CEAI, Vol.17, No.2 pp. 20-31, 2015 Printed in Romania

A New Robust Pole Placement Stabilization for a Class of Time-Varying Polytopic Uncertain Switched Nonlinear Systems under Arbitrary Switching

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Abstract: This paper is concerned with the problem of robust stabilization via state feedback control for a class of both continuous and discrete-time switched nonlinear systems with polytopic time-varying uncertainty. These studied systems are modeled by differential or difference equations. Therefore, a transformation of the systems representation under the arrow form is performed. Subsequently, by using a constructed common Lyapunov function and applying the Kotelyanski lemma associated with the $M-\text{matrix}$ properties. A new robust pole placement stabilization is proposed. These obtained results provide a solution to one of the basic problems for switched nonlinear systems which ensures asymptotic stability under arbitrary switching. Compared with the existing results of uncertain switched nonlinear systems, these proposed conditions are formulated in terms of the time-varying polytopic uncertain parameters and they allow us to avoid searching a common Lyapunov function which is a difficult matter. Finally, an application to stabilize a shunt DC motor with uncertain models under variable mechanical loads is performed to illustrate the effectiveness of the theoretical results.

Keywords: Switched nonlinear systems, polytopic time-varying uncertainty, common Lyapunov function, Kotelyanski lemma, state feedback control, pole placement, asymptotic stability, arbitrary switching.

1. INTRODUCTION

As one of the important class of hybrid systems, switched dynamic systems can be modeled by a family of continuous or discrete-time subsystems and a rule, called a switching signal that determines the switching manner between the subsystems rule. Mathematically, these subsystems are usually described by a collection of indexed differential or difference equations. The main motivation for studying switched systems comes from the fact that many practical systems evolve with switches and hybrid behaviors such as chemical processes (Putyrski et al., 2011), transportation systems (Hamdouch et al., 2007), power systems and power electronics (Sengupta et al., 2009), communication networks (Alnowibet et al., 2006), constrained robotics and robot manufacture (Back et al., 1993), computer disk drives (Gollu et al., 1989), and automated highways (Varaiya, 1993).

Stability and stabilization are two fundamental and important research issues in the control community. Several methods have been developed for solving these problems of switched systems (Vu et al., 2005; Araghi, et al., 2013; Zhao et al., 2012; Hespanha et al., 1999; Zhao et al., 2008), such as the common Lyapunov function approach (Vu et al., 2005) which is mainly investigated for stability under arbitrary switching, the average dwell time (Zhao *et al*., 2012), and the multiple Lyapunov functions method (Hespanha et al., 1999; Zhao et al., 2008) for studying stability under controlled switching. Hence, stability and stabilization under arbitrary switching which are considered in this work remain more performed when practical systems are involved. Indeed, the unique practically applicable approach to this problem is based on the construction of a common Lyapunov function for all the subsystems. Therefore, this method is usually very difficult to apply even for switched linear systems. However, it becomes more complicated when switched nonlinear systems are involved and relatively few results have been reported in this context (Yu et al., 2011; Yu et al., 2012; Dayawansa et al., 1999; Mancilla et al., 2000; Liberzon, 2004). So far, some attempts are presented to construct a general Lyapunov function for switched nonlinear systems, by using the Lyapunov converse theorems in (Dayawansa et al., 1999; Mancilla et al., 2000) and by recourse to some nilpotent Lie algebras (Liberzon, 2004).

On the other hand, from the practical viewpoint, it is of great importance to investigate uncertain switched systems. Norm-bounded and polytopic uncertainties are two commonly adopted schemes, and the latter one has been proved more general offering solution form any practical applications (Zhang et al., 2007; Daafouz et al., 2002; Zhang et al., 2008). In our investigation, considering switched nonlinear systems, we adopt time-varying polytopic uncertainties which depict strong practical significance of this work.

Based on the above discussion, switched nonlinear systems with polytopic uncertainties are worth studying. Up to now, due primarily to the complexity of this problem, the available results on stability analysis and stabilization of these systems are limited (Chiang et al., 2014; Shipei, et al., 2013; Weiming et al., 2014; Orani et al., 2011; Wang et al., 2009). These shortcomings motivate this study.

The purpose of this paper aims to solve the problem of state feedback stabilization for a class of both continuous and discrete-time switched nonlinear systems under arbitrary switching and subject to time-varying polytopic uncertainties. Indeed, by transforming the representation of these considered systems into the arrow form matrix (Kermani et al., 2014 a; Kermani et al., 2014 b; Elmadssia et al., 2013; Zhang et al., 2008; Borne et al., 1993; Kermani et al., 2012 a; Kermani et al., 2012 b; Benrejeb et al., 2008; Benrejeb et al., 2006; Borne et al., 2003; Borne et al., 2007; Benrejeb et al., 1982; Borne, 1987; Hahn; 1967; Grujc et al., 1987; Borne et al., 1972; Borne et al., 2008), *employing* a suitable constructed common Lyapunov function and applying the Kotelyanski lemma (Kotelyanski, 1952) combined with the $M-$matrix properties (Robert, 1966; Gantmacher, 1966), a new robust *pole placement stabilization* under arbitrary switching is deduced.

In contrast with some existing results on switched nonlinear systems with polytopic uncertainties, the contributions of this paper is twofold: First, the new pole placement design is given to guarantee the closed-loop system asymptotic stability under arbitrary switching and may overcome the conservatism of searching a common Lyapunov function. Second, the new stabilization conditions are formulated in terms of the time-varying polytopic uncertain parameters.

The rest of this paper is organized as follows. In section 2, we present the problem formulation and some preliminaries. Then, we give the main results on stabilization of the considered continuous-time switched nonlinear systems. The research problem formulations and the main results of the studied discrete-time switched systems are given in section 3. Two examples which model a shunt DC motor with uncertain models under variable mechanical loads are provided to show the effectiveness of the proposed approach. The conclusions are summarized in section 5.

Notations: The following notation will be used in this paper, $\Re^n$ denotes the n dimensional Euclidean space, I_n is the identity matrix with appropriate dimensions, $\|.\|$ denotes Euclidean vector norm. For any $u=(u_i)_{1\le i\le n}$, $v=(v_i)_{1\le i\le n}\in\Re^n$ We define the scalar product of the vector u and v as: $\langle u,v\rangle=\sum_{i=1}^{n}u_iv_i$. Denote by $\lambda(M)$ the set of eigenvalues of matrix M, M^T its transpose and M^{-1} its inverse. If $M=(m_{i,j})_{1\le i,j\le n}$, we denote $M^*=(m^*_{i,j})_{1\le i,j\le n}$ with $m^*_{i,j}=m_{i,j}$ if $i=j$ and $m^*_{i,j}=|m_{i,j}|$ if $i\ne j$, $|M|=|m_{i,j}|$, $\forall i,j$ and $\underline{N}=\{1,2,..,N\}$.

2. CONTINOUS-TIME SWITCHED SYSTEMS

2.1 Problem statement and preliminaries

Consider a class of continuous-time uncertain switched nonlinear systems of the form:

$$\dot{x}(t)=A_{\sigma(t)}(.)x(t)+B_{\sigma(t)}(.)u(t) \tag{1}$$

where $x(t)\in\Re^n$ is the state, $u(t)\in\Re$ is the control input, $A_{\sigma(t)}(.)$ and $B_{\sigma(t)}(.)$ are matrices with nonlinear elements of appropriate dimensions and $\sigma(t)$: $\Re^+\to\underline{N}=\{1,2,..,N\}$ is the switching signal assumed to be available in real time. Therefore, the continuous-time switched system is composed of N subsystems which are expressed as:

$$\dot{x}(t)=A_i(.)x(t)+B_i(.)u(t),\ i\in\underline{N} \tag{2}$$

where $A_i(.)$ and $B_i(.)$ are matrices with appropriate dimensions.

The system matrices are subject to polytopic uncertainties which can be modeled as: $A_i(.)=\sum_{l=1}^{N_l}\mu_{il}(t)A_{il}(.)$ and $B_i(.)=\sum_{l=1}^{N_l}\mu_{il}(t)B_{il}(.)$, where $A_{il}(.)(l=1,..., N_l)$ and $B_{il}(.)(l=1,..., N_l)$ are the vertex matrices denoting the extreme points of the polytope $A_i(.)$ and $B_i(.)$, N_l is the number of the extreme points and the weighting factors $\mu_{il}(t)$ $(l=1,..., N_l)$ are time-variant uncertainties which belong to:

$$\mu_i(t):\ \sum_{l=1}^{N_l}\mu_{il}(t)=1,\ \mu_{il}(t)\ge 0 \tag{3}$$

When the controllers are switched between the subsystems, the state feedback controllers are formed as:

$$u(t)=-K_{\sigma(t)}(.)x(t) \tag{4}$$

where $K_i(.)$ are the nonlinear controller gains.

Now, we first briefly review some preliminaries.

Kotelyanski lemma. (Kotelyanski, 1952) The real parts of the eigenvalues of matrix A, with non-negative off-diagonal elements, are less than a real number μ if and only if all those of matrix M ; $M=\mu I_n-A$ are positive, with I_n the n identity matrix.

The following definitions and remarks will be used in the sequel.

Definition 1. The matrix $A=(a_{ij})_{1\le i,j\le n}$ is called an $M-$matrix , if the following conditions are met:

- The principal minors of A are all positive:
$$(A)\begin{pmatrix}1\ 2\ \dots\ j\\ 1\ 2\ \dots\ j\end{pmatrix} > 0 \quad \forall j \in 1,\dots,n \tag{5}$$
- For any positive real numbers $\eta = (\eta_1,\dots,\eta_n)^T$ the algebraic equations $Ax = \eta$ have a positive solution $w = (w_1,\dots,w_n)^T$.

Remark 1. $A = (a_{ij})_{1\le i,j\le n}$ is the opposite of an $M-$matrix if $(-A)$ is an $M-$matrix.

Remark 2. A continuous-time system characterized by $A(.)$ is stable if $A(.)$ is the opposite of an $M-$matrix. In this case, the main minors of $A(.)$ are alternating sign with the first is negative and the Kotelyanski lemma allows us to conclude about stability of the system characterized by $A(.)$.

Below, we present the definition of an overvaluing system.

Definition 2. (Borne et al., 2008) The matrix $M_c(.)$ is said a pseudo-overvaluing matrix of the system $\dot{x}(t) = A_i(.)x(t)$ with respect to the vector norm $p(x) = [|x_1| \ \dots\ |x_n|]^T$, if the following inequality is met:
$$D^+ p(x) \le M_c(.)\,p(x) \tag{6}$$
where D^+ denotes the right hand derivative.

Consequently, the stability of the comparison system: $\dot{z}(t) = M_c(.)z(t)$ with the initial conditions such as $z_0 = p(x_0)$, implies the same property for the initial system.

2.2. Continuous-time switched systems: main results

In this section, we give the main result of the closed-loop system (1).

Theorem 1. System (1) with polytopic uncertainties (3) is globally robust asymptotically stablilizable with the state feedback controller (4) under arbitrary switching rule (1) if $M_c(.)$ is the opposite of an $M-$matrix, with:
$$M_c(.) = \max_{1\le i\le N}\left(\left(A_i^c(.)\right)^*\right) \tag{7}$$
where:
$$A_i^c(.) = \sum_{l=1}^{N_l}\mu_{il}(t)\left(A_{il}(.) - B_{il}(.)K_{il}(.)\right)$$
$$= \begin{bmatrix} \sum_{l=1}^{N_l}\mu_{il}(t)\left(a_i^{c(11)}(.)\right) & \dots & \dots & \sum_{l=1}^{N_l}\mu_{il}(t)\left(a_i^{c(1n)}(.)\right)\\ \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots\\ \sum_{l=1}^{N_l}\mu_{il}(t)\left(a_i^{c(n1)}(.)\right) & \dots & \dots & \sum_{l=1}^{N_l}\mu_{il}(t)\left(a_i^{c(nn)}(.)\right)\end{bmatrix} \tag{8}$$

Proof. Let $w \in \Re^n$ with components $(w_m > 0, \forall m = 1,\dots,n)$ and $x(t) \in \Re^n$ is the state vector.

Define the radially unbound common Lyapunov function below for the closed-loop system (1) with polytopic uncertainties (3):
$$V(x(t),t) = \langle |x(t)|, w\rangle \tag{9}$$

It is clear that $V(x(t_0),t_0) < \infty$.

The right hand derivative of $V(x(t),t)$ along the trajectory of the closed-loop system (1) under the switching signal $\sigma(t)$ is given as follows:
$$D^+V(x(t),t) = \left\langle \frac{d^+|x(t)|}{d^+t}, w\right\rangle = \left\langle \operatorname{sgn}(x(t))\frac{d^+x(t)}{d^+t}, w\right\rangle \tag{10}$$
where:
$$\operatorname{sgn}(x(t)) = \begin{pmatrix}\operatorname{sgn}(x_1(t)) & & \\ & \ddots & \\ & & \operatorname{sgn}(x_n(t))\end{pmatrix} \tag{11}$$

In this way, we obtain:
$$D^+V(x(t),t) = \left\langle \operatorname{sgn}(x(t))\left(A_{\sigma(t)}^c(.)\right)x(t), w\right\rangle$$
$$\le \left\langle \left(A_{\sigma(t)}^c(.)\right)^* |x(t)|, w\right\rangle < \langle M_c(.)|x(t)|, w\rangle \tag{12}$$

To complete this proof, we assume that $M_c(.)$ is the opposite of an $M-$matrix. Therefore, we can find a vector $\rho \in \Re_+^{*n}\ (\rho_m \in \Re_+^*\ \ m = 1,\dots,n)$ satisfying that $(M_c(.))^T w = -\rho$, $\forall w \in \Re_+^{*n}$.

Hence, we obtain:
$$\langle (M_c(.))|x(t)|, w\rangle = \left\langle (M_c(.))^T w, |x(k)|\right\rangle = \langle -\rho, |x(t)|\rangle \tag{13}$$

Taking into account (13), relation (12) becomes:
$$D^+V(x(t),t) < \langle -\rho, |x(t)|\rangle = -\sum_{m=1}^{n}\rho_m |x(t)| < 0 \tag{14}$$

This completes the proof of Theorem 2.

2.3. Application to uncertain switched systems defined by differential equations

In this section, a new state feedback stabilization design is presented for a class of continuous-time uncertain switched nonlinear systems described by N subsystems. All the subsystems are modeled by a family of differential equations given as below:
$$y^n(t) + \left(\sum_{j=0}^{n-1}\sum_{l=1}^{N_l}\mu_{il}(t)a_{il}^{n-j}(.)\,y^{(j)}(t)\right) = u(t) \tag{15}$$

where $y(t) \in \Re^n$, $a_{il}^j(.)$ are nonlinear coefficients for each $i \in \underline{N}$, $(l = 1,..., N_l)$ and $(j = 0,..., n-1)$. $\mu_{il}(t)$ are time-varying polytopic uncertain parameters given in (3) and $u(t) \in \Re$ is the control input.

Consider the following state variables for system (15):

$$x_{j+1}(t) = \frac{dy^{(j)}}{dt^{(j)}}, \; j = 0,...,n-1 \tag{16}$$

By substituting (16) into (15), we obtain:

$$\begin{cases} \dot{x}_j(t) = x_{j+1}(t), \; j = 0,...,n-1 \\ \dot{x}_n(t) = \left(-\sum_{p=0}^{n-1} \sum_{l=1}^{N_l} \mu_{il}(t) a_{il}^{n-j}(.) x_{j+1}(t) \right) + u(t) \end{cases} \tag{17}$$

or under matrix representation, we obtain:

$$\dot{x}(t) = \sum_{l=1}^{N_l} \mu_{il}(t) \big(A_i(.) x(t) + B_i(.) u(t) \big), \; i \in \underline{N} \tag{18}$$

Due to (17), it is easy to see that system (18) is given in the controllable form such as:

$$A_{il}(.) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_{il}^n(.) & -a_{il}^{n-1}(.) & \cdots & -a_{il}^1(.) \end{bmatrix}, \; B_{il} = B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{19}$$

Led to the following state feedback controller:

$$u(t) = -\sum_{l=1}^{N_l} \mu_{il}(t) K_{il}(.) x(t) \tag{20}$$

where $K_{il}(.)$ are the vectors gains of the controller for each $i \in \underline{N}$, and $(l = 1,..., N_l)$.

So, all the closed-loop subsystems are characterized by the following state space description:

$$\begin{aligned} \dot{x}(t) &= \sum_{l=1}^{N_l} \mu_{il}(t) \big(A_{il}(.) - B_{il}(.) K_{il}(.) \big) x(t) \\ &= \sum_{l=1}^{N_l} \mu_{il}(t) A_{il}^c(.) x(t), \; i \in \underline{N} \end{aligned} \tag{21}$$

with:

$$A_{il}^c(.) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_{il}^{c(n)}(.) & -a_{il}^{c(n-1)}(.) & \cdots & -a_{il}^{c(1)}(.) \end{bmatrix} \tag{22}$$

where:

$$a_{il}^{c(j)}(.) = a_{il}^j(.) + k_{il}^{n+1-j}(.), \; (j = 1,..., n) \tag{23}$$

and $a_{il}^{c(j)}(.)$ are the coefficients of the instantaneous characteristic polynomial $P_{A_{il}(.)}(\lambda)$ of the vertex matrix $A_{il}^c(.)$. It is given by:

$$P_{A_{il}(.)}(\lambda) = \lambda^n + \sum_{p=0}^{n-1} a_{il}^{c(n-p)}(.) \lambda^p \tag{24}$$

Now, by considering the switching signal (1), the closed-loop switched nonlinear system is deduced as below:

$$\dot{x}(t) = A_{\sigma(t)}^c(.) x(t) \tag{25}$$

Next, the following basic change will be adopted in order to simplify the application of the Kotelyanski lemma:

$$\dot{z}(t) = M_i(.) z(t), \; i \in \underline{N} \tag{26}$$

where $z(t) = Px(t)$ is the new state vector, P is the corresponding passage matrix and $M_i(.) = \sum_{l=1}^{N_l} \mu_{il}(t) M_{il}(.)$ where $M_{il}(.)$ are vertex matrices in the arrow form given as follows:

$$M_{il}(.) = P^{-1} A_{il}^c(.) P = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma_{il}^1(.) & \cdots & \cdots & \gamma_{il}^{n-1}(.) & \gamma_{il}^n(.) \end{bmatrix} \tag{27}$$

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ \alpha_1 & \alpha_2 & \dots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \dots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \dots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \dots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix} \tag{28}$$

$$\begin{cases} \beta_j = \prod_{\substack{q=1 \\ q \neq j}}^{n-1} (\alpha_j - \alpha_q)^{-1} \quad \forall \; j = 1,...,n-1 \\ \gamma_{il}^j(.) = -P_{A_{il}^c(.)}(\alpha_j) \quad \forall \; j = 1,..,n-1 \\ \gamma_{il}^n(.) = -a_{il}^{c(1)}(.) - \sum_{j=1}^{n-1} \alpha_j \end{cases} \tag{29}$$

and α_j, $j = 1...n-1$ are distinct arbitrary constant parameters.

Next, the comparison system associated to the vector norm $p(z) = [|z_1| \; ... \; |z_n|]^T$ is defined by:

$$\dot{z}(t) = M_c(.) z(t) \tag{30}$$

where $M_c(.)$ is the comparison matrix relative to system (25), it is deduced as below:

$$M_c(.)=\begin{bmatrix}\alpha_1 & 0 & \cdots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & |\beta_{n-1}| \\ \overline{t}(.)^1 & \cdots & \cdots & \overline{t}(.)^{n-1} & \overline{t}(.)^n\end{bmatrix} \tag{31}$$

with:

$$\begin{cases}\overline{t}^n(.)=\max\limits_{1\le i\le N}\left(\sum\limits_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^n(.)\right) \\ \overline{t}^j(.)=\max\limits_{1\le i\le N}\left(\left|\sum\limits_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^j(.)\right|\right),\ j=1,...,n-1\end{cases} \tag{32}$$

The aim of this work consists to design a state feedback controller (20) by using the pole placement control which guarantees asymptotic stability of the closed loop-system (25).

Next, the following theorem presents a new pole placement control stabilization for system (25).

Theorem 2. If all the n poles $\{p_1,...,p_n\}$ are chosen to be real, distinct and negative. Then, the closed-loop system (25) with polytopic uncertainties (3) is robust stabilizing by the controller (20) under arbitrary switching (1) and the following conditions are satisfied:

$$\begin{cases}\overline{t}^j(.)=\max\limits_{1\le i\le N}\left(\left|\sum\limits_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^j(.)\right|\right)=0,\ j=1,...,n-1 \\ \overline{t}^n(.)=\max\limits_{1\le i\le N}\left(\sum\limits_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^n(.)\right)=p_n<0\end{cases} \tag{33}$$

where:

$$\begin{cases}\beta_j=\prod\limits_{\substack{q=1\\ q\ne j}}^{n-1}\left(p_j-p_q\right)^{-1}\quad \forall\ j=1,...,n-1 \\ \gamma_{il}^j(.)=-P_{A_{il}(.)}\left(p_j\right)\quad \forall\ j=1,..,n-1 \\ \gamma_{il}^n(.)=-a_{il}^{c(1)}(.)-\sum\limits_{j=1}^{n-1}p_j\end{cases} \tag{34}$$

Proof of this theorem is given in appendix A.

3. DISCRETE-TIME SWITCHED SYSTEMS

3.1 Problem formulation and Preliminaries

Consider the following discrete-time switched nonlinear systems:

$$x(k+1)=A_{\sigma(k)}(.)x(k)+B_{\sigma(k)}(.)u(k) \tag{35}$$

where $x(k)\in\Re^n$ is the state vector of the system at time k, $u(k)\in\Re$ is the control input, $A_{\sigma(k)}(.)$ and $B_{\sigma(k)}(.)$ are matrices with have nonlinear elements of appropriate dimensions and $\sigma(k):\Re^+\to \underline{N}=\{1,2,..,N\}$ is the switching signal.

The switched system is composed of N discrete-time subsystems which are given by:

$$\dot{x}(t)=A_i(.)x(k)+B_i(.)u(k),\ i\in\underline{N} \tag{36}$$

when the uncertain model is presented, the matrices $A_i(.)$ and $B_i(.)$ are given by $A_i(.)=\sum\limits_{l=1}^{N_l}\mu_{il}(k)A_{il}(.)$ and $B_i(.)=\sum\limits_{l=1}^{N_l}\mu_{il}(k)B_{il}(.)$, where $\mu_{il}(k)$ $(l=1,...,N_l)$ are time-variant uncertainties given in (3).

The state feedback controller is given as below:

$$u(k)=-K_{\sigma(k)}x(k) \tag{37}$$

Now, we present some definitions and remarks which will play important roles in deriving our main results for discrete-time switched systems subsequently.

Remark 3. (Borne, et al., 2007) A discrete-time system given by a matrix $A(.)$ is stable if matrix $(I_n-A(.))$ verified the Kotelyanski conditions. In this case $(I_n-A(.))$ is an $M-$matrix and all the principal minors of $(I_n-A(.))$ are positive.

Definition 3. (Benrejeb, et al., 2006) The matrix $M_D(.)$ is the comparison matrix of the system given by a matrix $A(.)$ with respect to the vector norm p if the inequality below is satisfied:

$$p(x(k+1))\le M_D(.)\,p(x(k)) \tag{38}$$

Then, the stability of the comparison system: $z(k+1)=M_D(.)z(k)$ with the initial conditions such as $z_0=p(x_0)$ implies the same property for the initial system.

3.2 Discrete-time switched systems-main results

This subsection presents a new state feedback stabilization of the closed-loop system (35).

Theorem 3. The closed-loop system (35) is robustly asymptotically stablilizable with the state feedback controller (37) under arbitrary switching (35) and all admissible uncertainties (3) if (I_n-M_D) is an $M-$matrix, with:

$$M_D(.)=\left(\max_{1\le i\le N}\left(\left|A_i^c(.)\right|\right)\right) \tag{39}$$

and:

$$A_i^c(.) = \sum_{l=1}^{N_l} \mu_{il}(k)\left(A_{il}(.) - B_{il}(.)K_{il}(.)\right)$$

$$= \begin{bmatrix} \sum_{l=1}^{N_l} \mu_{il}(k)\left(a_i^{c(11)}(.)\right) & \cdots & \cdots & \sum_{l=1}^{N_l} \mu_{il}(k)\left(a_i^{c(1n)}(.)\right) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{l=1}^{N_l} \mu_{il}(k)\left(a_i^{c(n1)}(.)\right) & \cdots & \cdots & \sum_{l=1}^{N_l} \mu_{il}(k)\left(a_i^{c(nn)}(.)\right) \end{bmatrix} \quad (40)$$

Proof. Consider the following common Lyapunov function for the closed-loop system (35):

$$V\left(x(k),k\right) = \left\langle \left|x(k)\right|, w\right\rangle \quad (41)$$

where $w \in \Re^n$ with components $\left(w_m > 0, \forall m = 1,...,n\right)$ and $x(k) \in \Re^n$ is the state vector.

Along the trajectories of system (35), the difference of the Lyapunov function is given as below:

$$\Delta v\left(x(k),k\right) = v\left(x(k+1),k+1\right) - v\left(x(k),k\right)$$
$$= \left\langle \left|x(k+1)\right|, w\right\rangle - \left\langle \left|x(k)\right|, w\right\rangle \quad (42)$$

Subsequently, under arbitrary switching $\sigma(k)$, it is easy to verify that:

$$\left\langle \left|A_{\sigma(k)}^c(.)x(k)\right|, w\right\rangle - \left\langle \left|x(k)\right|, w\right\rangle$$
$$\le \left\langle \max_{1\le i\le N}\left(\left|A_i^c(.)\right|\right)\left|x(k)\right|, w\right\rangle - \left\langle \left|x(k)\right|, w\right\rangle$$
$$\le \left\langle \left(\max_{1\le i\le N}\left(\left|A_i^c(.)\right|\right) - I_n\right)\left|x(k)\right|, w\right\rangle$$
$$= \left\langle -\left(I_n - M_D(.)\right)\left|x(k)\right|, w\right\rangle \quad (43)$$

where $M_D(.)$ is introduced in (39).

We assume now that $\left(I_n - M_D(.)\right)$ is an $M-$matrix . In such condition, we can find a vector $\rho \in \Re_+^{*n}\left(\rho_p \in \Re_+^* \;\; p = 1,...,n\right)$ satisfying the following relation $\left(I_n - M_D(.)\right)^T w = \rho, \forall w \in \Re_+^{*n}$, then we can write:

$$\left\langle \left(I_n - M_D(.)\right)\left|x(k)\right|, w\right\rangle$$
$$= \left\langle \left(I_n - M_D(.)\right)^T w, \left|x(k)\right|\right\rangle = \left\langle \rho, \left|x(k)\right|\right\rangle \quad (44)$$

According to (44) it holds that:

$$\left\langle -\left(I_n - M_D(.)\right)\left|x(k)\right|, w\right\rangle = \left\langle -\rho, \left|x(k)\right|\right\rangle \quad (45)$$

Finally, we have:

$$\Delta V(k) \le \left\langle -\left(I_n - M_D(.)\right)\left|x(k)\right|, w\right\rangle$$
$$\le -\sum_{p=1}^{n} \rho_p \left|x_p(k)\right| < 0 \quad (46)$$

This completes the proof of Theorem 3.

3.3. Application to switched systems defined by difference equations

In this section, we consider a class of discrete-time uncertain switched nonlinear systems composed of N subsystems, all of them are modeled by the following difference equation:

$$y(k+n) + \left(\sum_{l=1}^{N_l} \mu_{il}(k) \sum_{j=0}^{n-1} a_{il}^{n-j}(.)\, y(k+j)\right) = u(k) \quad (47)$$

where $y(k) \in \Re^n$, $a_{il}^j(.)$ are nonlinear coefficients for each $i \in \underline{N}$, $\left(l = 1,..., N_l\right)$ and $\left(j = 1,..., n-1\right)$, $\mu_{il}(k)$ are time-varying polytopic uncertain parameters given in (3) and $u(k) \in \Re$ is the control input.

To solve this problem, we introduce the following state variables:

$$x_{j+1}(k) = y(k+j), \; j = 0,...,n-1 \quad (48)$$

Combining (47) and (48) yields for each $i \in \underline{N}$:

$$\begin{cases} x_j(k+1) = x_{j+1}(k), \; j = 1,...,n-1 \\ x_n(k+1) = -\left(\sum_{l=1}^{N_l} \mu_{il}(k) \sum_{j=0}^{n-1} a_{il}^{n-j} x_{j+1}(k)\right) + u(k) \end{cases} \quad (49)$$

or under matrix representation:

$$x(k+1) = \sum_{l=1}^{N_l} \mu_{il}(k)\left(A_{il}(.)x(k) + B_{il}(.)u(k)\right), \; i \in \underline{N} \quad (50)$$

where $x(k)$ is the state, $A_{il}(.)$ and $B_{il}(.)$ are vertex matrices that have nonlinear elements of appropriate dimension.

It is clear to see that all the models are given in the controllable form given in (19).

The feedback controller is given as follows:

$$u(k) = -\sum_{l=1}^{N_l} \mu_{il}(k) K_{il}(.)x(k), \; i \in \underline{N} \quad (51)$$

So, all the closed-loop subsystems can be written as follows:

$$x(k+1) = \sum_{l=1}^{N_l} \mu_{il}(k)\left(A_{il}(.) - BK_{il}(.)\right)x(k)$$
$$= \sum_{l=1}^{N_l} \mu_{il}(k) A_{il}^c(.)x(k), \; i \in \underline{N} \quad (52)$$

where the vertex matrix $A_{il}^c(.)$ is defined in (22).

Therefore, by considering the switching law (35), the closed-loop switched nonlinear is given as follows:

$$x(k+1) = A_{\sigma(k)}^c(.)x(k) \quad (53)$$

As done for the continuous time case a change of base for all the subsystems defined in (52) under the arrow form yields to:

$$z(k+1)=\sum_{l=1}^{N_l}\mu_{il}(k)M_{il}(.)z(k),\ i\in\underline{N} \tag{54}$$

where $M_{il}(.)$ is given in (28).

Finally, the comparison discrete-time system $z(k)\in\Re^n$ is given by:

$$z(k+1)=M_D(.)z(k) \tag{55}$$

where the comparison matrix $M_D(.)$ for discrete-time is given as below, it is deduced from the matrices $M_i(.)=\sum_{l=1}^{N_l}\mu_{il}(k)M_{il}(.)$ by substituting all their elements by their absolute values:

$$M_D(.)=\begin{bmatrix}|\alpha_1| & 0 & \cdots & 0 & |\beta_1|\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & |\alpha_{n-1}| & |\beta_{n-1}|\\ \overline{t}_D^{\,1}(.) & \cdots & \cdots & \overline{t}_D^{\,n-1}(.) & \overline{t}_D^{\,n}(.)\end{bmatrix} \tag{56}$$

with:

$$\overline{t}_D^{\,j}(.)=\max_{1\le i\le N}\left(\left|\sum_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^{j}(.)\right|\right),\ j=1,...,n \tag{57}$$

After this formulation, now we are in position to present a new robust stabilization of the closed-loop system (53) by using pole assignment control.

Theorem 4. If all the n poles $\{z_1,...,z_n\}$ imposed by the control law (51) of the closed-loop system (53) assumed to be real and have modules inferior to the unit, then system (53) is robust stabilizing by the control law (51) under arbitrary switching rule (35) and all admissible uncertainties (3) and the conditions below are met:

$$\begin{cases}\overline{t}_D^{\,j}(.)=\max\limits_{1\le i\le N}\left(\left|\sum_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^{j}(.)\right|\right)=0,\ j=1,...,n-1\\ \overline{t}_D^{\,n}(.)=\max\limits_{1\le i\le N}\left(\left|\sum_{l=1}^{N_l}\mu_{il}(t)\gamma_{il}^{n}(.)\right|\right)=|z_n|<1\end{cases} \tag{58}$$

with:

$$\begin{cases}\beta_j=\prod\limits_{\substack{q=1\\ q\ne j}}^{n-1}\left(z_j-z_q\right)^{-1}\ \ \forall\ j=1,...,n-1\\ \gamma_{il}^{j}(.)=-P_{A_{il}(.)}\left(z_j\right)\ \ \forall\ j=1,..,n-1\\ \gamma_{il}^{n}(.)=-a_{il}^{c(1)}(.)-\sum\limits_{j=1}^{n-1}z_j\end{cases} \tag{59}$$

Proof of this theorem is given in Appendix B.

4. ILLUSTRATIVE EXAMPLES

In the following section, an application to stabilize a real system is provided to demonstrate the effectiveness of the proposed methods.

Example 1. (Benrejeb et al., 2008) Consider a switched system which model a DC motor with shunt excitation under variable mechanical loads. All the subsystems are characterized by a transfer function which is preceded by a nonlinear element $\varphi(.)$ corresponding to the nonlinear characteristic of the magnetic flux (Lur'e Postnikov problem).

All the subsystems are given by:

$A_1(.)=\mu_{11}(t)A_{11}+\mu_{12}(t)A_{12}$;

$A_2(.)=\mu_{21}(t)A_{21}+\mu_{22}(t)A_{22}$

and: $A_3(.)=\mu_{31}(t)A_{31}+\mu_{32}(t)A_{32}$

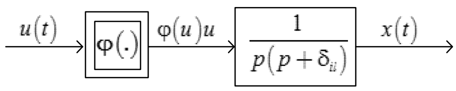

Fig. 1. Model of DC motor with shunt excitation.

According to Figure 1, the state equation for all the subsystems $i\in\{1,2,3\}$ can be written such as:

$$\dot{x}(t)=\sum_{l=1}^{2}\mu_{il}(t)\left(A_{il}(.)x(t)+Bv\right),\ v(.)=\varphi(.)u$$

and: $u(t)=-K_{il}(.)x(t)$, where the vertex matrices are defined as follows:

$A_{11}=\begin{bmatrix}0 & 1\\ 0 & -2.5\end{bmatrix}$, $A_{12}=\begin{bmatrix}0 & 1\\ 0 & -2\end{bmatrix}$, $A_{21}=\begin{bmatrix}0 & 1\\ 0 & -1{,}66\end{bmatrix}$, $A_{22}=\begin{bmatrix}0 & 1\\ 0 & -1.42\end{bmatrix}$, $A_{31}=\begin{bmatrix}0 & 1\\ 0 & -1.11\end{bmatrix}$, $A_{32}=\begin{bmatrix}0 & 1\\ 0 & -0.76\end{bmatrix}$ and $B=\begin{bmatrix}0\\ 1\end{bmatrix}$.

Consider the following time-varying uncertainties parameters:

$\mu_{11}(t)=\rho(.)$, $\mu_{12}(t)=1-\rho(.)$, $\mu_{21}(t)=\rho(.)$, $\mu_{22}(t)=1-\rho(.)$, $\mu_{31}(t)=\rho(.)$ and $\mu_{32}(t)=1-\rho(.)$

with $\rho(.)$ being a general nonlinearity such as $0\le\rho(.)\le 1$.

In this application, we aim to design a state feedback controller which guarantees that the resulting closed-loop system is asymptotically stable under any selected mechanical load. Indeed, the state feedback controller is characterized by the following parameters:

$K_{11}(.)=\left[k_{11}^{1}(.)\ k_{11}^{2}(.)\right]$, $K_{12}(.)=\left[k_{12}^{1}(.)\ k_{12}^{2}(.)\right]$,

$K_{21}(.)=\left[k_{21}^{1}(.)\ k_{21}^{2}(.)\right]$, $K_{22}(.)=\left[k_{22}^{1}(.)\ k_{22}^{2}(.)\right]$,

$K_{31}(.)=\left[k_{31}^{1}(.)\ k_{31}^{2}(.)\right]$ and. $K_{32}(.)=\left[k_{32}^{1}(.)\ k_{32}^{2}(.)\right]$

So, the closed-loop system is given by:

$$A_{11}^{c}(.)=\begin{bmatrix}0 & 1\\ -K_{11}^{1}(.)\varphi(.) & -2.5-\varphi(.)K_{11}^{2}(.)\end{bmatrix},$$

$$A_{12}^{c}(.)=\begin{bmatrix}0 & 1\\ -K_{12}^{1}(.)\varphi(.) & -2-\varphi(.)K_{12}^{2}(.)\end{bmatrix},$$

$$A_{21}^{c}(.)=\begin{bmatrix}0 & 1\\ -K_{21}^{1}(.)\varphi(.) & -1.66-\varphi(.)K_{21}^{2}(.)\end{bmatrix},$$

$$A_{22}^{c}(.)=\begin{bmatrix}0 & 1\\ -K_{22}^{1}(.)\varphi(.) & -1.42-\varphi(.)K_{22}^{2}(.)\end{bmatrix}$$

$$A_{31}^{c}(.)=\begin{bmatrix}0 & 1\\ -K_{31}^{1}(.)\varphi(.) & -1.11-\varphi(.)K_{31}^{2}(.)\end{bmatrix}$$

and: $A_{32}^{c}(.)=\begin{bmatrix}0 & 1\\ -K_{32}^{1}(.)\varphi(.) & -0.76-\varphi(.)K_{32}^{2}(.)\end{bmatrix}$

The vertex matrices in the arrow form are the following:

$$M_{11}(.)=P^{-1}A_{11}^{c}(.)P=\begin{bmatrix}\alpha & 1\\ \gamma(.)_{11}^{1} & \gamma(.)_{11}^{2}\end{bmatrix},$$

$$M_{12}(.)=P^{-1}A_{12}^{c}(.)P=\begin{bmatrix}\alpha & 1\\ \gamma(.)_{12}^{1} & \gamma(.)_{12}^{2}\end{bmatrix},$$

$$M_{21}(.)=P^{-1}A_{21}^{c}(.)P=\begin{bmatrix}\alpha & 1\\ \gamma_{21}^{1}(.) & \gamma_{21}^{2}(.)\end{bmatrix},$$

$$M_{22}(.)=P^{-1}A_{22}^{c}(.)P=\begin{bmatrix}\alpha & 1\\ \gamma_{22}^{1}(.) & \gamma_{22}^{2}(.)\end{bmatrix},$$

$$M_{31}(.)=P^{-1}A_{31}^{c}(.)P=\begin{bmatrix}\alpha & 1\\ \gamma_{31}^{1}(.) & \gamma_{31}^{2}(.)\end{bmatrix}$$

and: $M_{32}(.)=P^{-1}A_{32}^{c}(.)P=\begin{bmatrix}\alpha & 1\\ \gamma_{32}^{1}(.) & \gamma_{32}^{2}(.)\end{bmatrix}$

with:

$$\begin{cases}\gamma_{11}^{1}(.)=-P_{11}(\alpha)=-\left[\alpha^{2}+\left(2.5+\varphi(.)k_{11}^{2}(.)\right)\alpha+\varphi(.)k_{11}^{1}(.)\right]\\ \gamma_{11}^{2}(.)=-\left(2.5+\varphi(.)k_{11}^{2}(.)+\alpha\right)\end{cases}$$

$$\begin{cases}\gamma_{12}^{1}(.)=-P_{12}(\alpha)=-\left[\alpha^{2}+\left(2+\varphi(.)k_{12}^{2}(.)\right)\alpha+\varphi(.)k_{12}^{1}(.)\right]\\ \gamma_{12}^{2}(.)=-\left(2+\varphi(.)k_{12}^{2}(.)+\alpha\right)\end{cases};$$

$$\begin{cases}\gamma_{21}^{1}(.)=-P_{21}(\alpha)=-\left[\alpha^{2}+\left(1.66+\varphi(.)k_{21}^{2}(.)\right)\alpha+\varphi(.)k_{21}^{1}(.)\right]\\ \gamma_{21}^{2}(.)=-\left(1.66+\varphi(.)k_{21}^{2}(.)+\alpha\right)\end{cases}$$

$$\begin{cases}\gamma_{22}^{1}(.)=-P_{22}(\alpha)=-\left[\alpha^{2}+\left(1.42+\varphi(.)k_{22}^{2}(.)\right)\alpha+\varphi(.)k_{22}^{1}(.)\right]\\ \gamma_{22}^{2}(.)=-\left(1.42+\varphi(.)k_{22}^{2}(.)+\alpha\right)\end{cases}$$

$$\begin{cases}\gamma_{31}^{1}(.)=-P_{31}(\alpha)=-\left[\alpha^{2}+\left(1.11+\varphi(.)k_{31}^{2}(.)\right)\alpha+\varphi(.)k_{31}^{1}(.)\right]\\ \gamma_{31}^{2}(.)=-\left(1.11+\varphi(.)k_{31}^{2}(.)+\alpha\right)\end{cases}$$

and:

$$\begin{cases}\gamma_{32}^{1}(.)=-P_{32}(\alpha)=-\left[\alpha^{2}+\left(0.76+\varphi(.)k_{32}^{2}(.)\right)\alpha+\varphi(.)k_{32}^{1}(.)\right]\\ \gamma_{32}^{2}(.)=-\left(0.76+\varphi(.)k_{32}^{2}(.)+\alpha\right)\end{cases}$$

Next, by choosing the two poles $p_1=-1$ and $p_2=-3$ are real negative, this implies that $\alpha=-1$ and $\beta=1$.

According to Theorem 2, for all admissible uncertainties and under any switching law (1), we can deduce the following stabilization conditions:

i) $\alpha=p_1=-1<0$

ii) $\left|\mu_{11}(t)\gamma_{11}^{1}(.)+\mu_{12}(t)\gamma_{12}^{1}(.)\right|$
$=\left|\mu_{21}(t)\gamma_{21}^{1}(.)+\mu_{22}(t)\gamma_{22}^{1}(.)\right|$
$=\left|\mu_{31}(t)\gamma_{31}^{1}(.)+\mu_{32}(t)\gamma_{32}^{1}(.)\right|=0$

iii) $p_2=-3$

$$=\max\begin{pmatrix}\mu_{11}(t)\gamma_{11}^{1}(.)+\mu_{12}(t)\gamma_{12}^{1}(.),\\ \mu_{21}(t)\gamma_{21}^{1}(.)+\mu_{22}(t)\gamma_{22}^{1}(.),\\ \mu_{31}(t)\gamma_{31}^{1}(.)+\mu_{32}(t)\gamma_{32}^{1}(.)\end{pmatrix}<0$$

For a particular choice $k_{11}^{1}(.)=k_{12}^{1}(.)$, $k_{11}^{2}(.)=k_{12}^{2}(.)$, $k_{21}^{1}(.)=k_{22}^{1}(.)$, $k_{21}^{2}(.)=k_{22}^{2}(.)$; $k_{31}^{1}(.)=k_{32}^{1}(.)$ and $k_{31}^{2}(.)=k_{32}^{2}(.)$ relations (ii) and (iii) allow us to deduce the following stabilization conditions:

$$\begin{cases}k_{11}^{1}(.)\varphi(.)=k_{12}^{1}(.)\varphi(.)=3\\ k_{11}^{2}(.)\varphi(.)=k_{12}^{2}(.)\varphi(.)=2-0.5\rho(.)\end{cases},$$

$$\begin{cases}k_{21}^{1}(.)\varphi(.)=k_{22}^{1}(.)\varphi(.)=3+0.03\rho(.)\\ k_{21}^{2}(.)\varphi(.)=k_{22}^{2}(.)\varphi(.)=2.58-0.24\rho(.)\end{cases}$$

and:

$$\begin{cases}k_{31}^{1}(.)\varphi(.)=k_{32}^{1}(.)\varphi(.)=2.99\\ k_{31}^{2}(.)\varphi(.)=k_{32}^{2}(.)\varphi(.)=3.24-0.35\rho(.)\end{cases}$$

Now, we assume that the nonlinear gain $\varphi(.)$ is given by the following relationship: $\varphi(.)=\varphi(u)=\dfrac{0.5u}{1+0.4u^{2}}$; where the nonlinear gain $\varphi(.)$ is represented in the shape of Figure 2.

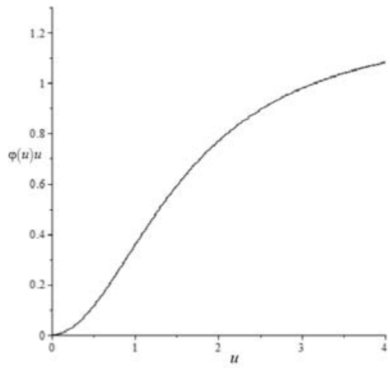

Fig. 2. Characteristic of the nonlinear gain $\varphi(.)$.

If we choose the uncertain parameters $\rho(.) = 0.4$, the initial state vector $x(t_0 = 0) = [2 \;\; 1]^T$ and the switching sequence given in Figure 3, the simulation result of the close-loop system are shown in Figures 4 and 5**,** respectively which correspond to the evolution of states with respect to time and the control input evolution.

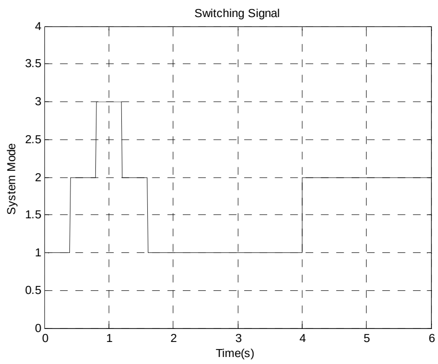

Fig. 3. Switching function $\sigma(t)$ between subsystems.

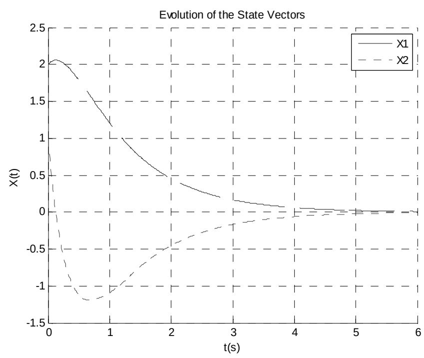

Fig 4. State responses of the close-loop system.

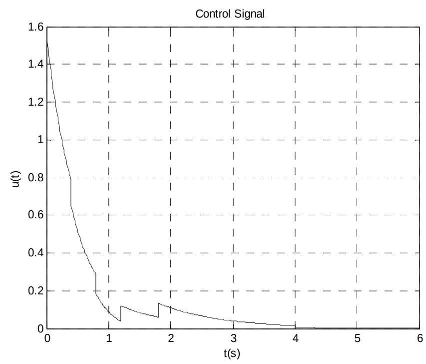

Fig. 5. Control signal.

Example 2. (Benrejeb et al., 2006) This example is deduced by the discretization of the linear part of the continuous-time switched system of example 1 by a zero-order holder with a sampling time $T_e = 0.2s$.

So, the vertex matrices are defined as:

$$A_{d11} = \begin{bmatrix} 0 & 1 \\ -0.6 & 1.6 \end{bmatrix},\; A_{d12} = \begin{bmatrix} 0 & 1 \\ -0.67 & 1.67 \end{bmatrix},$$

$$A_{d21} = \begin{bmatrix} 0 & 1 \\ -0.71 & 1.71 \end{bmatrix},\; A_{d22} = \begin{bmatrix} 0 & 1 \\ -0.75 & 1.75 \end{bmatrix},$$

$$A_{d31} = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.8 \end{bmatrix} \text{ and } A_{d32} = \begin{bmatrix} 0 & 1 \\ -0.85 & 1.85 \end{bmatrix}.$$

The varying-time uncertainties parameters are:
$\mu_{11}(k) = \rho(.)$, $\mu_{12}(k) = 1 - \rho(.)$, $\mu_{21}(k) = \rho(.)$, $\mu_{22}(k) = 1 - \rho(.)$, $\mu_{31}(k) = \rho(.)$ and $\mu_{32}(k) = 1 - \rho(.)$.

By considering the state feedback controller given in (51) characterized by the following gains vectors:
$K_{11}(.) = [k_{11}^1(.) \; k_{11}^2(.)]$, $K_{12}(.) = [k_{12}^1(.) \; k_{12}^2(.)]$,
$K_{21}(.) = [k_{21}^1(.) \; k_{21}^2(.)]$, $K_{22}(.) = [k_{22}^1(.) \; k_{22}^2(.)]$,
$K_{31}(.) = [k_{31}^1(.) \; k_{31}^2(.)]$ and. $K_{32}(.) = [k_{32}^1(.) \; k_{32}^2(.)]$

then, the closed-loop system is given by:

$$A_{11d}^c(.) = \begin{bmatrix} 0 & 1 \\ -0.6 - K_{11}^1(.)\varphi(.) & 1.6 - \varphi(.)K_{11}^2(.) \end{bmatrix},$$

$$A_{12d}^c(.) = \begin{bmatrix} 0 & 1 \\ -0.67 - K_{12}^1(.)\varphi(.) & 1.6 - \varphi(.)K_{12}^2(.) \end{bmatrix},$$

$$A_{21d}^c(.) = \begin{bmatrix} 0 & 1 \\ -0.71 - K_{21}^1(.)\varphi(.) & 1.71 - \varphi(.)K_{21}^2(.) \end{bmatrix},$$

$$A_{22d}^c(.) = \begin{bmatrix} 0 & 1 \\ -0.75 - K_{22}^1(.)\varphi(.) & 1.75 - \varphi(.)K_{22}^2(.) \end{bmatrix},$$

$$A_{31d}^c(.) = \begin{bmatrix} 0 & 1 \\ -0.8 - K_{31}^1(.)\varphi(.) & 1.8 - \varphi(.)K_{31}^2(.) \end{bmatrix}$$

and: $A_{32}^c(.) = \begin{bmatrix} 0 & 1 \\ 0.85 - K_{32}^1(.)\varphi(.) & 1.85 - \varphi(.)K_{32}^2(.) \end{bmatrix}$

The vertex matrices in arrow form are the following:

$$M_{11}(.) = P^{-1}A_{11d}^c(.)P = \begin{bmatrix} \alpha & 1 \\ \gamma_{11}^1(.) & \gamma_{11}^2(.) \end{bmatrix},$$

$$M_{12}(.) = P^{-1}A_{12d}^c(.)P = \begin{bmatrix} \alpha & 1 \\ \gamma_{12}^1(.) & \gamma_{12}^2(.) \end{bmatrix},$$

$$M_{21}(.) = P^{-1}A_{21d}^c(.)P = \begin{bmatrix} \alpha & 1 \\ \gamma_{21}^1(.) & \gamma_{21}^2(.) \end{bmatrix},$$

$$M_{22}(.) = P^{-1}A_{22d}^c(.)P = \begin{bmatrix} \alpha & 1 \\ \gamma_{22}^1(.) & \gamma_{22}^2(.) \end{bmatrix},$$

$$M_{31}(.) = P^{-1}A_{31d}^c(.)P = \begin{bmatrix} \alpha & 1 \\ \gamma_{31}^1(.) & \gamma_{31}^2(.) \end{bmatrix}$$

and: $M_{32}(.) = P^{-1} A^{c}_{32d}(.) P = \begin{bmatrix} \alpha & 1 \\ \gamma^{1}_{32}(.) & \gamma^{2}_{32}(.) \end{bmatrix}$

with:

$$\begin{cases} \gamma^{1}_{11}(.) = -\left[\alpha^2 + \left(-1.6 + \varphi(.) k^{2}_{11}(.)\right)\alpha + 0.6 + \varphi(.) k^{1}_{11}(.)\right] \\ \gamma^{2}_{11}(.) = -\left(1.6 + \varphi(.) k^{2}_{11}(.) + \alpha\right) \end{cases}$$

$$\begin{cases} \gamma^{1}_{12}(.) = -\left[\alpha^2 + \left(-1.67 + \varphi(.) k^{2}_{12}(.)\right)\alpha + 0.67 + \varphi(.) k^{1}_{12}(.)\right] \\ \gamma^{2}_{12}(.) = -\left(-1.67 + \varphi(.) k^{2}_{12}(.) + \alpha\right) \end{cases};$$

$$\begin{cases} \gamma^{1}_{21}(.) = -\left[\alpha^2 + \left(-1.77 + \varphi(.) k^{2}_{21}(.)\right)\alpha + 0.71 + \varphi(.) k^{1}_{21}(.)\right] \\ \gamma^{2}_{21}(.) = -\left(-1.71 + \varphi(.) k^{2}_{21}(.) + \alpha\right) \end{cases}$$

$$\begin{cases} \gamma^{1}_{22}(.) = -\left[\alpha^2 + \left(-1.75 + \varphi(.) k^{2}_{22}(.)\right)\alpha + 0.75 + \varphi(.) k^{1}_{22}(.)\right] \\ \gamma^{2}_{22}(.) = -\left(-1.75 + \varphi(.) k^{2}_{22}(.) + \alpha\right) \end{cases}$$

$$\begin{cases} \gamma^{1}_{31}(.) = -\left[\alpha^2 + \left(-1.8 + \varphi(.) k^{2}_{31}(.)\right)\alpha + 0.8 + \varphi(.) k^{1}_{31}(.)\right] \\ \gamma^{2}_{31}(.) = -\left(-1.8 + \varphi(.) k^{2}_{31}(.) + \alpha\right) \end{cases}$$

and:

$$\begin{cases} \gamma^{1}_{32}(.) = -\left[\alpha^2 + \left(-1.85 + \varphi(.) k^{2}_{32}(.)\right)\alpha + 0.85 + \varphi(.) k^{1}_{32}(.)\right] \\ \gamma^{2}_{32}(.) = -\left(-1.85 + \varphi(.) k^{2}_{32}(.) + \alpha\right) \end{cases}$$

Now, according to theorem 4, by choosing the two poles $z_1 = e^{P_1 T_e} = 0.81$ and $z_2 = e^{P_2 T_e} = 0.54$, this implies the following stabilization conditions for all admissible uncertainties and under any switching law (35):

i) $\quad |z_1| = 0.81 < 1$

ii) $\quad \left|\mu_{11}(t)\gamma^{1}_{11}(.) + \mu_{12}(t)\gamma^{1}_{12}(.)\right|$

$= \left|\mu_{21}(t)\gamma^{1}_{21}(.) + \mu_{22}(t)\gamma^{1}_{22}(.)\right|$

$= \left|\mu_{31}(t)\gamma^{1}_{31}(.) + \mu_{32}(t)\gamma^{1}_{32}(.)\right| = 0$

iii) $\quad |z_2| = 0.54$

$$\max \begin{pmatrix} \left|\mu_{11}(t)\gamma^{1}_{11}(.) + \mu_{12}(t)\gamma^{1}_{12}(.)\right|, \\ \left|\mu_{21}(t)\gamma^{1}_{21}(.) + \mu_{22}(t)\gamma^{1}_{22}(.)\right|, \\ \left|\mu_{31}(t)\gamma^{1}_{31}(.) + \mu_{32}(t)\gamma^{1}_{32}(.)\right| \end{pmatrix} < 1$$

For a particular choice $k^{1}_{11}(.) = k^{1}_{12}(.)$, $k^{2}_{11}(.) = k^{2}_{12}(.)$, $k^{1}_{21}(.) = k^{1}_{22}(.)$, $k^{2}_{21}(.) = k^{2}_{22}(.)$; $k^{1}_{31}(.) = k^{1}_{32}(.)$ and $k^{2}_{31}(.) = k^{2}_{32}(.)$ conditions (ii) and (iii) allow us to deduce the following stabilization conditions:

$$\begin{cases} k^{1}_{11}(.)\varphi(.) = k^{1}_{12}(.)\varphi(.) = -1.102 + 0.068\rho(.) \\ k^{2}_{11}(.)\varphi(.) = k^{2}_{12}(.)\varphi(.) = 1.4 - 0.07\rho(.) \end{cases},$$

$$\begin{cases} k^{1}_{21}(.)\varphi(.) = k^{1}_{22}(.)\varphi(.) = -1.1813 + 0.0425\rho(.) \\ k^{2}_{21}(.)\varphi(.) = k^{2}_{22}(.)\varphi(.) = 1.48 - 0.04\rho(.) \end{cases}$$

and:

$$\begin{cases} k^{1}_{31}(.)\varphi(.) = k^{1}_{32}(.)\varphi(.) = -1.2874 + 0.04\rho(.) \\ k^{2}_{31}(.)\varphi(.) = k^{2}_{32}(.)\varphi(.) = 1.58 - 0.05\rho(.) \end{cases}$$

As done for the continuous-time case, the nonlinear gain $\varphi(.)$ is given by the following relationship: $\varphi(.) = \varphi(u) = \dfrac{0.5u}{1 + 0.4u^2}$ given in Figure 2.

For this example, in case when the uncertain parameter $\rho(.)$ is fixed at 0.4, the initial state vector $x(0) = [2 \;\; 1]^T$ and by considering the same switching sequence given in Figure 3, the evolution of the states and the control signal are given in Figures 6 and 7, respectively.

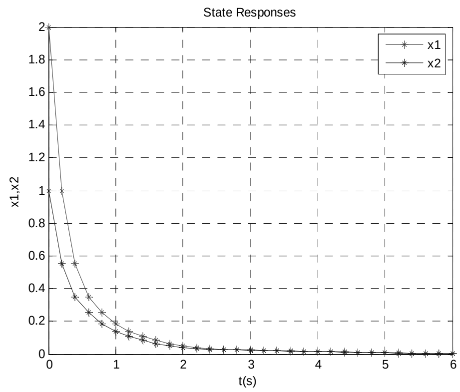

Fig. 6. Sate evolution of the closed-loop system.

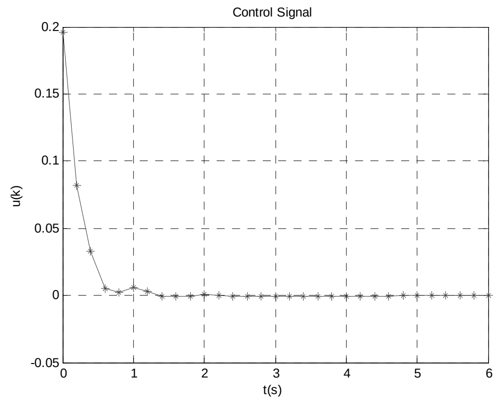

Fig. 7. Control signal of the closed-loop system.

These two examples were treated to show the effectiveness of the proposed method. In fact, the controllers designed in examples 1 and 2 can guarantee the robust stabilization of the closed-loop systems, as well to see that the proposed stabilization conditions are sufficient and closely necessary. By another side, according to the number of the subsystems and the complexity of those subsystems, it is very difficult to find a common Lyapunov function for all the subsystems given in the examples, in order to guarantee stability of the corresponding closed-loop systems under arbitrary switching.

5. CONCLUSIONS

This paper has investigated robust pole placement stabilization with state feedback controller for a class of both continuous and discrete-time switched nonlinear systems with time-varying polytopic uncertainties. These conditions were deduced by constructing a common Lyapunov function and applying the Kotelyanski lemma combined with the $M-$matrix properties.

Compared with the existing results of switched systems, the main advantages of this approach consist in that these obtained conditions are formulated in terms of the time-varying polytopic uncertain parameters and they allow us to avoid searching a common Lyapunov function.

As application, the effectiveness of the theoretical results is illustrated for a DC motor with shunt excitation and polytopic uncertainties models under variable mechanical loads for the both continuous and discrete-time cases.

REFERENCES

Aleksandrov, Y., Chen, A., Platonov, Y., and Zhang, A. L. (2012). Stability analysis and uniform ultimate boundedness control synthesis for a class of nonlinear switched difference systems. *J. Difference Equ. Appl*, Volume 18, 1545–1561.

Aleksandrov, Y., Chen, A., Platonov, V., and Zhang. L. (2011). Stability analysis for a class of switched nonlinear systems. *Automatica*, Volume 47, 2286–2291.

Alnowibet, K.A., and Perros, H. (2006). Nonstationary analysis of circuit-switched communication networks. *Perform Eval*; Volume 63, 892–909.

Araghi, L.F., Suratgar, A.A., and Feizi, E. (2013). Stability of a Class of Switched Linear Systems with Infinite Number of Subsystems**.** *Control Engineering and Applied Informatics***,** volume 15, 38-44.

Back, A., Guckenheimer, J., and Myers, M. (1993). A Dynamical Simulation Facility for Hybrid Systems, in: Hybrid Systems, volume. 736, 255–267, Springer-Verlag, New York.

Benrejeb, M., Borne, P., and Laurent. F. (1982). Sur une application de la représentation en flèche à l'analyse des processus. *Rairo-Autom-Syst*, volume 16, 133–146.

Benrejeb, M., Sakly, A., Ben Othman, K., and Borne, P. (2008). Choice of conjunctive operator of TSK fuzzy systems and stability domain study, *Math. Comput. Simulat*, volume 76, 410–421.

Benrejeb, M., Soudani, D., Sakly, A., and Borne, P. (2006). New Discrete Tanaka Sugeno Kang Fuzzy Systems Characterization and Stability Domain, *IJCCC*, volume 1, 9-19.

Borne, P., and Benrejeb, M. (2008).On the Representation and the Stability Study of Large Scale Systems. *Int. J. of Computers, Communications & Control*, volume 3, 55-66.

Borne, P., Dambrine, M., Perruquetti, W., and Richard, J.P. (2003). Vector Lyapunov functions: Nonlinear; time-varying; ordinary and functional diferantial equations; Advances in stability theory. *Stability and control: theory methods and applications*, *Taylors & Francis*, 49-73, London.

Borne, P., Gentina, J.C., and Laurent, F. (1972). Sur la stabilité des systèmes échantillonnés non linéaires. *REV.Fr.auto inf Rech.oper*, volume 2, 96-105.

Borne, P. (1987). Nonlinear system stability: Vector norm approach System and Control. *Encyclopedia*, volume 5, 3402-3406.

Borne, P., Richard, J.P., and Radhy, N.E. (1993). Stabilité stabilisation régulation: approche par les normes vectorielles Systèmes non linéaires, *tome 2: stabilité-stabilisation, coordonné par A.J. Fossard et D. Normand-Cyrot, Responsable du tome: P. Borne*, Editions Masson, 45–90, Paris.

Borne, P., Vanheeghe, P., and Duflos, E. (2007). Automatisation des processus dans l'espace d'état, *Ed. Technip*, Paris.

Chiang, M.L., and Fu, L.C. (2014). Robust output feedback stabilization of switched nonlinear systems with average dwell time. *Asian Journal of Control*, volume 16, 1–13.

Daafouz, J., Millerioux, G., and Iung, C. (2002). Apoly-quadratic stability based approach for linear switched systems, *Int. J. Control*, volume 75, 1302–1310.

Dayawansa, W., and Martin, C. (1999). A converse Lyapunov theorem for a class of dynamical systems which undergo switching, *IEEE Transactions on Automatic control*, volume 44, 751- 760.

Elmadssia, S., Saadaoui, K., and Benrejeb, M. (2013). New delay-dependent stability conditions for linear systems with delay, *Systems Science & Control Engineering*, volume 1, 37-41.

Gantmacher, F.R. (1966). Théorie des matrices, *T.1 et 2, Dunod*, Paris, France.

Gollu, A., and Varaiya, P. (1989). Hybrid dynamical systems, in: *Proceedings of 28th IEEE Conference on Decision and Control*, Tampa, FL, 2708–2712.

Grujc, T.L., Martynyuk, A.A., and Pavella, M.R. (1987). Large scale systems stability under structural and singular perturbations, *Springer-Verlag*, Berlin.

Hamdouch, Y., Florian, M., Hearn, D.W., and Lawphongpanich, S. (2007). Congestion pricing for multi-modal transportation systems. *Transport res B-meth*; volume 41, 275–91.

Hahn, W. (1967). Stability of Motion, *Springer-Verlag*, Berlin.

Hespanha, J., and Morse, A. (1999). Stabilization of nonholonomic integrators via logicbased switching, *Automatica*, volume *35*, 385–393.

Huang, S., and Xiang, Z. (2013) Robust L1 reliable control for uncertain switched nonlinear systems with time delay under asynchronous switching. *Applied Mathematics and Computation*, volume 222, 658-670.

Kermani, M., Jaballi, A., and Sakly, A. (2014). A New Stability Analysis for Discrete-time Switched Time-delay Systems. *International Journal of Control, Energy and Electrical Engineering,* volume 1, 23-30.

Kermani, M., Sakly, A., and M'sahli, F. (2013). A new stability analysis and stabilization of uncertain switched linear systems based on vector norms approach. *10th*

International Multi-Conference on Systems, Signals & Devices (SSD), Hammamet, Tunisia, 1-6.

Kermani, M., Sakly, A., and M'sahli, F. (2012). A new stability analysis and stabilization of discrete-time switched linear systems using vector norms approach. *World Academy of Science, Engineering and Technology*, volume 71, 1302–1307.

Kermani, M., and Sakly, A. (2014). Stability analysis for a class of switched nonlinear time-delay systems. *Systems Science & Control Engineering:* Taylor & Francis, volume 2, 80–89.

Kotelyanski, D.M. (1952). Some properties of matrices with positive elements. *Matematicheskii Sbornik*, volume 31, 497–505.

Liberzon, D. (2004). Lie algebras and stability of switched nonlinear systems, in: V. D. Blondel, A. Megretski (Eds.). *Unsolved Problems in Mathematical Systems and Control Theory*, Princeton University Press, Princeton, Oxford, 90–92.

Mancilla-Aguilar, J., and Garcia, R. (2000). A converse Lyapunov theorem for nonlinear switched systems. *systems & control letters*, volume 41, 67-71.

Min, W., and Jun, Z. (2009). L_2-gain Analysis and Control Synthesis for a Class of Uncertain Switched Nonlinear Systems. *Acta Automatica Sinica*, volume 35, 1459-1464.

Orani, N., Pisano, A., Franceschelli, M., Giua, A., and Usai, E. (2011). Robust reconstruction of the discrete state for a class of nonlinear uncertain switched systems. *Nonlinear Analysis: Hybrid Systems*, volume 5, 220-232.

Putyrski, M. and Schultz, C. (2011). Switching heterotrimericg protein subunits with a chemical dimerizer. *Chem Biol*, volume 18, 26–33.

Robert. F. (1966). Recherche d'une M-matrice parmi les minorantes d'un opérateur liéaire, *Numerische Mathematik*, volume 9, 188–199.

Sengupta, I., Barman, A.D., and Basu, P.K. (2009). Circuit model for analysis of SOA-based photonic switch. *Opt Quant Electron*; volume 41, 37–48.

Varaiya, P. (1993). Smart cars on smart roads: problems of control. *IEEE Trans. Automat. Control*, volume 38, 195–207.

Vu, L., and Liberzon, D. (2005). Common Lyapunov functions for families of commuting nonlinear systems, *Systems Control Lett*, volume 54, 405–416.

Xiang, W., Xiao, J., and Iqbal, M. (2014). Robust fault detection for a class of uncertain switched nonlinear systems via the state updating approach**.** *Nonlinear Analysis: Hybrid Systems*, volume 12, 132-146.

Zhang, L., Shi, P., Boukas, E.K., and Wang, C. (2008). H1 model reduction for uncertain switched linear discrete-time systems. *Automatica*, volume 44, 2944–2949.

Zhang, L., Shi, P., Boukas, E.K., and Wang, C. (2007). Robust l2-l1 filtering for switched linear discrete time-delay systems with polytopic uncertainties. *IET Control Theory Appl*, volume 1, 722–730.

Zhao, J., and Hill, D. (2008). On stability L2-gain and H∞ control for switched systems. *Automatica*, volume 44, 1220–1232.

Zhao, X., Zhang, L., Shi, P., and Liu, M. (2012). Stability and stabilization of switched linear systems with mode-dependent average dwell time. *IEEE Trans. Automat. Control*, volume 57, 1809–1815.

APPENDIX A

Proof of Theorem 2. (Borne et al., 2008) Consider the close loop system (26) given in the arrow form, by choosing the poles $p_j = \alpha_j$ for $j = 1,...,n-1$ are real and negative. Then, the closed-loop system is stable if $M_c(.)$ is the opposite of an $M-$matrix. In such conditions, the principal minors of $M_c(.)$ are alternating signs with the first is negative, it becomes $\overline{t}^{\,j}(.) = 0,\ j = 1,...,n-1$ where: $\overline{t}^{\,j}(.)$ are given in (33) and the last condition $j = n$ is:

$$(-1)^n \begin{vmatrix} p_1 & 0 & \cdots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & p_{n-1} & |\beta_{n-1}| \\ 0 & \cdots & \cdots & 0 & \overline{t}^{\,n}(.) \end{vmatrix} > 0\,;$$

That is $p_n = \overline{t}^{\,n}(.) < 0$. So, in such condition, the new dynamic is characterized by the distinct poles imposed on the system and the switched system is stable under arbitrary switching and the poles $p_j < 0 \quad j = 1,...,n$.

APPENDIX B

Proof of Theorem 4. (Borne et al., 2008) For system (53) with the control law (51), the dynamic of this system is characterized by the distinct poles imposed on the system by choosing the poles $z_j = \alpha_j$ for $j = 1,...,n-1$, it permits us to conclude that the system is stable if all the principal minors of $(I_n - M_D)$ are positive, it becomes that $\overline{t}_D^{\,j}(.) = 0$ for $j = 1,...,n-1$ and $\overline{t}_D^{\,n}(.) = |z_n|$, under these conditions, we have that switched system (54) is stable under arbitrary switching and $|z_j| < 1 \quad j = 1,...,n$.