Design of an optimized fractional order fuzzy PID controller for a piezoelectric actuator

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Abstract: This paper deals with piezoelectric actuators control. A fractional order fuzzy PID controller is designed for this class of systems with the help of particle swarm optimization (PSO) algorithm. Due to its special characteristics, the modeling and control of piezoelectric actuators has been one of the research topics for the last decade. In this work, the fractional order fuzzy PID controller parameters are defined as an optimization problem and the PSO is used to find their optimal values.

First, the dynamic model is introduced based on the Bouc-Wen model which is generally used with a second order linear model to describe piezoelectric actuators behavior. After giving its principle, the fractional order fuzzy PID is designed using the PSO algorithm. Finally, the proposed approach validation and their performances evaluation are done through simulation.

The major contribution of this paper is that the proposed controller structure is easy to implement, and has the same structure with the classical PID controller, but it gives better performances compared with classical PID and fuzzy PID controllers.

Keywords: Piezoelectric actuator control, Particle swarm optimization, Fractional order PID controller, fuzzy PID controller

1. INTRODUCTION

The piezoelectric actuators (PEA) based on the inverse piezoelectric effect are used in many fields due to their properties such as: high stiffness, small volume and fast response. They are widely used in the ultra-precision applications (Xu and Li, 2010; Zhang and Zhu, 1997; Haitjema, 1996). However, the hysteresis property, existing in piezoelectric materials, makes the modeling and the control of PEA difficult.

In order to simplify its analysis and control, many dynamic models are developed for piezoelectric actuators. In (Goldfarb and Celanovic, 1997; Adriaens et al., 2000), an electromechanical piezo-model is presented, where a first-order differential equation is adopted to describe the hysteresis effect, and a partial differential equation is used to describe the mechanical behavior.

In (Xie et al., 2009; Feng et al., 2010), the piezoelectric actuators behavior is identified as a second-order linear model preceded by hysteretic nonlinearity that described by the Duhem model.

In recent years, the PEA hysteresis modeling became the subject of many research tasks. In (Xiao and Li, 2013), a novel modified inverse Preisach model featured with weighed sum of $\mu$-density functions is proposed to compensate the hysteresis of a piezoelectric actuator at varying frequency ranges. In (Peng and Chen, 2012), a novel hysteresis operator modified from the Preisach hysteresis operator is presented where a rate-independent hysteresis model and a rate-dependent hysteresis model are developed with methods to estimate their parameters.

Authors in (Hui et al., 2011) present an identification method for dynamic hysteresis based on Duhem model. In (Bellmunt et al., 2009), the Bouc-Wen model is used to describe the PEA behavior and to control it where the proposed approach is tested by numerical simulations then experimentally. (Xiao and Li, 2014) proposes a novel modified inverse Bouc-Wen model for the dynamic compensation of the hysteresis nonlinearity. Furthermore, the experimentation showed that the hysteresis non-linearity in PEA is not symmetric and many models was proposed in (Aguirre et al., 2012; Zhua and Wang, 2012; Jiang et al., 2010) to describe the asymmetric hysteresis existing in PEA.

To compensate the hysteresis behavior of PEA, many intelligent techniques was used such as fuzzy logic (Li et al., 2013, 2010), neural networks (Zhang et al., 2009; Li and Chen, 2013), adaptive filter (Minase et al., 2010; Liu et al., 2013), minimum variance scheme(Rebai et al., 2014b), hybrid models (X. Zhanga, 2010), NARMAX models (Denga and Tan, 2009; Deng and Tan, 2008),
fractional order models (Rebai et al., 2014a) and iterative learning control (Liu et al., 2010).

To achieve the desired performances, many works in the literature deal with the problem of PEA control. The first category is the feed-forward and feedback techniques based on the inverse model of the PEA.

Indeed, authors in (Li et al., 2013) present a novel fuzzy system based method for modeling both rate-independent and rate-dependent hysteresis in piezoelectric actuators where a feed-forward controller is proposed.

In (Song et al., 2005), the inverse classical Preisach model is established and applied to cancel the hysteresis of PEA system for the real-time microposition tracking control. Also, a feedback controller is designed to improve the control accuracy and to increase damping of the actuator system. However, these strategies require the existence of the inverse model which is, generally, difficult to obtain.

The second category is based on the classical PID controller. In (Sung et al., 2008), A classical PID controller is designed and used to regulate the output displacement of a piezoelectric actuator that is linearized as second-order linear dynamic model.

Another research direction is based on sliding mode control (SMC). In (Wang and Liu, 2010), a fuzzy sliding mode controller for piezoelectric system with a sliding mode state estimator is introduced where the Bouc-Wen model is used to represent the hysteresis phenomenon.

(Liawa et al., 2007) presents an enhanced sliding mode motion tracking control methodology for an electromechanical model of PEA to track desired motion trajectories.

In (Liu et al., 2013), a second order sliding mode tracking controller is proposed where the simulation results showed the validity of the proposed method for this kind of nonlinear systems. In (Huang et al., 2009), An adaptive sliding mode controller for PEA with an experimental verification of the proposed technique is investigated.

Authors in (Peng and Chen, 2014) presents so-called PID sliding mode observer and its integration with PID sliding mode control for the PEA tracking control, simulations showed the performances of the proposed technique.

In (Huang et al., 2009), authors consider the adaptive control problem for piezoelectric actuators with nonlinear uncertainties, where the sliding mode control is used to achieve a good tracking performance of piezo-based motion systems in the presence of the uncertain hysteresis dynamics. Simulation and experimental results showed the effectiveness of the proposed controller.

Nevertheless, the major disadvantage of techniques based on the sliding mode control is the chattering phenomenon.

Other robust techniques are proposed in (Xiao and Li, 2014; Chuang and Petersen, 2010) based on $H\infty$. The main limit of the $H\infty$ strategies is obtaining ”an easy to implement” version of the control law.

To deal with this problem, fuzzy logic control (FLC) can be a solution. It has gained, during the last years, more attention and became a standard solution for wide variety of nonlinear and complex problems due to its design simplicity. A common used version of FLC is fuzzy PID which is characterized by a structure similar to classical PID and several works are published about this sort of controller. In (Pan et al., 2011), an optimal fuzzy PID is tuned using the genetic algorithm (GA) and the particle swarm optimization (PSO) techniques where the closed loop performances are compared. In (Saban and Volkan, 2010), a hybrid fuzzy PID controller with coupled rules is proposed and it illustrates more performances compared with classical PID, fuzzy logic and hybrid fuzzy PID controllers.

The use of fractional calculus in modeling and control of dynamical systems attracted, recently, more attention due to the advancements in computation power. This fact allows simulation and implementation of such systems with adequate precision. The control strategy using fractional calculus is based on the fractional order proportional integral derivative (FOPID) controller and is a generalization of a classical PID controller. Its output is a linear combination of the input and the fractional integrator derivative of the error.

Authors of (Das et al., 2012) combined FOPID controller with FLC to obtain fractional order fuzzy PID (FOFPID) where the parameters of the controller are obtained using the genetic algorithm technique and simulation results show that FOFPID produces better performances compared with conventional PID, fractional order PID, and fuzzy PID controllers.

Many optimization techniques have been proposed to find the optimal values of the FOFPID controller parameters. In (Sheng and Bao, 2013), fruit fly optimization algorithm is used to optimize a FOFPID controller for electronic throttle. In (Das et al., 2013a), a FOFPID is designed with real coded Genetic Algorithm to control the power level of a nuclear reactor at various operating conditions. In (Das et al., 2013b), Performance evaluation of optimal fractional order hybrid fuzzy PID controllers is presented. In this last study, the controller parameters are tuned using the genetic algorithm optimization technique. In (Sharma et al., 2014), Cuckoo Search Algorithm (CSA) optimization technique is used to tune the FOFPID for a robotic manipulator control purposes. Nevertheless, the implementation question remains the main problem of these techniques.

To deal with this drawback, others optimization techniques can be used to determine the controller gains. One of these techniques is the so-called particle swarm optimization (PSO) known by its combination of simplicity (in terms of implementation) with low computational cost and good performances (Lazinica, 2009; Song and Gu, 2004). Indeed, compared to genetic algorithms, PSO has some attractive characteristics. First, PSO has memory which utilizes the knowledge of good solutions retained by all particles, whereas in genetic algorithms, previous knowledge on the problem is destroyed once the population is updated. Second, PSO has constructive cooperation between particles and particles in the swarm that share their information (Hung et al., 2008). These are precisely the main motivations that led us to choose the PSO for FOFPID controller optimization.
In (Pan et al., 2012; Pan and Das, 2015), the FOFPID controller gains are tuned using the PSO algorithm and results showed the effectiveness of the proposed approaches. However, in both studies a Mamdani type of fuzzy logic controller (FLC) is used. This type suffers from many limitations from a control point of view. Indeed, compared to Takagi-Sugeno (TS) type of FLC, the Mamdani one is more difficult to implement and requires additional effort to choice and to dispatch fuzzy sets, for the conclusion part, on the output universe of discourse. Furthermore, the output of TS FLC can be easily expressed analytically and hence concepts such as stability can be proven.

This paper deals with the control of piezoelectric actuators and an optimized fractional order TS fuzzy PID controller is proposed. The use of PSO algorithm with TS structure of FLC is the main contribution of this study. The choice of this structure is motivated by its simplicity and the possibility to obtain a mathematically manipulable expression of the fuzzy controller output. Furthermore, the proposed controller structure is easy to implement, has the same structure of classical PID and ensures better performances.

This paper is structured as follows: in sect. 2, the mathematical model of the PEA is introduced. Then, the principle of the fractional order fuzzy PID is described in sect. 3. In sect. 4, the FOFPID is designed using the PSO algorithm. The simulation results are illustrated in sect. 5 to validate the proposed approach and to evaluate their performances.

2. MATHEMATICAL MODEL OF THE PEA

The most used model to describe the PEA dynamic behavior is the one developed in (Low and Guo, 1995). This model has received an increasing attention due to its ability to capture in an analytical form the hysteresis shape in piezoelectric actuators.

In this model, the hysteresis phenomenon and the piezo effect are separated.

The first one is represented by the Bouc-Wen model which is initiated by R. Bouc (Bouc, 1971), and generalized by Y. K. Wen (Wen, 1980).

The second one is described by a linear second order model as shown in Fig. 1.

![Fig. 1. PEA used model: Bouc-Wen model for hysteresis behavior and second order linear model for piezo effect](image)

The dynamic model of the PEA can be described by the following expressions:

\[ m\ddot{x} + c\dot{x} + kx = k_d u - h \]  \hspace{1cm} (1)

\[ h = \alpha_1 \dot{u} - \alpha_2 |\dot{u}| h - \alpha_3 \ddot{u} |h| \]  \hspace{1cm} (2)

where \( x \) and \( u \) denote, respectively, the displacement of the PEA and the applied voltage, \( m \), \( c \), \( k \) and \( d_e \) are the mass, damping, stiffness and effective piezoelectric coefficient, respectively. \( h \) is the output of the Bouc-Wen hysteresis model. \( \alpha_1, \alpha_2 > 0 \) and \( \alpha_3 < 0 \) are parameters that affect the shape of the hysteresis nonlinearity. For more details on this model, the reader can refer to (Low and Guo, 1995).

The model parameters used in this paper are shown in Table 1. These parameters are obtained by the excitation of the PEA by a sinusoidal signal of frequency 10Hz: \( u(t) = 100\sin(10t) \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>1.595 x 10^{-2}</td>
<td>kg</td>
</tr>
<tr>
<td>( c )</td>
<td>1.169</td>
<td>Ns/m</td>
</tr>
<tr>
<td>( k )</td>
<td>3.197 x 10^{3}</td>
<td>N/m</td>
</tr>
<tr>
<td>( d_e )</td>
<td>1.014 x 10^{-6}</td>
<td>m/V</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>4.357 x 10^{-7}</td>
<td></td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>3.438 x 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>-2.865 x 10^{-3}</td>
<td></td>
</tr>
</tbody>
</table>

3. FRACTIONAL ORDER FUZZY PID

In this section, the principle of the proposed fractional order fuzzy PID (FOFPID) is described.

The proposed structure of the FOFPID is given in Fig. 2. The parameters \( \{K_e, K_{d_e}, \alpha, \beta, \mu, \lambda\} \) shown in Fig. 2 are to be determined and optimized, using the particle swarm optimization algorithm (PSO), in the next section.

As shown in Fig. 2, the input variables of the FLC are the error \( e \) and the error fractional differentiation \( \frac{de}{dt^\alpha} \). The error signal is defined as:

\[ e(t) = r(t) - x(t) \]  \hspace{1cm} (3)

where \( r(t) \) is the reference signal, \( x(t) \) is the displacement of the PEA.

The TS structure (Passino and Yurkovich, 1998) is adopted for the fuzzy controller. The choice of this structure is motivated by its simplicity and the possibility to obtain a manipulable mathematical expression of the FLC output. The control strategy can be defined by rules of the form

\[ \text{If } e \text{ is } A \text{ and } \frac{de}{dt^\alpha} \text{ is } B \text{ then } du \text{ is } C \]

where \( e \) and \( \frac{de}{dt^\alpha} \) are the inputs of the FLC, \( du \) is its output. \( A, B \) and \( C \) are the fuzzy sets.

The rules base of the fuzzy controller is shown in Fig. 3. The linguistic variables NVB, NB, NM, NS, ZE, PS, PM, PB, PVB shown in Fig. 3 mean Negative Very Big, Negative Big, Negative Medium, Negative small, Zero, Positive Small, Positive Medium, Positive Big and Positive Very Big, respectively. The membership functions of the fuzzy controller are illustrated in Fig. 4 and Fig. 5, respectively. For the input variables, the NVB and PVB linguistic variables are trapezoid membership functions and the others are symmetric triangles. For the FLC output, the membership functions are singletons. The control surface describing the input-output relationship of the FLC is given in Fig. 6.
4. DESIGN OF THE OPTIMAL FRACTIONAL ORDER FUZZY PID CONTROLLER

In this section the optimal values for parameters of the FOFPID are determined after the description of the PSO algorithm.

The particle swarm optimization is one of the modern heuristic algorithms which is developed by Kennedy and Eberhart in 1995 (Kennedy and Eberhart, 1995). It uses the metaphor of the flocking behavior of birds to solve optimization problems (Boussad et al., 2013). This technique is based on the following principle: a population of particles in the swarm is first initialized, then, the particle locations are updated using the position and velocity equations given by

\[ v_{id}(j + 1) = wv_{id}(j) + c_1r_1(P_{id} - x_{id}) + c_2r_2(P_{gd} - x_{id}) \]  \hspace{1cm} (4)

\[ x_{id}(j + 1) = x_{id}(j) + v_{id}(j + 1) \]  \hspace{1cm} (5)

where \( j = 1, \ldots, n \) and \( i = 1, \ldots, d \). The technique parameters and their correspond significations are summarized
in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Signification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>population size</td>
</tr>
<tr>
<td>$d$</td>
<td>problem dimension</td>
</tr>
<tr>
<td>$v_i$</td>
<td>velocity of the particle $i$ at the iteration $j$</td>
</tr>
<tr>
<td>$x_i$</td>
<td>position of the particle $i$ at the iteration $j$</td>
</tr>
<tr>
<td>$P_i$</td>
<td>local best position of the particle $i$</td>
</tr>
<tr>
<td>$P_g$</td>
<td>global best position of the swarm</td>
</tr>
<tr>
<td>$w$</td>
<td>inertia factor</td>
</tr>
<tr>
<td>$c_1, c_2$</td>
<td>acceleration factors</td>
</tr>
<tr>
<td>$r_1, r_2$</td>
<td>random numbers uniformly disturbed between 0 and 1</td>
</tr>
</tbody>
</table>

From the optimal FOFPID scheme given in Fig. 7, five parameters $\{K_p, K_{de}, \alpha, \beta, \mu, \lambda\}$ are required to be designed. The single input single output (SISO) systems can be transformed into the following form:

$$
\dot{z} = f(z, u, p) \\
x = g(x, p)
$$

(6)

where $u, x$ and $z$ denote the control input, the piezoelectric actuator displacement and state vector, respectively. $p$ is a vector which contain the unknown parameters of the controller.

The fitness function is defined as the Integral of the Squared Error (ISE) given by

$$
\min_{p} J_t = \frac{1}{t} \int_{0}^{t} e^2(t) dt = \sum_{i=0}^{t/h} (r(i) - x(i))^2
$$

(7)

where $r, x, t, h$ are the desired output displacement, actual output of the PEA, the integration time and the sampling period, respectively.

The steps of the PSO algorithm applied to the FOPID controller of a piezoelectric actuator system are given as follows:

- **Step 1**: choice of the algorithm parameters (search space dimension $d$, population size $n$, number of iterations...)
- **Step 2**: swarm initialization with a population of random solutions. Each particle in the swarm is a random possible set of the unknown parameters to be optimized.
- **Step 3**: fitness function evaluation to determine whether the best fitting solution is achieved.
- **Step 4**: set the best of the local best positions as the global best position.
- **Step 5**: update the particles position and velocity according to equations 4 and 5.
- **Step 6**: update the global best position from the obtained results in Step 5.
- **Step 7**: verify the stop criterion which is the iterations number. The steps 2 to 7 are repeated until the verification of the stop criterion.

The global PSO algorithm flowchart is shown in Fig. 8.

5. SIMULATION RESULTS

In this section the simulation results are presented. The FOFPID controller and the PSO algorithm are imple-
As can be shown in Table 6, the proposed approach has a fast response capability with a small fitness function and little overshoot. These performances can be improved using other fitness functions that take into account the overshoot and the rise time.

<table>
<thead>
<tr>
<th></th>
<th>ISE value</th>
<th>overshoot (%)</th>
<th>rise time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>$8.24 \times 10^{-4}$</td>
<td>-</td>
<td>0.64</td>
</tr>
<tr>
<td>FPID</td>
<td>$5.19 \times 10^{-7}$</td>
<td>25.6</td>
<td>0.077</td>
</tr>
<tr>
<td>FOFPID</td>
<td>$3.69 \times 10^{-7}$</td>
<td>5.84</td>
<td>0.079</td>
</tr>
</tbody>
</table>

6. CONCLUSION

A PSO-optimized fractional order fuzzy PID controller for piezoelectric actuators is presented in this paper. The controller parameters are determined using the PSO algorithm with the ISE fitness function. The simulations results validated the proposed approach and showed that
Step 1: PSO parameters selection

Step 2: Particles initialization

Step 3: Fitness function evaluation

Step 4: Set the best of the local best positions as the global best position

Step 5: Update the particles position and velocity

Step 6: Update the global best position

Step 7: The stop criterion

Step 8: Get the optimal values of the FOFPID parameters

Fig. 8. PSO algorithm flowchart
Fig. 14. Control input of the piezoelectric actuator system

the proposed controller presents better performances compared to well-tuned classical PID and fuzzy PID
controllers. The obtained performances can be enhanced more by taking into account other criteria in the fitness function and a trade-off between complexity and performances should be made.

REFERENCES


