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Improved Identification and Control of 2-by-2 MIMO System using Relay Feedback

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Abstract: Several groups[6-8] have analyzed Multi Input Multi Output (MIMO) systems by approximating all the individual transfer functions as First Order plus Dead Time (FOPDT), which may result in degraded closed loop performance. In this work, ideal relay feedback tests are conducted on transfer function models of 2-by-2 MIMO systems, which are having its individual transfer function elements without approximating them as FOPDT. A systematic approach for the identification and control of 2-by-2 MIMO (Wardle and Wood distillation column) system using Relay feedback test is proposed. The analytical expression is derived for the relay responses obtained from the relay feedback test and these expressions are used to estimate unknown system parameters. The model parameters are estimated by applying boundary conditions obtained from the limit cycle data on the derived expressions and the interaction is analyzed by using Relative Gain Array method. The identified model is used for the design of controllers to carryout closed loop studies. The work also includes: stability analysis, robustness and effect of measurement noise. The advantages of the proposed scheme are highlighted.

Keywords: MIMO, Modelling, Autotuning, Interaction, Distillation Column.

1. INTRODUCTION

Majority of the industrial plants are multivariable and higher order in nature where more than one input variable is coupled with outputs. Interactions between input and output cause the manipulated variable to affect more than one controlled variable and hence MIMO processes are difficult to control. In the control system design, loop interaction can therefore lead to instability, except they are taken into description. By a proper choice of input-output pairings, the problem of loop interaction can be minimized by a proper choice of input-output pairings. Relay feedback is one promising tool for sequential autotuning and parameter estimation.

(Astrom et al., 1984) proposed simple methods for tuning PID regulators using relay tests which were robust and easy to use. (Luyben et al., 2001) employed the relay feedback test to get additional information about the process dynamics using the so called shape factor. (Majhi et al., 1999) have worked on auto tuning of PID controller for low-order systems and concluded that more information as well as better performance of closed- loop system can be obtained. (Panda et al., 2002) derived mathematical equations representing relay responses in time domain contain ultimate properties of systems. (Shih-Haur Shen and Yu, 1994) proposed a sequential identification-design procedure for the autotuning of multivariable systems. (Thyagarajan et al., 2003) presented the identification procedure for different processes such as Fist Order plus Dead Time, Second Order plus Dead Time and Higher Order systems. Panda et al., 2003) derived time domain model equations for first and many higher order processes. (Selvakumar and Panda, 2010) presented the modeling of multivariable process using relay feedback test. Here all the individual transfer function models are approximated as FOPDT. (Wardle and Wood, 1969) distillation column was used for illustration. (Sujatha et al., 2012) presented the modeling of 2-by-2 MIMO systems using (Wood and Berry, 1973) Distillation Column. The main drawback in these methods is that the individual transfer function model of the MIMO process is approximated as FOPDT. However, in industries the individual transfer function model is not always FOPDT and it may be any higher order process.

The present work deals with implementation of relay feedback test on MIMO process by retaining the original structure of the individual transfer function models. The proposed scheme is illustrated on Wardle and Wood Distillation Column using simulation studies.

This paper is organized as follows: Application of relay feedback test for sequential autotuning on MIMO system is presented in section 2. The time domain analytical expressions are derived in section 3. In section 4, the details of Wardle and Wood distillation column are presented. The validation and parameter estimation are presented in section 5. The closed loop analysis is presented in section 6. The stability analysis and the robustness studies are presented in the section 7 and 8 respectively. The performance of the proposed scheme in the presence of noise is presented in section 9. The experimental studies are presented in section 10. The concluding remarks are presented at the end.

2. RELAY FEEDBACK TEST ON MIMO SYSTEMS

Let us consider a 2 x 2 MIMO system with a known pairing (y_1, u_1) and (y_2, u_2), under decentralized PI control (figure 1). Initially, relay is placed between y_1 and e_1, while loop 2 is on manual mode. A controller can be designed using the ultimate gain and ultimate frequency obtained from relay test, following the relay feedback test. Multiloop PI controller is used in figure 1 and figure 2. While loop 1 is on automatic mode, the second step is to make the relay-feedback test between y_2 and e_2 (figure 2). A controller is designed for loop 2 following the relay-feedback test. Once, the controller on the loop 2 is put on automatic mode, another relay-feedback test is performed between y_1 and e_1 (figure 3). Generally, a new set of tuning constants will be found for the controller placed in loop 1. The similar procedure is repeated until the parameters of controller converge. The controller parameter normally converges in 3-4 relay-feedback tests for 2 x 2 systems. Thus, relay feedback approach is used to perform sequential autotuning. The sequential design provides an attractive alternative in MIMO autotuning. By sequential design, each controller in a multivariable system is designed in sequence i.e. the MIMO process is treated as a sequence of SISO system.

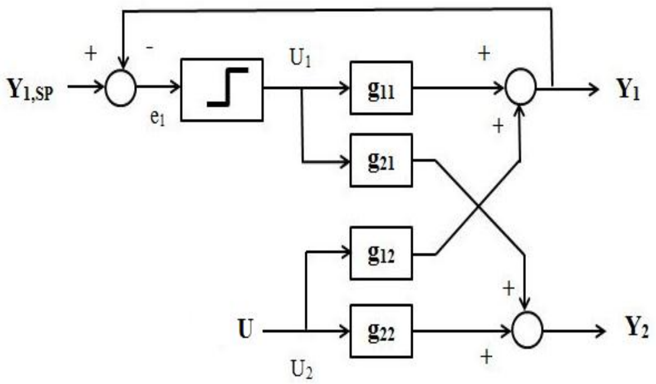

Fig. 1. Step 1 of sequential tuning for 2x2 MIMO system.

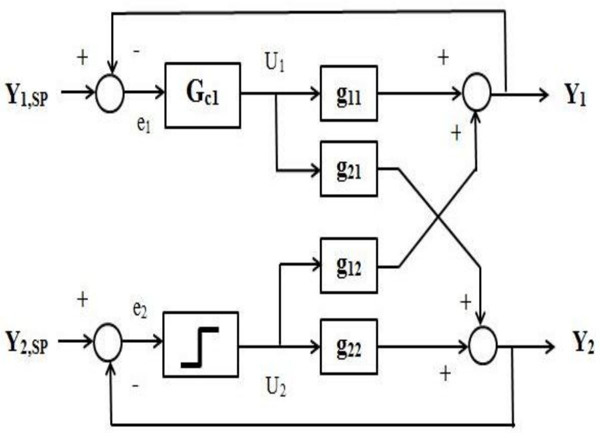

Fig. 2. Step 2 of sequential tuning for 2x2 MIMO system.

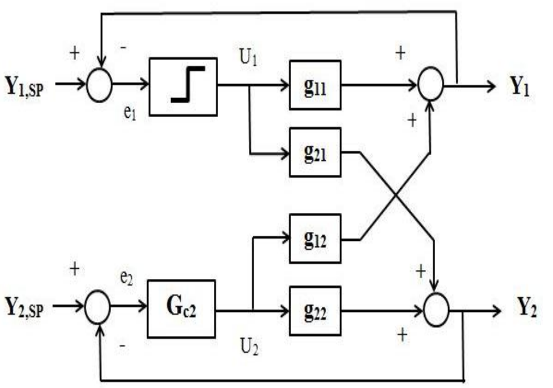

Fig. 3. Step 3 of sequential tuning for 2x2 MIMO system.

This procedure of autotuning can be extended to n-by-n MIMO system with n x 1 desired closed-loops (closed-loop response for set point tracking is termed as desired closed loops) and n x (n-1) undesired closed-loops (in the closed-loop, the load disturbances drive the system away from its desired behaviour is termed as undesired closed-loops).

3.DERIVATION OF ANALYTICAL EXPRESSIONS

3.1. Analytical Expression for the G_{p21} Interactive Transfer Function

Analytical expression for the off-diagonal closed loop transfer function (G_{p21}) of 2 x 2 MIMO system (Wardle and Wood distillation column) is derived in time domain using ideal relay feedback test. The relay response for G_{p21} is shown in figure 4. The response can be described as given in equation (1), at the first interval.

$$Y_2(s) = G_{p21,cl}(s).U_1(s) \tag{1}$$

Substituting the values of $G_{p21,cl}(s)$ and $U_1(s)$ in equation (1), we get

$$Y_2(s) = \frac{\frac{k_{21}e^{-D21s}}{\tau_{21}s+1}}{1+k_{c2}\left(1+\frac{1}{\tau_{i2}s}\right)\frac{k_{22}e^{-D22s}}{\tau_{22}s+1}}.\frac{1}{s} \tag{2}$$

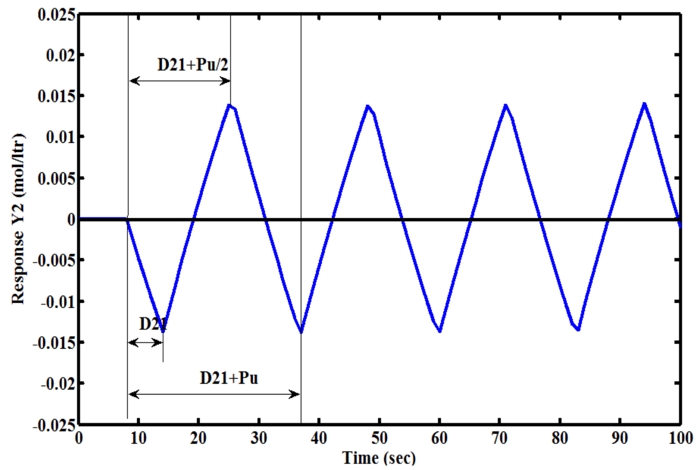

Fig.4. Relay response for G_{p21}

At first interval (once synchronizing the input with output by time shift), the relay response can be mathematically described as follows:

$$y_1(t) = k_{21}\left[1-e^{\frac{-t}{\tau_{21}}}\right] - \frac{k_{22}k_{c2}}{\tau_{i2}}\left[1+(\tau_{i2}-\tau_{22})e^{\frac{-t}{\tau_{22}}}\right]y_1(t-1) \tag{3}$$

At the second instant introducing a time shift by D amount in the equation (3), we get

$$y_2(t) = k_{21}\left\{[1-2]-e^{\frac{-t}{\tau_{21}}}\left[e^{\frac{-D_{21}}{\tau_{21}}}-2\right]\right\} - \frac{k_{22}k_{c2}}{\tau_{i2}}\left\{[1-2]+\left((\tau_{i2}-\tau_{22})e^{\frac{-t}{\tau_{22}}}\left[e^{\frac{-D_{22}}{\tau_{22}}}\right]\right)\right\}y_2(t-1) \tag{4}$$

The third interval lags by an amount $D_{21}+p_u/2$ and $D_{22}+p_u/2$ from input can be given as follows:

$$y_3(t) = k_{21}\left\{[1-2+2] - e^{\frac{-t}{\tau_{21}}}\left[e^{\frac{D_{21}+\frac{P_u}{2}}{\tau_{21}}} - 2e^{\frac{\frac{P_u}{2}}{\tau_{21}}} + 2\right]\right\} - \frac{k_{22}k_{c2}}{\tau_{i2}}\left\{\begin{matrix}[1-2+2]+\\ \left(\begin{matrix}(\tau_{i2}-\tau_{22})e^{\frac{-t}{\tau_{22}}}\times\\ \left[e^{\frac{-D_{22}+\frac{P_u}{2}}{\tau_{22}}} - 2e^{\frac{\frac{P_u}{2}}{\tau_{22}}} + 2\right]\end{matrix}\right)\end{matrix}\right\} y_3(t-1) \quad (5)$$

The equation (5) for $y_3(t)$ slowly forms a series, as time tends to infinity the response becomes stabilized and it can be described as in given in equation (6).

$$y_n(t) = k_{21}\left\{\begin{matrix}[1-2+2-\cdots] - e^{\frac{-t}{\tau_{21}}}\times\\ \left[e^{\frac{D_{21}+\sum_{n=1}^{n-1}\frac{P_u}{2}}{\tau_{21}}} - 2e^{\frac{\sum_{n=1}^{n-1}\frac{P_u}{2}}{\tau_{21}}} + \cdots + 2e^{\frac{\frac{P_u}{2}}{\tau_{22}}} - 2\right]\end{matrix}\right\} - \frac{k_{22}k_{c2}}{\tau_{i2}}\left\{\begin{matrix}[1-2+2-\cdots]+\\ \left(\begin{matrix}(\tau_{i2}-\tau_{22})e^{\frac{-t}{\tau_{22}}}\times\\ \left[e^{\frac{D_{22}+\frac{\sum_{n=1}^{n-1}\frac{P_u}{2}}{2}}{\tau_{22}}} - 2e^{\frac{\sum_{n=1}^{n-1}\frac{P_u}{2}}{\tau_{22}}} + \cdots + 2e^{\frac{\frac{P_u}{2}}{\tau_{22}}} - 2\right]\end{matrix}\right)\end{matrix}\right\}$$

$$\times[y_1(t-1)+y_2(t-1)+y_3(t-1)+\cdots+y_n(t-1)] \quad (6)$$

Let $r_1 = e^{\frac{P_u}{2\tau_{21}}}$ and $r_1 = e^{\frac{P_u}{2\tau_{22}}}$

$$y_n(t) = k_{21}\ \{\text{first part + second part}\} - \frac{k_{22}k_{c2}}{\tau_{i2}}\ \{\text{third part + fourth part}\}$$

First part = [1-2+2-2+...up to ∞] = $(1)^{2n-1}=1$

The summation of the series could be either +1(for positive step change) or -1(for negative step change). The generalized expression is derived for positive step change and hence, the summation is chosen as 1.

In similar way the third part = 1

Second part =

$$2\left[e^{\frac{-D_{21}}{\tau_{21}}} r_1^{\,n} - 2r_1^{\,n-1} + 2r_1^{\,n-2} - \cdots + 2r_1 - 2\right]$$

This second part can be formed into following series:

Second part =

$$2[1 - r_1 + r_1^{\,2} - r_1^{\,3}\ldots] = \frac{2}{1+r_1} = \frac{2}{1+e^{\frac{P_u}{2\tau_{21}}}}$$

Similarly the fourth part can be simplified as follows:

Fourth part =

$$\frac{2}{1+r_2} = \frac{2}{1+e^{\frac{P_u}{2\tau_{21}}}}$$

The generalized analytical expression G_{p21} relay response can be expressed as given in equation (7)

$$y_n(t) = k_{21}\left[1 - e^{\frac{-t}{\tau_{21}}}\left(\frac{2}{1+e^{\frac{\frac{P_u}{2}}{\tau_{21}}}}\right)\right] - \frac{k_{22}k_{c2}}{\tau_{i2}}\left[1+(\tau_{i2}-\tau_{22})e^{\frac{-t}{\tau_{22}}}\left(\frac{2}{1+e^{\frac{\frac{P_u}{2}}{\tau_{21}}}}\right)\right] y_n(t-1) \quad (7)$$

The term $y_n(t-1)$ in the above equation (7) is one step ahead prediction of $y_n(t)$.

3.2. Analytical Expression for G_{p12} Interactive Transfer Function

Analytical expression is derived for the off-diagonal closed loop transfer function (G_{p12}) of 2 x 2 MIMO system (Wardle and Wood distillation column) in time domain using ideal relay feedback test. The relay response for G_{p12} is shown in figure 5. The response can be described as given in equation (8), at the first interval.

$$Y_1(s) = G_{p12,cl}(s).\,U_2(s) \quad (8)$$

Substituting the values of $G_{p12,cl}(s)$ and $U_2(s)$ in equation (9), we get

$$Y_1(s) = \frac{\frac{k_{12}e^{-D12s}}{(\tau_a s+1)(\tau_b s+1)}}{1+k_{c1}\left(1+\frac{1}{\tau_{i1}s}\right)\frac{k_{11}e^{-D11s}}{\tau_{11}s+1}}.\frac{1}{s} \quad (9)$$

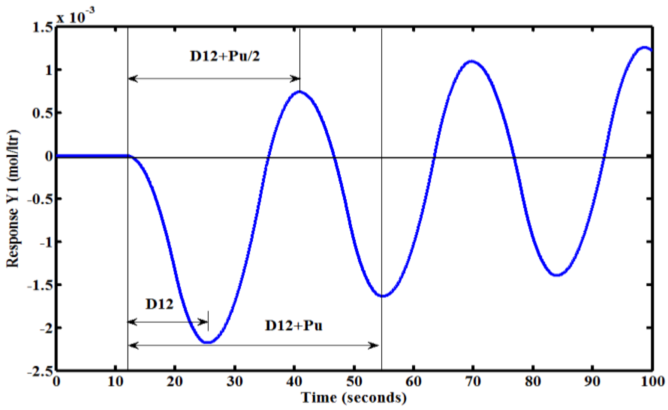

Fig. 5. Relay response for G_{p12}.

Similarly, the generalized analytical expression G_{p12} relay response can be expressed as given in equation (10)

$$y_n(t) = k_{12}\left[1 - a_1 e^{\frac{-t}{\tau_a}}\left(\frac{2}{1+e^{\frac{\frac{P_u}{2}}{\tau_a}}}\right) + b_1 e^{\frac{-t}{\tau_b}}\left(\frac{2}{1+e^{\frac{\frac{P_u}{2}}{\tau_b}}}\right)\right] - \frac{k_{11}k_{c1}}{\tau_{i1}}\left[1+\left(\frac{\tau_{i1}-\tau_{11}}{\tau_{11}}\right)e^{\frac{-t}{\tau_{11}}}\left(\frac{2}{1+e^{\frac{\frac{P_u}{2}}{\tau_{11}}}}\right)\right] y_n(t-1) \quad (10)$$

The term $y_n(t-1)$ in the above equation (10) is one step ahead prediction of $y_n(t)$.

4. WARDLE AND WOOD DISTILLATION COLUMN

Most of the unit operations in process engineering have more than one control loop. Each unit needs control of atleast two variables, product rate and quality, there by requiring two control loops. Some of these processes are concentration and temperature control of continuous stirred tank reactor, liquid level control of coupled storage tanks, heat exchangers, flash drums, boiler drums, etc. The need for multivariable control in case of a continuous distillation column is to reduce interactions between top products and feed as well as that between bottom product and feed. The proposed scheme is illustrated on a Wardle and Wood Distillation Column, using simulation studies.

(Wardle and Wood, 1969) reported a column for methanol-water separation with transfer function given in equation (11) is considered as an actual model for this work.

$$\begin{bmatrix} x_D \\ x_b \end{bmatrix} = \begin{bmatrix} \dfrac{0.126e^{-6s}}{60s+1} & \dfrac{-0.101e^{-12s}}{(48s+1)(45s+1)} \\ \dfrac{0.094e^{-7s}}{38s+1} & \dfrac{-0.1e^{-3s}}{35s+1} \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix} \tag{11}$$

The controlled variables are compositions of top (x_D) and bottom (x_B) products, expressed in wt % of methanol. The manipulated variables are reflux (L) and the reboiler (V) steam flow rates, expressed in lb/min, time constants are in minutes. Feed flow rate is the disturbances.

5. VALIDATION AND PARAMETER ESTIMATION

Validation of G_{p21} was carried out and is shown in figure 6. It is observed that the simulated relay response exactly matches with that obtained for theoretical value.

Validation of G_{p12} was also carried out on similar lines and is shown in figure 7. Here also it is observed that the simulated relay response exactly matches with that obtained for theoretical value.

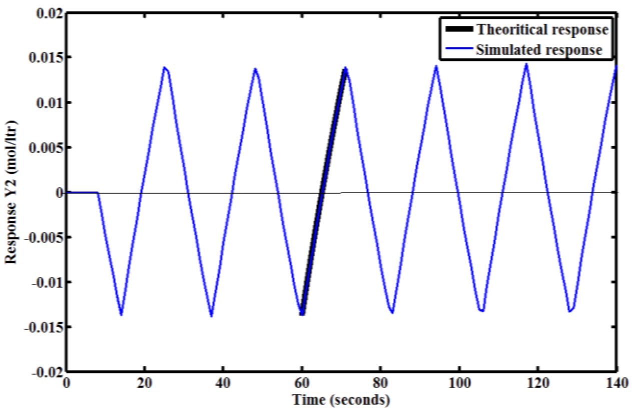

Fig. 6. Validation of analytical expression of relay response for G_{p21}.

From the equations (7 & 10), it is clear that there are twelve parameters to be estimated in the 2 x 2 MIMO process. Here,the objective is to identify those parameters from relay response by using the information obtained under the stable oscillating condition. The dead time D_{11}, D_{12}, D_{21}, D_{22} can be obtained straight away from the initial relay response. The residual eight parameters can be estimated by applying four different boundary conditions in equation (7) and (10) and solving them. The comparison of true process and estimated process is presented in Table 1.

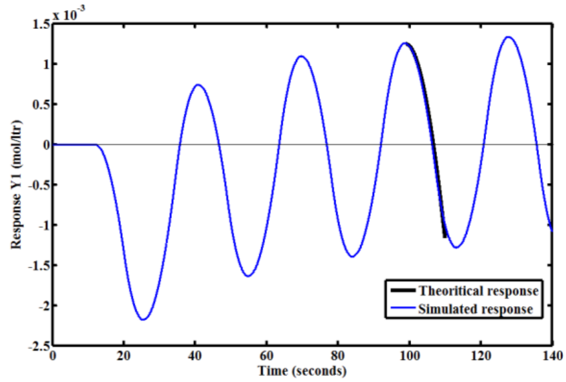

Fig.7. Validation of analytical expression of relay response for G_{p12}

Table 1. Comparison of True process and estimated process.

S.No	True process	Estimated process
1.	$\frac{0.126e^{-6s}}{60s+1}$	$\frac{0.1267}{60.35s+1}$
2.	$\frac{-0.101e^{-12s}}{(48s+1)(45s+1)}$	$\frac{-0.1018e^{-12s}}{(48.472s+1)(45.45s+1)}$
3.	$\frac{0.094e^{-7s}}{38s+1}$	$\frac{0.0948e^{-7s}}{38.2005s+1}$
4.	$\frac{-0.1e^{-3s}}{35s+1}$	$\frac{-0.1202e^{-3s}}{35.02s+1}$

From the table (1), it is found that the estimated parameters are matching with that of the true process.

6. CLOSED LOOP ANALYSIS

In the conventional closed loop control approach, the controller is tuned based on the limit cycle data K_u and P_u obtained from the relay response. However, this generalized Ziegler-Nichols (ZN) method which is independent of model structure and associated model parameters, is not suitable for many applications. In order to improve the closed loop performance, the controller must be tuned based on the model parameters. In this section, the performance of controller, tuned based on conventional ZN method is compared with that of the controller, tuned based on the estimated model parameters for 2 x 2 MIMO process. For illustration purpose, Biggest Log Modulus (BLT) tuning method is chosen to tune the controller parameters based on the estimated model parameters of 2 x 2 MIMO process. The controller settings of PI controller obtained from ZN method and BLT tuning methods are given in Table 2.

Table 2. PI controller Parameters.

Tuning Method	Loop	Kc	Ti
ZN	1	59	19.3
	2	-28.3	24.6
B L T	1	27.4	41.4
	2	-13.3	52.9

Using the selected PI tuning method, the closed loop responses are obtained for 2 x 2 MIMO systems for set point tracking and disturbances rejection. The servo responses for variation in top and bottom products for multiple set point change with PI tuned using ZN and BLT method is given in figures 8 & 10. The corresponding variations in top product and bottom product for manipulated variable are shown in figures 9 & 11. From the responses it is observed that for multiple set point change, the top and bottom product composition settle down faster.

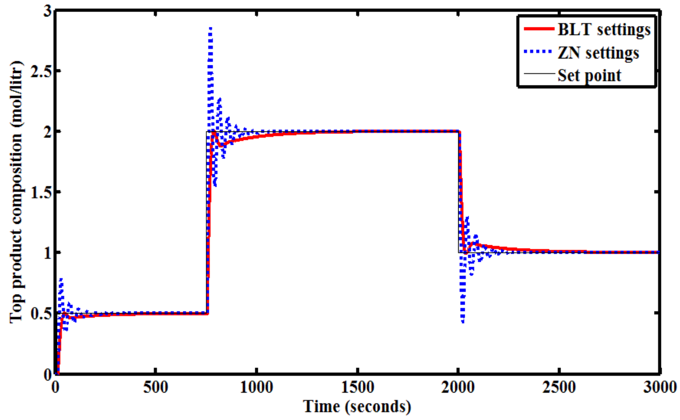

Fig. 8. Variation in top product for multiple setpoint tracking.

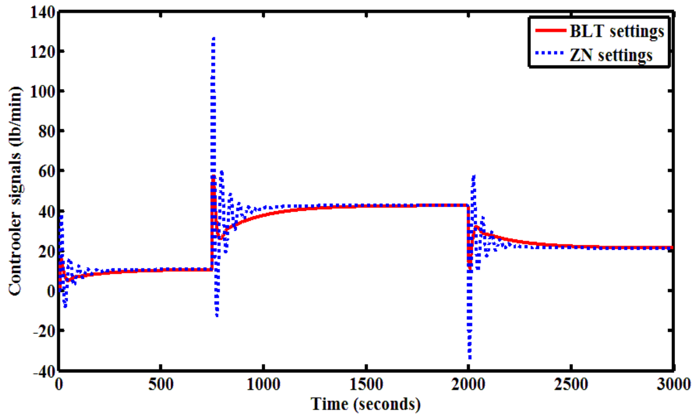

Fig. 9. Manipulated variable (reflux steam flow rate) for variation in top product for multiple setpoint tracking.

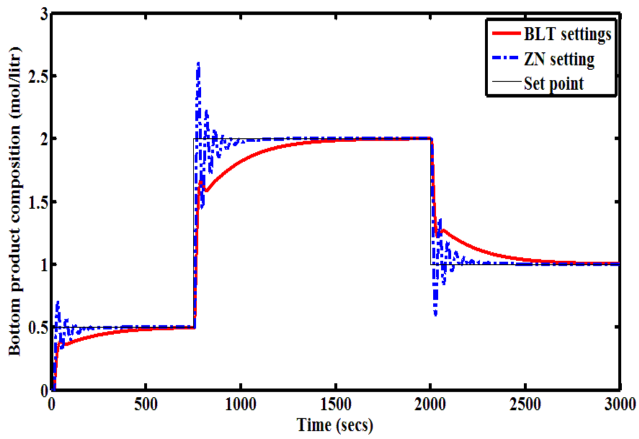

Fig. 10. Variation in bottom product for multiple setpoint tracking.

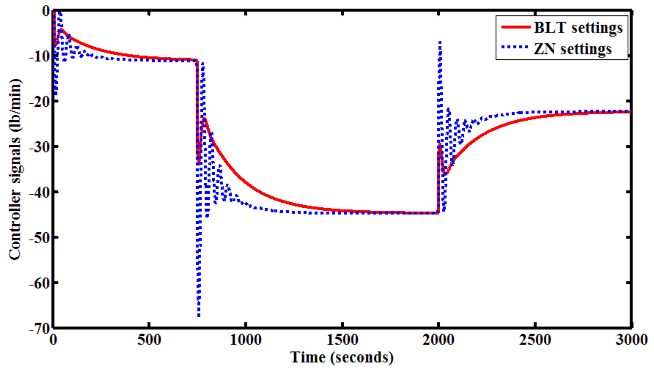

Fig. 11. Manipulated variable (reboiler steam flow rate) for variation in bottom product for multiple setpoint tracking.

The regulatory responses for variation in top and bottom product with PI tuned using ZN and BLT method are given in figure 12 & 14. From the response it is observed that for the single set point and multiple set point change BLT method settles down faster than ZN method. The corresponding variations in top product and bottom product for manipulated variable are shown in figure 13 & 15. From the responses, it is observed that the proposed method works well for disturbance rejection.

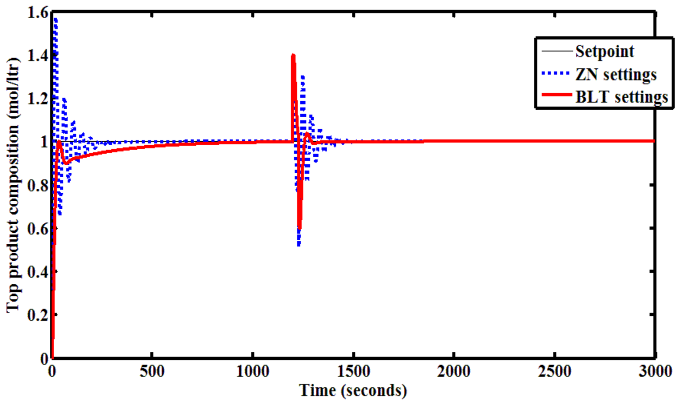

Fig. 12. Variations of top product for increase in feed flow rate at 1200s (Regulatory responses of PI controller tuning using ZN and BLT).

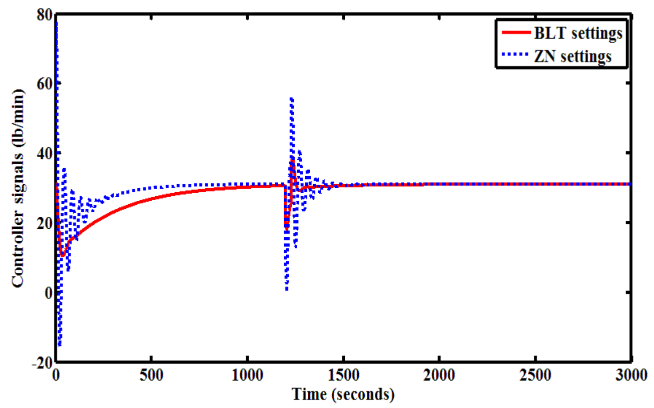

Fig. 13. Manipulated variable (reflux steam flow rate) for variations in top product for increase in feed flow rate at 1200 s (Regulatory responses of PI controller tuning using ZN and BLT).

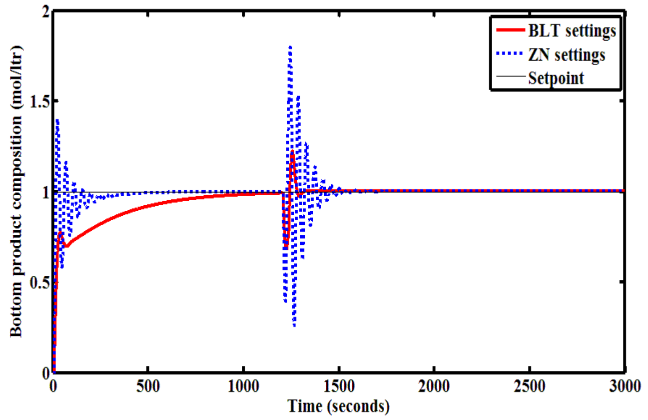

Fig. 14. Variations of bottom product for increase in feed flow rate at 1200s (Regulatory responses of PI controller tuning using ZN and BLT).

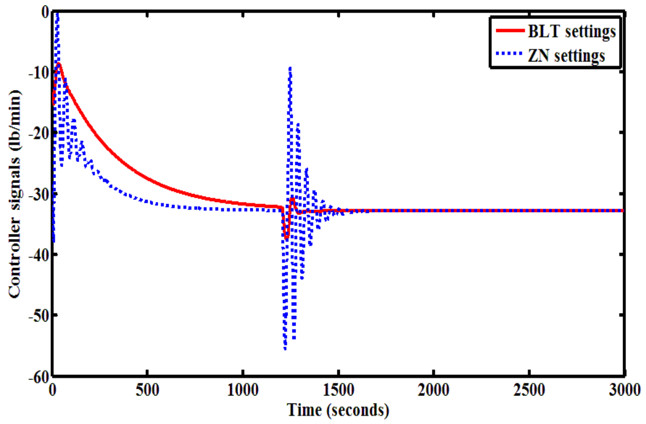

Fig. 15. Manipulated variable (reboiler steam flow rate) for variations in bottom product for increase in feed flow rate at 1200s (Regulatory responses of PI controller tuning using ZN and BLT).

The time domain specifications namely % overshoot, peak time and rise time and the performance indices namely ISE and IAE are used to evaluate the controller performance and are presented in Table 3.

Table 3. Comparison of performance indices for BLT tuning method with and without approximation of the entire transfer function model as FOPDT.

MIMO System Representation	IAE	% Over shoot	Peak time (sec)	Rise time (sec)
Transfer function approximated as FOPDT	81.15	6.73	42	18.38
Without approximation	98.18	0.6	38	18.13

From the Table 3 it is found that the closed loop performance of the controller is better on the MIMO system when its transfer function models not approximated as FOPDT. The performance of proposed scheme is compared with ZN tuning and the results are presented in Table 4.

Table 4. Comparison of controller performance for ZN and BLT Tuning.

Tuning rule	Controller Performance	
	ISE	IAE
ZN	69.96	104.2
BLT	30.41	96.16

From the Table 4 it is found that the controller tuned using proposed method (BLT) gives minimum ISE and IAE values compared with the ZN tuning method.

7. STABILITY ANALYSIS

The Nyquist stability criterion can be directly applied to multivariable processes. The procedure is based on a complex variable theorem that says that the difference between the number of zeros and poles of a function inside a closed contour can be found by plotting the function and looking at the number of times it encircles the origin. By using this theorem, it is easy to find if the closed loop characteristic equation has any roots or zeros in the right half of the s-plane. The closed contour can be followed by (s) variable that completely surrounds the entire right half of the s-plane. The closed loop characteristic equation is given as

$$F(s) = \mathrm{Det}\,[I+G_M(s)\,G_C(s)] \qquad (12)$$

In order to use a similar plot in multivariable systems, the function can be defined as follows:

$$W\,(i\omega) = -1 + \mathrm{Det}[I+G_M(i\omega)G_C(i\omega)] \qquad (13)$$

In the right half of the s plane, the number of encirclements of the (-1, 0) point made by $W_{(i\omega)}$ as ω varies from 0 to ∞ gives the number of zeros of the closed loop characteristic equation.

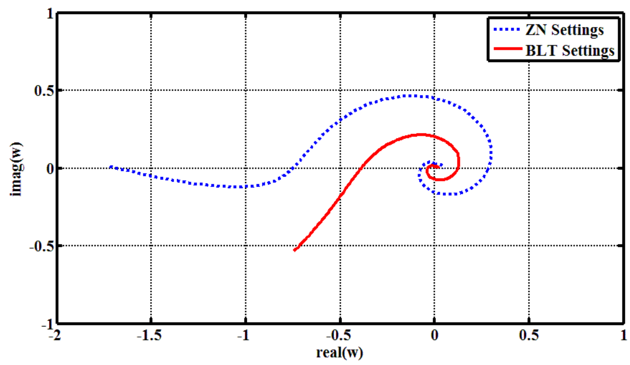

Fig. 16. Stability Analysis of MIMO system using Nyquist Plot.

In MIMO system, after forming the four transfer functions for the process and two transfer functions for the controllers, they are evaluated at each frequency using the polyval command. The ω plane curve given in figure 16 shows that the settings does not encircle the (-1, 0) point and therefore the system exhibits closed loop stability.

8. ROBUSTNESS STUDIES

The robustness studies were carried out for the MIMO system by increasing the individual element gain of the transfer function model by 10%, without changing the controller settings. The performance of the controller is shown in figures (17-20), from which, it is clear that even in the presence of variation in process gain, the controller does not result in degraded performance.

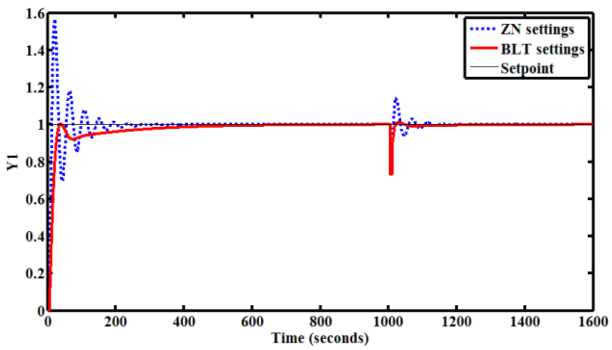

Fig. 17. Controller performance of perturbed system for unit step change $Y_{1(set)}$ in top product.

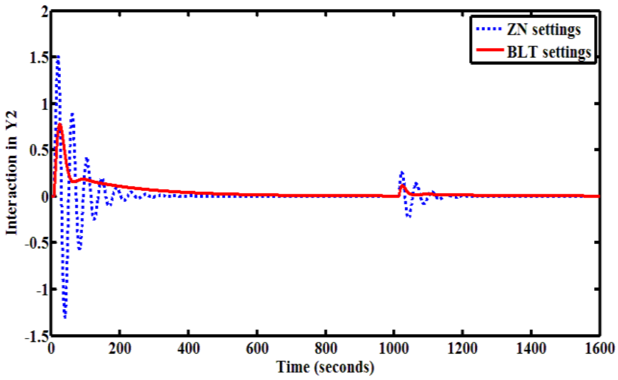

Fig. 18. Controller performance of perturbed system of interaction in Y2 for unit step change $Y_{1(set)}$ for bottom product.

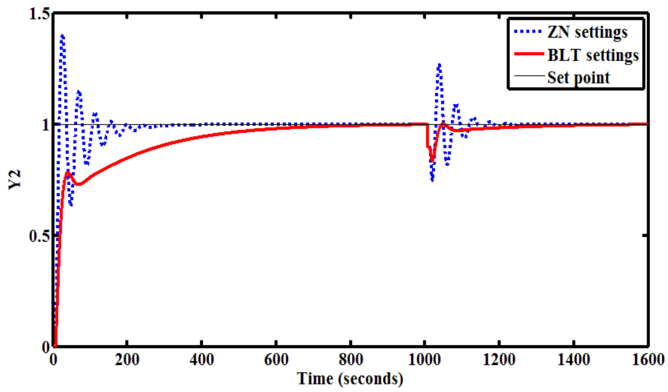

Fig. 19. Controller performance of perturbed system for unit step change $Y_{2(set)}$ in bottom product.

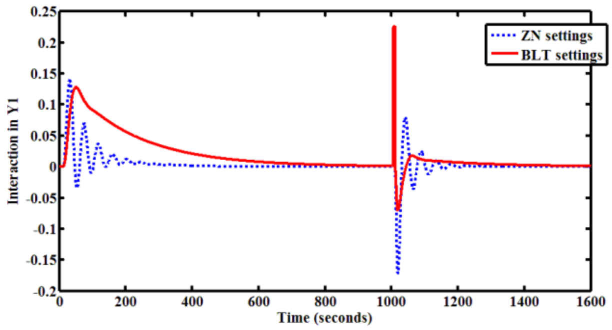

Fig. 20. Controller performance of perturbed system of interaction in Y1 for unit step change $Y_{2(set)}$ for top product.

The comparison of controller performance of the Wardle and Wood distillation column under ±10% parametric change in all the process gains is given in table 5.

Table 5. Comparison of controller performance for the Wardle and Wood distillation column under ±10% parametric change in all the process gains.

Tuning rule		BLT		ZN
+10%	**Set point**	**ISE**	26.6	101.7
		IAE	90.17	141.1
	Disturbance	**ISE**	35.69	135.6
		IAE	116.2	226.5
+10%	**Set point**	**ISE**	32.62	52.66
		IAE	106.6	63.6
	Disturbance	**ISE**	39.05	74.68
		IAE	137.6	144.5

From table 5, it is observed that the controller setting of the proposed method (BLT) provides superior robust performance for both set point tracking and disturbance rejection operation.

9. EFFECT OF MEASUREMENT NOISE

In real environment, the noise will play a major role and it cannot be preventable. Here, the measured process output is despoiled by adding distributed noise with the variance of 0.0005 as shown in figure 21 and figure 22.

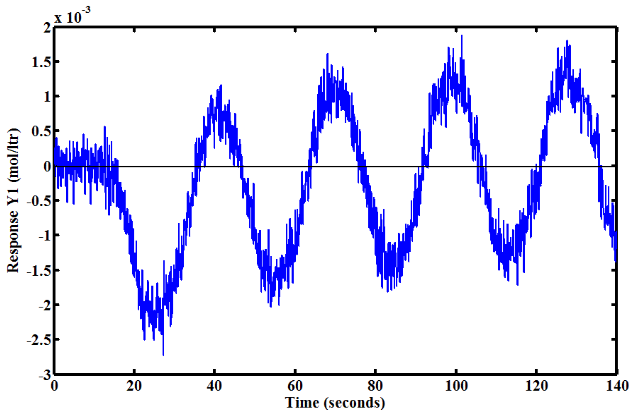

Fig. 21. Relay response for G_{p12} with measurement noise.

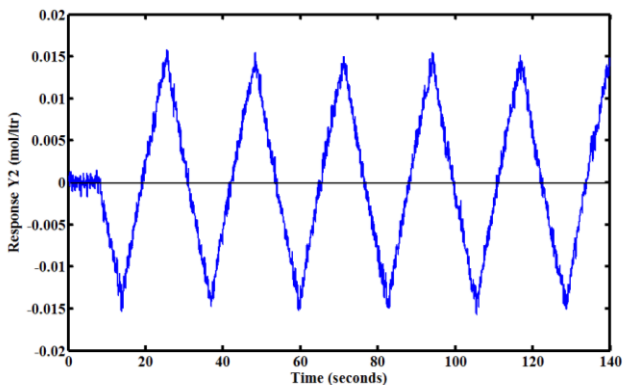

Fig. 22. Relay response for G_{p21} with measurement noise.

9.1 Parameter Estimation in the presence of noise

The parameter can be estimated in the presence of noise by means of taking the average of the fictitious peaks around the peak to compute the limit cycle data and the model parameters can be estimated using these data. The comparison of the true process with the estimated process is shown in Table 6. With the presence of noise it is noted that the percentage of error in estimated process when compared to that of the true process is not more than 6.38.

Table 6. Comparison of true and estimated parameters with presence of noise.

Process	True process	Estimated process	% Error
k_{11}	0.126	0.1279	1.5
k_{12}	-0.101	-0.0966	4.356
k_{21}	0.094	0.1130	6.38
k_{22}	-0.12	-0.1208	0.66
τ_{11}	60	59.66	0.562
τ_a	48	46.155	3.84
τ_b	45	44.478	1.159
τ_{21}	38	38.80	2.10
τ_{22}	35	35.08	0.22

9.2. Closed loop studies in the presence of noise

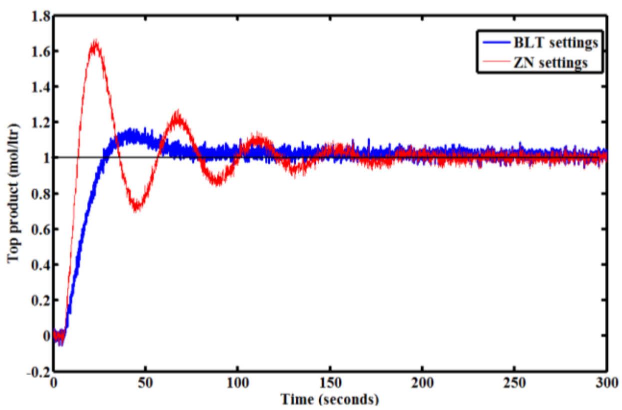

Fig. 23. Variation of top product with measurement noise for unit step change.

The closed loop studies were carried out by using ZN and BLT tuning based PI control schemes for set point tracking in the presence of measurement noise. The variation of top product and bottom product with measurement noise for unit step change are shown in figure 23 and 24. Here, the closed response is corrupted by adding noise. The PI controller parameters obtained in the presence of noise for ZN and BLT is given in table 7.

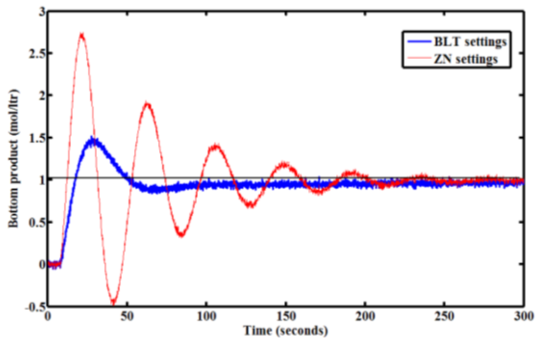

Fig. 24. Variation of bottom product with measurement noise for unit step change.

From figure (23 & 24), it is observed that the PI controller tuned using BLT tuning rule works well even in the presence of measurement noise, resulting in satisfactory closed loop performance.

Table 7. PI controller Parameters in the presence of noise.

Tuning Method	Loop	Kc	Ti
ZN	1	75.45	18.67
	2	-36.37	26.31
BLT	1	29.59	47.621
	2	-14.26	67.09

Table 8. Comparison of performance indices for BLT tuning method with and without approximation of the entire transfer function model as FOPDT.

MIMO System Representation	IAE	% Overshoot	Peak time (sec)	Rise time (sec)
Transfer function approximated as FOPDT	81.15	6.73	42	18.38
Without approximation	98.18	0.6	38	18.13

From the Table 8 it is found that the closed loop performance of the controller is better on the MIMO system when its transfer function models not approximated as FOPDT.

Table 9. Comparison of performance indices of PI controller in the presence measurement noise.

Tuning rule	Controller Performance			
	Without Noise		With Noise	
	ISE	IAE	ISE	IAE
ZN	69.96	104.2	245.9	176.2
BLT	30.41	96.16	109.9	31.68

From the Table 9 it is found that the controller tuned in the presence of measurement noise using proposed method (BLT) gives minimum ISE and IAE values compared with the ZN tuning method.

10. EXPERIMENTAL ANALYSIS

The MIMO process considered to demonstrate the proposed scheme is the coupled tank system (shown in figure 25). It contains two cylindrical tanks connected in interacting fashion at two different heights. The system is configured as a MIMO system with two input and two output variables.

10.1. Experimental Set-Up of Coupled Tanks

The schematic diagram of a Two Tank Cylindrical Interacting System (TTCIS) shown in figure 25 is based on the two-tank benchmark problem that has been used by number of researchers. The TANK1, TANK2 are two identical cylindrical tanks whose maximum height is ‘h’. These two tanks are interconnected at the bottom through a manually controlled valve, mv_3. F_{in1} and F_{in2} are the two input flows to TANK1 and TANK2 respectively. mv_1 and mv_2 are the two manual control valves which are placed at the outlet pipe. The two heights of the liquid in TANK 1 and TANK 2 are h1 and h2 respectively. The liquid heights are transmitted in the form of 4-20 mA. The A1 and A2 are the two areas of the TANK 1 and TANK 2 respectively. The Flow Transmitter (FT) is used to measure the flow of liquids in pipelines and convert the results into proportional electric signals that can be transmitted to distant receivers or controllers. The Differential Pressure Transmitter (DPT) is used to measure the level based on the pressure which is generated by the liquid in the tank.

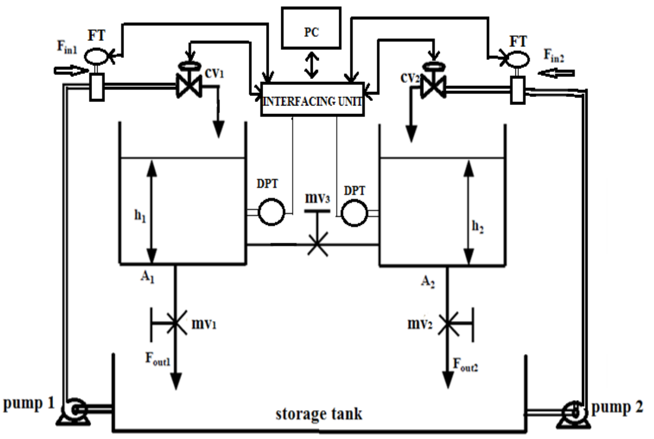

Fig. 25. Schematic diagram of two tank interacting system.

Level transmitters and control valves are interfaced with PC with the help of NI 6008 DAQ board which is USB compatible PC based data acquisition board. It has 8 analog input and 2 analog output channels, 16 digital inputs/outputs channels compatible with LabVIEW software. The signals in the form of 1-5 V from DAQ is converted to 4-20 mA by means of V/I converter and are supplied to control valve CV_1 and CV_2 respectively. The real time experimental set-up of TTCIS is shown in figure 26.

Fig. 26. Experimental setup of TTCIS.

10.2. Control of Two Tank Cylindrical Interacting System:

The PI controller designed from the sequential auto tuning is placed in loop 1 and loop 2 and implemented in real time. The PI controller is designed for the control of TTCIS. The controller output is given to the process through DAQ assistant block and process variable is acquired through the input DAQ assistant block.

According to the control action taken, the control variable (level in tank 1) is able to track the given set point (1.7). From the response it can be noted that the overshoot is not large. The input flow (u_1) to the tank 1 is regulated by the PI controller, whose parameters are computed by sequential auto tuning and the control signal is plotted in the figure 28.

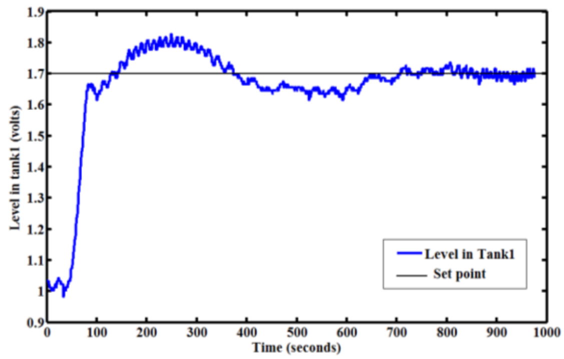

Fig. 27. Variation of Level in tank1.

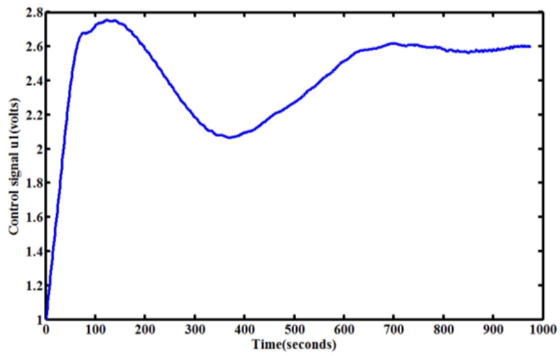

Fig. 28. Corresponding variation in Control signal, u_1.

According to the control action taken, the control variable (level in tank 2) is able to track the given set point (2.5). From the response it can be noted that the overshoot is not large.

The input flow (u_2) to the tank 2 is regulated by the PI controller, whose parameters are computed by sequential auto tuning and the control signal is plotted in the figure 29.

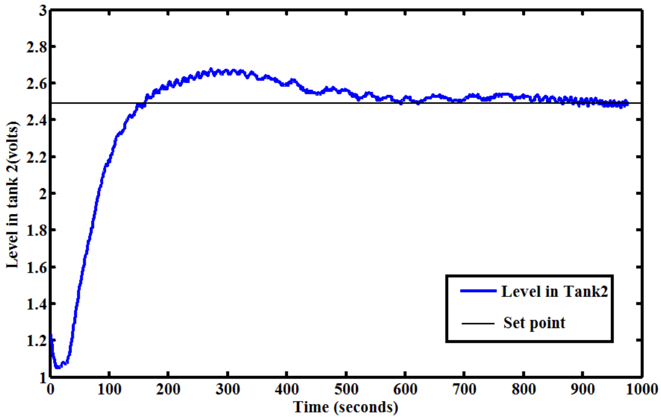

Fig. 29. Variation of Level in tank2.

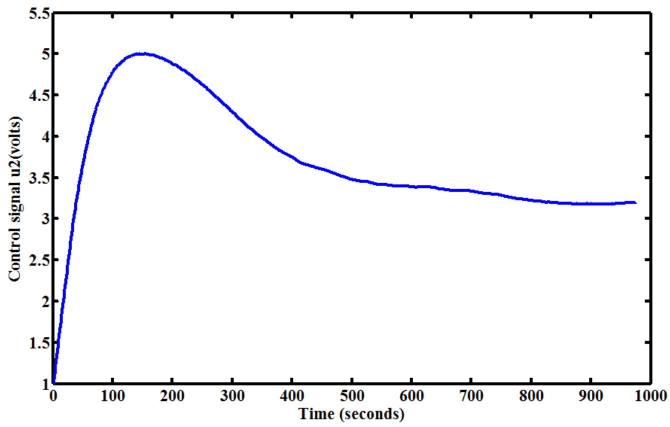

Fig. 30. Corresponding variation in Control signal u_2.

11. CONCLUSION

An efficient approach is proposed to derive the exact analytical expressions in time domain for the relay feedback responses of a 2 x 2 MIMO systems with different (FOPDT and SOPDT) structures. The model parameters are estimated by applying boundary conditions. Without approximating the system, the interaction transfer functions of closed loop response for 2 x 2 systems were modeled. The results are presented in a closed form solution. The effectiveness of the proposed scheme is illustrated by simulations on Wardle and Wood distillation column. Then, the closed loop studies were carried with PI controller tuned with ZN method and Model based Biggest Log Modulus (BLT) tuning method. The proposed approach treats the 2 x 2 MIMO system as a sequence of SISO systems for which the relay-feedback system is proven useful and reliable. In terms of identification, it is a more accurate approach.. The time domain expressions are obtained from relay responses for interactive transfer functions and also, the complexity due to presence of interaction is incorporated in formulating the model. The process parameters are identified without approximating the process transfer function as a FOPDT. As these inter-active transfer functions carry the information on interactions between inputs and outputs, these theoretical models will provide the information on measuring the interactions directly. This advance information will also help to assess interaction measures between inputs and outputs. Also, the robustness study was conducted by inserting a perturbation uncertainty of ±10% in all the process gains simultaneously. The proposed scheme provides superior robust performance compared with the ZN method both for the set point tracking and disturbance rejection. It also works well even in the presence of measurement noise, resulting in satisfactory closed loop performance and acceptable performance indices. The proposed method (BLT) gives minimum ISE and IAE values compared with the ZN tuning method. The usefulness of the scheme is also verified on a Laboratory bench mark two-tank laboratory set-up.

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