Event-triggered and Self-triggered Controllers Design with application of Piecewise Continuous Hybrid Systems

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Abstract: In recent years, networked control systems have been the research focus for its convenience, however, the problems of sampling and congestion caused by the networked transmission bring a great challenge to control design. To save networked bandwidth resources and reduce energy consumption via cutting down the number of transmissions, both of event-triggered and self-triggered output feedback controllers are designed and implemented with the application of piecewise continuous hybrid systems (PCHS). Also, their corresponding stabilities were demonstrated by using the Lyapunov stability theory. To validate the proposed methods, a cart system was applied to demonstrate the effectiveness of the proposed method.

Keywords: Networked Control Systems, Piecewise Continuous Hybrid Systems, Event-triggered Controller, Self-triggered Controller, Stability

1. INTRODUCTION

With the rapid development and wide application of networked technology, some new problems of signal transmission through network have brought new challenges to traditional networked control systems.

The control signal of traditional networked control systems is periodically updated (Heemels et al., 2011). But the periodic sampling is sometimes inappropriate since it usually wastes computation resources and increases the network load. In order to overcome the shortcomings of periodic sampling control method, in the late 90's, a new control method, namely event-triggered control was proposed (Årzén, 1999; Åström and Bernhardsson, 1999). But so far, both in the academic and industrial fields, very little research and applications of event-triggered mechanism have been done because related system theory and method haven't formed. A new class of event-triggered controller for Cyber-Physical Systems which guarantees better quadratic performance than traditional periodic time-triggered control has been proposed (Antunes and Heemels, 2014). A neural network (NN) approximation-based event-triggered control of multiple-input and multiple output (MIMO) nonlinear discrete-time systems also has been put forward (Sahoo et al., 2014). These two papers consider the time delays in event-triggered control systems (Wang et al., 2014; Wu et al., 2014). But in this paper, we stress on the problem of sampling and congestion regardless of the problem of time delays. And event-triggered schemes have been also designed for multi-agent systems which have extensive applications in the field of unmanned vehicle, formation control, sensor networks and many other areas.(Guang-Hui et al., 2014; Hao et al., 2014; Lulu et al., 2014; Nowzari and Cortes, 2014; Qian-Qian et al., 2014; Shi and Yuan, 2014; Wang et al., 2014b).

In addition, event-triggered mechanism usually requires an unreasonable hardware device. Therefore, an implementation method of software, namely self-triggered mechanism attracts the attention of many scholars. A new self-triggered coordination algorithm for multi-agent systems has been proposed (Fan et al., 2014). A quadratic programming (QP) problem is solved to compute the control input and sampling period for self-triggered control (Kobayashi and Hiraishi, 2014). And these papers proposed self-triggered schemes when considering the disturbances in control systems (Almeida et al., 2010, 2012; Brunner et al., 2014; Wang and Lemmon, 2009, 2010). Besides, the self-triggered and event-triggered control scheme have been introduced and compared together in (Heemels et al., 2012; Mazo and Tabuada, 2008; Mingyuan and Xia, 2013; Postoyan et al., 2011).

However, very little attention has been paid to design of event-triggered control based on PCHS. So, in this paper, an event-triggered controller with PCHS (Wang et al., 2014b; Wang et al., 2014c) based variable-period event generator would be designed. In addition, most of the prior works of the event-triggered and self-triggered control make contributions to the study of state-feedback controllers instead of the output-feedback one. So, based on it, the idea that a control task is executed when the difference between current output and latest sampled output of plant goes beyond a limit restricted by current output is considered. Under this event-triggered strategy, a condition is established by a feasibility problem of an LMI to ensure the asymptotic stability of the closed-loop control system, and the event condition to obtain longer task periods can be designed by solving this LMI. Moreover, as a development of event-triggered control, a self-triggered scheme is provided to overcome the shortcomings of event-triggered scheme in which the next control task release time is predicted based on the current sampled output. Also, an LMI which is obtained by the similar method in stability analysis of event-triggered control system can be solved to get a parameter setting in the predicted control task release period. Finally, simulations are shown to illustrate the efficiency of the results.

2. PIECEWISE CONTINUOUS HYBRID SYSTEMS

2.1 PCHS Definition

A PCHS which is piecewise continuous, finite-dimensional, strictly causal linear time-invariant system is denoted as $\sum_p (S, \mathbf{A}, \mathbf{B^c}, \mathbf{B^d}, \mathbf{C})$ with

$S = \{t \mid t = t_k, k \in \overline{0, K_t}, t_{k+1} > t_k \geq 0\}$: switching instants,

$\mathbf{A} \in R^{n\times n}, \mathbf{B^c} \in R^{n\times r}, \mathbf{B^d} \in R^{n\times s}, \mathbf{C} \in R^{m\times n}$: real, time-invariant matrices, can be described as follows

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B^c}\mathbf{u}(t), \forall t \in R_+ \setminus S \tag{1a}$$

$$\mathbf{x}(t) = \mathbf{B^d}\mathbf{v}(t), \forall t \in S \tag{1b}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \forall t \in R_+ \tag{1c}$$

where $\mathbf{x}(t) \in \sum^n$ is the state vector, $\mathbf{u}(t) \in U^r$ is the bounded input vector, $\mathbf{v}(t_k) \in V^s$ is the bounded discrete control vector, $\mathbf{y}(t) \in Y^m$ is the output vector, $\sum^n$ is n-dimensional state space, U^r is r-dimensional input space, V^s is s-dimensional condition space, and Y^m is m-dimensional output space. Equation (1a) describes the continuous evolution of system over the time intervals $(t_k, t_{k+1}), k \in \overline{0, K_t}$. Equation (1b) gives the right limit value of $\mathbf{x}(t)$ at switching instans S . Equation (1c) is the output equation. The architecture of the PCHS is shown in Fig.1.

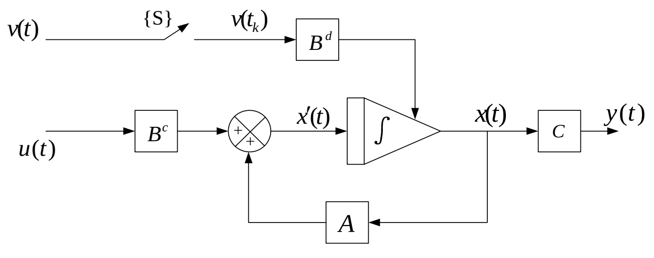

Fig. 1. Structure of PCHS controller.

2.2 Realization of PCHS based Variable-period Sampler

In this case, to realize a variable period sampler, the matrix in (1a-1c) can be designed as follows:

$\mathbf{A} = \mathbf{0}, \ \mathbf{B^d} = \mathbf{C} = \mathbf{I_n}(s = m = n)$, with $\mathbf{I_n}$ is an n-dimensional unit matrix, and $\mathbf{u}(t) \equiv \mathbf{0}$. So, one obtains

$$\mathbf{y}(t) = \mathbf{v}(t_k), \forall t \in [t_k, t_{k+1}), k \in \overline{0, K_t}$$

and the variable sampling period can be denoted as $T_k = t_{k+1} - t_k$ which is obtained by two successive sampling instants in S .

Note that switching instants S here can be generated by an external trigger signal in the form of rising edge. Therefore, an event-triggered signal generating circuit and a self-triggered signal generating circuit would be designed. And it will be introduced in chapter 3.3 and chapter 4.3.

3. EVENT-TRIGGERED OUTPUT-FEEDBACK CONTROLLER DESIGN

3.1 Event-triggered Output Feedback Controller

Considering the linear time invariant system (1a)-(1c) with the following conditions

$\mathbf{A} = \mathbf{A_x}, \mathbf{B^c} = \mathbf{B_x}, \mathbf{C} = \mathbf{C_x}, \mathbf{B^d} = \mathbf{I_n}, S = \{t \mid t = 0\},$
$\mathbf{v}(t) = \mathbf{x}_0.$

One obtains

$$\dot{\mathbf{x}}(t) = \mathbf{A_x}\mathbf{x}(t) + \mathbf{B_x}\mathbf{u}(t), t > 0 \tag{2a}$$

$$\mathbf{x}(t) = \mathbf{x}_0, t = 0 \tag{2b}$$

$$\mathbf{y}(t) = \mathbf{C_x}\mathbf{x}(t), \forall t \geq 0 \tag{2c}$$

The output feedback based controller is

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t)$$

For reducing number of transmissions, an event-triggered scheme is proposed in which the data of controller is updated only at instants t_k, that is to say

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t_k) \ \ , \forall t \in [t_k, t_{k+1})$$

For convenience of analysis, an output error between current output and latest sampling output is defined as

$$\mathbf{e_y}(t) = \mathbf{y}(t) - \mathbf{y}(t_k), t \in [t_k, t_{k+1})$$

For deciding the triggered instants, the event condition is designed as

$$\|\mathbf{e_y}(t)\|^2 < \sigma \|\mathbf{y}(t)\|^2$$

where $\sigma > 0$ is an appropriate constant. In the process of system operation, once the inequality is not satisfied, i.e. $\mathbf{e_y}(t)$ exceeds the limit, a transmission will be triggered, which means at the triggering instant t_k, the data of output feedback controller is updated, and

$$t_{k+1} = \min_t \left\{ t \,\middle|\, t > t_k, \|\mathbf{e_y}(t)\|^2 \geq \sigma \|\mathbf{y}(t)\|^2 \right\}$$

i.e. $\|\mathbf{e}_{\mathrm{y}}(t)\|^2 < \sigma \|\mathbf{y}(t)\|^2$, $t \in [t_k, t_{k+1})$

3.2 Calculation of σ

It's necessary to find a suitable value σ which makes the output feedback control system asymptotically stable.

According to $\mathbf{e}_{\mathrm{y}}(t) = \mathbf{y}(t) - \mathbf{y}(t_k)$ $t \in [t_k, t_{k+1})$, one gets

$$\dot{\mathbf{e}}_{\mathrm{y}}(t) = \dot{\mathbf{y}}(t) = \mathbf{C}_{\mathrm{x}}(\mathbf{A}_{\mathrm{x}}\mathbf{x} + \mathbf{B}_{\mathrm{x}}\mathbf{u})$$

$$= \mathbf{C}_{\mathrm{x}}(\mathbf{A}_{\mathrm{x}}\mathbf{x} + \mathbf{B}_{\mathrm{X}}\mathbf{K}\mathbf{y}(t_k))$$

$$\mathbf{y} = \mathbf{C}_{\mathrm{x}}\mathbf{x} \Rightarrow \mathbf{C}_{\mathrm{x}}^T\mathbf{y} = \mathbf{C}_{\mathrm{x}}^T\mathbf{C}_{\mathrm{x}}\mathbf{x} \Rightarrow \mathbf{x} = (\mathbf{C}_{\mathrm{x}}^T\mathbf{C}_{\mathrm{x}})^{-1}\mathbf{C}_{\mathrm{x}}^T\mathbf{y}$$

$$\dot{\mathbf{e}}_{\mathrm{y}}(t) = \mathbf{C}_{\mathrm{x}}\mathbf{A}_{\mathrm{x}}(\mathbf{C}_{\mathrm{x}}^T\mathbf{C}_{\mathrm{x}})^{-1}\mathbf{C}_{\mathrm{x}}^T\mathbf{y} + \mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}(\mathbf{y} - \mathbf{e}_{\mathrm{y}})$$

$$= -\mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}\mathbf{e}_{\mathrm{y}} + (\mathbf{C}_{\mathrm{x}}\mathbf{A}_{\mathrm{x}}(\mathbf{C}_{\mathrm{x}}^T\mathbf{C}_{\mathrm{x}})^{-1}\mathbf{C}_{\mathrm{x}}^T + \mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K})\mathbf{y}$$

$$= \mathbf{A}_{\mathrm{y}}\mathbf{e}_{\mathrm{y}} + \mathbf{B}_{\mathrm{y}}\mathbf{y}$$

Hence, the output error system can be written in the PCHS form as

$$\dot{\mathbf{e}}_{\mathrm{y}}(t) = \mathbf{A}_{\mathrm{y}}\mathbf{e}_{\mathrm{y}}(t) + \mathbf{B}_{\mathrm{y}}\mathbf{u}_{\mathrm{y}}(t), t \neq t_k, t \geq 0 \quad (3a)$$

$$\mathbf{e}_{\mathrm{y}}(t) = \mathbf{0}, t = t_k \quad (3b)$$

$$\mathbf{u}_{\mathrm{y}}(t) = \mathbf{y}(t) \quad (3c)$$

with $\mathbf{A}_{\mathrm{y}} = -\mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}$ and $\mathbf{B}_{\mathrm{y}} = \mathbf{C}_{\mathrm{x}}\mathbf{A}_{\mathrm{x}}(\mathbf{C}_{\mathrm{x}}^T\mathbf{C}_{\mathrm{x}})^{-1}\mathbf{C}_{\mathrm{x}}^T + \mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}$.

Theorem1: For the output feedback control system

$$\dot{\mathbf{x}}(t) = \mathbf{A}_{\mathrm{x}}\mathbf{x}(t) + \mathbf{B}_{\mathrm{x}}\mathbf{u}(t), t > 0 \quad (4a)$$

$$\mathbf{x}(t) = \mathbf{x}_0, t = 0 \quad (4b)$$

$$\mathbf{y}(t) = \mathbf{C}_{\mathrm{x}}\mathbf{x}(t), \forall t \geq 0 \quad (4c)$$

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t_k) \quad ,\forall t \in [t_k, t_{k+1}) \quad (4d)$$

$$t_{k+1} = \min_t \left\{ t \,\middle|\, t > t_k, \|\mathbf{e}_{\mathrm{y}}(t)\|^2 \geq \sigma \|\mathbf{y}(t)\|^2 \right\} \quad (4e)$$

$$\mathbf{e}_{\mathrm{y}}(t) = \mathbf{y}(t) - \mathbf{y}(t_k), t \in [t_k, t_{k+1}) \quad (4f)$$

If there exists a positive definite symmetric matrix $\mathbf{P} \in R^{n\times n}$ and a scalar constant $\sigma > 0$, satisfy the following inequality defined as follow

$$\mathbf{B}_{\mathrm{y}}^T\mathbf{P} + \mathbf{P}\mathbf{B}_{\mathrm{y}} + \mathbf{P}\mathbf{A}_{\mathrm{y}}\mathbf{A}_{\mathrm{y}}^T\mathbf{P}^T + \sigma\mathbf{I} < \mathbf{0} \quad (5)$$

with $\mathbf{A}_{\mathrm{y}} = -\mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}$ and $\mathbf{B}_{\mathrm{y}} = \mathbf{C}_{\mathrm{x}}\mathbf{A}_{\mathrm{x}}(\mathbf{C}_{\mathrm{x}}^{\mathrm{T}}\mathbf{C}_{\mathrm{x}})^{-1}\mathbf{C}_{\mathrm{x}}^T + \mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}$.

Then, the output feedback control system (4) is asymptotic stable.

Proof: A Lyapunov candidate equation $V(x) = y^T Py$ was defined, where P is a positive definite symmetric matrix, then the derivative of $V(\mathrm{x})$ is

$$\dot{\mathrm{V}}(x) = \dot{\mathbf{y}}^T\mathbf{P}\mathbf{y} + \mathbf{y}^T\mathbf{P}\dot{\mathbf{y}}$$

$$= \dot{\mathbf{e}}_{\mathrm{y}}^T\mathbf{P}\mathbf{y} + \mathbf{y}^T\mathbf{P}\dot{\mathbf{e}}_{\mathrm{y}}$$

$$= (\mathbf{A}_{\mathrm{y}}\mathbf{e}_{\mathrm{y}} + \mathbf{B}_{\mathrm{y}}\mathbf{y})^T\mathbf{P}\mathbf{y} + \mathbf{y}^T\mathbf{P}(\mathbf{A}_{\mathrm{y}}\mathbf{e}_{\mathrm{y}} + \mathbf{B}_{\mathrm{y}}\mathbf{y})$$

$$= \mathbf{y}^T(\mathbf{B}_{\mathrm{y}}^T\mathbf{P} + \mathbf{P}\mathbf{B}_{\mathrm{y}})\mathbf{y} + 2\mathbf{y}^T\mathbf{P}\mathbf{A}_{\mathrm{y}}\mathbf{e}_{\mathrm{y}}$$

$$\leq \mathbf{y}^T(\mathbf{B}_{\mathrm{y}}^T\mathbf{P} + \mathbf{P}\mathbf{B}_{\mathrm{y}})\mathbf{y} + \mathbf{y}^T\mathbf{P}\mathbf{A}_{\mathrm{y}}\mathbf{A}_{\mathrm{y}}^T\mathbf{P}^T\mathbf{y} + \mathbf{e}_{\mathrm{y}}^T\mathbf{e}_{\mathrm{y}}$$
$$\leq \mathbf{y}^T(\mathbf{B}_{\mathrm{y}}^T\mathbf{P} + \mathbf{P}\mathbf{B}_{\mathrm{y}} + \mathbf{P}\mathbf{A}_{\mathrm{y}}\mathbf{A}_{\mathrm{y}}^T\mathbf{P}^T + \sigma\mathbf{I})\mathbf{y}$$

Because of the following defined relationship

$$\mathbf{B}_{\mathrm{y}}^T\mathbf{P} + \mathbf{P}\mathbf{B}_{\mathrm{y}} + \mathbf{P}\mathbf{A}_{\mathrm{y}}\mathbf{A}_{\mathrm{y}}^T\mathbf{P}^T + \sigma\mathbf{I} < \mathbf{0}$$

one gets $\dot{\mathbf{V}}(\mathbf{x}) < \mathbf{0}$ .According to the Lyapunov stability theorem, the output feedback system (4) is ensured stable asymptotically.

3.3 Event-triggered Signal Generating Circuit: used as actuating input for PCHS based variable sampling period sampler

According to the above event triggered condition, the result $e_r(t) = \|\mathbf{e}_{\mathrm{y}}(t)\|^2 - \sigma\|\mathbf{y}(t)\|^2$ can be used as the input signal, and followed by a sign function, one gets consequently

$$f_1(e_r) = \begin{cases} 1 & e_r > 0 \\ -1 & e_r < 0 \end{cases}$$

Then, one has

$$f_2(e_r) = (f_1(e_r) + 1) * 0.5 = \begin{cases} 1 & e_r > 0 \\ 0 & e_r < 0 \end{cases}$$

Through a data type conversion and a logic module circuit, the actuating input signal for the PCHS based variable sampling period sampler can be generated, that is to say when e_r is greater than zero, a short rectangular pulse can be obtained and its amplitude is set to 1. Under a rising front of rectangular pulse for the variable period sampler, a new transmission will be done. Its corresponding realized circuit is shown in Fig.2.

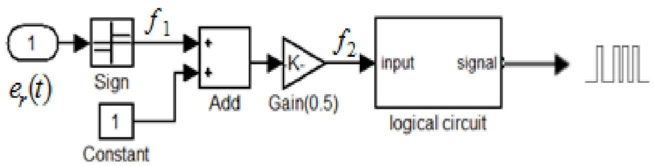

Fig. 2. Event-triggered signal generating circuit.

4. SELF-TRIGGERED OUTPUT-FEEDBACK CONTROLLER DESIGN

4.1 Comparison between Event-triggered and Self-triggered Mechanism

Generally speaking, both event-triggered and self-triggered control systems consist of two elements: one is a feedback controller used to calculate control input, another is an event-triggered mechanism to decide whether the control input data should be updated or not. The difference between event-triggered and self-triggered mechanism is that the former is real-time, while the latter is predicted in advance. In fact, in event-triggered control system, a trigger condition based on the current value is continuously monitored, that is to say, only when the trigger condition is not satisfied, a transmission will be generated. In self-triggered control system, the next update time is calculated by the data and information of the dynamic system accepted in advance.

Based on the last chapter of event-triggered scheme, a self-triggered control method is developed in this chapter.

4.2 Self-triggered Output Feedback Controller

Define a new event condition

$$\|\mathbf{e}_{\mathrm{y}}(t)\|^2 < \sigma \|\mathbf{y}(t_k)\|^2, \forall t \in [t_k, t_{k+1}) \quad (6)$$

With $0<\sigma<0.5$. According to condition (6), one obtains

$$\begin{aligned}\mathbf{e}_{\mathrm{y}}(t)^T\mathbf{e}_{\mathrm{y}}(t) &< \sigma(\mathbf{y}(t)-\mathbf{e}_{\mathrm{y}}(t))^T(\mathbf{y}(t)-\mathbf{e}_{\mathrm{y}}(t))\\ &= \sigma\mathbf{e}_{\mathrm{y}}(t)^T\mathbf{e}_{\mathrm{y}}(t)+\sigma\mathbf{y}(t)^T\mathbf{y}(t)-2\sigma\mathbf{e}_{\mathrm{y}}(t)^T\mathbf{y}(t)\\ &\le 2\sigma\mathbf{e}_{\mathrm{y}}(t)^T\mathbf{e}_{\mathrm{y}}(t)+2\sigma\mathbf{y}(t)^T\mathbf{y}(t)\end{aligned}$$

$$\Rightarrow (1-2\sigma)\mathbf{e}_{\mathrm{y}}(t)^T\mathbf{e}_{\mathrm{y}}(t) \le 2\sigma\mathbf{y}(t)^T\mathbf{y}(t)$$

$$\Rightarrow \mathbf{e}_{\mathrm{y}}(t)^T\mathbf{e}_{\mathrm{y}}(t) \le \frac{2\sigma}{(1-2\sigma)}\mathbf{y}(t)^T\mathbf{y}(t)$$

In a similar way as the proof of Theorem 1, Theorem 2 can be presented as follow.

Theorem 2: For the output feedback control system

$$\dot{\mathbf{x}}(t) = \mathbf{A}_{\mathrm{x}}\mathbf{x}(t)+\mathbf{B}_{\mathrm{x}}\mathbf{u}(t), t>0 \quad (7a)$$

$$\mathbf{x}(t) = \mathbf{x}_0, t=0 \quad (7b)$$

$$\mathbf{y}(t) = \mathbf{C}_{\mathrm{x}}\mathbf{x}(t), \forall t \ge 0 \quad (7c)$$

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t_k) \;, \forall t \in [t_k, t_{k+1}) \quad (7d)$$

$$t_{k+1} = \min_t\left\{t \,\middle|\, t>t_k, \|\mathbf{e}_{\mathrm{y}}(t)\|^2 \ge \sigma\|\mathbf{y}(t_k)\|^2\right\} \quad (7e)$$

$$\mathbf{e}_{\mathrm{y}}(t) = \mathbf{y}(t)-\mathbf{y}(t_k), t \in [t_k, t_{k+1}) \quad (7f)$$

If there exists a positive definite symmetric matrix $\mathbf{P} \in R^{n\times n}$ and a scalar constant $0<\sigma<0.5$, satisfy the following inequality

$$\mathbf{B}_{\mathrm{y}}{}^T\mathbf{P}+\mathbf{P}\mathbf{B}_{\mathrm{y}}+\mathbf{P}\mathbf{A}_{\mathrm{y}}\mathbf{A}_{\mathrm{y}}{}^T\mathbf{P}^T+\frac{2\sigma}{(1-2\sigma)}\mathbf{I}<0 \quad (8)$$

with

$$\mathbf{A}_{\mathrm{y}} = -\mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}$$

$$\mathbf{B}_{\mathrm{y}} = \mathbf{C}_{\mathrm{x}}\mathbf{A}_{\mathrm{x}}(\mathbf{C}_{\mathrm{x}}{}^T\mathbf{C}_{\mathrm{x}})^{-1}\mathbf{C}_{\mathrm{x}}{}^T+\mathbf{C}_{\mathrm{x}}\mathbf{B}_{\mathrm{x}}\mathbf{K}$$

Then, the output feedback control system is asymptotic stable.

The referred event condition (6) is equivalent to

$$\|\mathbf{e}_{\mathrm{y}}(t)\| = \|\mathbf{y}-\mathbf{y}(t_k))\| < \sqrt{\sigma}\|\mathbf{y}(t_k)\|$$

Then with the following relationships

$$\|\mathbf{y}\| < (\sqrt{\sigma}+1)\|\mathbf{y}(t_k)\|$$

$$\mathbf{y} = \mathbf{C}_{\mathrm{x}}\mathbf{x} \Rightarrow \|\mathbf{y}\| \le \|\mathbf{C}_{\mathrm{x}}\|\|\mathbf{x}\|$$

If one has

$$\|\mathbf{C}_{\mathrm{x}}\|\|\mathbf{x}\| < (\sqrt{\sigma}+1)\|\mathbf{y}(t_k)\|, t \in [t_k, t_{k+1})$$

That is to say, with the following formula

$$\|\mathbf{x}\| < ((\sqrt{\sigma}+1)/\|\mathbf{C}_{\mathrm{x}}\|)\|\mathbf{y}(t_k)\|, t \in [t_k, t_{k+1})$$

One obtains that system (7) is asymptotically stable.

Theorem 3: If there exists a positive definite symmetric matrix $\mathbf{P} \in R^{n\times n}$ and a scalar constant $0<\sigma<0.5$ satisfies (8), the output feedback control system (9) expressed as follows is asymptotic stable:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_{\mathrm{x}}\mathbf{x}(t)+\mathbf{B}_{\mathrm{x}}\mathbf{u}(t), t>0 \quad (9a)$$

$$\mathbf{x}(t) = \mathbf{x}_0, t=0 \quad (9b)$$

$$\mathbf{y}(t) = \mathbf{C}_{\mathrm{x}}\mathbf{x}(t), \forall t \ge 0 \quad (9c)$$

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t_k) \;, \forall t \in [t_k, t_{k+1}) \quad (9d)$$

$$t_{k+1} = t_k + \alpha_1(\mathbf{y}(t_k))/\alpha_2(\|\mathbf{y}(t_k)\|) \quad (9e)$$

with $\alpha_1(\mathbf{y}) = \sqrt{\sigma \mathbf{y}^T \mathbf{y}}$ and

$$\alpha_2 = \|\mathbf{C_x B_x K y}(t_k)\| + (\sqrt{\sigma}+1) \cdot (\|\mathbf{C_x A_x}\| / \|\mathbf{C_x}\|) \|\mathbf{y}(t_k)\|$$

Proof: Define $\boldsymbol{\beta}(t) = \mathbf{e_y}(t)$, the event condition $\|\mathbf{e_y}(t)\|^2 < \sigma \|\mathbf{y}(t_k)\|^2$ can be written as $\|\boldsymbol{\beta}(t)\| < \alpha_1(\mathbf{y}(t_k))$. This also means

$$\|\boldsymbol{\beta}(t_{k+1})\| = \alpha_1(\mathbf{y}(t_k)),$$

The derivation of $\|\boldsymbol{\beta}(t)\|$ is obtained as

$$\begin{aligned} &\frac{d}{dt}\|\boldsymbol{\beta}(t)\| \le \|\dot{\mathbf{e}}_\mathbf{y}\| \\ &= \|\mathbf{C_x A_x x} + \mathbf{C_x B_x K y}(t_k)\| \\ &\le \|\mathbf{C_x B_x K y}(t_k)\| + \|\mathbf{C_x A_x}\| \|\mathbf{x}\| \end{aligned}$$

By following relationship of

$$\|\mathbf{x}\| < ((\sqrt{\sigma}+1) / \|\mathbf{C_x}\|) \|\mathbf{y}(t_k)\|, t \in [t_k, t_{k+1})$$

one has

$$\frac{d}{dt}\|\boldsymbol{\beta}(t)\| < \|\mathbf{C_x B_x K y}(t_k)\| + (\sqrt{\sigma}+1) \cdot (\|\mathbf{C_x A_x}\| / \|\mathbf{C_x}\|) \|\mathbf{y}(t_k)\| = \alpha_2(\|\mathbf{y}(t_k)\|).$$

Then with $\|\boldsymbol{\beta}(t_k)\| = 0$, one gets

$$\|\boldsymbol{\beta}(t)\| \le \alpha_2(\|\mathbf{y}(t_k)\|)(t - t_k), t \in [t_k, t_{k+1}].$$

Moreover, according to the following relationship

$$\alpha_1(\mathbf{y}(t_k)) = \|\boldsymbol{\beta}(t_{k+1})\| \le \alpha_2(\|\mathbf{y}(t_k)\|)(t_{k+1} - t_k),$$

the next updating sampling period can be calculated as follow

$$T_k = t_{k+1} - t_k \ge \alpha_1(\mathbf{y}(t_k)) / \alpha_2(\|\mathbf{y}(t_k)\|) \tag{10}$$

It's reasonable to take the minimum time interval which is able to guarantee the system (9) asymptotically stable.

4.3 Self-triggered Signal Generating Circuit

According to the formula $t_{k+1} = t_k + \alpha_1(\mathbf{y}(t_k)) / \alpha_2(\|\mathbf{y}(t_k)\|)$, the sampling interval T_k can be computed, and if the condition of $t_k + T_k - t \le 0$ is satisfied, a short pulse signal will be given to generate the variable period sampler to do a new sampling. The corresponding self-triggered signal generating circuit structure is shown in Fig.3.

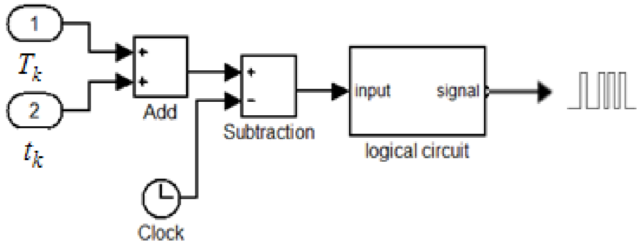

Fig. 3. Self-triggered signal generating circuit.

5. NUMERICAL SIMULATION RESULTS

To validate the proposed event-triggered and self-triggered control methods, in this section a motorized cart system which moves along a horizontal and straight line segment is employed. Actually, the cart is moved through a notched belt powered by an electric motor. This referred motorized cart system can be modeled as

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ 0 & -1/\tau \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ k_c/\tau \end{bmatrix} \mathbf{u} \\ \mathbf{y} &= \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \mathbf{x} \\ \mathbf{x}(0) &= \begin{bmatrix} 1 & 0 \end{bmatrix}^T \end{aligned} \tag{11}$$

With $\tau = 8.3s$ and $k_c = 2.9s/V$ are the system time constant and overall gain. One selects the appropriate poles of the closed-loop system, and the corresponding matrix K can be obtained through the pole placement technique $\mathbf{K} = \begin{bmatrix} -6 & -18 \end{bmatrix}$. Through application of LMI, one gets $\sigma_{\max} = 0.1016$ in event-triggered control. For comparing the number of samplings in the same performance, $\sigma_{\max}$ are not used in self-triggered control, and it is setted as $\sigma = 0.2308$. The entire closed-loop control system structure is shown in the following Fig.4.

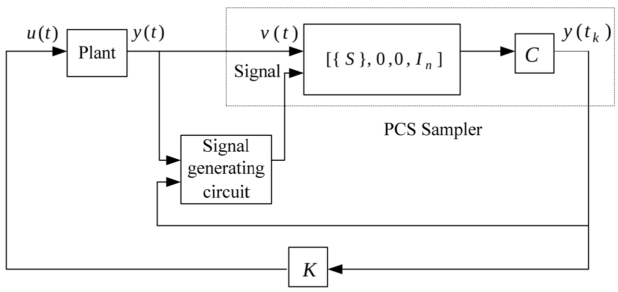

Fig. 4. Closed-loop control system structure.

5.1 Event-triggered Controller Results

With the proposed PCHS based event triggered controller for the considered plant, one obtains corresponding results in Fig. 5-7, which represent respectively the controlled systems state trajectory in Fig. 5, event-triggered signal (rising edge) in Fig.6 and the output error $\mathbf{e_y}(t)$ between real and sampled value in Fig.7.

Through the calculation by MATLAB/Simulink, it can be get that the controller samples 17 times in 5 seconds, and the average sampling period is about 0.294 seconds. Through the state trajectory curve, it can be obtained that the settling time is 2.26 seconds in an allowed bound of 0.05, and the peak values of two state components are 10.15 and -13.23.

5.2 Self-triggered Controller Results

Then with the proposed PCHS based self-triggered controller for the considered plant, one obtains the corresponding results in Fig. 8-10 which represent

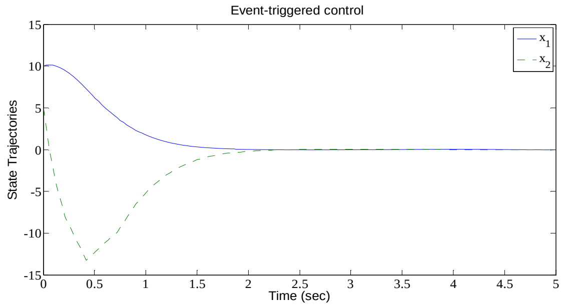

Fig. 5.State evolution under event-triggered control.

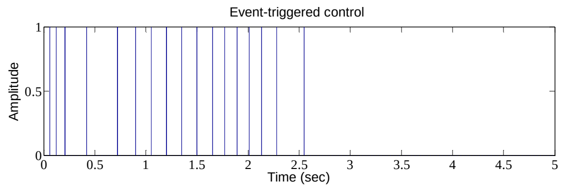

Fig. 6. Event triggered signal.

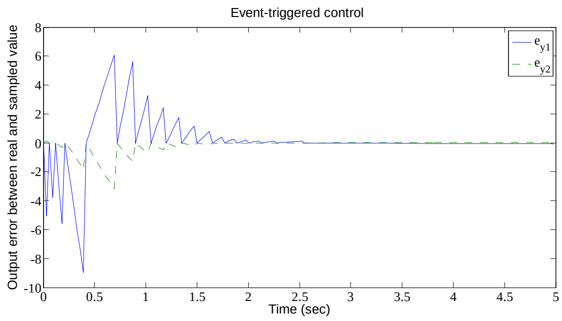

Fig. 7. $\mathbf{e}_\mathbf{y}(t)$ under event-triggered control.

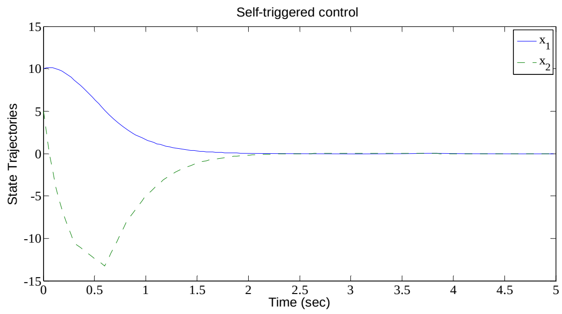

Fig. 8. State evolution under self-triggered control.

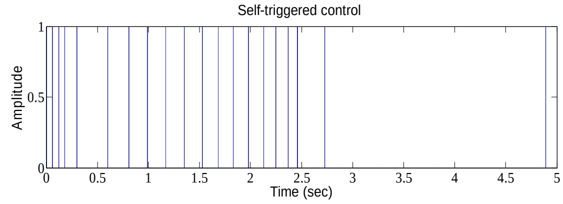

Fig. 9. Self-triggered signal.

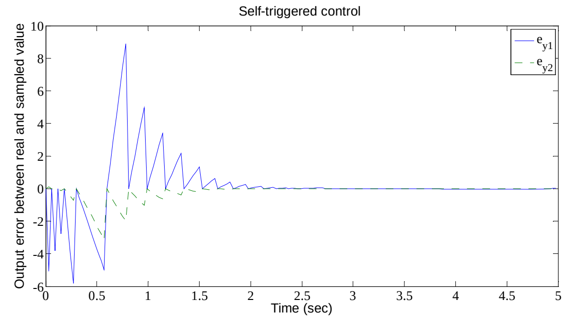

Fig. 10. $\mathbf{e}_\mathbf{y}(t)$ under self-triggered control.

respectively the state trajectory, the self-triggered signal and the output error $\mathbf{e}_\mathbf{y}(t)$ between real and sampled value of the self-triggered output feedback control system. From these figures through MATLAB/Simulink, it can be obtained that the controller samples 19 times in 5 seconds, and the average sampling period is about 0.263 seconds. Through the state trajectory curve, it can be obtained that the settling time is 2.29 seconds in an allowed bound of 0.05, and the peak values of two state components are 10.15 and -13.25.

5.3 Periodic Controller Results

To validate the proposed event triggered and self-triggered controller, the periodic output feedback control is implemented under the period of

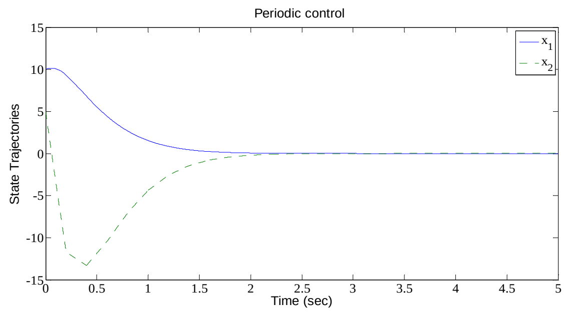

Fig. 11. State evolution under periodic control.

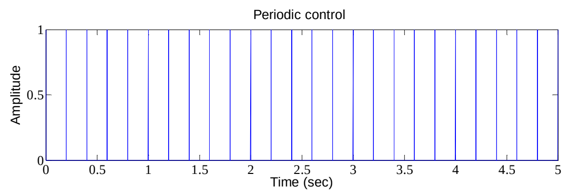

Fig. 12. Periodic-triggered signal.

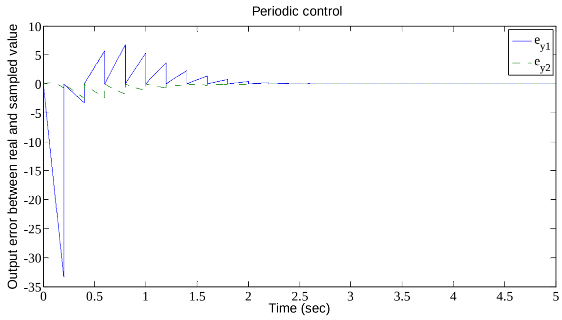

Fig. 13. $\mathbf{e}_y(t)$ under periodic control.

T=0.2 seconds. Its corresponding results are illustrated in Fig.11-13 which represent respectively the state trajectory the periodic-triggered signal and the output error $\mathbf{e}_y(t)$ between real and sampled value. It can be seen that the periodic controller samples 25 times in 5 seconds. Through the state trajectory curve, it can be obtained that the settling time is 2.38 seconds in an allowed bound of 0.05, and the peak values of two state components are 10.15 and -13.31.

Table 1. The comparison of the three mechanisms

Mechanism	Event trigger	Self trigger	Periodic trigger
Sampling times/5s	17	19	25
Average sampling period (second)	0.294	0.263	0.2
Settling time (second)	2.26	2.29	2.38
Peak value of state x_1 / x_2	10.15 /-13.23	10.15 /-13.25	10.15 /-13.31

6. CONCLUSIONS

In this paper, the event-triggered controller and self-triggered controller are designed with the application of PCHS system. As shown in Table.1, it can be seen that the number of samplings under event-triggered mechanism is less than the periodic sampling mechanism when its settling time is still a litte shorter and peak value is still slightly smaller, saving 32% bandwidth resource. And the number of samplings under the self-trigger mechanism is also less than the periodic sampling mechanism when its settling time is also a litte shorter and peak value is slightly smaller, saving 24% bandwidth resource. Thus compared with the periodic sampling, the advantage of the event-triggered and self-triggered mechanisms in saving the sampling times is obvious.

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