Adaptive Friction Compensation In The Presence Of Backlash

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Abstract: *The behavior of motion control systems with adaptive friction and load compensation algorithm in the presence of backlash is investigated. The applied controller is developed for trajectory tracking tasks and it estimates on-line the nonlinear Stribeck friction. A simulation environment was developed to analyze the effect of the backlash type nonlinearity on control performances. The transient behavior and steady state tracking performances were determined for different backlash gap sizes.*

Keywords: Adaptive control, non-smooth nonlinearities, motion control

1. INTRODUCTION

In many mechanical control systems the drive motor is connected to the load trough a gear mechanism. In these mechanical connections, due to mechanical imperfections, the effect of friction and the backlash appear overlapped. There are special types of gears, such as harmonic drives, in which the effect of the backlash is reduced but its effect can hardly be eliminated for good.

The friction is the force resisting to relative motion of two surfaces in contact. When contacting surfaces move relative to each other, the friction between two objects converts kinetic energy into thermal energy. It is not a fundamental force; its value cannot be determined analytically, only by experimental measurements. The applied friction models are experimental; they are developed in such way to fit experimental data. Moreover the parameters of the friction models are time varying, they depend on external factors such as temperature, humidity of the environment, dwell time (the time a junction spends in rest), normal forces. It is why the adaptive control methods are popular for friction compensation.

The gear machines are lubricated with oil or grace (hydrodynamic lubrication). Tribological experiments showed that in the case of lubricated contacts in the low velocity regime the friction force decrease with velocity (Stribeck effect, see Figure 1). In the high velocity regime the friction force increases with velocity (viscous friction). The decreasing slope is more accentuated than the increasing slope. Both static (Armstrong, 1991) and dynamic (Canudas et al., 1995) models were introduced to explain this phenomena. One of the most common one is the exponential model:

$$\tau_f = \begin{cases} \tau, \text{ for } \dot{q} = 0 \text{ and } |\tau| \le F_S \\ \left(F_C + (F_S - F_C)e^{-|\dot{q}|/\dot{q}_S}\right)sign(\dot{q}) + F_V\dot{q}, \text{ otherwise} \end{cases} \quad (1)$$

where τ denotes the generalized tangential control force, $\dot{q}$ denotes the velocity (or angular velocity in the case of rotational motion). The parameters are: $F_C > 0$ denote the Coulomb friction coefficient, $F_S > 0$ is the static friction coefficient, $F_V > 0$ is the viscous friction coefficient, $\dot{q}_S > 0$ is the Stribeck velocity. The friction can be described with switching model, since when the velocity is zero the machine does not start moving until the value of the control force reaches the level of the static friction.

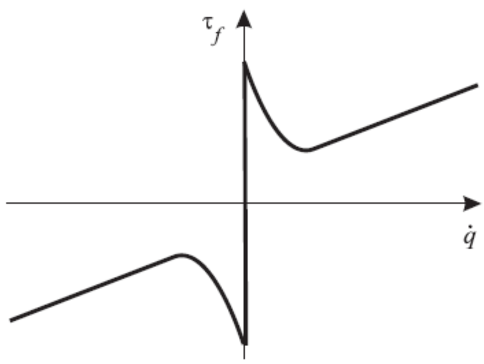

Fig. 1. Stribeck friction

The backlash, sometimes called play, is the clearance between mating components, the amount of lost motion due to slackness when movement is reserved and contact is reestablished. In a pair of gears the backlash is the amount of clearance between mated gear teeth.

Both the friction and backlash have disadvantageous effects on control characteristics. The under compensation of friction leads to steady state error, its overcompensation leads to limit cycles (Márton, 2008; Putra et. al., 2007).

The backlash may also introduce oscillating behavior in the control system response. It is why the compensation of these non-smooth nonlinearities is a widely studied topic of control engineering. A good review of the friction compensation algorithms can be found in (Basilio et al., 2005). Results in backlash modeling analysis and compensation are summarized in (Nordin et al., 2002).

If the friction and backlash appear simultaneously in the controlled mechanical system, the control algorithm should deal with both of these nonlinearities. Early results can be found in (Brandenburg et al., 1989) where the limit cycles generated by Coulomb friction and backlash are analyzed in speed control systems with integral component in the control law. A simple robust friction and backlash compensation scheme is proposed in the paper (Cadiou et al., 2001). In (Menon et al., 1999) a switching controller is proposed for simultaneous friction and backlash compensation.

The paper (Huang et al., 1998) proposes a model free method based on Fourier series for adaptive compensation of friction and backlash. The neural and fuzzy systems are also applied in feedforward compensators to reduce the effect of friction and backlash, see e.g. (Jun et al., 2004) and (Suraneni et al., 2005).

The rest of this paper is organized as follows: In Section 2 the adaptive friction compensation law for Stribeck friction is presented. Section 3 describes how the backlash modifies the model of the controlled mechanical system. In Section 4 it is showed trough simulations how the backlash phenomena influences the adaptive control system performances. Finally, Section 5 sums up the conclusions of this study.

2. ADAPTIVE FRICTION COMPENSATION

2.1 Modeling for control

To develop an adaptive friction compensation algorithm, a classical mechanical positioning system is considered, driven by a servo motor, which electrical time constant can be neglected as compared to mechanical time constant of the controlled system. The position and the velocity of the plant is measured.

Consider that the dynamics of the controlled system is described by

$$J\ddot{q} = \tau - \tau_f(\dot{q}) + d \tag{2}$$

where $J>0$ denotes the inertia (or mass) of the load, τ represents the control input and d is unmeasurable bounded additive disturbance, $|d|<D_F$. The parameters of the system (mass/inertia of the load and the frictional parameters) are considered unknown.

To apply the well known adaptive control schemes for friction compensation it is desirable that the friction force/torque of the mechanical system could be written in a linearly parameterized form, namely as a scalar product between a known regressor vector $\xi_F(\dot{q})$ and an unknown parameter vector θ_f ($\tau_f = \theta_f^T \xi_F(\dot{q})$ In the other hand the friction parameters could change even in the function of the sign of velocity. Hence it is recommended to use different friction parameters in positive and negative velocity regimes.

The friction model for adaptive compensation is developed based on (1). Assume that the mechanical system moves in $(0, \dot{q}_{max}]$ velocity domain. Consider a linear approximation for the exponential curve with two lines: The first line crosses through the $(0, \tau_f(0))$ point and it is tangential to curve and the second line passes through the $(\dot{q}_{max}, \tau_f(\dot{q}_{max}))$ point and it is also tangential to curve. These two lines meet each other at the $\dot{q}_S$ velocity. In the domain $(0, \dot{q}_S]$ the first line can be used for the linearization of the curve and the second line is used in the domain $(\dot{q}_S, \dot{q}_{max}]$. The maximum approximation error occurs at the velocity $\dot{q}_S$ for both linearizations. Same study can be made for the negative velocity regime. For details see (Márton et al., 2007; Márton, 2008).

The resulting friction model is a piecewise linearly parameterized form of the model (1):

$$\tau_f = \begin{cases} a_{1+} + b_{1+}\dot{q}, & if\ \dot{q} \in (0, \dot{q}_S] \\ a_{2+} + b_{2+}\dot{q}, & if\ \dot{q} \in (\dot{q}_S, \infty) \\ a_{1-} + b_{1-}\dot{q}, & if\ \dot{q} \in [\dot{q}_S, 0) \\ a_{2-} + b_{2-}\dot{q}, & if\ \dot{q} \in (-\infty, -\dot{q}_S) \end{cases} \tag{3}$$

There are some relations between the parameters of the original friction model (1) and the linearized model (3). The parameter a_{1+} represents the static friction term, $a_{1+} = a_{1-} = F_S$. The parameter b_{1+} is strongly connected to the Stribeck effect, it gives the slope of the Stribeck friction at low velocities, $b_{1+} = b_{1-} = F_V - (F_S - F_C)/\dot{q}_S$. The parameter a_{2+} is equal to Coulomb friction parameter, $a_{2+} = a_{2-} = F_C$ and b_{2+} represents the viscous friction parameter, $b_{2+} = b_{2-} = F_V$. Note that it was assumed that the parameters in the positive and negative velocity regime are equal. However in real a mechanical system their values may differ, depending on the sign of velocity.

Since this model has linearly parameterized parts, its parameters can simply be estimated on-line using classical adaptation laws. The switching between different parts of the model happens in function of velocity. There is a switching at zero velocities. Another switching occurs at the $\dot{q}_S$ velocity which corresponds to Stribeck velocity. If $|\dot{q}| < \dot{q}_S$ the friction decreases in function of velocity ($b_{1+}, b_{1-} < 0$), otherwise the value of the friction force increases in function of velocity ($b_{2+}, b_{2-} > 0$).

2.2 Position tracking control algorithm

To formulate the control law, define the tracking error

$e(t) = q_d(t) - q(t)$

and the tracking error metric

$S(t) = (d/dt + \lambda)e(t)$

where $\lambda > 0$. Here q_d is the prescribed trajectory, a smooth, twice differentiable function in time.

The control problem can be formulated as follows: design a control law τ in such way that the tracking error metric $S(t)$ satisfies $|S(t)| < \Phi$ for $t \to \infty$, where $\Phi > 0$ is the given precision.

In order to obtain a control law which satisfies this requirement, define S_Δ as $S_\Delta(t) = S(t) - \Phi sat(S(t)/\Phi)$ where $sat(\cdot)$ denotes the saturation function.

To solve the control problem consider the following control law that depends on the estimated parameters:

$$\tau = \hat{J}(\ddot{q}_d + \lambda \dot{e}(t)) + \underline{\hat{\theta}}_f \underline{\xi}_f(\dot{q}) + k_S S \tag{4}$$

with $k_S > D_F/\Phi$. The unknown friction vector parameter and regressor vector are:

$$\underline{\hat{\theta}}_f = (\hat{a}_i \; \hat{b}_i), \underline{\xi}_f(\dot{q}) = (1 \quad \dot{q})$$

The subindex i depends on velocity regime, where the machine moves.

The values of the friction and inertial parameters can be obtained using adaptive techniques. Define the adaptation rules as follows:

$$\begin{aligned} \dot{\underline{\hat{\theta}}}_f &= S_\Delta(t)\Gamma_f \underline{\xi}_f(\dot{q}) \\ \dot{\underline{\hat{J}}} &= S_\Delta(t)\gamma_{JM}(\ddot{q}_d + \lambda \dot{e}(t)) \end{aligned} \tag{5}$$

with Γ_f positive definite diagonal matrix and $J > 0$.

To examine the behavior of the closed loop system, consider the following Lyapunov function candidate:

$$V(t) = \frac{J}{2} S_\Delta(t)^2 + \frac{1}{2\gamma_J}\tilde{J}^2 + \frac{1}{2}\underline{\tilde{\theta}}_f^T \Gamma_f \underline{\tilde{\theta}}_f \tag{6}$$

$\tilde{J} = J - \hat{J}, \underline{\tilde{\theta}}_f = \underline{\theta}_f - \underline{\hat{\theta}}_f$ denote the estimation errors.

The time derivative of $V(t)$ is given by:

$$\dot{V}(t) = J\dot{S}_\Delta(t)S_\Delta(t) - \frac{1}{\gamma_J}\tilde{J}\dot{\hat{J}} - \underline{\tilde{\theta}}_f^T \Gamma_f^{-1} \dot{\underline{\hat{\theta}}}_f \tag{7}$$

Due to the definition of S_Δ and the adaptation laws (5), $\dot{V}(t) = 0$ for $|S(t)| \le \Phi$.

Outside the boundary layer Φ the tracking error dynamics can be written as:

$$\begin{aligned} J\dot{S}_\Delta(t) = J\dot{S}(t) &= J(\ddot{\alpha}_d + \lambda \dot{e}(t)) + \tau_f - \tau + d = \\ &= \tilde{J}(\ddot{q}_d + \lambda \dot{e}(t)) + \underline{\tilde{\theta}}_f^T \underline{\xi}_f(\omega) - k_S S + d_f \end{aligned} \tag{8}$$

By substituting (8) and the adaptation laws (5) into (7), it yields:

$$\dot{V}(t) \le -k_S S_\Delta(t) S(t) + d S_\Delta(t) \tag{9}$$

Outside the boundary layer (Φ) $sign(S) = sign(S_\Delta)$, hence $SS_\Delta = |S||S_\Delta|$. Because the disturbance d is bounded $|d| \le D_F$ the following relation holds: $dS_\Delta(t) \le D_F |S_\Delta(t)|$. It results:

$$\dot{V}(t) \le -k_S |S_\Delta(t)||S_\Delta(t)| + (-k_S \Phi + D_F)|S_\Delta(t)| \tag{10}$$

According to assumption $k_S > D_F/\Phi$ the second term in the inequality (10) is always negative. Hence, it yields:

$$\dot{V}(t) \le -k_S S_\Delta(t)^2 \tag{11}$$

Notice that (11) is also valid for $|S(t)| \le \Phi$.

Since $V(t)$ is a positive and non-increasing function, therefore $V(\infty)$ is finite. It is assumed that the initial values of the estimated parameters and the initial value of the tracking error metric are finite. By applying the Barbalat lemma (see (Márton et al., 2007)) yields that the control law (4) with the adaptation laws (5) solves the formulated control problem, guarantee the convergence of the tracking error S inside the boundary layer Φ and it also guarantees the boundedness of the estimated parameters.

3. BACKLASH IN MECHANICAL CONTROL SYSTEMS

The so called inertia driven model (see (Nordin et al., 2002)) is applied to describe the backlash phenomenon in the control system. In this model two regimes are separated. In contact mode (*CM*), when the load is in contact with the motor shaft, the torque developed by the motor acts directly on the load. In backlash mode (*BM*), that occurs when the direction of motion changes, there is no contact between the motor shaft and the load.

In order to determine the condition for contact mode denote with K_G the gear ratio ($\dot{q}_L = \dot{q}/K_G$) and with δ the backlash gap size. For contact mode the difference between the motor shaft position and load position, modified with the gear ratio, should be equal with the backlash gap size if the machine moves in negative direction. Otherwise the position difference should be equal with the negative of the gap.

Contact mode (CM):

$$\begin{aligned} &if \; ((\dot{q} > 0) \; and \; (q_L K_G - q = -\beta)) \\ &or \; ((\dot{q} < 0) \; and \; (q_L K_G - q = \beta)) \end{aligned}$$

Backlash mode (CM):

otherwise (12)

In backlash mode the dynamics is given by:

$$(BM)\begin{cases} J\ddot{q} = \tau - \tau_f \\ J_L \ddot{q}_L = 0 \end{cases} \tag{13}$$

J_L denote the inertia on the load side.

In contact mode the load velocity will be equal with the motor velocity, modified with the gear ratio, and the motion on the motor side is directly influenced by the load:

$$(JM)\begin{cases} \left(J + J_L / K_G^2\right)\ddot{q} = \tau - \tau_f \\ \dot{q}_L = \dot{q} / K_G \end{cases} \tag{14}$$

Note that due to the backlash nonlinearity, the inertia of the mechanical system is different in backlash mode and in contact mode.

4. SIMULATION RESULTS

In order to analyze the effect of the backlash on the presented adaptive friction compensation algorithm, simulations have been performed. In the controlled system the friction is described by the model (1). It was considered, that the inertia on the motor side is $J = 0.1$ *[kg]*, the inertia on the load side is $J_L = 10$ *[kg]*. The value of the gear amplification was considered $K_G = 1$. The friction model parameters were taken as follows: $F_S = 1.5$ *[N]*, $F_C = 1$ *[N]*, $F_V = 14$ *[Nsec/m]*, $\dot{q}_S = 0.001$*[m/sec]*.

In the controller the introduced friction model has been applied, given by the relation (3). The friction parameters in the linearly parameterized model can be determined as: $a_{1+} = -a_{1-} = 1.5$, $b_{1+} = b_{1-} = -480$, $a_{2+} = -a_{2-} = 1$, $b_{2+} = b_{2-} = 14$. For the simulations, in the control law all friction parameters have been departed with 50% from their real values. The prescribed trajectory is a sinusoidal one with small amplitude which assures that the mechanical system moves both in positive and negative velocity regime near zero velocities.

$$\dot{q}_d(t) = 0.02\sin\left(\frac{2\pi}{5}t\right)\left[\frac{m}{\sec}\right]$$

$$q_d(t) = \int_0^t \omega_d(\tau)d\tau\ [m] \quad \ddot{q}_d(t) = \frac{d\omega_d(t)}{dt}\left[\frac{m}{\sec^2}\right] \tag{15}$$

The control objective is to track this prescribed position, such that the tracking error metrics $S(t) \le 10^{-3}$. The parameters of the controller have been chosen as follows: $\lambda = 10$, $k_S = 0.1$, $D_M = 1$, $\Phi = 10^{-3}$.

Two types of measurements were performed. In the first measurement the estimated load inertia at the beginning of the adaptation were taken as zero ($\hat{J}(0) = 0$). In the second one the initial estimate of the inertia was taken $\hat{J}(0) = 15$ which is *150%* of its real value.

In the first case (Figures 3 and 4) it can be observed that during the transient state the position error and tracking error metric takes larger values without backlash. It is because at the very beginning of the adaptation the machine is in backlash mode and the small value of the estimated parameter is nearer to the inertia of the controlled mechanical system ($J = 0.1$). However the duration of the transients are larger if the backlash is present in the system and the steady state value of the inertia estimation error is greater with backlash. In the second case (Figures 5 and 6) the simulations show similar transient behavior with and without backlash. To analyze the simulation results, the mean value of the absolute position error was calculated for different backlash gap sizes during the first *3.5* seconds of the control. The results can be seen in the Figure 2. With increasing backlash gap size the mean error also increases.

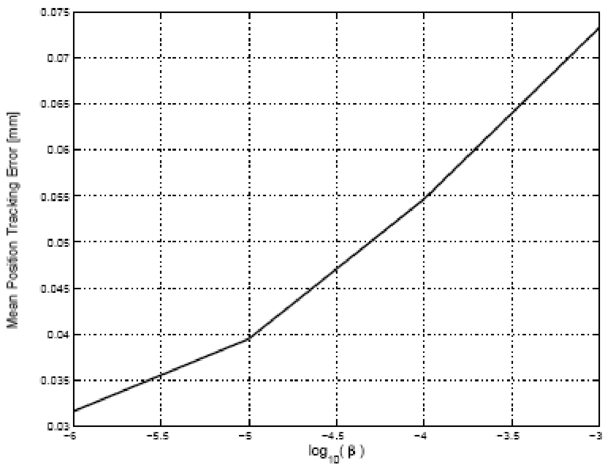

Fig. 2. Mean position tracking error in function of backlash gap size

5. CONCLUSIONS

The adaptive friction compensation performances were tested for the case when the controlled mechanical system contains backlash element as well. The adaptive control law under investigation was designed for trajectory tracking tasks and it can also handle unmeasurable additive disturbances. It was found that the influence of backlash is conditioned by the initial value of the inertia estimate. If the backlash is present, the time of the transient state increases. The mean tracking error during the transients also increases, if at the beginning of the adaptation the inertia is overestimated. Hence the development of such adaptive friction compensation algorithms that takes into account the backlash gap size is necessary for better tracking control performances.

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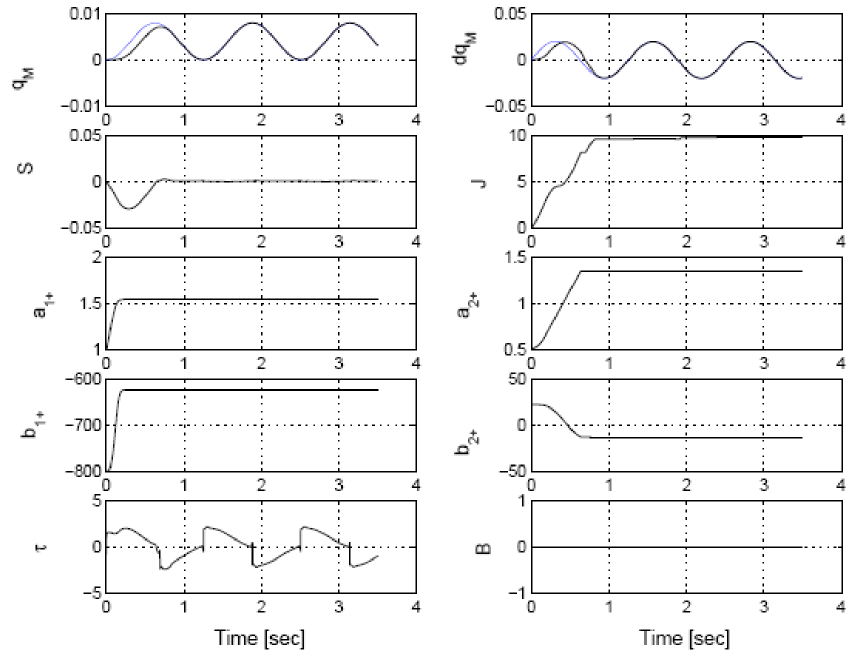

Fig. 3. Simulation results $\hat{J}(0) = 0$

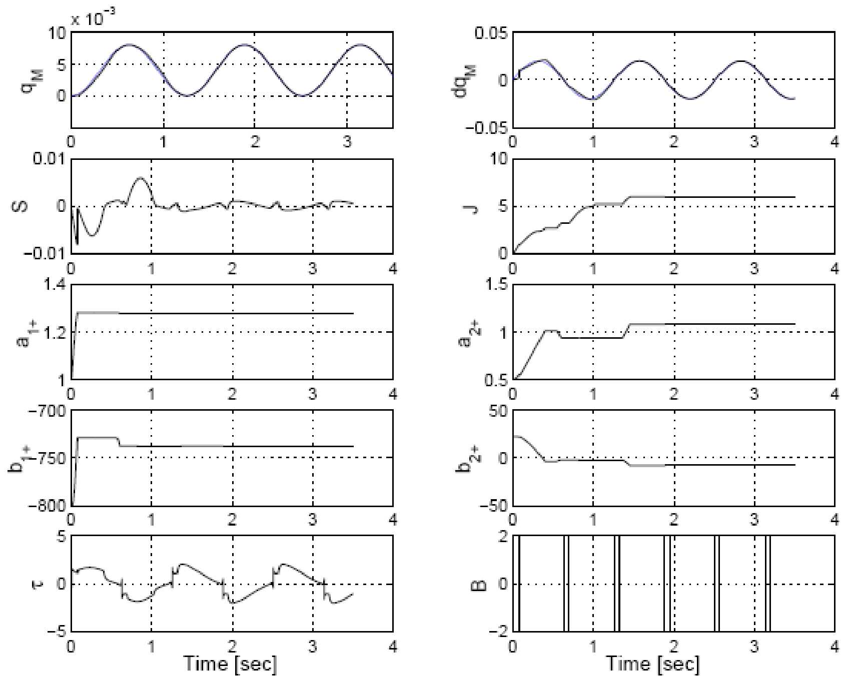

Fig. 4. Simulation results – backlash with $\beta = 10^{-4}$, $\hat{J}(0) = 0$

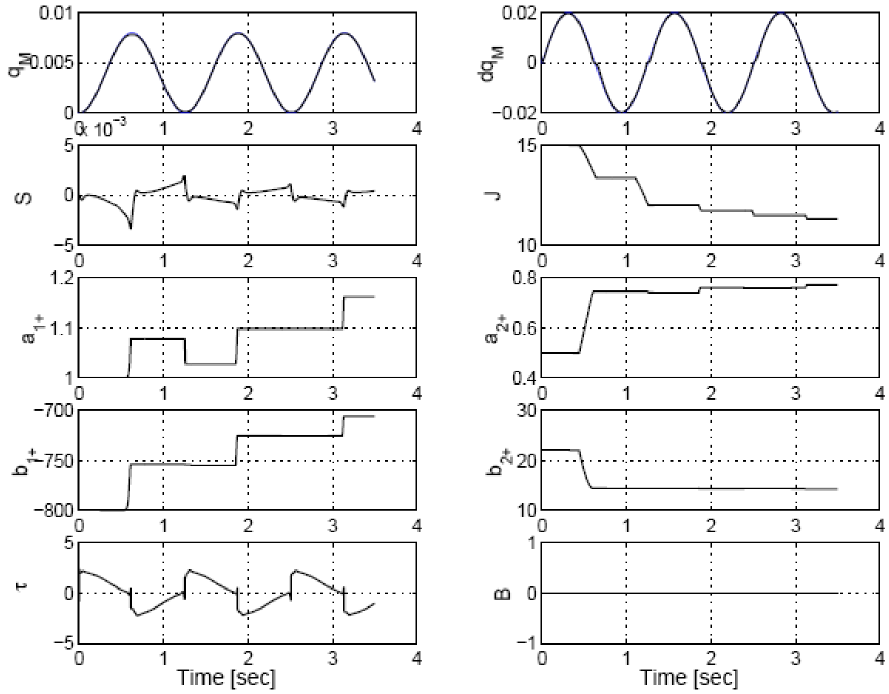

Fig. 5. Simulation results – no backlash, $\hat{J}(0) = 15$

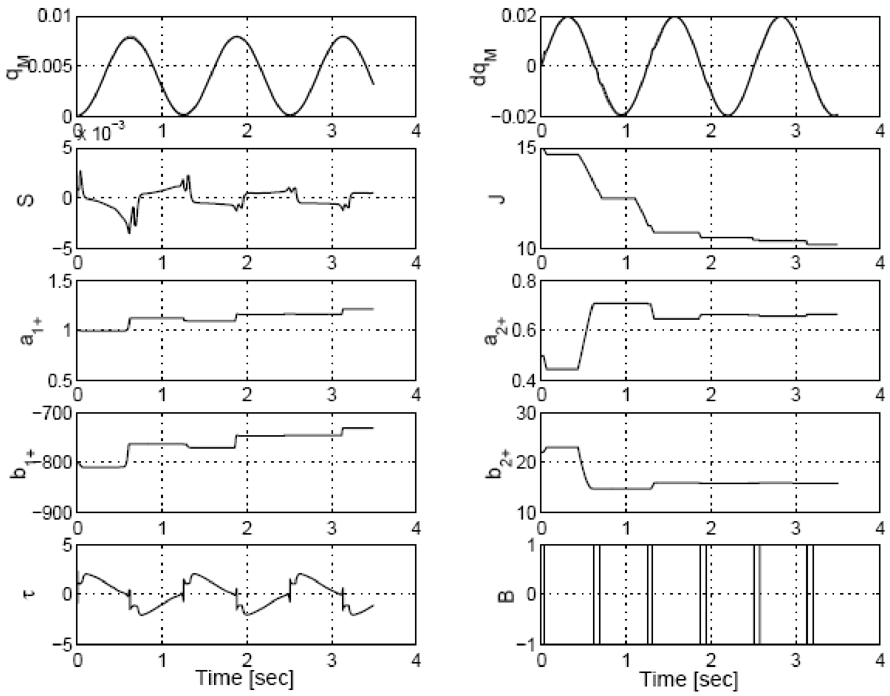

Fig. 6. Simulation results – backlash with $\beta = 10^{-4}$, $\hat{J}(0) = 15$