Partially Decentralized Control for a Benchmark Boiler

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Abstract: Partially decentralized control for a Benchmark Boiler is proposed. First a partially decentralized control structure selection method is proposed based on the gap metric. Since the boiler contains integrating action due to the drum level dynamics, most interaction measures are not applicable here. The gap metric can be used for both stable and unstable systems, so the proposed method can be used for selecting suitable control structure for the boiler. The design of partially decentralized controllers is based on the generalized predictive control method, which overcomes the drawback of the IMC method that requires computing the pseudo-inverse of a non-square matrix. Simulation results show that the proposed partially decentralized control can achieve good performance with a simpler structure.

Keywords: Partially Decentralized Control, Gap Metric, Generalized Predictive Control, Boiler.

1. INTRODUCTION

Boilers are very common in power plants. The boiler control system is a multivariable process that shows great interactions and is subjected to input constraints under a wide range of operating conditions (Åström and Bell, 2000). In order to achieve good control performances, multivariable control strategies are usually required (Tan et al., 2002).

To propagate the PID control technique, a benchmark boiler with a full non-linear model in Matlab/Simulink was proposed in an IFAC meeting in 2012 (Morilla, 2011a, b). Many control methods have been proposed for this boiler system, e.g., inverted decoupling (Garrido et al., 2012), Data-Driven Loop-Shaping (Saeki et al., 2012), balanced truncation to integral-type optimal servomechanism (Ochi and Yokoyama, 2012), Model-Free Adaptive PID Controllers (Silveir et al., 2012), Virtual Reference Feedback Tuning (Rojas et al., 2012), H_∞ loop shaping (Damiran et al., 2014a), centralized PID by decoupling (Damiran et al., 2014b), etc. For the benchmark problem, the baseline controller provided with the benchmark (Morilla, 2011a, b) is a decentralized one, which is easy to implement and tune, but the performance is not very satisfactory.

For a multivariable process, it is generally practical to first try decentralized control due to its simpler control structure and fewer tuning parameters, and it is easier for control engineers to understand the key concepts behind the decentralized control so that they can design and re-tune the controllers when necessary. However, the use of decentralized controllers may not be suitable for all processes. When a process has strong coupling effects among controlled variables, decentralized controllers might have poor performance, even cause instability. When this happens, one possible solution is to resort to the more sophisticated and costly fully centralized controllers. Another possible solution is to use partially decentralized control (PDC) (Lee et al., 1998; Lee and Chiu, 2001; Lee et al., 2001). Partially decentralized controllers are structures that lie in between the two extreme control structures. For example, for a 2×2 system, the partially centralized controllers have the four structures

$$C_1=\begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22}\end{bmatrix},\quad or\quad C_2=\begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22}\end{bmatrix},\quad or$$
$$C_3=\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & 0\end{bmatrix},\quad or\quad C_4=\begin{bmatrix} 0 & c_{12} \\ c_{21} & c_{22}\end{bmatrix}. \tag{1}$$

So it is an attractive option for the situations where the stability or performance requirement cannot be met by decentralized controllers while the complexity in the design and high cost in the installation of centralized controllers are to be avoided.

To design a partially decentralized controller, the first step is to select the PDC structure, and then synthesize the controller with the specified structure.

For PDC structure selection, it is very important in the design of the partially decentralized control systems, which directly affects the stability and the performance of the control systems. Up to now, an effective method to select the PDC structure is the gramian-based interaction measure (Conley and Salgado, 2000). The measure uses the observability and the controllability gramians to form the Participation Matrix (PM). The elements of the PM encode the information of the channel interactions. PM is used for pairing and the controller structure selection. The Hankel Interaction Index Array (HIIA) is a similar interaction measure (Salgado and Conley, 2004). This family of the interaction measures suggests suitable pairing for the decentralized control, but also allows selecting more complicated control structures (Conley and Salgado, 2000; Wittenmark and Salgado, 2002; Salgado and Conley, 2004; Halvarsson, 2008).

An alternative method for PDC structure selection is considered in (Tan et al., 2011) based on a singular analysis method, where the channel interaction measure is built upon the steady-state gain of the MIMO process.

However, the above-mentioned methods are not applicable to the benchmark boiler, which involves integral action due to the drum level dynamics. The observability and the controllability gramians and the steady-state gain cannot be obtained for the boiler. In this paper, we will propose a method based on gap metric to select PDC structure. An important feature of the gap metric is that it is applicable not only to stable systems, but also to integrating and unstable systems. This is a new idea different from the above interaction measures. Roughly speaking, gap metric measures the 'distance' between the partially decentralized systems and the original system, the one having the smallest gap to the original system can be selected as the optimal model for partially decentralized controller design.

A widely adopted method to design PDC is to expand the original system to a non-square system, and treat PDC as the decentralized controller for the non-square system. Internal model control (IMC) technique is used to parameterize the non-square controllers (Lee et al., 1998; Lee et al., 2001). This method requires computing the pseudo-inverse of a non-square matrix with a pseudo-inverse factor, which is not unique. The pseudo-inverse factor can affect the stability of the designed system. A sufficient condition on the pseudo-inverse factor for the stability of the control system is derived in (Lee and Chiu, 2001), and an 'ideal' pseudo-inverse factor is obtained using the notion of non-square block relative gain. However, the results only apply to 2×2 systems. It is difficult to extend the results to higher dimensional systems since the design of these control structures involves two or more pseudo-inverse factors.

In this paper, generalized predictive control (GPC) is used to design PID controllers for non-square subsystems, overcoming the drawback of the IMC method that requires computing the pseudo-inverse of a non-square matrix. The rest of the paper is as follows: In section 2, a method to select control configuration based on the gap metric is introduced, then a PDC design via GPC is studied. In section 3, the above method is applied to the benchmark boiler and compared with the reference controllers. Finally conclusions are given in section 4.

2. PDC DESIGN

2.1 Control Structure Selection Based on the Gap Metric

Control system design includes two aspects: control structure selection and controller design. An important issue in PDC design is to determine the suitable PDC structures. Improper PDC structures will probably not be able to improve the performance of the decentralized controller.

In this paper, the selection of the 'best' control structure for a multivariable system is studied in the sense of robustness. Usually we need to choose a model for the multivariable system for controller design. If the model is close to the original system, then the controller designed for the model would be guaranteed to stabilize the original system. If the design model is of special structure, then the designed controller can have a corresponding structure. For example, if the model is chosen to be diagonal, then a decentralized controller can be obtained for the original system if the diagonal model is close to it. The idea can be extended to partially decentralized controller. For example, for a 2x2 system,

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \tag{2}$$

if the design model is chosen as

$$G_1 = \begin{bmatrix} g_{11} & 0 \\ g_{21} & g_{22} \end{bmatrix} \tag{3}$$

then a controller of the following form can be designed using the procedure described below.

$$C_1 = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \tag{4}$$

So choosing the 'best' PDC structure in the robustness sense amounts to finding a model that is closest to the original system. It is found that the gap metric is the best tool to measure the closeness of two linear systems.

Let $P_1 = N_1 M_1^{-1}$, $P_2 = N_2 M_2^{-1}$ be the normalized right coprime factorizations of P_1 and P_2. Then the gap between two linear systems can be defined by (Zhou and Doyle, 1998; Georgiou and Smith, 1990; Galan et al., 2003)

$$\delta(P_1, P_2) = \max\left\{\vec{\delta}(P_1, P_2), \vec{\delta}(P_2, P_1)\right\}, \tag{5}$$

where $\vec{\delta}(P_1, P_2)$ is the directed gap and can be computed by

$$\vec{\delta}(P_1, P_2) = \inf_{Q \in H_\infty} \left\| \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} - \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} Q \right\|_\infty . \tag{6}$$

For any two linear systems, the gap is bounded as

$$0 \le \delta(P_1, P_2) \le 1 \tag{7}$$

The gap can be regarded as the 'distance' between two linear systems, and it is a generalization of the conventional distance expressed by the ∞-norm. An important feature of the gap metric is that it is applicable not only to stable systems, but also to integrating and unstable systems.

In this paper, we will design PDC for a benchmark boiler (Morilla, 2011a, b). This model contains an integrating action in the dynamics of the water level, so interaction measures like RGA, gramian, and SVD do not apply here, while the gap is still applicable.

Control structure selection process based on the gap metric is as follows: Analyze the gap of the partially decentralized models to the original system, and if the gap of a (partially decentralized) model is close to the original linear model, then it will be expected to have much information as the original, and it will be a good candidate for PDC control design.

2.2 Expansion Design for PDC

This section briefly presents some background on the expansion design procedure for PDC. For detail, refer to (Lee et al., 1998; Lee et al., 2001). Take a 2×2 system (2) as an example.

Suppose we would like to design a PDC with the structure (4). The first controller's output, u_1, depends on both of the controller inputs, e_1 and e_2, and the second controller's output, u_2, is determined only by the second controller's input, e_2, as shown in the top diagram of Figure 1.

The controller output u_1 can be decomposed as

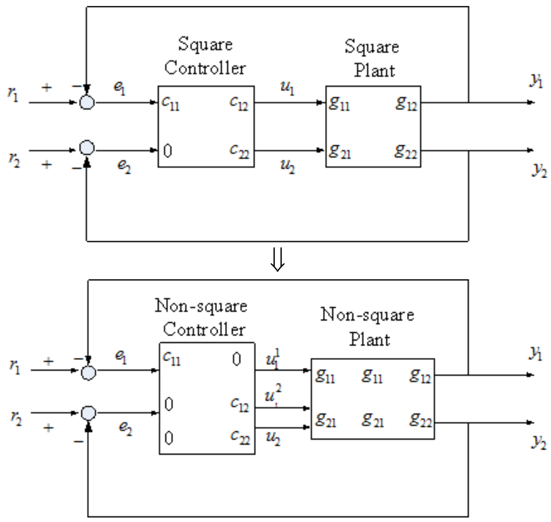

Fig. 1. Expansion of a 2 × 2 PDC system.

$$u_1 = c_{11}e_1 + c_{12}e_2 \overset{def}{=} u_1^1 + u_1^2 \tag{8}$$

If we regard u_1^1 and u_1^2 as separate signals, then the controller (4) can be expanded in the non-square decentralized form as

$$C' = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{12} \\ 0 & c_{22} \end{bmatrix} \tag{9}$$

and the plant outputs y_1 and y_2 can be expressed as:

$$\begin{aligned} y_1 &= g_{11}u_1 + g_{12}u_2 = g_{11}u_1^1 + g_{11}u_1^2 + g_{12}u_2 \\ y_2 &= g_{21}u_1 + g_{22}u_2 = g_{21}u_1^1 + g_{21}u_1^2 + g_{22}u_1 \end{aligned} \tag{10}$$

or

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = G' \begin{bmatrix} u_1^1 \\ u_1^2 \\ u_2 \end{bmatrix} \tag{11}$$

where G' is the corresponding non-square plant

$$G' = \begin{bmatrix} g_{11} & g_{11} & g_{12} \\ g_{21} & g_{21} & g_{22} \end{bmatrix} \tag{12}$$

So design of controller (9) is transformed to the design of a decentralized controller for a non-square plant (12) as shown in the bottom diagram of Fig. 1. If independent design is assumed, then it amounts to design a controller for the following model

$$\begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{21} & g_{22} \end{bmatrix} \tag{13}$$

It is clear that model G_1 (3) contains the same information as model (13), so it is reasonable to choose PDC structure via gap metric.

It is straightforward to apply the above expansion procedure to n×n partially decentralized control systems.

2.3 PDC design via GPC

It is clear that the expansion design of PDC relies on the controller design for non-square systems. IMC method tries to solve the problem by utilizing the pseudo-inverse of the plant model instead of the exact inverse. However, the pseudo-inverse depends on a factor that has direct effect on the performance of the final controller. So it is desired to find a method to overcome the drawback.

Model predictive control has found wide applications in industry due to its simplicity and robustness, especially for multivariable processes (Maciejowski, 2002; Camacho and Bordons, 2004). It is found that MPC is readily applicable to non-square systems, so we would like to apply MPC to design partially decentralized controllers for non-square systems.

GPC for Non-square Systems

To establish the GPC algorithm for non-square systems, we start from a SISO system. It is supposed that a model of the linearized plant is expressed in terms of the following CARIMA (Controlled Auto-Regressive Integrated Moving Average) form:

$$A(z^{-1})y(t) = B(z^{-1})u(t) + C(z^{-1})v(t)/\Delta \tag{14}$$

where $\Delta := 1 - z$, $v(t)$ is a white noise, and

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$
$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_m z^{-m}$$

The objective function of GPC is

$$\min J(P,M) = \sum_{k=1}^{P} \left[\hat{y}(t+k|t) - \hat{r}(t+k|t) \right]^2 + \sum_{k=1}^{M} \left[\lambda \Delta \hat{u}(t+k-1|t) \right]^2 \tag{15}$$

where P is the output prediction horizon and M is the control horizon. The signal $\hat{r}(t+k|t)$ is a reference trajectory along which the system output $y(t)$ is set to follow. The constant λ adds weight to the relative importance of the control and the tracking errors.

To solve problem (15), k-step prediction $\hat{y}(t+k|t)$ of the output should be obtained for k = 1, · · · , P based on the information known at time t and on the future values of the control increments. To derive $\hat{y}(t+k|t)$, introduce the following Diophantine equations:

$$E_k(z^{-1})A\Delta + z^{-k}F_k(z^{-1}) = C(z^{-1}) \tag{16}$$

with the degree of $E_k(z\text{-}1)$ no more than k-1 and that of $F_k(z\text{-}1)$ no more than n, and

$$R_k(z^{-1})C(z^{-1}) + z^{-k}L_k(z^{-1}) = E_k(z^{-1})B(z^{-1}) \tag{17}$$

with the degree of $R_k(z\text{-}1)$ no more than k, then we have

$$\begin{aligned}\hat{y}(t+k|t) &= R_k(z^{-1})\Delta\hat{u}(t+k|t) + \frac{L_k(z^{-1})}{C(z^{-1})}\Delta u(t-1) + \frac{F_k(z^{-1})}{C(z^{-1})}y(t) \\ &= R_k(z^{-1})\Delta\hat{u}(t+k|t) + L_k(z^{-1})\Delta u^f(t-1) + F_k(z^{-1})y^f(t)\end{aligned} \tag{18}$$

where $u^f(t) = \frac{1}{C(z^{-1})}u(t)$, $y^f(t) = \frac{1}{C(z^{-1})}y(t)$ are the filtered input and the filtered output, respectively. If the output horizon is P and the control horizon is M, then the predictions can be written in the vector form as

$$Y = \Theta U + Y_f(t) \tag{19}$$

where

$$Y = \left[\hat{y}(t+1|t), \hat{y}(t+2t)\cdots, \hat{y}(t+P|t)\right]^T$$
$$U = \left[\Delta\hat{u}(t|t), \Delta\hat{u}(t+1|t), \cdots, \Delta\hat{u}(t+M-1|t)\right]^T$$

and Θ is the system's dynamic matrix with dimension $P\times M$, and each row is

$$\Theta_k = \begin{bmatrix} r_k & r_{k-1} & \cdots & r_1 & \cdots & 0\end{bmatrix}, (k = 1, \cdots, P) \tag{20}$$

where $r_i, (i = 1, \cdots, k)$ are the coefficients of polynomial $R_k(z\text{-}1)$. If $k > M$, then just use the first M coefficients of $R_k(z\text{-}1)$. $Y_f(t)$ is the free response of the system due to past inputs.

$$Y_f(t) = \Psi Y^f(t) + \Upsilon U^f(t-1) \tag{21}$$

where

$$Y^f(t) = \frac{1}{C(z^{-1})}\left[y(t), y(t-1), \cdots, y(t-n+1)\right]^T$$
$$U^f(t-1) = \frac{1}{C(z^{-1})}\left[u(t-1), u(t-2), \cdots, u(t-m)\right]^T$$

and Ψ is a matrix with dimension $P \times (n+1)$ and each row is composed of the coefficients of $F_k(z^{-1})$. Υ is a matrix with dimension $P \times m$ and each row is composed of the coefficients of $L_k(z^{-1})\Delta$.

The quadratic cost function (15) has a least-squared solution.

$$\begin{aligned}\hat{U} &= (\Theta^T\Theta + \lambda^2 I)^{-1}\Theta^T(\Gamma(t) - Y_f(t)) \\ &=: K_{gpc}(\Gamma(t) - Y_f(t))\end{aligned} \tag{22}$$

where

$$K_{gpc} = (\Theta^T\Theta + \lambda^2 I)^{-1}\Theta^T \tag{23}$$

and $\Gamma(t)$ is a vector composed of future trajectory to be tracked.

$$\Gamma(t) = \left[\hat{r}(t+1|t),\quad \hat{r}(t+2|t),\quad \cdots,\quad \hat{r}(t+P|t)\right]^T \tag{24}$$

The optimal control $\Delta u(t)$ at time t is

$$\Delta u(t) = k_1^T(\Gamma(t) - \Psi Y^f(t) - \Upsilon U^f(t-1)) \tag{25}$$

where k_1 is the first row of K_{gpc}. The optimal control is then applied to the system at time t and the same procedure is repeated at the next time instant.

For a 1×2 non-square system, the CARIMA model becomes:

$$A(z^{-1})y(t) = \begin{bmatrix} B_1(z^{-1}) & B_2(z^{-1})\end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t)\end{bmatrix} + \frac{C(z^{-1})v(t)}{\Delta} \tag{26}$$

and accordingly the objective function becomes

$$\begin{aligned}&\min J(P, M) = \sum_{k=1}^{P}\left[\hat{y}(t+k|t) - \hat{r}(t+k|t)\right]^2 + \\ &\sum_{k=1}^{M}\left[\lambda_1\Delta\hat{u}_1(t+k-1|t)\right]^2 + \sum_{k=1}^{M}\left[\lambda_2\Delta\hat{u}_2(t+k-1|t)\right]^2\end{aligned} \tag{27}$$

The Diophantine (16) is the same as the SISO case, but the Diophantine equation (17) becomes two equations,

$$R_{k1}(z^{-1})C(z^{-1}) + z^{-k}L_{k1}(z^{-1}) = E_k(z^{-1})B_1(z^{-1}) \tag{28}$$
$$R_{k2}(z^{-1})C(z^{-1}) + z^{-k}L_{k2}(z^{-1}) = E_k(z^{-1})B_2(z^{-1}) \tag{29}$$

and the k-step prediction becomes

$$\begin{aligned}\hat{y}(t+k|t) &= \begin{bmatrix} R_{k1}(z^{-1}) & R_{k2}(z^{-1})\end{bmatrix}\begin{bmatrix}\Delta\hat{u}_1(t+k|t) \\ \Delta\hat{u}_2(t+k|t)\end{bmatrix} \\ &+ \begin{bmatrix} L_{k1}(z^{-1}) & L_{k2}(z^{-1})\end{bmatrix}\begin{bmatrix}\Delta u_1^f(t-1) \\ \Delta u_2^f(t-1)\end{bmatrix} \\ &+ F_k(z^{-1})y^f(t)\end{aligned} \tag{30}$$

The system's dynamic matrix Θ and the free response $Y_f(t)$ can be obtained accordingly from the k-step prediction.

The optimal control $\left[\Delta u_1(t) \quad \Delta u(t)_2\right]^T$ at sample time t can be computed similarly as the SISO case. For simplicity, the procedure is not repeated here, and it is straightforward to extend to any non-square system, as long as the prediction horizon P, the control horizon M, and the control weight λ_i are tuned. No pseudo-inverse is used in the design, so it is a good candidate method to design non-square decentralized controllers for non-square systems, an important step in the expansion design of PDC.

GPC is usually implemented as a real-time optimizing algorithm in practice. However, here we are designing GPC for the expanded system instead of the original system. To take advantage of the GPC method for non-square system, we need to obtain an explicit form of GPC controllers so that we can retain the original partially control structure.

The explicit form of a GPC control can be obtained from (25). However, the explicit GPC controller may be of high order, so model reduction is needed to implement it. PID controllers are the widely adopted control structure in industrial process control, so we would like to reduce the high-order GPC controller to a PID controller for implementation purpose. An approximation method (Tan et al., 2002) will be used in this paper.

In summary, PDC can be designed via GPC by tuning P, M, and λ_i for each non-square subsystem. They can be tuned separately. However, due to the interaction of the multivariable system, the parameters cannot be tuned arbitrarily. Since we are interested in the final PDC controller, the robustness of the PDC will be checked before simulating it in the time domain. We can use the method proposed in Tan et al. (2004) to evaluate the robustness of the designed PDC system to make sure that enough robustness is achieved by the PDC.

3. APPLICATION TO THE BENCHMARK BOILER

3.1 Benchmark Boiler

The benchmark boiler was proposed in an IFAC conference to test the multivariable PID controllers. A schematic diagram of a typical drum boiler is shown in Figure 2. The water that is to be evaporated is added to a drum. From the drum, the water goes down through the downcomers. The water then goes into the risers. Here, the water evaporates, and the steam rises and flows back up to the drum. The combustible, fuel in this case, is burned with air in the furnace.

The function of a boiler is to deliver steam of a given quality (temperature and pressure) either to a single user, such as a steam turbine, or to a network of many users. A properly functioning boiler must satisfy the following basic requirements:

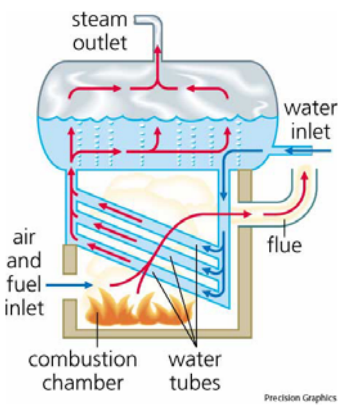

Fig. 2. Schematic picture of an industrial drum boiler.

(a) The ratio of air to fuel must be carefully controlled in order to obtain good, safe, and efficient combustion.

(b) The level of water in the drum must be controlled at the desired level to prevent overheating of drum components or flooding of steam lines.

(c) A desired steam pressure must be maintained at the outlet of the drum despite variations in the quantity of steam demanded by users.

To fulfil the control objectives listed above, the control system for a drum boiler is usually divided into several subsystems. Here, assuming air flow rate is regulated well by the air control subsystem. The boiler system becomes a multivariable system with two variables (steam pressure and water level) that are to be controlled by two manipulated variables (fuel flow and water flow). Additionally, there is a measurable disturbance variable (load level), and an indirect controlled variable (oxygen level) used as quality performance variable. All of these variables are expressed in percentage. The input variables are subjected to the range of [0-100] %, and the fuel flow has a slew-rate limit of ±1%.

Three types of experiments in the Benchmark PID 2012 have been considered:

(a) Standard test. A new operating point is reached due to a 20% load level step change at t=100 s.

(b) Type1 test. Several operating points are tested. First the load increased in ramp from 46.36% at t=100s until 70% in t=500s, and remains constant till t=2000s. Then it is decreased in ramp until reaching the initial operating point at t=2400s, where it remained until t=4200s.

(c) Type2 test. The system is subject to a sudden change of 5% of the steam pressure set point at t=100 s.

More information about the boiler model can be found in (Morilla, 2011a, b). In order to carry out the proposed design in this work, it is necessary to start from a linear model of the plant. Using the MATLAB identification toolbox, a linearized model of the boiler system has been obtained around the normal operation point (Garrido et al., 2012): fuel flow ≌35.21%, water flow ≌57.57 %, load level ≌46.36 %, steam pressure ≌60 %, oxygen level ≌50 %, and water level ≌50 %. The obtained continuous model is given by (31), which is the transfer matrix relating the controlled variables to manipulated variables, the oxygen level is not shown because it will not be taken into account in the design.

$$G(s) = \begin{bmatrix} \dfrac{0.308}{28.96s+1} & \dfrac{-0.159}{183.7s+1} \\ \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s} \end{bmatrix} \tag{31}$$

In the following, $G(s)$ will be used to design the PDC for the benchmark boiler. It can be shown that the first output (steam pressure) response is stable for the two input signals (fuel flow and water flow). There is a non-minimum phase behaviour in the second output (water level) associated with the first input (fuel level). Moreover, the water level shows an integrating response for all of input signals. The main control difficulties in this multivariable process are caused by the coupling, the non-minimum phase, the integration and the load disturbance.

3.2 Selection of the Partially Decentralized Structure

For the scaled boiler model, partially decentralized models for the boiler system may take the following structures:

$$G_1(s)=\begin{bmatrix}\dfrac{0.308}{28.96s+1} & 0\\ \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s}\end{bmatrix},$$

$$G_2(s)=\begin{bmatrix}\dfrac{0.308}{28.96s+1} & \dfrac{-0.159}{183.7s+1}\\ 0 & \dfrac{0.010645}{s}\end{bmatrix},$$

$$G_3(s)=\begin{bmatrix}0 & \dfrac{-0.159}{183.7s+1}\\ \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s}\end{bmatrix},$$

$$G_4(s)=\begin{bmatrix}\dfrac{0.308}{28.96s+1} & \dfrac{-0.159}{183.7s+1}\\ \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & 0\end{bmatrix}. \tag{32}$$

The gaps between the four partially decentralized models and the original linear model ($G(s)$) are shown in Table 1. It is shown that $G_1(s)$ has the smallest gap to the original model ($G(s)$), and a controller designed for this model will be suited for the original multivariable system.

Table 1. Gaps between the four partially decentralized models and G(s).

Gap	$G_1(s)$	$G_2(s)$	$G_3(s)$	$G_4(s)$
$G(s)$	0.0722	0.5922	0.2673	0.9483

3.2 PDC Design for the Benchmark Boiler

Based on the above selected optimal partially decentralized model $G_1(s)$, a partially decentralized controller for the boiler system take the following structures:

$$C_1=\begin{bmatrix}c_{11} & c_{12}\\ 0 & c_{22}\end{bmatrix} \tag{33}$$

To design a PDC via the proposed GPC method, we expand the controller in (33) as

$$C'=\begin{bmatrix}c_{11} & 0\\ 0 & c_{12}\\ 0 & c_{22}\end{bmatrix} \tag{34}$$

For this controller, the corresponding expanded non-square system is obtained as

$$G'(s)=\begin{bmatrix}\dfrac{0.308}{28.96s+1} & \dfrac{0.308}{28.96s+1} & \dfrac{-0.159}{183.7s+1}\\ \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s}\end{bmatrix} \tag{35}$$

and the following non-square model is used to design the decentralized non-square controllers.

$$\tilde{G}'(s)=\begin{bmatrix}\dfrac{0.308}{28.96s+1} & 0 & 0\\ 0 & \dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s}\end{bmatrix} \tag{36}$$

Decentralized GPC controller is designed for $\tilde{G}'(s)$. For subsystem

$$\tilde{G}_1'(s)=\frac{0.308}{28.96s+1} \tag{37}$$

By trial and error we choose Ts =5, P = 20, M = 4, and $\lambda_{1-1}=0.8$ in the GPC design. We reduce it to a PID controller using the method proposed in Tan et al. (2002). The final PID controller is

$$C_{11}(s)=5.693+\frac{0.2418}{s} \tag{38}$$

For subsystem

$$\tilde{G}_2'(s)=\begin{bmatrix}\dfrac{-0.0055872(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s}\end{bmatrix} \tag{39}$$

we choose Ts =5, P = 20, M = 4, and $\lambda_{2-1}=20, \lambda_{2-2}=6.5$ in the GPC design, and two PID controllers are obtained using the method proposed in Tan et al. (2002).

$$C_{12}(s)=5.837+\frac{0.0937}{s}, C_{22}(s)=3.853+\frac{0.0564}{s} \tag{40}$$

The final partially decentralized controller is then

$$C_1(s)=\begin{bmatrix}5.693+\dfrac{0.2418}{s} & 5.837+\dfrac{0.0937}{s}\\ 0 & 3.853+\dfrac{0.0564}{s}\end{bmatrix} \tag{41}$$

The robustness measure for the designed PDC is shown in Figure 3. The maximum is less than 3.7, indicating that the designed system is very robust (can tolerate at least 27% simultaneous input and inverse output uncertainties (Tan et al., 2004).

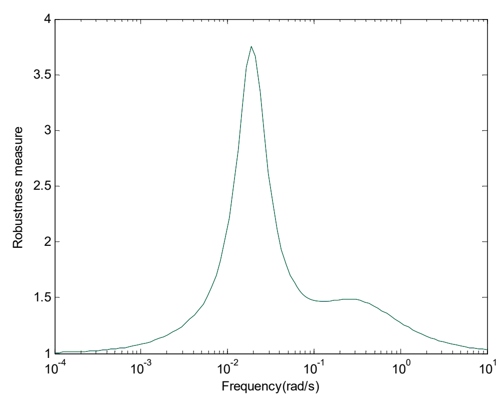

Fig. 3. Robustness measure of the proposed PDC for boiler system.

3.3 Simulation Results

In this section, the proposed PDC is tested for three types of experiments in the benchmark PID 2012, and results are compared with the reference controller 1 presented in the benchmark (Morilla, 2011a, b).

Figure 4 is the standard test case, which includes a step change in the load level. It is shown that the steam pressure and the water level recover their set points in about 1800s in the reference control, while under the proposed PDC, the steam pressure recovers its set point in about 800 s and the water level about 1000 s. Meanwhile, the proposed PDC also achieves smaller deviation in the water level from its set-point than the reference decentralized controller. During the experiment the oxygen level remains indirectly controlled by the fuel/air ratio, affected only by the noise.

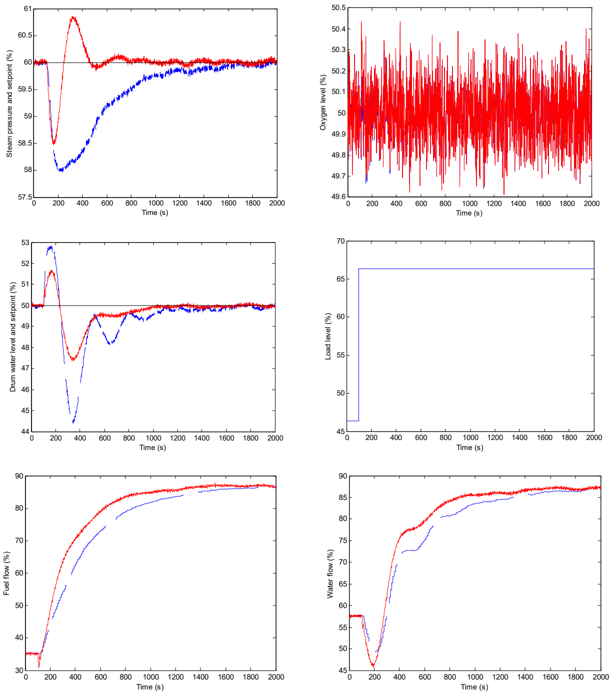

Fig. 4. Standard test.

Figure 5 is the Type1 test case, which includes a profile of load level. It is shown that the proposed PDC achieves faster responses and smaller deviations for the steam pressure and the water level from their respective set-points than the reference decentralized controller, just like in the standard test.

Finally, Figure 6 shows the simulation results for the Type2 test, which includes a step change in the steam pressure set point. The proposed PDC reaches the new steam pressure set-point with larger overshoot but faster than the reference controller. The water level showed minor oscillation and faster response in the proposed PDC compared with the reference controller. So the coupling is reduced with the PDC structure, just as we expect.

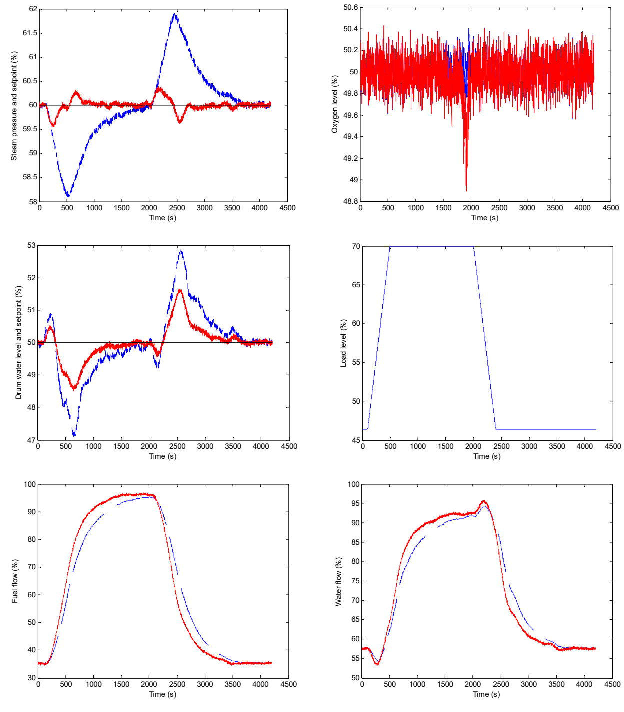

Fig. 5. Type1 test.

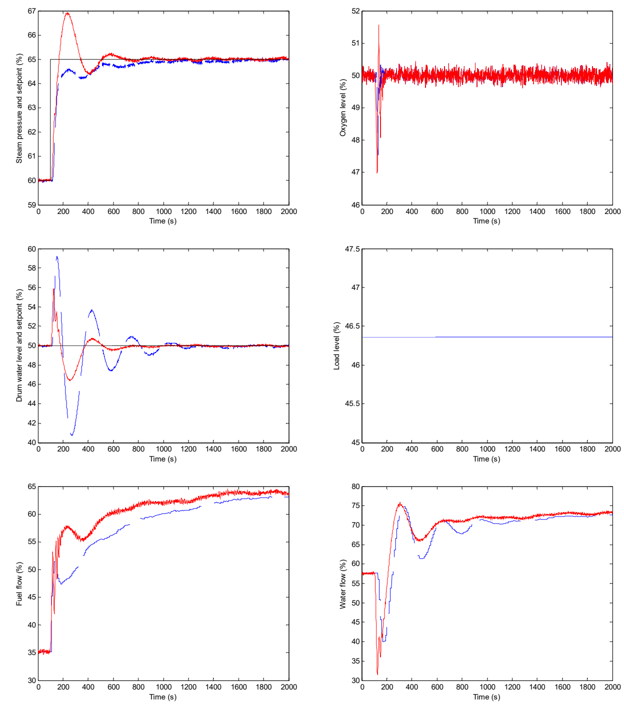

Fig. 6. Type2 test.

4. CONCLUSIONS

In this paper, partially decentralized control for a benchmark boiler is proposed. In the proposed design method, a partially decentralized control structure is selected based on the gap metric. Compared with the former interaction measures that are only applicable to stable systems, the gap metric method is applicable not only to stable systems, but also to integrating systems. The design of PDC is based on the GPC method, which overcomes the drawback of the IMC method that requires computing the pseudo-inverse of a non-square matrix via pseudo-inverse factors. Simulation results show that the proposed PDC perform well for the nonlinear boiler system. Interactions are reduced, zero tracking error is achieved and they can operate well in a wide operating range. Compared with the reference controller for the benchmark boiler, the proposed design achieves better performance.

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