Active Fault Tolerant Controller Design using Model Predictive Control

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Abstract: This paper presents an approach for active fault-tolerant control (FTC) design of constrained linear multi input-multi output (MIMO) systems. The proposed approach is based on model predictive control (MPC) and fault estimation scheme. This method uses two optimal observers to estimate the plant states and loss of effectiveness factor of actuators and sensors. A supervisor unit also uses the fault modelling and correction of plant model per sampling time to accommodate simultaneous partial actuator and sensor faults. Also, by using embedded integrator and feedback compensation in MPC formulation, actuator and sensor faults are compensated automatically. The most important advantages of the proposed approach is ability to deal with the constraints and all types of faults in control system simultaneously low on-line computational load, and its simplicity for real applications. Simulation results on active magnetic bearing system show the effectiveness of the proposed approach.

Keywords: fault-tolerant control (FTC), model predictive control (MPC), fault detection and isolation (FDI).

1. INTRODUCTION

In recent years, fault-tolerant control (FTC) systems have been gaining an increasing insert among researchers. The main motivation behind such an increasing insert is high performance requirements of modern industries and demand for higher reliability and safety in control systems.

A control system that can accommodate faults among system components automatically while maintaining system stability along with a desired level of overall performance is denoted as an FTC system (Zhang and Jiang, 2008; Jiang and Yu, 2012; Mirzaee and Salahshoor, 2012). FTC system design techniques can be classified in two types: passive approach (PFTC) such as robust fault accommodation approach (Yang et al., 2001; Veillrtte et al., 1992) and active approach (AFTC) such as adaptive approach (Tao et al., 2001; Bodson and Groszkiewicz, 1997); model following (Huang and Stengel, 1990); eigenstructure assignment (Jiang, 1994); and multiple model (Zhang and Jiang, 2001; Boskovic et al., 1998). In PFTC, the controller structure is constant and system can tolerate only a limited number of faults which are assumed to be known prior to controller design. On the other hand, AFTC can accommodate faults by reconfiguration or restructuring the controller based on information provided by of fault detection and isolation (FDI) module (Jiang and Yu, 2012).

Model predictive control (MPC) has been widely adopted by process industry as an effective practical control technique, specially, in chemical process (Prakash et al., 2010; Puig et al., 2008; Gambier et al., 2010). This is mainly due to its unique advantages in dealing with hard constraints on inputs and states, and complex process dynamics such as time varying, unstable and multivariable behaviors (Camacho et al., 2010). In MPC at each sample time, starting at the current state, an open loop optimal control problem is solved over finite horizon. At the next time step, the computation is repeated starting from the new state and over a shifted horizon, leading to a moving horizon policy (Bemporad et al., 2002).

The idea of using MPC in FTC is firstly discussed in (Maciejowiski, 1997; Maciejowiski and Jones, 2003); both references show that MPC provides suitable implementation architecture for FTC. MPC based on FTC system design technique can be classified in two types: passive approaches and active approaches. Passive methods such as (Abdel-Geliel et al., 2006; Mahmood and Mhaskar, 2012) compensate the faults that are known and used in MPC design by coping the extra constraints. On the other hand, this fact that MPC is a discrete model-based approach that can handle constraints, makes it a serious candidate for AFTC approaches (Joosten et al., 2008). In these approaches, MPC module can be reconfigured by using the information provided by FDI module. Active approaches can be classified in two types: multiple MPCs such as (Kanthalakshmi and Manikandan, 2011; Kargar et al., 2013; Ichtev et al., 2002; Mendonça et al., 2012; Chilin et al., 2012) and adaptive approaches such as (Salahshoor et al., 2010; Martínez et al., 2005; Qi Sun et al., 2008; Menighed et al., 2011; Chilin et al., 2012). In adaptive approaches, faults can be compensated by modifying the constraints in MPC problem such as (Joosten et al., 2008; Salahshoor et al., 2010; Martínez et al., 2005); other faults can be compensated by modifying in internal model used by MPC such as (Qi Sun et al., 2008; Menighed et al., 2011; Chilin et al., 2012).

In this paper, an AFTC scheme based on combination MPC with fault estimation is presented to accommodate actuator and sensor faults of a linear time-invariant system with some constraints on control inputs. Figure 1 show the architecture of the proposed FTC in this paper where controller composed of MPC, fault/state estimator and supervisor. The fault/state estimator is based on the observers that estimate the loss of effectiveness of actuators and sensors ($\hat{\gamma}_a$,$\hat{\gamma}_s$) and the states of plant ($\hat{x}_P$). The fault information provided by fault estimator is then used in supervisor that modifies the internal model of MPC. Thus, the proposed controller can compensate actuator and sensor faults. By using embedded integrator (EI) model in MPC design, bias faults in actuators and sensors can also be compensated, and by using feedback compensation (FC) in MPC, the plant faults can be compensated automatically.

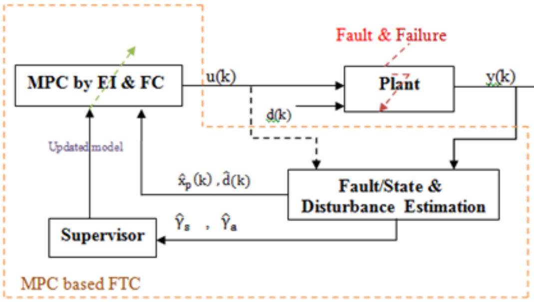

Fig. 1. Block diagram of the proposed FTC system.

Compared to existing works, the contributions of this paper are in four aspects: 1) Actuator and sensor faults as well as control constraints can be compensated simultaneously in the FTC design approach; 2) The proposed approach has low computational load because of using MPC based on Laguerre functions and dealing with actuator and sensor faults by correction of internal model instead of changing constraints; 3) An fault estimator is explicitly designed to provide fault information for MPC; and 4) Simplicity and effectiveness of the proposed FTC approach is significant .

The paper is organized as follows: in Section 2, the general MPC formulation is discussed. Section 3 presents the proposed FTC based on MPC. Section 4 illustrates the simulation results to show the effectiveness of the proposed method on the active magnetic bearing system. Finally, conclusions are given in Section 5.

2. MODEL PREDICTIVE CONTROL

MPC has been known as an effective solution for constrained MIMO control system design problem in the process industries. The basic structure of MPC is depicted in Figure 2 (Camacho and Bordons, 2004).

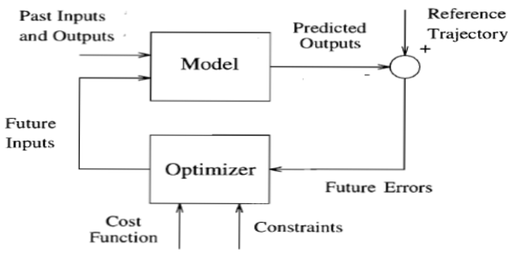

Fig. 2. Basic structure of MPC.

At each sample time, the process model is used to predict the future plant outputs on the prediction horizon N_P, based on past and current values and proposed optimal future inputs. These inputs are calculated by solving a constrained optimization problem. The optimization yields an optimal input sequence, but only the first element of the sequence is applied to the plant and the other elements removed. In the next sample time, the complete calculation is repeated. This policy is called the receding horizon control principle (RHC), (Camacho and Bordons, 2004; van den Boom and Stoorvogel, 2010).

Consider the discrete- time linear system

$$x_P(k+1) = A_P\, x_P(k) + B_P\, u(k) + B_1 d(k) \tag{1}$$

$$y(k) = C_P\, x_P(k) \tag{2}$$

While fulfilling the constraints

$$\Delta u^{min} \le \Delta u(k) \le \Delta u^{max}, \quad u^{min} \le u(k) \le u^{max}$$
$$y^{min} \le y(k) \le y^{max} \tag{3}$$

at all time instant $k \ge 0$, where $x_P \in R^{n_1}$, $u(k) \in R^m$ and $y \in R^q$ are the state, input and output vectors respectively. Also $d(k)$ is the known disturbance and $\Delta u(k) = u(k) - u(k-1)$ is the increment of input vector.

Taking a difference operation on both sides of (1) and denoting the variables below

$$\Delta x_P(k+1) = x_P(k+1) - x_P(k)$$
$$\Delta x_P(k) = x_P(k) - x_P(k-1)$$
$$\Delta u(k) = u(k) - u(k-1), \qquad \Delta d(k) = d(k) - d(k-1)$$

and choosing new state variable vector $x(k) = [\Delta x_P(k)^T \ y(k)]^T$, leads to the state-space model with embedded integrator (EI), (Grimble and Johnson, 2009):

$$\overbrace{\begin{bmatrix} \Delta x_P(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} A_P & 0_P^T \\ C_P A_P & 1 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \Delta x_P(k) \\ y(k) \end{bmatrix}}^{x(k)}$$
$$+ \overbrace{\begin{bmatrix} B_P \\ C_P B_P \end{bmatrix}}^{B} \Delta u(k) + \overbrace{\begin{bmatrix} B_1 \\ C_P B_1 \end{bmatrix}}^{Bd} \Delta d(k) \tag{4}$$

$$y(k) = \overbrace{[0_P \quad 1]}^{C} \begin{bmatrix} \Delta x_P(k) \\ y(k) \end{bmatrix} \tag{5}$$

where $0_m = \overbrace{[0 \ \ 0 \ldots \ 0]}^{n_1}$ and $\Delta u(k)$ is the input vector of the system. The triple (A, B, C) is called the augmented model, which will be used in the design of MPC. When the state-space model with EI in MPC formulation is used, MPC can compensate the disturbance with step dynamic in control system, automatically.

In general, MPC solves the following constrained optimization problem

$$min_{u_0,\ldots,u_{N_u-1}} \sum_{m=0}^{N_p} \left(x(k+m|k)\right)^T Q\left(x(k+m|k)\right) +$$
$$\sum_{m=0}^{N_u-1} (u(k+m))^T R\, (u(k+m)) \tag{6}$$

Subject to

$$\Delta u^{min} \le \Delta u(k) \le \Delta u^{max}, \quad k = 1,2,\dots,N_c \tag{7}$$

$$u^{min} \le u(k) \le u^{max}, \quad k = 1,2,\dots,N_c \tag{8}$$

$$y^{min} \le y(k) \le y^{max}, \quad k = 1,2,\dots,N_p \tag{9}$$

$$x(k+1) = A\,x(k) + B\,\Delta u(k) + B_d(k)\Delta d(k) \tag{10}$$

$$y(k) = C\,x(k) \tag{11}$$

$$\Delta u(k) = K_{mpc}\,x(k) + K_d D(k) \tag{12}$$

at each sampling instant, where $Q = C^T C \ge 0$, $R = R^T > 0$ and N_p , N_u , N_c are the output, input and constraint horizon, respectively with $N_p \ge N_u$ and $N_c \ge N_p - 1$.

The optimization problem (6-12) is solved by quadratic programming which gives the desired control sequence $\Delta u(k) = [\Delta u^T(k)\ \Delta u^T(k+1) \dots \Delta u^T(k+N_u-1)]^T$ that fulfilled in (12) where $D(k)$ is known statement of $d(k)$. Then by RHC principle, the first vector of control sequence is injected to the plant. The parameters of above optimization problem are N_p , N_u and N_c and computational load in MPC is related to them. One of the MPC formulation is classic approach presented in Bemporad et al., 2002. By this approach, in the case of rapid sampling, complicated process dynamics and/or high demands on closed-loop performance, satisfactory approximation of the control signal Δu may require a very large number of parameters (large N_u), leading to poorly numerically conditioned solutions and heavy computational load when implemented on-line. Instead, a more appropriate approach is to use Laguerre networks in the design of MPC presented in Grimble and Johnson, 2009.

A set of discrete-time orthonormal basis Laguerre functions can be obtained by

$$\Gamma_n(z) = \Gamma_{n-1}(z)\,\frac{z^{-1}-a}{1-az^{-1}}, \qquad n=2, 3, \dots, N. \tag{13}$$

where $\Gamma_1(z)\ \frac{\sqrt{1-a^2}}{1-az^{-1}}$, N is the number of Laguerre functions in the network and a is the pole of the Laguerre network. the set of Laguerre functions can be expressed as $L(k) = [l_1(k)\ l_2(k) \dots l_N(k)]^T$, where $l_i(k)$ denote the inverse Z-transform of $\Gamma_i(z,a)$. The main idea in MPC based on Laguerre functions is the approximating member of control sequence by a set of Laguerre functions as

$$u(k+i) = L^T(k)\eta, \quad i = 1,2,\dots,N_u - 1 \tag{14}$$

where the parameter vector η comprises N Laguerre coefficients: $\eta = [c_1\ c_2 \dots c_N]^T$. By using this approximation, the optimization problem (6-12) is expressed in terms of coefficient vector η , instead of Δu as in the classic approach. Thus, the coefficient vector η will be optimized and computed in the design. With this design framework, the control horizon N_u from the classic MPC approach has vanished. Instead, the number of terms N ($N < N_u$) is used to describe the complexity of the trajectory in conjuction with the parameter a. Furthermore, a long control horizon (N_u) can be realized without using a large number of parameters, thus the computation load is decreased. In this paper, the MPC based on Laguerre functions is used in proposed FTC scheme.

Remark 1: For tracking objective, when the reference signal $r(k) \ne 0$, $x(k+m|k)$ can be chosen as $x(k+m|k) = [\Delta x_P(k+m|k)^T\ y(k+m|k) - r(k)]^T$ and the optimization problem is solved.

In the most cases, there are not explicit model of the system and there are some uncertainties in process model. In these cases, model of process is achieved by model mismatch. When MPC uses this model, it cannot achieve control objectives. For this purpose, feedback compensation (FC) could be used to solve this problem by marking error $e(k)$ at time k as follows

$$e(k) = x(k) - x(k|k-1) \tag{15}$$

where $x(k)$ can be obtained by system feedback or state observer at time k, and $x(k|k-1)$ is the predictive value of $x(k)$ at time $k-1$. By adding error in MPC formulation, equation (12) is rewritten as follows

$$\Delta u(k) = K_{mpc}\,x(k) + K_d D(k) + K_e E(k) \tag{16}$$

where $E(k)$ is the known statement of $e(k)$. By using FC in MPC formulation, the control system has robustness against model mismatch.

Remark 2: When the fault occurs in plant, it can be modeled as model mismatch; then by using FC in MPC formulation, the control system can accommodate the fault in plant.

In addition, MPC can compensate the failure in the components of the control system relatively (van den Boom and Stoorvogel, 2010). This property of MPC is due to this fact that in MPC, the control signal is recomputed at each sample time by solving an open-loop problem, then it is easy to make changes in the problem formulation. In Section 4, this property of MPC on the active magnetic bearing system control will be shown.

3. FTC BASED ON MPC

In this section, the fault description is introduced in subsection 3.1, and the proposed fault estimator and supervisory method are presented in subsection 3.2 and 3.3 respectively, and finally the architecture of proposed FTC scheme is shown in 3.4.

3.1 Fault Description in Control System Components

During the system operation, faults or failures may be occur in the closed-loop control system components such as actuators, sensors and plant. These faults can be modeled as additive or multiplicative due to malfunction or equipment aging.

The fault that changes the dynamics of the plant is the plant fault and modeled as bellow

$$A_{Pf} = A_P + \delta A_P \tag{17}$$

For example, the tank system that is pierced and its flow rate is changed, has the plant fault.

The faults that may be occurred in actuators are bias and partial actuator failures that is reduction of control effectiveness. The i'th actuator fault can be shown as follow

$$u_{fi} = (1+\gamma_{ai})u_i + u_{f_{0i}}$$
$$i = 1,2,\dots,m\,, \quad -1 \leq \gamma_{ai} \leq 0 \tag{18}$$

where γ_{ai} is the control effectiveness factor, m is the number of actuators in the control system and $u_{f_{0i}}$is the bias of i'th actuator. Different types of actuator faults are shown in Table 1.

Table 1. Actuator fault type

	$u_{foi} = 0$	$u_{foi} \neq 0$
$\gamma_{ai} = 0$	Fault free	bias
$-1 < \gamma_{ai} < 0$	Partial failure	Partial failure and bias
$\gamma_{ai} = -1$	failure	Stuck

The actuator fault can also be represented in a control system by the following compact form

$$u_f = (I+\gamma_a)u + u_{f_0} \tag{19}$$

where, $\gamma_a = diag(\gamma_{a1}, \gamma_{a2}, \dots, \gamma_{am})$, I is the identity matrix and $u_{f_0} = [u_{f_{01}}\ u_{f_{02}} \dots u_{f_{0m}}\ ]^T$.

In a similar way, sensor faults can be represented as bellow

$$y_f = (I+\gamma_s)\, y + y_{f_0} \tag{20}$$

where,
$\gamma_s = diag(\gamma_{s1}, \gamma_{s2}, \dots, \gamma_{sq})$ *and* $y_{f_0} = [y_{f_{01}}\ y_{f_{02}} \dots y_{f_{0q}}\ ]^T$.

3.2 Fault/ State Estimator

As presented in Section 1, AFTC method is used to compensate actuator and sensor faults. For AFTC design, designing of fault estimator is necessary. For estimator design, this paper uses the idea that presented in Qi Sun et al., 2008, and presents a fault/state estimator that estimates the control and output effectiveness factors ($\gamma_a\,, \gamma_s$) and states of the plant (x_P) that are used in MPC. Consider the discrete-time linear system

$$x_P(k+1) = A_P\, x_P(k) + B_P\, u(k) + B_1 d(k) \tag{21}$$

$$y(k) = C_P\, x_P(k) \tag{22}$$

Regarding (18), the state equation with actuator fault is given by

$$x_P(k+1) = A_P\, x_P(k) + B_P\, u(k)$$
$$+[b_1\gamma_{a1} \quad b_2\gamma_{a2} \quad \cdots \quad b_m\gamma_{am}]\begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_m(k) \end{bmatrix} + B_1 d(k) \tag{23}$$

or in a compact form

$$x_P(k+1) = A_P\, x_P(k) + B_P u(k)$$
$$+E(k)\,\gamma_a(k) + B_1 d(k) \tag{24}$$

where

$$E(k) = B_P U(k)\ ,\ U(k) = \begin{bmatrix} u_1(k) & 0 & \cdots & 0 \\ 0 & u_2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & u_m(k) \end{bmatrix}$$

$$\gamma_a(k) = \begin{bmatrix} \gamma_{a1}(k) \\ \gamma_{a2}(k) \\ \vdots \\ \gamma_{am}(k) \end{bmatrix}$$

In the absence of the knowledge on the evolution of the effectiveness factors, the control effectiveness can be modeled as a random bias vector

$$\gamma_a(k+1) = \gamma_a(k) + w(k) \tag{25}$$

where $w(k)$ denote the white noise sequence. By defining the new state such as $z_a(k) = [x_P(k)\ \gamma_a(k)]^T$, the augmented model is

$$z_a(k+1) = \tilde{A}_a\, z_a(k) + \tilde{B}_a\, u(k)\ + \tilde{B}_1 d(k)$$
$$+B_w w(k) \tag{26}$$

$$y(k) = \tilde{C}_a\, z_a(k) \tag{27}$$

where

$$\tilde{A}_a = \begin{bmatrix} A_P & E(k) \\ 0 & I \end{bmatrix}, \tilde{B}_a = \begin{bmatrix} B_P \\ 0 \end{bmatrix}, \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

$$B_w = \begin{bmatrix} 0 \\ I \end{bmatrix}, \tilde{C}_a = [C_P\ 0]$$

If the pair ($\tilde{A}_a\ , \tilde{C}_a$) is observable, by an optimal observer such as kalman filter (van den Boom and Stoorvogel, 2010), both state vector of plant and control effectiveness factors can be estimated.

In a similar way, the output equation with sensor faults is given by

$$y(k) = C_P x_P(k) + F(k)\,\gamma_s(k) \tag{28}$$

where

$$F(k) = -\,Y(k), \qquad Y(k) = \begin{bmatrix} y_1(k) & 0 & \cdots & 0 \\ 0 & y_2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & y_q(k) \end{bmatrix}$$

$$\gamma_s(k) = \begin{bmatrix} \gamma_{s1}(k) \\ \gamma_{s2}(k) \\ \vdots \\ \gamma_{sq}(k) \end{bmatrix}$$

In the absence of the knowledge on the evolution of the effectiveness factor, the sensor effectiveness can be model as

$$\gamma_s(k+1) = \gamma_s(k) + v(k) \tag{29}$$

where $v(k)$ denote the white noise sequence. By defining the new state such as $z_s(k) = [x_P(k)\ \gamma_s(k)]^T$, the augmented model is

$$z_s(k+1) = \tilde{A}_s\, z_s(k) + \tilde{B}_s\, u(k)\ + \tilde{B}_1 d(k)$$
$$+B_v v(k) \tag{30}$$

$$y(k) = \tilde{C}_s\, z_s(k) \tag{31}$$

where

$$\tilde{A}_s = \begin{bmatrix} A_P & 0 \\ 0 & I \end{bmatrix}, \tilde{B}_s = \begin{bmatrix} B_P \\ 0 \end{bmatrix}, \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, B_v = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\tilde{C}_s = [C_P\ F(k)]$$

If the pair $(\tilde{A}_s\ , \tilde{C}_s)$ is observable, by an optimal observer such as kalman filter, both state vector of plant and effectiveness factors of sensors can also be estimated.

Thus, the fault estimator composed of two observer that its existence conditions are the observability of $(\tilde{A}_a\ , \tilde{C}_a)$ and $(\tilde{A}_s\ , \tilde{C}_s)$, and the number of faults must be lower than or equal to the measurable outputs of the plant.

It is worth mentioning that disturbance estimation method is similar to the proposed fault estimation method. In the absence of the knowledge on the evolution of the disturbance, it can be modeled as

$$d(k+1) = d(k) + e_d(k) \tag{32}$$

where $e_d(k)$ denote the white noise sequence. By defining the new state such as $s(k) = [x_P(k)\ d(k)]^T$, the augmented model is given by

$$s(k+1) = \tilde{A}_d\, s(k) + \tilde{B}_d\, u(k) + B_e e(k) \tag{33}$$

$$y(k) = \tilde{C}_d\, s(k) \tag{34}$$

where

$$\tilde{A}_d = \begin{bmatrix} A_P & B_1 \\ 0 & I \end{bmatrix}, \tilde{B}_d = \begin{bmatrix} B_P \\ 0 \end{bmatrix}, \tilde{C}_d = [C_P\ 0], B_e = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

If the pair $(\tilde{A}_d\ , \tilde{C}_d\ )$ is observable, by an observer, both state vector of plant and disturbance can also be estimate.

3.3 Supervisory Scheme

As shown in Figure 1, the supervisor is unit that gives the fault information from fault/state estimator, then by modifies the internal model of plant, and the MPC controller is then reconfigured. The technique of modeling the all types of faults in actuators and sensors is presented in the following.

Consider the fault-free model of system

$$x_P(k+1) = A_P\, x_P(k) + B_P\, u(k) + B_1 d(k) \tag{35}$$

$$y(k) = C_P\, x_P(k) \tag{36}$$

when actuator fault occurs, by replacing $u(k)$ with $u_f(k)$, presented in (19), the state equation (35) is changed by

$$x_P(k+1) = A_P\, x_P(k) + B_P(I+\gamma_a)u(k)$$
$$+B_P u_{f_0} + B_1 d(k) \tag{37}$$

By defining $B_{Pnew} = B_P(I+\gamma_a)$, the equation (37) can be written as

$$x_P(k+1) = A_P\, x_P(k) + B_{Pnew} u(k)$$
$$+B_P u_{f_0} + B_1 d(k) \tag{38}$$

For compensating the actuators fault, by using the information about γ_a , B_{Pnew} is constructed, and MPC is then redesigned. It should be noted, in most cases the bias of actuator (u_{f_0}) has a step dynamic; then as discussed in Section 2, because of using EI in MPC, this fault can be compensated automatically. Thus, it is not necessary to consider the part $B_P u_{f_0}$ in internal model. Therefore, for compensating the actuator faults, the new model represented in (39) and (40), will be replaced by the old fault-free model in the MPC formulation

$$x_P(k+1) = A_P\, x_P(k) + B_{Pnew} u(k) + B_1 d(k) \tag{39}$$

$$y(k) = C_P\, x_P(k) \tag{40}$$

Remark 3: If the actuator bias has different dynamic, for compensating, it can be treated as disturbance in MPC formulation. In this case, the control low in optimization problem of MPC can be modified as bellow

$$\Delta u(k) = K_{mpc}\, x(k) + K_d D(k) + K_b U_{f0}(k) \tag{41}$$

Where $U_{f0}(k)$ is the known statement of u_{f_0}. Certainly, it is necessary that amplitude of u_{f_0} is known.

In a similar way, when sensor fault occur, by replacing $y(k)$ with $y_f(k)$ in (20), the output equation (36) is obtained as follows

$$y(k) = (I+\gamma_s)^{-1}\, C_P\, x_P(k) - (I+\gamma_s)^{-1}\, y_{f_0} \tag{42}$$

By defining $C_{Pnew} = (I+\gamma_s)^{-1} C_P$, the equation (42) can be written as

$$y(k) = C_{Pnew}\, x_P(k) - (I+\gamma_s)^{-1}\, y_{f_0} \tag{43}$$

For compensating the sensor fault, by using of information about γ_s , C_{Pnew} is constructed, then MPC is redesigned. It should be noted, in most cases the bias of sensor (y_{f_0}) have a step dynamic and because of using EI in MPC, this fault can be compensated automatically. So it is not necessary to consider the part $(I+\gamma_s)^{-1}\, y_{f_0}$in internal model. Thus for compensating the sensor faults the new model that represented in (44) and (45), will be replaced by the old fault free-model in the MPC formulation.

$$x_P(k+1) = A_P x_P(k) + B_P u(k) + B_1 d(k) \tag{44}$$

$$y(k) = C_{Pnew} x_P(k) \tag{45}$$

In general, when actuator and sensor fault occur simultaneously, the corrected internal model is

$$x_P(k+1) = A_P\, x_P(k) + B_{Pnew} u(k) + B_1 d(k) \tag{46}$$

$$y(k) = C_{Pnew} x_P(k) \tag{47}$$

where $B_{Pnew} = B_P(I+\gamma_a)$ and $C_{Pnew} = (I+\gamma_s)^{-1} C_P$.

3.4 FTC Architecture

The architecture of the proposed FTC scheme based on MPC is shown in Figure 3.

In every sampling instant, the fault/state estimator estimates the control and output effectiveness factors $(\hat{\gamma}_a\, , \hat{\gamma}_s)$ and states of plant $(\hat{x}_P)$. Then $\hat{\gamma}_a\, , \hat{\gamma}_s$ passes to supervisor and new faulty model will take the place of the old fault-free model in the MPC formulation by the supervisor. In addition, the estimated states of plant also used in MPC formulation when the states are not measurable. Also, disturbance estimator estimates $d(k)$ and sends it to MPC. Then, MPC using this new information, updates the optimization problem and compute k_{mpc}, k_d, k_e in equation (16). Then the actuator and sensor faults are compensated by an AFTC system. Also, by using EI and FC in MPC, the actuator and sensor bias and also plant faults are compensated automatically without their information.

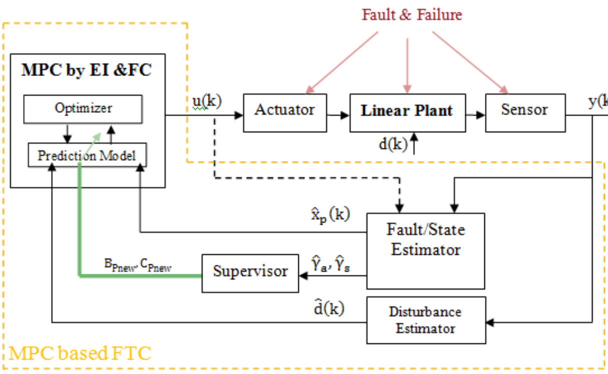

Fig. 3. Architecture of proposed FTC based on MPC.

In brief, the whole procedure of implementation the proposed FTC approach can be summarized in the following algorithm:

(1) Develop the nonlinear model of plant.

(2) Linearize the obtained nonlinear model around operating points.

(3) Construct MPC controller by EI and FC based on the linear model of the process and considering the real constraints (this step needs some trial and error for choosing the MPC weight matrices, control and prediction horizon).

(4) Check the observability of $(\tilde{A}_a, \tilde{C}_a)$, $(\tilde{A}_s, \tilde{C}_s)$ and $(\tilde{A}_d, \tilde{C}_d)$, if they are observable go to step 5.

(5) Construct the fault/state estimator and disturbance estimator and supervisor unit.

(6) Construct the control system as shown in Figure 3.

As seen in the above algorithm, the proposed approach however needs some observability conditions, for example, it cannot be used for the cases that $(\tilde{A}_a, \tilde{C}_a)$, $(\tilde{A}_s, \tilde{C}_s)$ and $(\tilde{A}_d, \tilde{C}_d)$ are not observable. Similar to other MPC approaches, the proposed approach needs an exact modelling for obtaining A_P and B_P in (1) and (2). Therefore, robustness can be a critical issue in this approach. Finally, it needs some trials and errors for choosing the parameters of MPC to guarantee the closed-loop stability and desired performance.

4. SIMULATION RESULTS

Simulation results are carried out to evaluate the proposed FTC approach based on MPC by using active magnetic bearing (AMB) system. AMB system is a nonlinear unstable two input-two output system that used in high speed motors and centrifuges that used for enrichment. Figure 4 shows the AMB system.

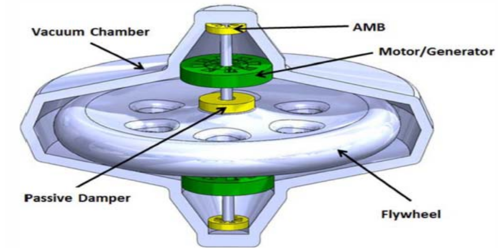

Fig. 4. Structure of Active Magnetic Bearing system.

The main objective is the control of axis and regulation beyond reference while avoiding instability due to input. The linear discrete-time model of AMB system with sampling interval 0.2 ms is identified as (Grimble and Johnson, 2009)

$$x_P(k+1) = A_P\, x_P(k) + B_P\, u(k) + B_1\, d(k) \quad (48)$$

$$y(k) = C_P\, x_P(k) \quad (49)$$

subject to $|\Delta u_i(k)| \leq 1$ and $|u_i(k)| \leq 2,\ i = 1,2$

where

$$A_P = \begin{bmatrix} 4.387 & 6.077 & 0.503 & -0.01 \\ -1.87 & -2.36 & -0.27 & -0.62 \\ -0.12 & -0.24 & 0.875 & 7.950 \\ 0.0004 & -0.000 & -0.0005 & 1.094 \end{bmatrix}$$

$$B_P = \begin{bmatrix} 0.031 & -0.00 \\ 0.054 & -0.00 \\ 0.004 & -0.00 \\ 0.00 & 0.06 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.002 & 0 \\ 0 & -0.002 \end{bmatrix}$$

$$C_P = \begin{bmatrix} 0.0265 & 0.0785 & 0.0009 & -0.024 \\ -0.049 & -0.073 & -0.024 & 0.110 \end{bmatrix}$$

and $d(k)$ is disturbance by step dynamic.

The simulation results of control of AMB system by MPC are shown in Figure 5, and the performance and computational load of classic MPC and MPC based on Laguerre functions are compared. In the classic MPC for reaching stability and acceptable performance, the parameters by trial and error are chosen as $N_u = 7$, $N_p = 15$, and in MPC based on Laguerre functions parameters are chosen as *N=2, a=0.9,* $N_p = 15$.

From Figure 5, it can be seen that classic MPC can achieve stability and acceptable performance by optimization of 7 parameters, while MPC based on Laguerre functions can achieve stability and desirable performance by optimization of 2 parameters. This can show low computational load in MPC based on Laguerre functions.

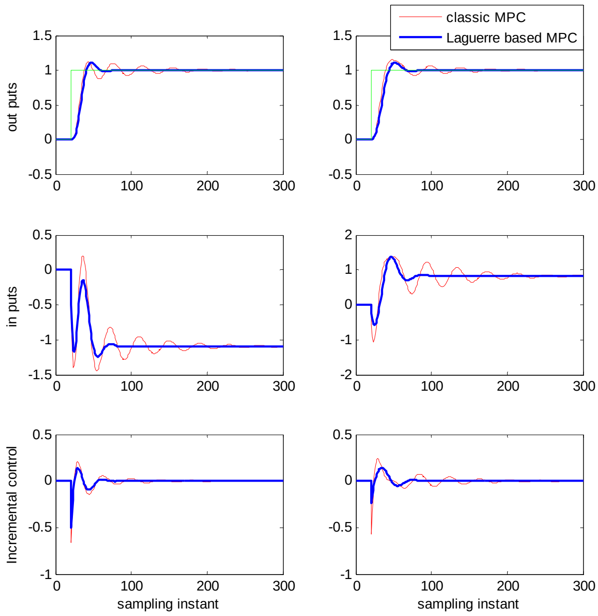

Fig. 5. Control of AMB system by MPC based on Laguerre functions and classic MPC.

In the following, two FTC approaches are compared. For this purpose, the performance of the proposed FTC approach based on MPC using Laguerre functions is compared with FTC based on classic MPC as presented in Qi Sun et al., 2008 under different fault scenarios.

Fault scenario #1

In the first fault scenario, simultaneous faults in actuator, sensor and plant are considered such that bias in actuator 1 and sensor 1 is equal to $u_{f_0} = 0.7$ and $y_{f_0} = 0.5$, fault in actuator 1 and sensor 1 is 50% and 60%, respectively and dynamic change is given by

$$A_{Pf} = A_P + \delta A_P \quad , \quad \delta A_P = \begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This change is occurred at instant *k=300*. The simulation results are shown in Figure 6. This figure compares FTC based on Laguerre based MPC with FTC based on classic MPC as presented in Qi Sun et al., 2008. As shown in Figure 6, the performance of the proposed FTC is more better than other FTC approach. Also, Figure 7 shows the loss of effectiveness factor of actuator and sensor estimated by fault/state estimator.

Fault scenario #2

In the second fault scenario, the failure of the actuator 1 is considered to show the capability of the proposed FTC approach in dealing with actuator failure. Suppose that the actuator 1 fails at instant *k=300*. The simulation results are shown in Figure 8. Also, Figure 9 shows the control loss of effectiveness factor of actuator 1 that estimated by fault/state estimator.

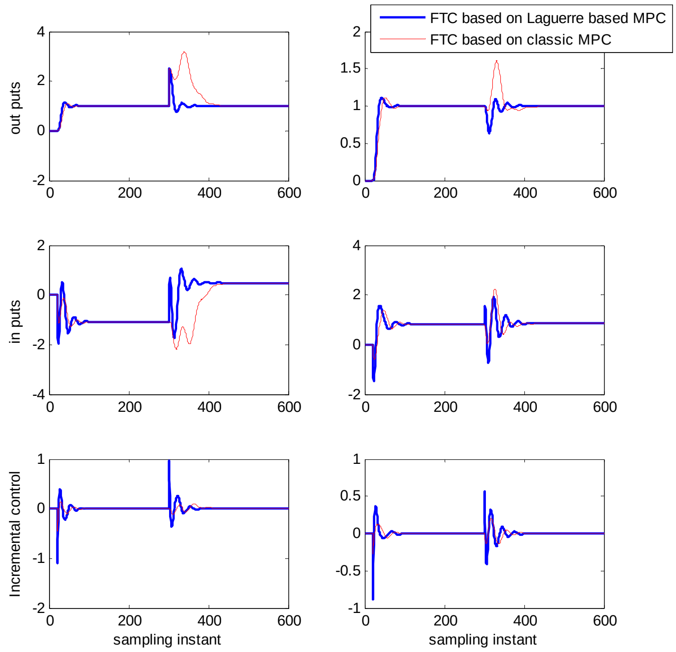

Fig. 6. Control of AMB system by FTC based on MPC in fault scenario #1.

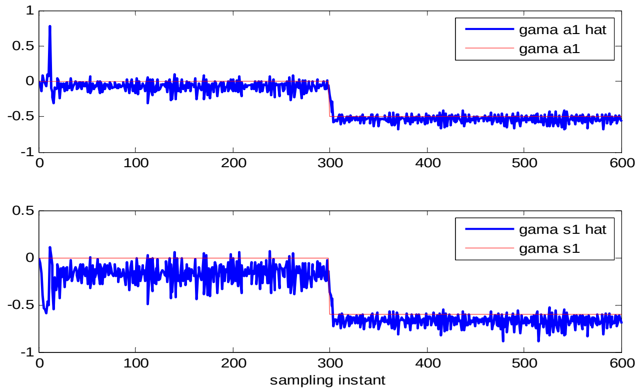

Fig. 7. Control and output loss of effectiveness factor estimated by Fault/State estimator in fault scenario #1.

Fault scenario #3

In the third fault scenario, the failure of the sensor 2 is considered. Assume that failure of the sensor 2 occurs at instant *k=300*. The simulation results are shown in Figure 10. By using proposed FTC, system is stable in the presence of the fault in sensor 2. It should be note that when fault sensor occurs, the observability of $(\tilde{A}_s\ ,\tilde{C}_s)$ is destroyed. In this scenario, we can assume that γ_s is known.

Simulation results in all fault scenarios show that the proposed FTC based on MPC using Laguerre functions accommodates the simultaneous faults in the closed-loop control system and can deal with constraints on control and incremental control with low computational load in comparison with other FTC based on classic MPC.

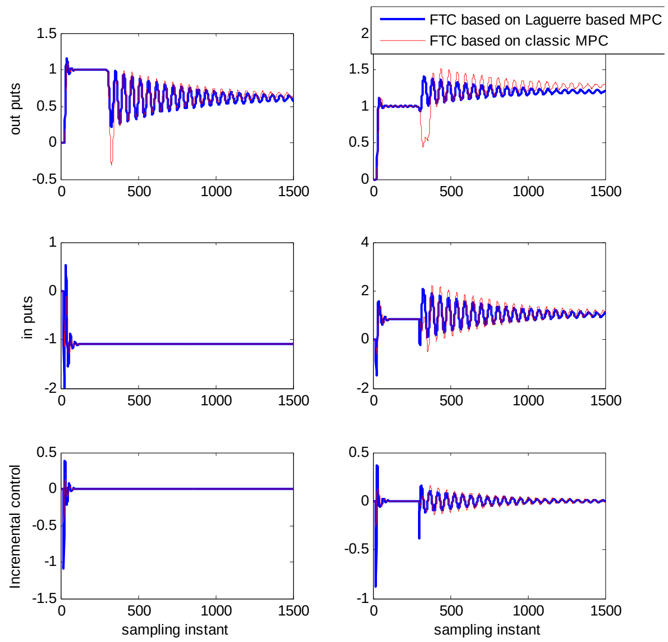

Fig. 8. Control of AMB system by FTC based on MPC in fault scenario #2.

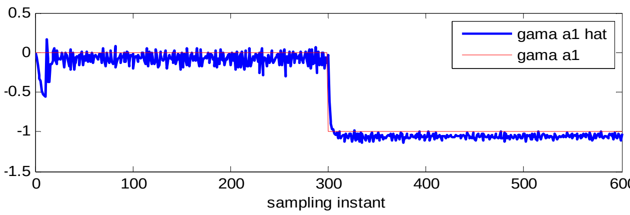

Fig. 9. Control effectiveness factor of actuator 1 estimated by Fault/State Estimator in fault scenario #2.

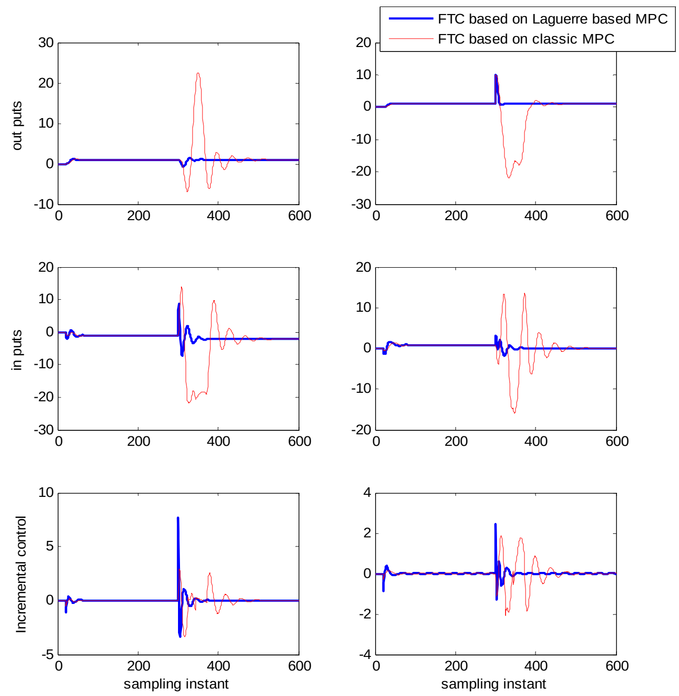

Fig. 10. Control of AMB system by FTC based on MPC in fault scenario #3.

5. CONCLUSIONS

In this paper, an active FTC approach based on MPC for a class of MIMO constrained linear systems has been presented. Based on fault estimation approach, the internal model of MPC is corrected in each sampling instant and then actuator and sensor faults are compensated. Also, by using EI and FC in MPC formulation, the bias faults of actuators and sensors can be compensated automatically. The advantages of the proposed FTC approach are that it is comprehensive in fault accommodation point of view because it is able to compensate all types of faults in control systems simultaneously, it has low computational load because of using MPC based on Laguerre functions and can handle control constraints to prevent of actuator saturations, it is able to deal with failures in actuators and sensors as well as bias fault without any additional work in comparison with other FTC approaches, and finally simplicity and effectiveness of the proposed FTC approach for real applications are more significant. In the future work, this approach will be extended to general MIMO constrained nonlinear systems.

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