-0.4	8/3-0.2	8/3	8/3+0.2	8/3+0.4	8/3+0.6	8/3+0.8	8/3+1	8/3+1.2
$\|N_3\|$	3.7341	3.7110	3.8548	4.1019	3.5795	3.7194	3.8815	3.7964	4.4686	4.5644	3.7546	4.3534	3.8984

The second part of the numerical simulation is focused on the estimation of the unknown parameters of both systems so as to check out the performance of the adaptive scheme.

To appreciate the importance of the LI condition, the authors, in (Sun *et al.* 2013), introduced multi- switching synchronization between two uncertain Lorenz systems (Wang & Sun 2011) as an example. They indicated the differences between the first and the second switches in the estimation of unknown parameters sense. They found that the estimation of the unknown parameters was failed in the first switch Fig.9; however, in the second switch, the estimation was successful. The failure to estimate of the unknown parameters was referred to the function terms $-(x_2 - x_1), x_3$ and $-x_1$ which are linearly dependent to $-(y_2 - y_1), y_3, -y_1$; as a result, all the six unknown parameters have been failed to be identified.

Many modifications have been occurred to avoid the problem of linear dependence of the function terms such as altering the synchronization orbits, adding extra signals to the state function, changing the structures of the function groups and adjusting the persistent time and transient time.

In the next part, the performance of the proposed adaptive approach is checked with the same case studied in (Sun *et al.* 2013). It is possible to identify the unknown parameters using (47). However, in the presence of the unknown parameters in both systems, (47) gives only the error between the parameters of the systems and becomes

$$\begin{cases} e_a(t) = \dfrac{-\varphi_1(t)}{y_2(t) - y_1(t)} & y_2 - y_1 \neq 0 \\ e_c(t) = \dfrac{-\varphi_2(t)}{y_1(t)} & y_1 \neq 0. \\ e_b(t) = \dfrac{\varphi_3(t)}{y_3(t)} & y_3 \neq 0 \end{cases} \tag{48}$$

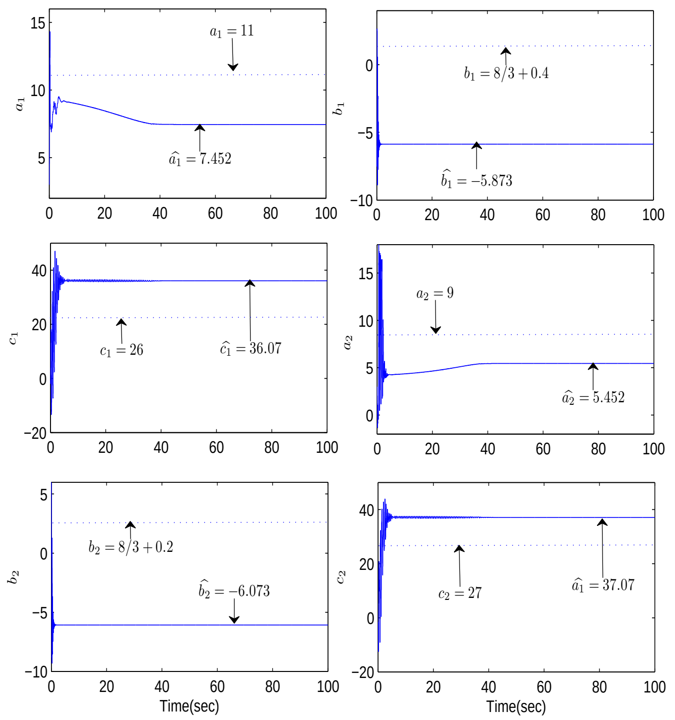

Fig. 9. The wrong estimation of the unknown parameters of the first switch.

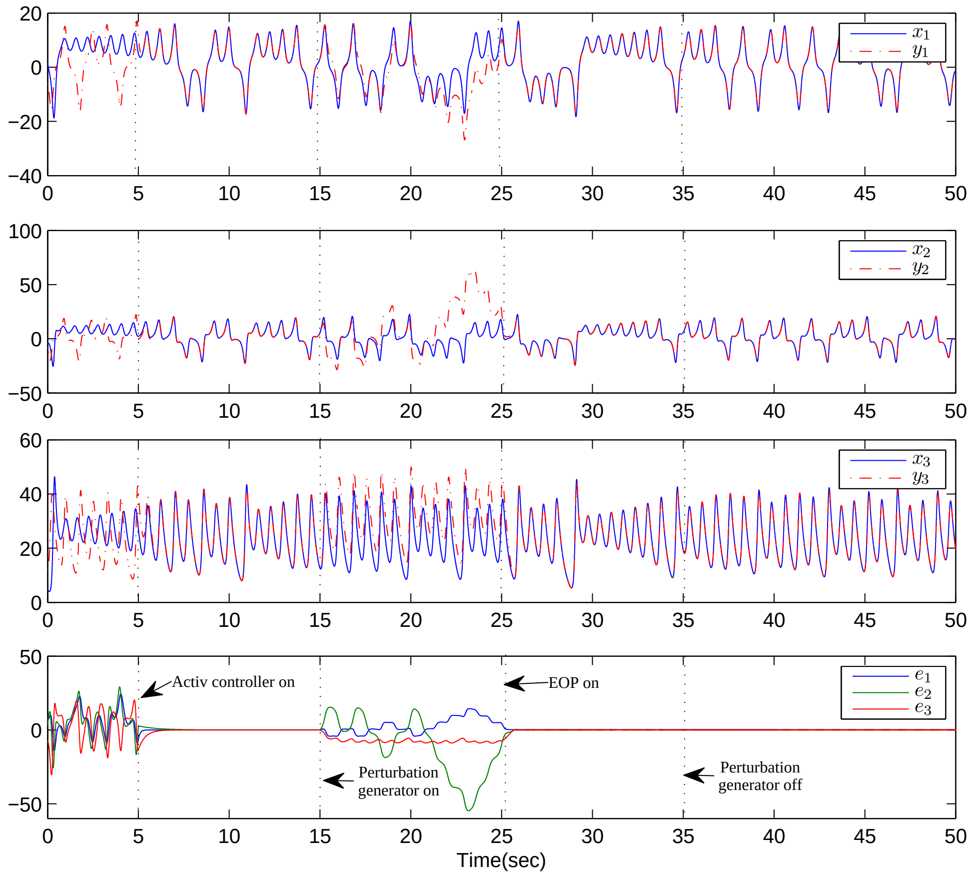

Fig. 7. The Synchronization behavior in each case.

Fig. 10 shows the error between the parameters of both systems in the case when initial values of the unknown parameters are chosen as: $\widehat{a}_1(0) = 2, \widehat{b}_1(0) = 10, \widehat{c}_1(0) = -6, \widehat{a}_2(0) = -2, \widehat{b}_2(0) = 5, \widehat{c}_2(0) = 1$. The true values of the parameters of the master system are $a_1 = 11, b_1 = 8/3+0.4, c_1 = 26$, while the parameters of the salve system are $a_2 = 9, b_2 = 8/3 + 0.2, c_2 = 27$.

To find the true values of the parameters, the unknown parameters of the master or the slave system must be estimated at least. To solve this problem, a reference Lorenz system with known parameters must be designed. The reference system allows to estimate the parameters of the master or the slave system. Hence, the unknown parameters of the slave system can be estimated using (47) as follows:

$$\begin{cases} \widehat{a}_2(t) = \dfrac{-\varphi_1(t)}{y_2(t) - y_1(t)} + a_r & y_2 - y_1 \neq 0 \\ \widehat{c}_2(t) = \dfrac{-\varphi_2(t)}{y_1(t)} + c_r & y_1 \neq 0, \\ \widehat{b}_2(t) = \dfrac{\varphi_3(t)}{y_3(t)} + b_r & y_3 \neq 0 \end{cases} \tag{49}$$

where $a_r = 10, b_r = 8/3, c_r = 28$ are the parameters of the reference system.

The estimation of the unknown parameters of the master system can be derived from (48) and (49) as follows:

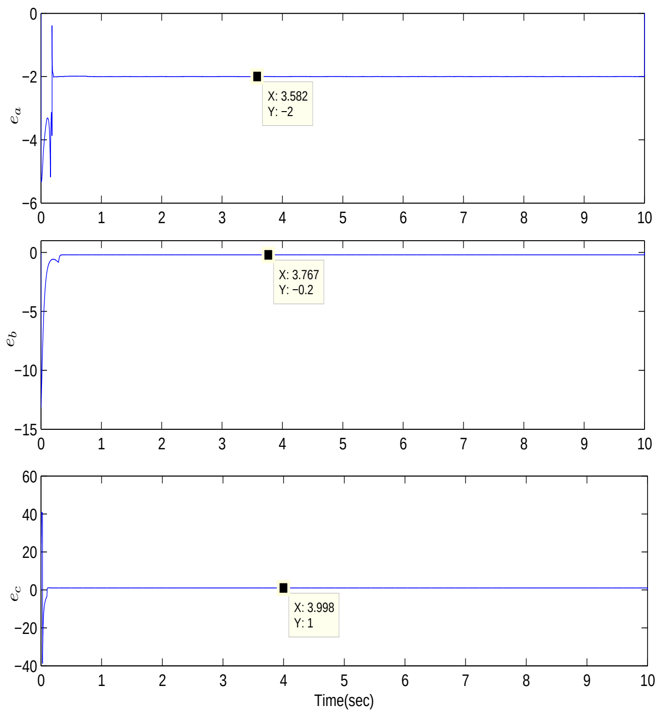

Fig. 10. The parameter errors using (48).

$$\begin{cases} \widehat{a}_1(t) = \widehat{a}_2(t) - e_a(t) \\ \widehat{c}_1(t) = \widehat{c}_2(t) - e_c(t). \\ \widehat{b}_1(t) = \widehat{b}_2(t) - e_b(t) \end{cases} \tag{50}$$

Fig.11 shows the estimation of the parameters of the master and the slave systems using (48),(49) and (50).

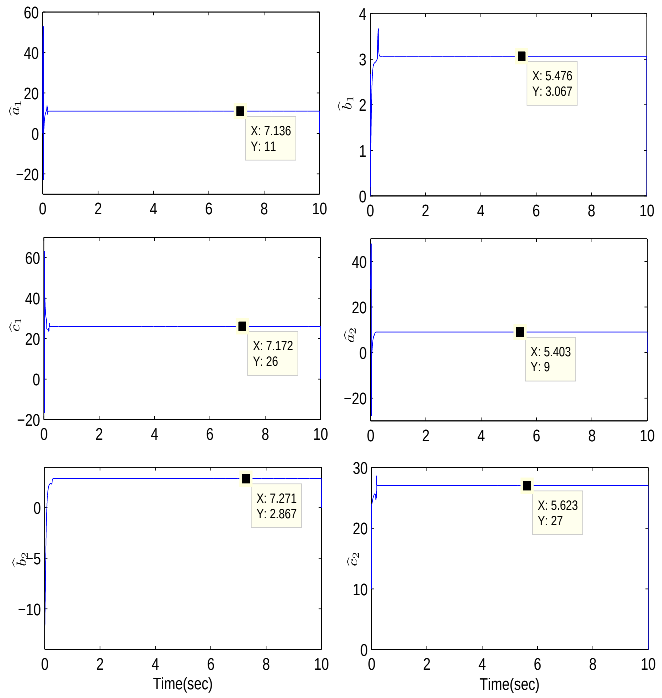

Fig. 11. The identification of the parameters.

6. CONCLUSION

The defects which have appeared in many adaptive synchronization approaches have given many questions about the way that should be taken to avoid them. In this paper, a novel adaptive synchronization scheme is introduced in order to develop an easy and efficient method for chaos synchronization and estimation of unknown parameters. The synchronization and the estimation of unknown parameters are performed by combining the fuzzy set theory and Lyapunov theory. The proposed approach is applied successfully without using many mathematical formulas and without taking into account the linear independence condition which makes it easier to use. The problem of LI condition is solved by involving fuzzy PI controller in a part of the controller as an eliminator of perturbation terms, and this reconfirm that FLC is a powerful tool that concerns the control of very complex and complicated nonlinear systems like chaotic systems. The effectiveness of the proposed approach is verified by numerical simulations.

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