

# Experimental Validation of an enhanced Optimal Adaptive Control Scheme using Dominant Poles for a Variable Area Process

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**Abstract:** This paper describes an efficient control scheme to enhance the adaptive control in real time and an approach to Optimal Model Reference Adaptive Control (OMRAC). The conventional MRAC method has its difficulty in choosing the reference model and adaptation gain ' $\gamma$ '. An OMRAC adaptive controller is used to identify the process dynamics automatically. The selection of reference models in OMRAC scheme is based on the Multiple Models (MM) depending on the operating regime of the process. In this proposed work, the reference model considered is a second order system having fixed roots in denominator polynomial representing the most dominant poles of the process, determined using Dominant Pole Algorithm (DPA). The adaptive controller employs an optimal search algorithm for tuning ' $\gamma$ ' using Particle Swarm Optimization (PSO), that best fits the observed response. Experimental validation is performed in a variable area conical tank process and comparative performance evaluations are analyzed for standard MRAC, conventional gain scheduled PID and proposed OMRAC method. The result gives consistently better setpoint tracking mechanism and error minimization.

**Keywords:** Adaptive control, Multiple model, Optimization, Dominant pole.

## 1. INTRODUCTION

Conical tanks in hydro-metallurgical industries, concrete mixing industries, food process industries, fertilizer industries and wastewater treatment industries find exceptionally tight control for the best product quality. It is important to keep the desired level in a conical tank because the difference in shape gives rise to the nonlinearity in the process. An adaptive control scheme was used to modify the behaviour in response to the changes in dynamics of the processes and the disturbances acting on the process (Swarnkar P. et al., 2010). Hence, a modified adaptive control technique is necessary to obtain the complete adaptive nature. Adaptive control will change the control parameters in real time and practical applications use MRAC scheme, to compensate for variations in the system environment and its nonlinearity (Samah A. M., 2010). The MRAC scheme should make the process follow a reference model. When designing an MRAC using the MIT rule, the designer needs to choose the reference model, controller structure and the tuning gains for the adjustment mechanism. In literature, many efforts have been attempted to formulate the reference model (Jang J. et al., 2008 and Tao G, 2003). The improved MRAC method is based on a recently proposed simulated work in control schemes for a non-linear dynamical system design (Anuj Abraham et al., 2014). In the proposed method, the reference model is replaced by a group of weighted reference models combined together for operating in the process regime. The focus of the work described in this paper is to retain two maximum dominant poles of the process in the reference model using DPA algorithm. The static sensitivities determined in each linearized process model of numerator polynomial are chosen as the dynamic sensitivity in the reference model, which changes depending on the operating region. An optimal search algorithm using PSO (Praveen Kumar Tripathi et al.,

2007), is also used to find the optimal value of the adaptation gain in the MIT rule for the design (Bergh F. et al., 2006). In this work, the real time implementation is carried out for three different control schemes, namely standard MRAC, conventional gain scheduled PID and proposed OMRAC. The servo and regulatory responses are obtained in the wide operating range of the conical tank process and a detailed comparison is made on the performance criteria like ISE and IAE.

## 2. PROCESS DESCRIPTION

### 2.1 Mathematical modelling

The process considered is a conical tank system and the schematic diagram of the process is shown in Fig. 1. The various process parameters used in the mathematical modelling are listed in Table 1.

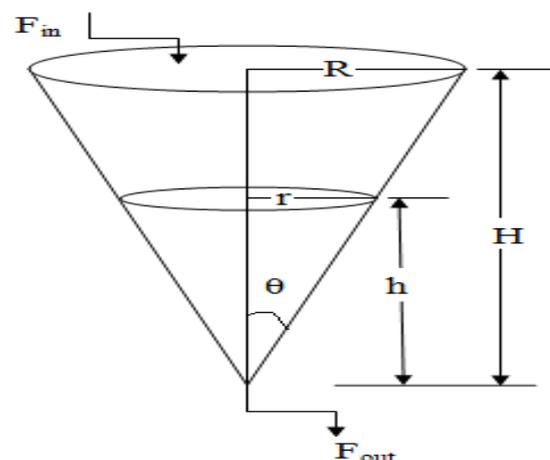


Fig. 1. Schematic diagram of conical tank process.

**Table 1. Variables and parameters of conical tank process.**

| Variable  | Variable Name                              | Unit    |
|-----------|--|---------|
| $F_{in}$  | Volumetric flow rate of the inlet stream   | LPH     |
| $F_{out}$ | Volumetric flow rate of the outlet stream  | LPH     |
| R         | Top radius of the conical tank             | cm      |
| r         | Radius of the conical tank at steady state | cm      |
| H         | Maximum level of the conical tank          | cm      |
| h         | Level of the conical tank at steady state  | cm      |
| $\theta$  | Half apex angle                            | degrees |

The Ordinary Differential Equation (ODE) representing the mathematical model and simplified expression model of a conical tank process are given by Eq.(1) and Eq.(2) respectively.

$$\frac{dh}{dt} = \frac{[F_{in} - K\sqrt{h}] H^2}{\pi R^2 h^2} \tag{1}$$

$$F_{in} - F_{out} = \frac{1}{3} \left[ A \frac{dh}{dt} + \frac{2\pi R^2 h^2}{H^2} \frac{dh}{dt} \right] \tag{2}$$

2.2 Experimental setup

The experimental setup of the conical tank process available at the process control laboratory is shown in Fig. 2.



Fig. 2. Experimental setup of a conical tank process.

The level of liquid in the tank is measured by EMERSON make capacitive differential pressure transmitter whose output is in the form of 4-20 mA current signal. The control valve is fitted with EMERSON make smart valve positioner

which will take 3–15 psi as an input signal. The level transmitter and the control valve are interfaced to a PC with the help of USB 6008 DAQ device. It has eight analog input channels (AI0-AI7) and two analog output channels (AO0-AO1). The current signal from transmitter is converted into voltage signal by a current to voltage (I-V) converter so that it could be fed directly into the interfacing device unit. Similarly, the voltage signal from the interfacing device unit is converted into current signal by a voltage to current (V-I) converter and then to pressure signal by a current to pressure (I-P) converter so that it could be fed to the control valve to take corresponding control action. A detailed schematic diagram of the conical tank process available at the process control laboratory is shown in Fig. 3.

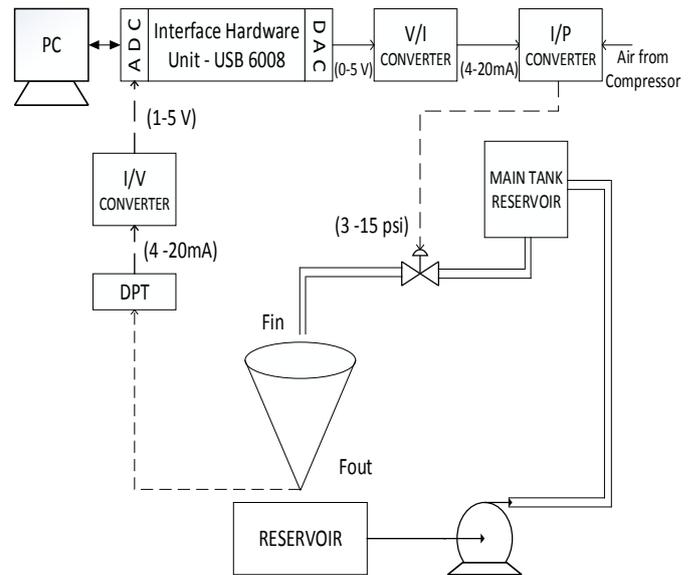


Fig. 3. Schematic diagram for liquid level control of a conical tank process.

In the conical tank process the outflow from the reservoir is pumped continuously for maintaining a constant head at the main tank reservoir. The inflow into the conical tank is constant from the main tank reservoir, whereas the inflow into the conical tank is a function of control valve position to have a negligible effect on the pump flow. The nominal values for process parameters of experimental conical tank process are listed in Table 2.

3. OPTIMAL MODEL REFERENCE ADAPTIVE CONTROL SCHEME

3.1 The Proposed Method

The proposed OMRAC can switch the reference models instantly depending upon the operating region based on the weights scheduled in accordance with the given input flow (Han Z. et al., 2012, Kuipers M. et al., 2010; Hespanha J. et al., 2001). The schematic diagram of the proposed optimal MRAC scheme with multiple reference models is shown in Fig.4, in which the weight scheduler will select the proper linearized model corresponding to the operating regime of the process (Anderson B. D. O. et al., 2001 and Narendra K.S. et al., 1995).

**Table 2. Nominal values for process parameters of conical tank process.**

| Part Name                               | Details                          |  |
|---|----------------------------------|--|
| Conical tank                            | Material                         | : Stainless steel                            |
|   | Height                           | : 60 cm                                      |
|   | Top diameter                     | : 43.68 cm                                   |
|   | Total Capacity                   | : 24 litres                                  |
|   | Half apex angle                  | : 20 degrees                                 |
| Pump                                    | Type                             | : Centrifugal, 0.5HP                         |
|   | Make                             | : Kirloskar                                  |
| Differential Pressure Transmitter (DPT) | Type                             | : Capacitance                                |
|   | Make                             | : Rosemount (EMERSON)                        |
|   | Supply                           | : 12 to 45 VDC Max                           |
|   | Output                           | : 4 - 20 mA                                  |
|   | Max W.P                          | : 140 kg/cm <sup>2</sup>                     |
| Control valve                           | Type                             | : Air to Open                                |
|   | Size                             | : Body: 3/4", Trim: 1/2", Pneumatic actuated |
|   | Flow coefficient, C <sub>v</sub> | : 5  |
|   | Input                            | : 3 -15 psi                                  |
|   | Actuator pressure                | : 35 psig (max)                              |
| I/P Converter                           | Make                             | : ABB  |
|   | Input                            | : 4 - 20 mA                                  |
|   | Output                           | : 3-15 psi                                   |
|   | Supply                           | : 20 ± 1.5 psi                               |
| Air filter regulator                    | Input pressure                   | : 18 kg/cm <sup>2</sup> (max)                |
|   | Output pressure                  | : 2.1 kg/cm <sup>2</sup>                     |

In the OMRAC control scheme, the reference model considered is a second order system having fixed the roots of the denominator polynomial representing the most dominant poles of the process, determined using Dominant Pole Algorithm (DPA).

A detailed description of calculating the dominant poles of the system using Dominant Pole Algorithm is shown in section 3.2. The static sensitivities determined in each

linearized process model of numerator polynomial are chosen as the dynamic sensitivity in the reference model which changes depending on the operating region.

When there is a step change in setpoint, an appropriate reference model is chosen based on which the controller parameters are adjusted quickly with optimized adaptation gain. The adjustment mechanism is not iterative and also, the proposed OMRAC scheme is not computationally intensive. Hence, it can be used for online applications. The time taken by the optimal adjustment mechanism is found to be very small when it is compared to the sampling time.

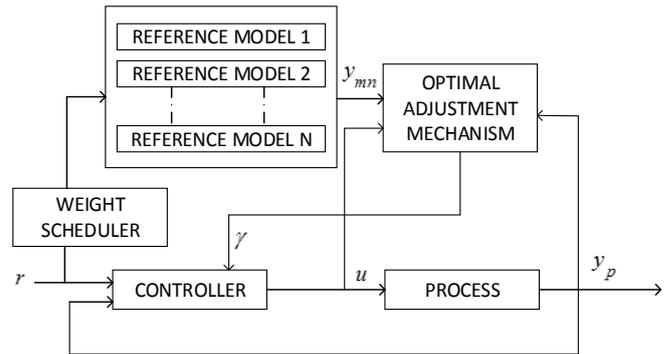


Fig. 4. Optimal MRAC Scheme with Multiple Models.

The multi reference model defines the desired system dynamics and produces a model output  $y_{mn}$ .

$$Y_{mn}(s) = G(s) R(s) = \frac{K_n}{\left( s \prod_{j=1}^q (s+p_j) \prod_{k=1}^r (s^2 + 2\xi\omega_k s + \omega_k^2) \right)} R(s) \quad (3)$$

where,  $n=1,2,\dots,N$ ;  $N$  is the total number of linear models. The one which has the dominant effect in the system transients is known as dominant closed loop pole and determined using DPA algorithm. Assume, the dominant complex for  $k=1$  as  $s^2 + 2\xi\omega_1 s + \omega_1^2$ .

where,  $\xi$  is the damping ratio, and  $\omega_1$  is the natural frequency.

The value of  $K$  which is known as dynamic sensitivity in the referenced model is selected from the each static sensitivities obtained from the linearized model in the various operating regime given in Table 4. Due to dynamic sensitivities  $K_n$ , there is multiple reference models present in the proposed scheme (Anderson B. D. O. et al., 2001).

For simplicity, let us consider the nonlinear plant dynamics assumed to behave as described in Eq. (4),

$$Y_p(s) = G_p(s) U(s) = \frac{b}{s^2 + s + a} \quad (4)$$

The process dynamics of the considered process is given in Eq.(5)

$$\ddot{y}_p + \dot{y}_p + ay_p = bu \quad (5)$$

Also, the process dynamics should follow the reference dynamics as given by Eq.(6),

$$\ddot{y}_{mn} + 2\xi\omega_k\dot{y}_{mn} + \omega_k^2 y_{mn} = K_n r \quad (6)$$

Now, assume the control law to be in the form as shown in Eq. (7),

$$u = r \theta_{1n} - y_p \theta_{2n} \quad (7)$$

where,  $\theta_{1n}$  and  $\theta_{2n}$  represent two parameters to be estimated from the control law.

Now substituting Eq. (7) in Eq. (5), the expression obtained is given in Eq. (8).

$$\ddot{y}_p + \dot{y}_p + (a+b\theta_{2n})y_p = b\theta_{1n}r \quad (8)$$

On comparing Eq.(8) and Eq.(6), the assumed estimated values of  $\theta_{1n}$  and  $\theta_{2n}$  are obtained as shown in Eq.(9) and Eq.(10) respectively.

$$\theta_{1n} = \left( \frac{K_n}{b} \right) \quad (9)$$

$$\theta_{2n} = \left( \frac{\omega_k^2 - a}{b} \right) \quad (10)$$

for all values of  $\xi\omega_k = 0.5$

However, as the true values 'a' and 'b' are unknown, adaptation mechanism estimation for  $\theta_{1n}$  and  $\theta_{2n}$  are to be determined. The experiments are conducted using the MIT rule. The cost function J is defined as shown in Eq.(11).

$$J = \frac{1}{2} e^2 \quad (11)$$

where 'e' is the error signal defined by,  $e = (y_p - y_{mn})$ .

The main idea is to change  $\theta$  along the steepest descent of the cost function J. Cost function will be reduced if a small step along  $\theta$ ; that is, the steepest descent condition is satisfied. Therefore, the time derivative is proportional to the negative gradient with an adaptation gain  $\gamma$  as given in Eq.(12).

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial t} = -\gamma e \frac{\partial e(\theta)}{\partial \theta} \quad (12)$$

$$\text{Now, } y_p = \left( \frac{b\theta_{1n}}{s+a+b\theta_{2n}} \right) r \quad (13)$$

The sensitivity derivative  $\frac{\partial e(\theta)}{\partial \theta}$ , is calculated as shown in Eq.(14).

$$\begin{aligned} \frac{\partial e}{\partial \theta_{1n}} &= \frac{\partial}{\partial \theta_{1n}} (y_p - y_{mn}) \\ &= \frac{\partial}{\partial \theta_{1n}} \left( \frac{b\theta_{1n}}{s+a+b\theta_{2n}} r - y_{mn} \right) \\ &= \left( \frac{b}{s+a+b\theta_{2n}} r \right) \end{aligned} \quad (14)$$

Similarly,

$$\begin{aligned} \frac{\partial e}{\partial \theta_{2n}} &= \frac{\partial}{\partial \theta_{2n}} (y_p - y_{mn}) \\ &= \frac{\partial}{\partial \theta_{2n}} \left( \frac{b\theta_{1n}}{s+a+b\theta_{2n}} r - y_{mn} \right) \\ &= \left( \frac{-b}{s+a+b\theta_{2n}} y_p \right) \end{aligned} \quad (15)$$

The sensitive derivatives found in Eq.(14) and Eq.(15) are substituted in Eq.(12), to obtain the updated law with estimated parameters  $\theta_{1n}$  and  $\theta_{2n}$  as given in Eq.(16) and Eq.(17) respectively.

$$\begin{aligned} \frac{d\theta_{1n}}{dt} &= -\gamma e \left( \frac{b}{s^2 + 2\xi\omega_1 s + \omega_1^2} \right) r \\ \theta_{1n} &= -\gamma \int e \left( \frac{b}{s^2 + 2\xi\omega_1 s + \omega_1^2} \right) r \end{aligned} \quad (16)$$

Similarly,

$$\begin{aligned} \frac{d\theta_{2n}}{dt} &= \gamma e \left( \frac{b}{s^2 + 2\xi\omega_1 s + \omega_1^2} \right) y_p \\ \theta_{2n} &= \gamma \int e \left( \frac{b}{s^2 + 2\xi\omega_1 s + \omega_1^2} \right) y_p \end{aligned} \quad (17)$$

The obtained response from open loop test which represents first order transfer function with zero dead time  $G_{mn}(s)$  is given in Eq. (18). The reference models are categorized into six regions as indicated in Table 4.

$$G_{mn}(s) = \frac{K_p}{1+s\tau} \quad (18)$$

An OMRAC adaptive controller is used to identify the process dynamics (reference model) automatically for the step changes. The adaptive controller employs an optimal search algorithm that finds the reference model based on the operational region of interest by optimally tuning for an error

minimization that best fits the observed response is given in section 3.3.

### 3.2 Dominant Pole Algorithm (DPA)

DPA computes dominant poles of the transfer function  $G(s)$  based on Newton method [7]. A pole  $\lambda_i$  that corresponds to a residue  $|R_i|$  with large magnitude  $|R_j|$  is called a dominant pole. A dominant pole is well observable and controllable in the transfer function. This can also be observed from the corresponding Bode plot of  $G(s)$ , where peaks occur at frequencies close to the imaginary parts of the dominant poles of  $G(s)$ .

$$G_i(s) = C(sI - A)^{-1} B = \sum_{i=1}^n \frac{R_i}{s - \lambda_i} \quad (19)$$

where, residue  $R_i = (C x_i) \times (y_i B)$  and  $\lambda_i, x_i, y_i$  are eigen triplets ( $i=1,2,..n$ ).

Consider a pole  $\lambda = \alpha + j\beta$ , with residue  $R$  then it is shown that,

$$\lim_{\omega \rightarrow \beta} G(j\omega) = \lim_{\omega \rightarrow \beta} \frac{R}{j\omega - (\alpha + j\beta)} + \sum_{j=1}^{n-1} \frac{C}{(j\omega - \lambda_j)} = \frac{R}{\alpha} + G_{n-1}(j\beta) \quad (20)$$

Hence pole  $\lambda_j$  is dominant if  $\left| \frac{R_j}{\text{Re}(\lambda_j)} \right|$  is large and causes

peak in the Bode plot. Model identification of the conical nonlinear process is accomplished by means of open loop procedure. With a specified variation in the input variable  $F_{in}$ , the output level  $H$  for the system is recorded. The equivalent transfer function  $G_1(s)$  given in Eq.(21) is obtained by mentioning 3 poles and 0 zeros in the system identification toolbox.

$$G_1(s) = \left[ \frac{0.2086}{s^3 + 49.3327s^2 + 58.4490s + 5.844 \times 10^{-5}} \right] \quad (21)$$

The various poles obtained are  $p_1 = -1 \times 10^{-6}$ ,  $p_2 = -1.2147$ ,  $p_3 = -48.1180$ .

System identification of the nonlinear process is done using black box modelling. Among the 3 poles determined, two most dominant poles in the process are selected using DPA algorithm and are fixed as the poles in the multiple reference models.

### 3.3 Heuristic Search Algorithm

Particle Swarm Optimization is one of the bio-inspired computation techniques, based on the social behaviors of birds flocking or fish schooling, biologically inspired computational search (Chang WookAhn et al., 2006; Zwe-

Lee Gaing, 2004). This optimization method is an Artificial Intelligence (AI) technique that can be used to find approximate solutions to extremely difficult or impossible numeric maximization and minimization problems (Kennedy, J. et al., 1995). The PSO search optimization algorithm is mathematically described using equations from Eq.(22) to Eq.(26).

$$V_{i,D}^{t+1} = W V_{i,D}^t + C_1 R_1 \left[ P_{i,D}^t - S_{i,D}^t \right] + C_2 R_2 \left[ G_{i,D}^t - S_{i,D}^t \right] \quad (22)$$

$$W = W_{\max} - \frac{W_{\max} - W_{\min}}{\text{iter}_{\max}} \text{iter} \quad (23)$$

$$V_{i,D}^{t+1} = \Psi \left[ V_{i,D}^t + C_1 R_1 \left[ P_{i,D}^t - S_{i,D}^t \right] \right] + \Psi \left[ C_2 R_2 \left[ G_{i,D}^t - S_{i,D}^t \right] \right] \quad (24)$$

$$\Psi = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|}, \quad \text{where, } \varphi = C_1 + C_2; \varphi > 4 \quad (25)$$

$$S_{i,D}^{t+1} = S_{i,D}^t + V_{i,D}^{t+1} \quad (26)$$

where,  $R_1$  and  $R_2$  are the random numbers in the range (0-1),  $C_1$  and  $C_2$  are the cognitive and global learning rate.

The parameters and variables used in PSO optimization algorithm are listed in Table 3.

**Table 3. Variables and Parameters of PSO.**

| Variable            | Variable Name                       |
|---------------------|-------------------------------------|
| $W$                 | Inertia weight                      |
| $V_{i,D}^{t+1}$     | Current velocity of the particle    |
| $S_{i,D}^t$         | Current position of the particle    |
| $V_{i,D}^{t+1}$     | Updated velocity                    |
| $S_{i,D}^{t+1}$     | Updated position                    |
| $W_{\max}$          | Maximum iteration number            |
| $W_{\min}$          | Minimum iteration number            |
| $P_{i,D}^t$         | pbest, $i = 1, 2 \dots N$ particles |
| $G_{i,D}^t$         | gbest, $i = 1, 2 \dots N$ particles |
| iter                | Current iteration                   |
| iter <sub>max</sub> | Maximum iteration                   |
| $\Psi$              | Construction factor                 |

It is the velocity vector that drives the optimization process, and reflects both the experiential knowledge of the particle and socially exchanged information from the particle's neighbourhood.

#### 4. CONVENTIONAL METHODS

##### 4.1 Standard MRAC method

The MRAC method is a direct adaptive control technique which uses a reference model and adjustment mechanism strategy, so that the process output tracks the output of a reference model. In this paper, a standard MRAC scheme is designed using MIT rule. The reference model chosen is first order with a constant gain of 2.5 and time constant of 50 s for the entire operating regime.

##### 4.2 Gain Scheduled PID method

The conventional gain scheduled PID method is very easy to apply and parameters can be changed quickly in response to changes in plant dynamics, but it is an open-loop adaptation scheme with no real learning or intelligence (Pakmehr M. et al., 2014). For the lower regions of operation, the controller settings will not work properly and more multiple models are required for a tight control (Stefanovic M. et al., 2008).

The nonlinear conical tank process is split into 6 linear regions and their respective PI controller settings are calculated using Direct Synthesis method given by Eq. (27),

$$K_c = \frac{1}{K_p} \quad ; \quad \tau_c = \tau, \text{ (min)} \quad (27)$$

where,  $K_c$  is Proportional gain and  $\tau_c$  is Integral time.

**Table 4. Operating points of the conical tank process.**

| Operating regime zones | Inflow Rate     |           | Height |     |
|------------------------|-----------------|-----------|--------|-----|
|                        | (LPH)           | $V_{max}$ | (cm)   | (V) |
| 0-10                   | 0 - 520.10      | 2.5       | 4.2    | 1.3 |
| 11-27                  | 520.10 - 647.40 | 3.0       | 20.0   | 2.3 |
| 28-38                  | 647.40 - 750.57 | 3.5       | 32.9   | 3.2 |
| 39-48                  | 750.57 - 830.30 | 4.0       | 44.0   | 3.9 |
| 49-56                  | 830.30 - 880.64 | 4.5       | 52.5   | 4.5 |
| 56-57                  | 880.64 - 950.00 | 5.0       | 57.0   | 4.8 |

Process model for various zones of conical tank process and the controller parameters obtained for a gain scheduling method are summarized in Table 5.

**Table 5. Process model and controller parameters of conical tank process for gain scheduling method.**

| Zones | Process model parameters |               | Controller parameters settings |                |
|-------|--------------------------|---------------|--------------------------------|----------------|
|       | Proportional gain        | Time Constant | Proportional gain              | Integral time  |
|       | $K_p$                    | $\tau$ (min)  | $K_c$                          | $\tau_c$ (min) |
| 1     | 0.399                    | 0.067         | 2.500                          | 0.067          |
| 2     | 2.100                    | 0.500         | 0.476                          | 0.500          |
| 3     | 1.727                    | 0.520         | 0.579                          | 0.520          |
| 4     | 1.475                    | 1.310         | 0.678                          | 1.310          |
| 5     | 1.140                    | 1.800         | 0.877                          | 1.800          |
| 6     | 0.599                    | 2.910         | 1.670                          | 2.910          |

#### 5. RESULTS AND DISCUSSION

##### 5.1 Multiple reference model scheme validation

The response of multiple reference models for stepping reference input in the various operating regime is shown in Fig.5. It is observed that the reference model has good tracking capabilities in each region of operation with stepping reference input. The sampling time chosen for the simulation and real process is 0.5 s.

The gain characteristics of the system  $G_1(s)$  is compared with the model equivalent using DPA algorithm as shown in Fig.6. It is observed that the essential dynamics of the system lie in the frequency range of 0.5 to 1 rad/s from the frequency response. The magnitude drops in both the very low and the high frequency ranges.

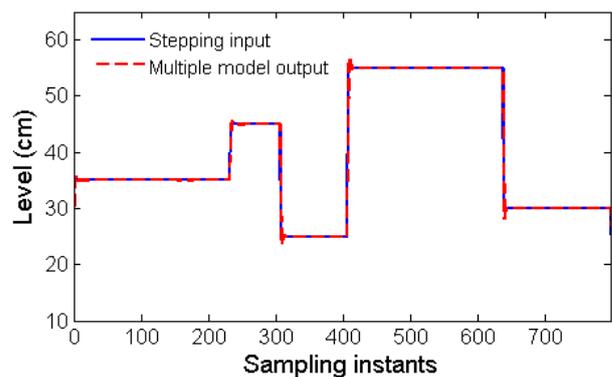


Fig. 5. Response of multiple reference model scheme for stepping reference input variation.

The two most dominant poles among  $p_1$ ,  $p_2$  and  $p_3$  in the process are obtained as  $p_1$  and  $p_2$ , using DPA algorithm and are fixed as the poles in the multiple reference models. The denominator polynomial of the multiple reference models is of the form as shown in Eq.(28).

$$[(s - p_1)(s - p_2)] = [s^2 + 1.2147s + 0.1215 \times 10^{-5}] \quad (28)$$

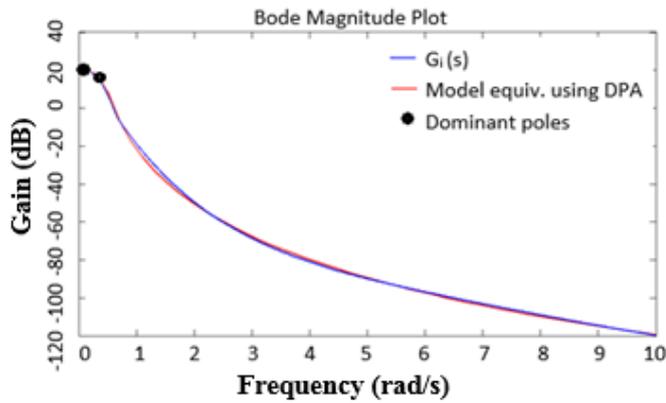


Fig. 6. Comparison of gain characteristics of  $G_i(s)$  and model equivalent using DPA.

### 5.2 Controller parameter estimation

The two estimated parameters  $\theta_1$  and  $\theta_2$  of the controller represented in control law, are generally updated at each sampling instant based on the current estimates of the model parameters, so that its performance can be made similar to the reference model.

The algorithm ensures a better parameter estimates for control design and the variation in controller parameter  $\theta_1$  and  $\theta_2$ , obtained using PSO for stepping reference inputs is shown in Fig. 7 and Fig. 8 respectively.

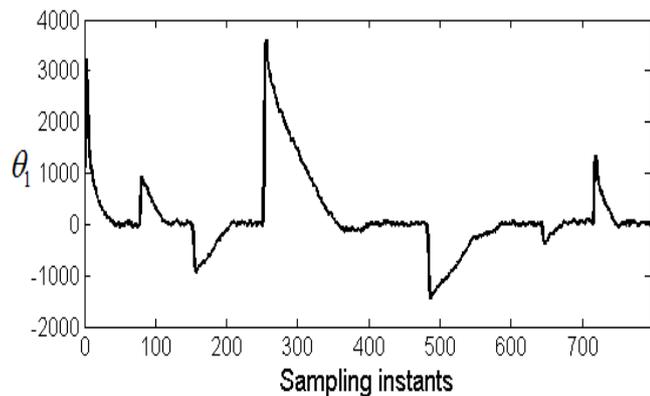


Fig. 7. Variation in controller parameter  $\theta_1$  obtained using PSO for stepping reference input.

It is observed from the Fig. 7 and Fig. 8, that atleast one estimated parameter will converge to zero and provides a required nominal value of inflow to the conical tank. When the inflow balances with the outflow in the conical tank, then  $\theta_1$  and  $\theta_2$  variations are constant. The controller parameters are adjusted to compensate for the changes in dynamics of the plant for desired closed loop performances.

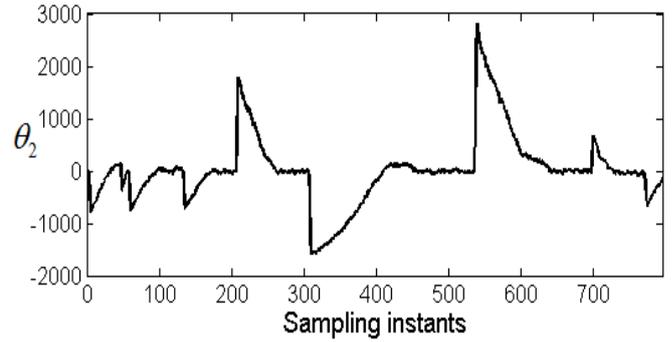


Fig. 8. Variation in controller parameter  $\theta_2$  obtained using PSO for stepping reference input.

### 5.3 Closed loop response of OMRAC scheme

In the proposed work, the values of PSO parameters chosen are listed in Table 6. The sum of cognitive learning rate ( $C_1$ ) and global learning rate ( $C_2$ ) is chosen to be a constant value which is greater than 4, (i.e.  $C_1 + C_2 = \varphi > 4$ ) (Kennedy, J. et al., 1995). Total number of iteration during the search is equal to  $N$  multiplied by the number of swarm steps. The objective function for the optimization algorithm is error minimization. The optimal adaptation gain of MRAC using PSO algorithm is obtained as 0.2773.

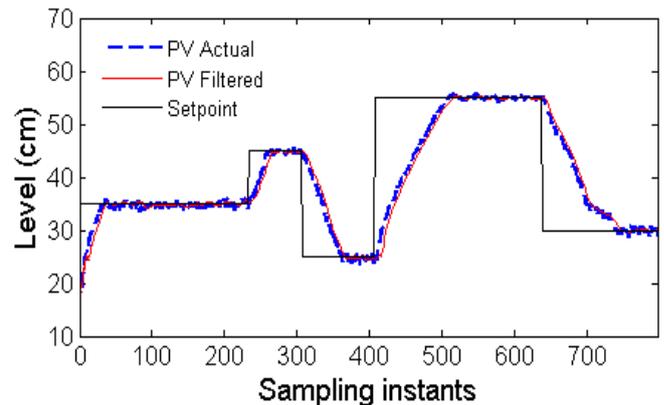


Fig. 9. Setpoint tracking of level in a conical tank for proposed OMRAC method.

Fig. 9 gives the real time closed loop results for setpoint tracking of level in a conical tank for the proposed optimal MRAC method. The response in Fig.9 shows for an actual and filtered process variable of level in the conical tank system. The filtering or smoothing of process variable values is done based on an averaging method using ten previous data sets and indicates good tracking performances.

The level in the conical tank goes to ramp when an unbalanced state occurs at the inflow and outflow. For any change in setpoint, the manipulated flow must drive past the equilibrium for the level to reach the new setpoint. When the volume of the inflow and outflow are equal, then the ramp stops. When a decrease in setpoint is given, then the feed

flow must be driven lower than the exit flow. A higher level does not force out more flow and a lower level does not force out less force. The response for proposed control scheme shows a better performance for good tracking capabilities. The error curve of proposed optimal MRAC control scheme is plotted in Fig.10.

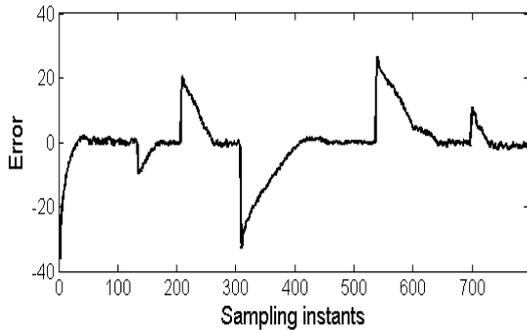


Fig. 10. Error variation of proposed OMRAC control scheme.

**Table 6. PSO parameters.**

| Parameters                        | Value |
|-----------------------------------|-------|
| Dimension of the search (D)       | 3     |
| Total number of swarm (N)         | 50    |
| Number of swarm steps             | 50    |
| Cognitive learning rate ( $C_1$ ) | 2.2   |
| Global learning rate ( $C_2$ )    | 2.2   |

It is observed in Fig. 10 that the proposed optimal MRAC method has minimum error and converges to zero very quickly. The quantitative analysis of error response for the proposed method is listed in Table 7.

The control force  $u$  depends on the plant output, reference input and the estimated parameters  $\theta_1$  and  $\theta_2$ . The maximum inflow rate of the tank ( $F_{in}$ ) is calculated as 950 LPH. The control force variation of proposed control scheme is plotted in Fig.11. The cluster of points represents the control force at various sampling instants of proposed OMRAC control scheme.

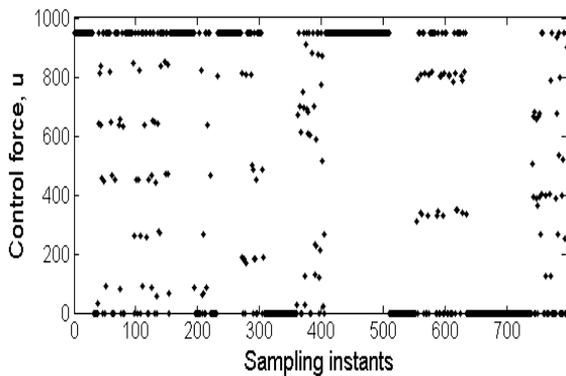


Fig. 11. Control force variation of proposed OMRAC control scheme.

It is observed from Fig. 11, that when a decrease in setpoint is given, then the feed flow is driven lower than the exit flow.

Similarly, for an increase in setpoint, the feed flow is driven to maximum inflow rate.

*5.4 Comparison of gain scheduled PID and proposed OMRAC scheme*

The response to set point changes in the level for a conical tank system with standard MRAC, gain scheduled PID control structure and proposed optimal MRAC control structure is shown in Fig. 12. The responses indicate good tracking capabilities of the proposed method than conventional methods. It is obvious that the proposed control scheme has better responses to track the setpoint with less settling time and minimum overshoot. As a measure of assessing control system performance for the control schemes, ISE and IAE values are calculated.

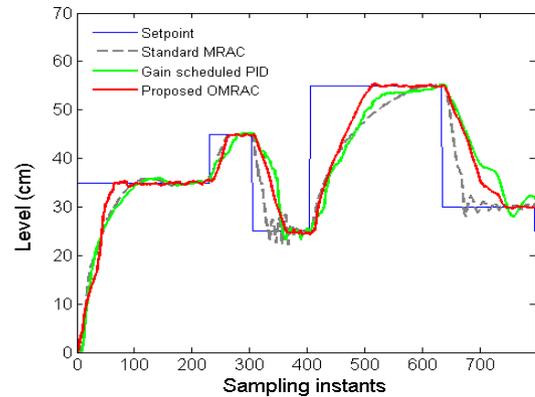


Fig. 12. Setpoint tracking of level in a conical tank for standard MRAC, gain scheduled PID and OMRAC method.

**Table 7. Performance comparison of standard MRAC, gain scheduled PID and OMRAC method.**

| Sampling Interval | Control schemes      |                      |                      |                      |                      |                      |
|-------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                   | Gain Scheduled PID   |                      | Standard MRAC        |                      | OMRAC Method         |                      |
|                   | ISE                  | IAE                  | ISE                  | IAE                  | ISE                  | IAE                  |
| 0 - 232           | $2.5880 \times 10^4$ | $1.2757 \times 10^3$ | $2.1311 \times 10^4$ | $1.3245 \times 10^3$ | $2.3554 \times 10^3$ | $3.1085 \times 10^2$ |
| 233- 306          | $1.1435 \times 10^3$ | $2.0896 \times 10^2$ | $1.8954 \times 10^3$ | $2.7402 \times 10^2$ | $1.1148 \times 10^3$ | $1.7047 \times 10^2$ |
| 307- 406          | $1.0536 \times 10^4$ | $7.0922 \times 10^2$ | $0.4796 \times 10^4$ | $6.8372 \times 10^2$ | $7.3661 \times 10^3$ | $5.6822 \times 10^2$ |
| 407- 636          | $3.2352 \times 10^4$ | $1.8176 \times 10^3$ | $3.4895 \times 10^4$ | $2.056 \times 10^3$  | $2.3986 \times 10^4$ | $1.3335 \times 10^3$ |
| 637- 797          | $2.2758 \times 10^4$ | $1.4645 \times 10^3$ | $1.5731 \times 10^4$ | $1.3198 \times 10^3$ | $1.7524 \times 10^4$ | $1.1171 \times 10^3$ |

Table 6 summarizes the performance indices for conventional gain scheduled PID control structure and proposed optimal MRAC method at different sampling intervals. The performance indices such as ISE and IAE are significantly lower in proposed OMRAC method for the chosen conical tank process in all the regions of operation.

Around sampling instant of 700, a small disturbance in the form of slightly closing the outlet valve was applied and

observed that the response has gone sluggish due to the closing of outlet valve and the controller is able to track the setpoint in the presence of disturbance.

## 6. CONCLUSION

In this paper, an approach to adaptive control for a class of SISO systems using a dominant pole technique has been proposed. An optimal MRAC algorithm utilizes multiple models and results in improving the transient performances when compared to a standard MRAC algorithm, which has only a single reference model reported in the literature. The efficacy of the proposed control scheme has been demonstrated by carrying out real time validation of a conical tank process which exhibits nonlinear dynamics. The heuristic based optimization using the PSO algorithm shows improved performance of the process in terms of time domain specification, error minimization, setpoint, and multiple setpoint tracking. This method creates a closed loop controller with parameters that can be updated to change the response of the system or process. It is observed that the performance indices mainly, ISE and IAE values are much lower for proposed optimal MRAC control structure compared to both the standard MRAC and conventional gain scheduled PID control structure. The results indicate that the proposed scheme has better parameter convergence, hence leads to an improvement in transient performance for good tracking capabilities.

Formulation of suitable reference model is a difficult process in standard MRAC which is overcome by automating the reference model selection in the proposed OMRAC scheme. It is simple and also an alternative method for gain scheduled PID owing to the complexity in the design procedure. The proposed control scheme can be applied to other similar application areas such as higher order complex systems and highly nonlinear processes.

## ACKNOWLEDGMENT

The authors gratefully acknowledge Anna University, Chennai for providing financial support to carry out this research work under Anna Centenary Research Fellowship (ACRF) scheme.

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