ON MODELLING AND SIMULATING NATURAL GAS TRANSMISSION SYSTEMS (Part I)

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Abstract: *Natural gas transmission systems stand for mathematical modelled complex systems as systems with distributed parameters having the form of differential equations with partial derivatives. In view of studying some practical situations, such as the process of flow along a natural gas pipeline, a simplified mathematical model may be used. Practical operation with this model demands a spatial and temporal discretization of functions of interest (pressure and weight/flow rate). The paper develops one such model and analyses the necessary discretization processes needed for numeric integration. Using an implicitly numeric integration method more case studies suitable to some flow processes through pipelines in distinct situations are developed. As the results obtained by computation are very close to the experimental ones, the case studies acknowledge both the validity of the developed simplified model and those of the calculation procedures used.*

Keywords: *natural gas transmission systems, modelling, numeric integration methods, transitory regime, case studies.*

1. INTRODUCTION

A natural gas transmission system (STG) represents the interconnected result of the many gas transmission pipe segments of various lengths and cross sections of passage, according to the flow and pressure requested by gas consumers. Figure 1 illustrates a schematic diagram of such a gas transmission system, marked by *STG*(1), the system's pipe segments, of *Lxy* lengths and *Dxy* diameters, are functionally interconnected by means of the so called internal *technological nodes* (valve control stations, gas compression stations) N_1, N_2 and N_s. The *STG*(1) system, under consideration, is in its turn interconnected by means of N_0 and, respectively N_4 technological nodes with the adjacent *STG(0)* and *STG(2)* gas

transmission systems. Consumers of the *STG(1)* system are represented by *SDG(3), SDG(6)* and respectively *SDG(7)* gas distribution systems connected to the *STG(1)* system, through the N_3, N_6, and respectively N_7 outlet nodes.

The exploitation of such a system assumes its integration within a control system which has to implement several functions out of which the most important one refers to its *physical balancing* [1], that is, to the establishment and maintenance of a balance between the incoming and respectively outgoing gas flows. We deal here with an automatic control issue, of stabilizing the P_3, P_6 and, respectively P_7 intake pressures in the *SDG(3), SDG(6)* and respectively *SDG(7)* gas distribution systems, in conditions of gas flow fluctuations required from within the system by these ones. From a technological point of view this problem is solved by performing a set of *predictive actions* (internal and of entrance or interconnection with other systems) which will lead to the permanent pressure remodelling for the whole system, so as to compensate for the gas flow fluctuations required by consumers connected to the system [4]. The predictive character of the automatic control algorithm is required by the process of wave propagation at the level of the transmission system, characterized by propagation times of action.

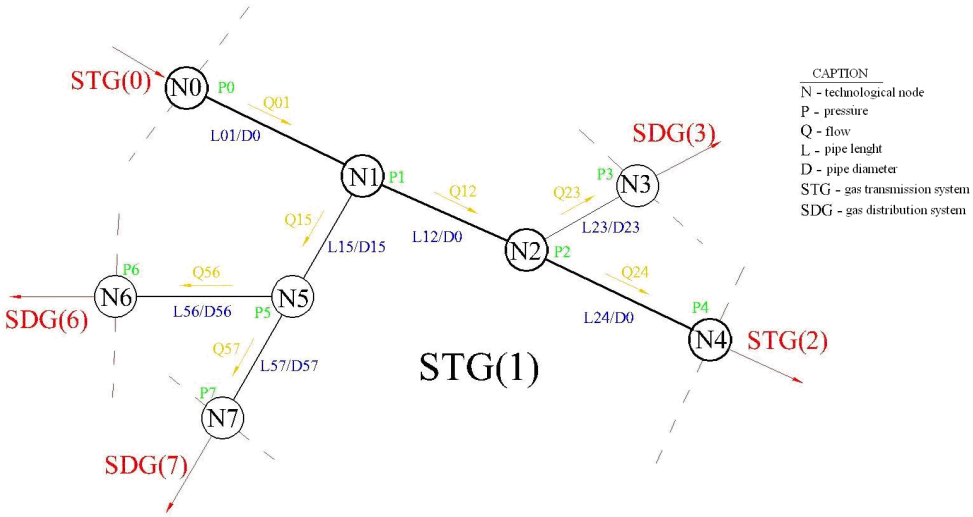

Fig.1. Schematic diagram of a gas transmission system

The systematic approach of these practical problems and the assurance on some other *auxiliary functions* is possible only with adequate *mathematical models*, from *the class of systems with distributed parameters*. The use of these systemic instruments in view of solving the tangible exploitation problems requires numerical methods rather than analytical ones. The complexity of the problems is partly diminished by the fact that pipeline segments between two junctions or assemblies of segments limited by junctions are displayed as *separable, independent modelled subsystems.*

The exploitation of natural gas transmission system from the implementation perspective of specific automatic control concepts has begun to apply in a *systemic content*, starting with the 1960s, through research programs supported by "The Pipeline Research Council International" (www.prci.org). The resulted mathematical models were used for the exploitation, optimal inspection, and more efficiency of engineering activity. Regarding the framework of this paper, the researches referred in [8] and [7] deal, for the first time, with the problem of modelling the transient behaviour of the flow process of natural gas through transmission systems, with simplifications which can be brought to the mathematical model, and the determination of some linear approximating models. For the usage of transient regime models numerical computation techniques have been used, based on finite differences, without the detailed analysis of the convergence properties of the method. Paper [13] (www.psig.org) is representative from this point of view and shows that the systems arising from discretization through the finite difference method (of the gas flow process models through a transmission system), ranks, starting with a certain required computational accuracy, within the class of

inflexible equation systems respectively ("stiff equations") which needs specific numerical solving methods.

Going on from a complex model of the gas flow process through a pipeline section, in [15] and [16] were studied the advantages and disadvantages of using simplified mathematical forms.

At European level, "SIMONE Research Group" (www.ercim.org) has developed, beginning with 1975 a research project focussed on the properties, the behaviour and the control of complex networks consisting of a great number of dynamic elements and non-linear restrictions. By means of specialized SIMONE software, numerical simulation algorithms for solving some network structures, which come up to more than 10.000 non-linear differential equations with partial derivatives dependent on several types of restrictions were implemented.

The systemic approach of natural gas transmission systems constitutes the object of some recent papers, [19] and [2]. In the first, by using a simplified linearized mathematical model with constant parameters, are designed some structures of state observers, while in the second, a linearized mathematical models of the flow process is used to implement some control algorithms of a natural gas transmission system.

The present paper intends to offer a systemic interpretation of fundamental equations of the flow process (chapter 2), to present assumptions which allow the use of some simplified mathematical models of the process in certain conditions of technological exploitation (chapter 3), to define and to simulate the systemic model of the process, from the perspective of a process with distributed parameters (chapter 4), and finally, in the absence of some analytical solutions of the mathematical model of the flow process, to validate the latter one by numerical simulation and by comparison with data measured in real operating conditions (chapter 5).

2. FUNDAMENTAL EQUATIONS OF THE FLOW PROCESS. SYSTEMIC INTERPRETATION

According to the previous chapter it is important to note two aspects:

- Through a transmission system, natural gas flow is a process with distributed parameters, described by a complex system of differential equations with partial derivatives, which incorporate mechanical, kinematics and thermodynamics aspects of the real viscous fluid flow [11] and [9].
- For modelling, one applies the separation hypothesis relative to the component pipeline segments of the system, i.e. starting from the functional point of view, the pipeline segments are considered in the developing of models, as separable elements.

In this context, as a separable subsystem, a segment of straight pipeline of length L, pipe bore D<< L and the area of the inner cross-section $A=\frac{\pi D^2}{4}$ is considered. The longitudinal axis is marked *Ox*. With respect to the horizontal direction the pipeline segment displays the elevation *z(x)* and the elevation angle *α(x)*. The D<<L hypothesis allows to use as independent variables of the model only the longitudinal coordinate *x* and the time *t*.

Thus, the flowing process through the pipeline section of a gas of $\rho(x,t)$ density, carried out at $p(x,t)$ pressure and $T(x,t)$ temperature, at a velocity $v(x,t)$ will be described by the system of differential equations with partial derivatives which describe mass, momentum and energy conservation laws [10], [6]:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0 \tag{1.a}$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial\left(\rho v^2\right)}{\partial x}+\rho g\sin\alpha+\frac{\partial p}{\partial x}+\frac{\lambda}{D}\frac{\rho v|v|}{2}=0 \tag{1.b}$$

$$\frac{\partial}{\partial t}\left[\rho\left(c_vT+\frac{v^2}{2}+gz\right)\right]+\frac{\partial}{\partial x}\left[\rho v\left(c_vT+\frac{v^2}{2}+gz+\frac{p}{\rho}\right)\right]=\rho q \tag{1.c}$$

to which the thermodynamics equation of state [14] is added:

$$\rho=\frac{p}{ZRT} \tag{1.d}$$

The space-temporal domain of flow is $D\left(L,\tau_f\right)=[0,L]\times[0,\tau_f]$, with τ_f the final time.

In (1), the meaning of the other quantities is: c_v –specific gas heat at constant volume; $Z=Z(p,T)$ – compressibility factor which expresses natural gas deviation from the thermodynamic behaviour of ideal gas; q – heat quantity changed into unit of time by the unity of gas mass; R – specific constant of gas; $g=9{,}81\frac{m}{s^2}$ -

gravitational acceleration; $\lambda = \lambda(Re)$ - the hydraulic loss factor (dimensionless).

System (1) has a *unique solution* when *terminal conditions* of the form (CI) and (CB) are imposed [17] [3]:

- initial conditions (CI): $p(x,0)$, $v(x,0)$, $T(x,0)$ correlated with $\rho(x,0)$ through the equality (1.d);
- boundary conditions (CF): any of the combinations of three distributions of the $\{p(0,t), p(L,t), v(0,t), v(L,t), T(0,t), T(L,t)\}$ set correlated with $\{\rho(0,t), \rho(L,t)\}$, also through (1.d) equality.

Because the gas flow $Q = \rho v A$ (mass or volume) is most often measured in technological applications, the system (1) of equations is rewritten [4] considering $Q(x,t)$ variables instead of $\rho(x,t)$. The boundary conditions change accordingly.

From a systemic point of view the assignation of terminal conditions means in fact the mathematical model orientation in view of assigning the input, state and output quantities [5]. In this context, Figure 2 exemplifies a *systemic realization* of the mathematical model consisting of equations (1) rewritten in $\{p,Q,T\}$ variables, and the boundary conditions $\{p(0,L), Q\{L,t\}, T(0,t)\}$.

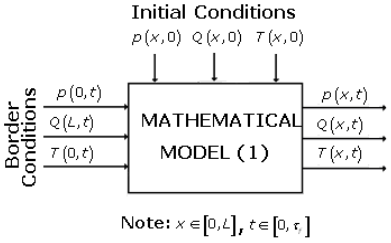

Fig. 2. Systemic interpretation of the flow eqs.(1) and associated terminal conditions

For the systemic realization in figure 3 we have:

- The input variables represented by the imposed boundary conditions: $p(0,t)$, $Q(L,t)$ and $T(0,t)$ for $t \in [0, \tau_f]$;
- The output variables represented by distributions of characteristic variables for the flow process: $p(x,t)$, $Q(x,t)$, and $T(x,t)$ for $(x,t) \in D = [0,L] \times [0,\tau_f]$, and in particular by the boundary values: $p(L,t)$, $Q(0,t)$ and $T(L,t)$ for $t \in [0,\tau_f]$;

At any point (x,t) the state of flow process within the flow domain $D = [0,L] \times [0,\tau_f]$ is represented by the $\underline{\Omega} = [p \quad Q \quad T]^t$ vector. Particularly, the initial state at the time moment $t = 0$ is rendered by the initial conditions.

Obtaining of an analytical solution for the mathematical model (1) at any combination of boundary conditions is possible only for some *simplifying hypotheses*. In change, through numerical computations, by respecting some quality criteria, one may obtain approximate solutions of the system of equations for any set of boundary conditions.

3. A SIMPLIFIED MATHEMATICAL MODEL OF THE FLOW PROCESS

Real operating conditions of the natural gas transmission pipelines allow the adoption of some simplification hypotheses of the mathematical model represented in system (1). General references with regard to this problem may be found in [7], [15], and [16].

A first, important, simplifying hypothesis is of thermodynamics nature and regards the process of flow in isothermal behaviour since the entrenchment of the pipeline in the ground: the temperature variation along the pipeline, on the largest part of the track, is negligible. Gas temperature will rank to ground temperature level $T(x,t) = T_{average} = T_{sol} = const$ and the practical consequence of this hypothesis consists in omitting equation (1.c) from the model. Comparative studies based on numerical simulations, performed by using this hypothesis for other situations, have proved that the deviations of the characteristic variables of flow ranks under 2% in comparison with values calculated with model (1) [7], [16].

A second simplification can be operated in (1.b). *The analysis of the order of magnitude* of the compound terms in (1.b) has shown that the values for $\frac{\partial(\rho v)}{\partial t}$ and $\frac{\partial(\rho v^2)}{\partial x}$ are about 1% from that of the $\frac{\lambda}{D}\frac{\rho v^2}{2}$, which express the friction loss during the flow process. Consequently, their influence on the dynamic behaviour is more reduced and, at a first approximation, they can be omitted [7], [16].

Thirdly, on the basis of some observations relative to the parameters $\alpha(x)$, $\lambda(\mathrm{Re})$ and $Z(p,T)$, simplifications which "*reduce" the non-linear character* of system (1) can be performed. Thus:

- The elevation angle of the pipe segment α can be considered null (horizontal pipe segment) or constant (equal with the total elevation Δh between the ends of the pipe segment $\alpha = \arcsin\frac{\Delta h}{L} = const$).
- Studies undertaken through numerical simulations have shown variations of the *loss pressure factor* λ under 20% within large enough ranges of variation (1:4) of the gas flow. At the same time, a dependence of pressure, density and velocity on squared root of λ has been noticed, so that within a range of [0.9 – 1.1] from the nominal value of λ, again for a first approximation one can consider $\lambda = const$ [7].
- At temperature $T = T_{average} = const$, in thermodynamics isotherm regime, the compressibility factor *Z* may be considered linear, dependent on pressure *p,* according to equation:

$$Z(p,T) = Z(p)\big|_{T=T_{sol}} = 1 - \tau p\,, \tag{2}$$

where τ is a parameter dependent on soil temperature and on natural gas characteristics [7]. For a pressure variation of $\pm 10 bar$ around an average value, the relative deviation of *Z* is less than 3%. The expression (2) is important in terms of equation (1.d).

In the above mentioned hypothesis, in [4] is shown that the flow process model (1), without direction change, may be brought to the simplified form (3), by using equation (1.c) and as variable the mass flow *Q*:

$$\frac{\partial p}{\partial t} + \xi_1(p)\frac{\partial Q}{\partial x} = 0 \tag{3.a}$$

$$\frac{\partial p}{\partial x} + \xi_2(p)\frac{Q^2}{p} + p\xi_3(p) = 0 \tag{3.b}$$

The coefficients in (3) are:

$$\xi_1(p) = \frac{(1-\tau p)^2 RT_{average}}{A} \tag{4.a}$$

$$\xi_2(p) = \frac{(1-\tau p)\lambda RT_{average}}{2DA^2} \tag{4.b}$$

$$\xi_3(p) = \frac{g}{(1-\tau p)RT_{average}}\sin\alpha \tag{4.c}$$

In its turn, in certain operating conditions, the system (3) can be reduced to a more simplified form, which allow for the acquiring of some analytical solutions which represent a first step in modelling the gas flow process through a pipe segment. Thus, by considering in a first step constant average values for some of the process parameters [6], [7], by integrating the steady-state regime equations one can assume a batch of computation relationships which allow for a complete description of the flow process in the operating conditions under consideration. Than, in a second step, by the analytical solving of the simplified form of (3) for initial uniform distributed conditions $\{p_{stat}, Q_{stat}\}$ and boundary conditions $\{p(0,t), p(L,t)\}$, with $t > 0$, one can conclude that the transitory dynamics of the process of flow represents the linear combination of some individual modes of dynamic behaviour described in terms as [13], [4]:

$$\left\{ M_j(t) = e^{-\frac{t}{\tau_j}}, j = \overline{1,m} \right\} \tag{5}$$

The time constants of the process are $\tau_j = \frac{\alpha}{\beta\cdot(\gamma + j^2)}$ where $\alpha, \beta, \gamma > 0$, $j = \overline{1,m}$.

The larger the number *m* of spatial discretization points is, the more their values differ.

4. THE PROBLEM OF NUMERICAL SIMULATION WITH THE SIMPLIFIED MODEL

In the following we refer to the use of the mathematical model (3) in order to *compute its response*, by appealing to the spatial discretization based on *finite differences* and on several approaches of numeric integration of the resulted dynamic systems with lumped parameters.

In perform it, a *discretization grid* of the space-temporal domain $D(L,\tau_f)$ is to be built on which, the values of continuous and differenttiable functions $p(x,t)$ and $Q(x,t)$ are computed. The discretization grid will be characterised by the $r = \frac{\Delta t}{\Delta x}$ parameter, where Δt is the discrete-time step and Δx the discrete spatial step. The discrete step time Δt and parameter *r* respectively can have, depending on the imposed rated numeric accuracy and considerations of assuring the convergence of the numeric method, different values on each iteration steps, operating in general with an irregular discretization grid [10]. The parameter *r* value selection to each iteration step is therefore, directly linked to the numeric method performances used.

By marking any of the variable {*p, Q*}, with *y*, in a first phase a *spatial discretization* of the mathematical model (4) is performed relative to *x* variable. This is done through approximation of the $\frac{\partial y}{\partial x}(x_j,t)$ derivates with the $\delta y_j(t,\Delta x)$, $j=\overline{1,m}$ finite differences, with formulas obtained from Taylor series with truncation errors $\varepsilon_{tr}(q,\Delta x)=0(\Delta x^q)$. The q parameter characterizes the development order.

Corresponding to the m spatial discretization points, from (3) we obtain the system (6), built up of $j=\overline{1,m}$ subsystems, each of two non-linear differential equations:

$$\begin{cases} \frac{dp_j}{dt}(t)+\xi_1(p_j(t))\cdot\delta Q_j(t,\Delta x)=0 \\ \delta p_j(t,\Delta x)+\xi_2(p_j(t))\frac{Q_j^2(t)}{p_j(t)}+p_j(t)\xi_3(p_j(t))=0 \end{cases} \quad (6)$$

Further on, system (6), having $2\cdot m$ equations, is restructured through removal of equations which describe the dynamics of variables known by specified boundary conditions.

Particularly, assuming that one knows the pressure in the point $j=1$: $p_1(t)=p(0,t)$, respectively, the flow in the point $j=m$: $Q_m(t)=Q(L,t)$, at level $j=1$, the first equation which expresses the pressure dynamics is omitted, respectively, at level $j=m$, the second equation which expresses the flow dynamics is omitted. By introducing the notation:

- $\underline{u}(t)=\left[p_1(t)\quad Q_m(t)\right]^t\in R^2$ - the vector of input quantities;
- $\underline{X}_p(t)=\left[p_2(t)\quad p_3(t)\quad \ldots\quad p_m(t)\right]^t\in R^{m-1}$, $\underline{X}_Q(t)=\left[Q_1(t)\quad Q_2(t)\quad \ldots\quad Q_{m-1}(t)\right]^t\in R^{m-1}$ - the vectors of state quantities, associated with pressure distribution, respectively with flow distribution,

the restructured system (6) for the given boundary conditions, are written as follows:

$$\frac{d\underline{X}_p}{dt}(t)=\underline{\Omega}\left(\underline{X}_p(t),\underline{X}_Q(t),Q_m(t),\Delta x\right) \quad (7.1)$$

$$\underline{X}_Q(t)=\underline{\Xi}\left(\underline{X}_p(t),p_1(t),\Delta x\right) \quad (7.2)$$

where: $\underline{\Omega}:R^{2m}\to R^{m-1}$, $\underline{\Xi}:R^{m+1}\to R^{m-1}$ are non-linear vector functions.

By replacing the flow distribution vector $\underline{X}_Q(t)$ from (7.2) into (7.1) the matrix differential equation (8) is obtained. This models, like a process with lumped parameters, only the pressure dynamic:

$$\frac{d\underline{X}_p}{dt}(t)=\underline{\Psi}\left(\underline{X}_p(t),\underline{u}(t),\Delta x\right). \quad (8)$$

In (8), $\underline{\Psi}:R^{m+2}\to R^{m-1}$ is a non-linear vector function. Obviously, (8) is used in association with (7.2).

As starting point for a second stage, the discretization of the derivative $\frac{d\underline{X}_p}{dt}(t_k)$ relative to time is used. This is done again through a finite difference, of expression $\delta\underline{X}_p(k,\Delta t)$ with $k\geq 0$ and Δt with the discrete-time coordinate step relying again on a Taylor series with a truncation error equal to the residue $\varepsilon_{tr}(s,\Delta t)=0(\Delta t^s)$. The s parameter characterizes the order of series development. This way, the continuous-time system (8) associated with the spatial discretization points $j=\overline{1,m}$ is approximate through the discrete-time system (9.1). It describes the $\underline{X}_p$ transition between discrete moments of time $k\to k+1$ with the variable discrete-time step Δt [4]:

$$\begin{aligned}\delta\underline{X}_p(k+1,\Delta t)=\theta\underline{\Psi}\left(\underline{X}_p(k+1),\underline{u}(k+1),\Delta x\right)+\\ (1-\theta)\underline{\Psi}\left(\underline{X}_p(k),\underline{u}_k(k),\Delta x\right)\end{aligned} \quad (9.1)$$

In (9.1) θ represents the parameter of the discretization method through approximation. It takes value 1 for the so-called implicit method and value 0 for the so-called explicit method. Thus, (9.1) is a recurrent algebraically system which for $\theta=0$ requires for $\underline{X}_p(k+1)$ determination only successive substitutions (with an explicit character), while for $\theta\neq 0$ requires solving, for each computation iteration, a non-linear system of algebraically equations (implicit character). Obviously, for $\theta\neq 0$, speed of the numerical method will be linked directly to the manner of finding the solution for this algebraically system of equations.

The knowledge of the state vector associated to pressure distribution $\underline{X}_p(k+1)$ at the discrete-time t_{k+1} (iteration k+1) allows the calculation of flow distribution according to relation (9.2), acquired from (7.2):

$$\underline{X}_Q(k+1)=\underline{\Xi}\left(\underline{X}_p(k+1),p_1(k+1),\Delta x,\Delta t\right) \quad (9.2)$$

To be able to use system (9) one must determine the initial distributions of pressure $\underline{X}_p(0)$ and of flow $\underline{X}_Q(0)$. They will be calculated using model (10) resulted from model (3) for a

stationary regime. Equations (10) are numerically integrated for boundary conditions: $p_1(0)$ and $Q_m(0)$, extracted from the boundary conditions under consideration:

$$\xi_1(p)\frac{dQ}{dx}=0 \tag{10.1}$$

$$\frac{dp}{dx}+\xi_2(p)\frac{Q^2}{p}+p\xi_3(p)=0 \tag{10.2}$$

Since $\xi_1(p)\neq 0$, from (10.1) it follows that $\frac{dQ}{dx}(x)=0$. Consequently:

$$Q_1(0)=Q_1(0)=...=Q_m(0)=Q_{stat}=const \tag{11.1}$$

By replacing the result in (10.2), we obtain the differential equation

$$\frac{dp_{j+1}}{dx}=-\xi_2(p_j)\frac{Q_{stat}^2}{p_j}-p_j\xi_3(p_j) \tag{11.2}$$

Through numerical integration for the boundary condition $p_1=p_1(0)$, (11.2) provides the initial pressure distribution $\{p_j(0)/\ j=\overline{2,m}\}$, (vector $\underline{X}_p(0)$) components).

In the following, we refer to the convergence problem for the method of integration of systems type (9.1). According to theory of numerical integration of differential equations the convergence affects the solution quality [17], [3].

Natural, when the discretization grid norm aims to zero the numeric approximation solution converges towards the accurate solution. From a computational point of view such a way is prohibitive, being difficult to implement.

By compensation, all important is Lax theorem according to which the partial check up of some weaker characteristics of the numeric method, that is the stability and consistency, together imply the convergence [5].

Since, the approximations achieved through finite differences represent the result of using some developments in Taylor series, whose truncation error aims to nullify when the norm of the discretization grid drops towards zero, $\varepsilon_{tr}(q,\Delta x)\xrightarrow{\Delta x\to 0}0$ and $\varepsilon_{tr}(s,\Delta t)\xrightarrow{\Delta t\to 0}0$ respectively, one can immediately state that they are consistent. At the same time, because the smaller the truncation errors, the greater the development orders in Taylor series are, respectively *q* and *s*, one can obtain an increase in the accuracy of the discretization formulas, without decreasing the discretization step, by increasing the development order in the used Taylor series [17].

It is more difficult to demonstrate the stability of the numerical integrating method of the non-linear discrete model with variable parameters (9.1). Thereby, owing to the variant character, to each *k* rated iteration, a stability analysis of the equation

$$\underline{X}_p(k+1)=\underline{\Phi}(k,\Delta x,\Delta t)\underline{X}_p(k)+\underline{\Gamma}_1(k,\Delta x,\Delta t)\underline{u}(k+1)+\underline{\Gamma}_2(k,\Delta x,\Delta t)\underline{u}(k)+\underline{\Lambda}(k,\Delta x,\Delta t) \tag{12.1}$$

obtained by linearization function $\underline{\Psi}$ around $\underline{X}_p(k)$ point, must be performed. At the same time, in the context developed further on, stability analysis of the linearized form, around points $\underline{X}_p(t_k)$, of the continuous-time model (8) must be also performed:

$$\frac{d\underline{X}_p}{dt}(t)=\underline{A}(t,\Delta x)\underline{X}_p(t)+\underline{B}(t,\Delta x)\underline{u}(t)+\underline{D}(t,\Delta x) \tag{12.2}$$

The discrete-time model (11) is (asymptotic) stable (in Lyapunov sense) if the spectre $\lambda(\underline{\Phi})$ (set of eigenvalues) of matrix $\underline{\Phi}$, lies within the unity radius circle, centred in the complex plane origin [12] [5]. We formally write:

$$\left|\lambda(\underline{\Phi})\right|<1 \tag{13}$$

Considering that $\underline{\Phi}$ depend on the discretization steps Δx, Δt, and that the discretization points are fixed, for each iterated calculation we should choose the value of discrete-time step Δt so that the eigenvalues of matrix $\underline{\Phi}$ lie within the unit circle.

By using an *implicit numerical method* of solving, when $\theta=1$, model (9.1) becomes:

$$\delta\underline{X}_p(k+1,\Delta t)=\underline{\Psi}\left(\underline{X}_p(k+1),\underline{u}(k+1),\Delta x\right). \tag{14}$$

Its linearized form is:

$$\underline{X}_p(k+1)=\underline{\Phi}_i(k,\Delta x,\Delta t)\underline{X}_p(k)+\underline{\Gamma}_i(k,\Delta x,\Delta t)\underline{u}(k+1)+\underline{\Lambda}_i(k,\Delta x,\Delta t) \tag{15}$$

Further on, a series of peculiarities which allow some simplifications in the analysis of model stability in discrete-time (15) shall be presented.

One should observe that the simplified form (14) represents a non-linear system of implicitly algebraically equations with the unknown variables grouped under vector $\underline{X}_p(k+1)$. The system (14) shall be solved, for each of the

iterations, by an adequate numerical method, for example Newton-Raphson method [3].

When the finite difference used in system (14) is in form:

$$\delta \underline{X}_p(k+1,\Delta t)=\frac{\underline{X}_p(k+1)-\underline{X}_p(k)}{\Delta t} \tag{16}$$

suitable to an Euler type numerical method, with the truncation error $0(\Delta t)$ [17], between matrix $\underline{\Phi}_i$ of the discrete model (15) and the matrix $\underline{A}$ of the continuous model (12.2) holds the relationship:

$$\underline{\Phi}_i=\left(\underline{I}-\Delta t\cdot\underline{A}\right)^{-1} \tag{17}$$

By taking the next theorem [12] into account:

If $\lambda(\underline{A})=\{\lambda_1,\lambda_2,...,\lambda_n\}$ *is the matrix spectrum* $A\in C^{n\times n}$ *than for each finite function f defined on* $\lambda(\underline{A})$ *holds* $\lambda\left(f(\underline{A})\right)=\left\{f(\lambda_1),f(\lambda_2),...,f(\lambda_n)\right\}$,

we particularly obtain the relationship:

$$\lambda\left(\underline{\Phi}_i\right)=\frac{1}{1-\Delta t\cdot\lambda\left(\underline{A}\right)}. \tag{18}$$

Here the notation on the left side signifies whatever element of $\lambda(\underline{A})$ and the notation on the right side that of the corresponding element in set $\lambda(f(\underline{A}))$. Equality (18) allows upholding that regardless the value of discrete-time step Δt, if $\mathrm{Re}\,al\left[\lambda(\underline{A})\right]<0$ then $|\lambda(\underline{\Phi})|<1$, and the stability condition of the numerical method described by the discrete model (14) is carried out. Consequently, the convergence of the implicit numerical method will only be conditioned by the structure of matrix $\underline{A}(t,\Delta x)$ resulted after the spatial discretization, and the value of discrete-time step Δt will influence only the numeric *accuracy* of computation.

The disadvantage of the method arises from the implicit shape of the equations of model (14) which, for the recurrence implementation, needs for each iteration additional computation of explicitness (i.e. the solving of a non-linear system of algebraically equations).

In case of an *explicit numerical method*, when $\theta=0$, the model (9.1) becomes:

$$\delta \underline{X}_p(k+1,\Delta t)=\underline{\Psi}\left(\underline{X}_p(k),\underline{u}_k(k),\Delta x\right), \tag{19}$$

having the following linearized form:

$$\begin{aligned}\underline{X}_p(k+1)=&\ \underline{\Phi}_e(k,\Delta x,\Delta t)\,\underline{X}_p(k)+\\ &\underline{\Gamma}_e(k,\Delta x,\Delta t)\,\underline{u}(k+1)+\underline{\Delta}_e(k,\Delta x,\Delta t)\end{aligned} \tag{20}$$

By expressing the finite difference $\delta \underline{X}_p(k+1,\Delta t)$ again through formula (16), between matrix $\underline{\Phi}_e$ of the discrete model (20) and the matrix $\underline{A}$ of the continuous model (12.2), the following relationship holds:

$$\underline{\Phi}_e=\underline{I}+\Delta t\cdot\underline{A} \tag{21}$$

Based on the same theorem, between the eigenvalues of the two matrixes the following relationship will be established:

$$\lambda\left(\underline{\Phi}_e\right)=1+\Delta t\cdot\lambda\left(\underline{A}\right) \tag{22}$$

Now, the stability of the explicit numerical method is conditioned both by respecting the stability condition of the continuous-time model $\mathrm{Re}\,al\left[\lambda(\underline{A})\right]<0$ and by choosing the value of discrete-time step Δt because it is possible to have $\mathrm{Real}(1+\Delta t\cdot\lambda_i)<-1$. In this case we talk about locating the values of the discrete-time step Δt under a maximum value called stability limit [5], [3] which have to be calculated for each computation.

The advantages of the explicit numerical method not necessitating the solving of a non-linear system of algebraically equations for each repeated computation like in case of the implicit numerical method is diminished by the disadvantages of necessitating to choose the values for discrete-time step Δt for each repeated computation. The difficulty of the problem arises from the following context.

Thus, according to relationships (5) in the general solution component of the flow process model *the fastest mode of dynamic behaviour* will be characterized by a time constant whose order of magnitude is directly proportional with $1/m^2$. For the discussed explicit numerical methods, this time constant will practically represent the stability limit implied in choosing the discrete-time step Δt. If high computational accuracy is required which, among others, imposes a high number m of spatial discretization points, automatically the range of choosing the discrete-time step Δt practically decreases with the square number m of spatial discretization points, implicitly increasing the computational effort, too. In the same time, the slowest mode of dynamic behaviour, which according to relationship (5) is obtained for $j=1$ will determine the value of transitory time.

This aspects are important both from a qualitative point of view (different orders of magnitude

of the extreme dynamic modes of behaviour) and from a quantitative point of view (proper values of time constants) because the resulted solutions will simultaneously present both the extremely slow dynamic components and the extremely fast dynamic components. From a mathematical point of view, such models are included in the category of inflexible systems of equations (stiff equations) [13].

For this type of systems of differential equations a special class of explicit numerical solving methods called Runge-Kutta-Chebyshev methods have been developed. These are based on the principle of numeric computation accuracy relaxation in favour of increasing the stability limit [20] [21]. In this case, the formulas for the finite difference $\delta \underline{X}_p(k+1,\Delta t)$ permit to expand the stability domain of the discrete equation system should towards the unitary radius circle used in case of applying formula (16).

From those presented above, it turns out that *the selection mechanism* of a numerical method in order to simulate the mathematical model of the gas flow process through a transmission system is not an easy task, in different well-defined situations being compulsory to take the discussed aspects into consideration.

The case studies presented in the next chapter refer to numerical simulations performed with the discrete model (6) related to the simplified mathematical model (3), relative to a pipe segment, for experimental validation of the model. Implicit numerical methods [2], selected on the convergence criterion, have been used, in inverse ratio to the number of differential equations and in direct ratio to the step value of time discretization.

Of course, this criterion may be inadequate when the question of simulating, on the whole, of a gas transmission system arises, situation in which the mathematical model resulted after mathematical models aggregation of the component elements tends to become extremely complex. And in this case, the presented systemic aspects, correlative with the experimental results to be presented, may constitute starting points in the selection and development of some numerical methods of simulation for these complex mathematical models.

5. EXPERIMENTAL TESTS

5.1 Preliminaries

For the experimental validation of the discrete model (6), comparisons between some characteristic variables of the gas flow process (pressures and flows) resulted after numerical simulations and the same characteristic quantity values measured in real operating conditions, were made.

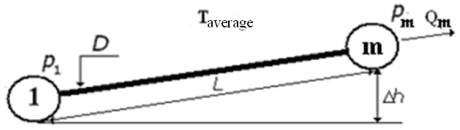

Fig. 3. Pipe segment and afferent parameters

The following *working procedure* was adopted [4]:

a) It is considered an equivalent pipe segments (figure 3), bounded by an inlet node *1* and an outlet node *m*, having the pipe bore *D*, pipe length *L* and the elevation between input and output Δh. For flow direction in figure 4 the value of elevation Δh is positive.

b) The variations $p_1(t)_{measured}$, $p_m(t)_{measured}$, $Q_m(t)_{measured}$ are measured on a time interval $[0,\tau_f]$ with the discrete-time step $\Delta t_{measurement}$.

c) There are identified some steady-state regime of the gas flow process through the pipe segment, characterized by the measured values for the stationary regime of the inlet pressure p_1, outlet pressure p_m and the outlet flow Q_m, which allow the computation of the pressure loss (friction) factor λ and the estimation of an interior roughness of the pipe segment based on the established calculus formula (Colebrook-White) [18]. The average temperature $T_{average}$ of gas flow which characterizes the isothermal behaviour of flow is estimated to be that of point level of measurement of the gas flow.

d) It is choose a flow stationary regime and it is computed, based on equation (10.2), the pressure distribution in the established *m* of points of spatial discretization. The results are used for initialization of the numerical simulation algorithm at the time $t=0$.

e) Accomplishment on the time interval $t\in[0,\tau_f]$ of the numerical simulation, taking as input variables the inlet pressure $p_1(t)_{measured}$

and the outlet flow $Q_m(t)_{measured}$, respectively, as output variable the outlet pressure $p_m(t)_{computed}$.

f) There are plotted and interpreted $p_1(t)_{measured}$, $p_m(t)_{measured}$, $p_m(t)_{computed}$, and the relative percentage error respectively $\varepsilon(t) = \frac{p_m(t)_{measured} - p_m(t)_{computed}}{p_m(t)_{measured}} \times 100$.

Numerical computations were made in the hypothesis of considering the methane gas as natural gas.

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