Grey Fuzzy sliding Mode controller for Vehicle Suspension System

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Abstract: This paper presents a Grey Fuzzy Sliding Mode controller (GFSMC) for enhancing the ride comfort of Vehicle Suspension System (VSS). Grey Prediction algorithm is used to predict the output or the error of the system with small amount of data. The prediction error in Grey Model (GM) is minimized by changing the initial condition and optimizing the weight factor in the data matrix of grey prediction algorithm using Particle Swarm Optimization (PSO). The aim of this paper is to design a Sliding Mode Controller (SMC) for vehicle suspension system. The chattering phenomena that occur in SMC, is eliminated by combining Fuzzy Logic with SMC. For further enhancement the Fuzzy Sliding Mode Controller is combined with Grey Prediction algorithm to develop GFSMC. The proposed controller is simulated for a Quarter Car model of VSS. Simulation results show that the proposed controller enhances ride comfort. Even under perturbed conditions the proposed controller offers robust performance. Power Spectral Density also proves the effectiveness of GFSMC in terms of ride comfort.

Keywords: Fuzzy sliding mode control, Grey Prediction algorithm, Particle Swarm Optimization, Quarter Car model, Ride comfort.

1. INTRODUCTION

Suspension system has been widely applied to vehicles and is an active area of research. The growing competition in automobile sectors has forced the industries to enhance the luxury especially the ride comfort of vehicles. Suspension system plays a crucial role in handling the vehicle performance, road holding ability, safety and ride comfort of passengers. Ride comfort is related to the vibrations sensed by the vehicle subjected to road disturbances such as bumps and rough terrains. Road handling is the degree to which a vehicle maintains contact with the road surface in various types of directional changes and speed. Conventional passive suspension system consists of static springs and dampers whose characteristics are fixed. Semi-active suspension system consists of passive elements like spring, variable damper and does not add energy to the suspension system. Active suspension system uses separate actuators which exert an independent force on the system to improve the ride characteristics (Alexandru.C and Alexandru.P, 2011).

Fuzzy Logic Controller (FLC) is a promising approach and it proves to be a superior control strategy even in the presence of model uncertainties and disturbances. FLC (Rajeswari and Lakshmi, 2010a), Self-Organizing Fuzzy Logic Controller (SOFLC) (Jeen Lin and Ruey-Jing Lian, 2008), Adaptive Fuzzy controller based on traffic condition (Soleymani et al., 2012). Adaptive Fuzzy-PID controllers (Juing-Shian Chiou et al., 2012) are some of the FLCs developed for Vehicle Suspension System (VSS). Since some state variables are unavailable for measurement Observer based Preview control (Hyoun-Surk and Teramura,1998), Single input rule modules fuzzy reasoning and disturbance observer (Yoshimura Toshio and Teramura, 2004) are some of the Look-Ahead control strategies employed for VSS. Grey prediction is used to describe and analyze the future development according to the past and present data for dynamic system. Grey Prediction Fuzzy control strategy has been developed for Magnetic Rheological (MR) damper based semi-active suspension system (Shao-BoLu et al., 2010). Grey Model has been proposed to minimize the prediction error (Wen and Chang, 2005). Trial and error technique was used in grey model to identify the optimal solution and an optimal weighting factor was determined for each sampling instant and the error vector was optimized using Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) (Niu Dong-xiao et al., 2007, 2008).

Further changes in the grey model like the sampling instant $\Delta t=1$ has been replaced by integration term, which is termed as optimization of background value and the initial value y(1)(0) has been replaced by accumulated value y(1)(N) (Cai-yun GAO and Ning GAO, 2009). These changes improve the prediction precision. PSO technique has also been used to minimize the mean absolute percentage error and improve the prediction accuracy (Dan Ma et al., 2011).

Model free Sliding Mode Controller (SMC) (Hung-Yi Chen and Shiuh-Jer Huang, 2008), Fuzzy sliding mode controller (FSMC) (Rajeswari and Lakshmi, 2009), Grey sliding mode controller (Erdal Kayacan et al., 2009), Enhanced Fuzzy Sliding Mode Controller (Jeen Lin et al., 2009), Enhanced Adaptive Self-Organizing Fuzzy Sliding Mode Controller

(Ruey-Jing Lian, 2013) are some of the Fuzzy sliding mode techniques developed for the suspension system. A Look ahead Fuzzy logic controller (Lavanya and Rajeswari, 2013) has been designed for Vehicle suspension system to improve the ride comfort. Grey prediction algorithm was employed for the Look-Ahead control strategy. Modified Grey Fuzzy logic controller (Rajeswari and Lakshmi, 2014) has been proposed for enhancing the ride comfort and road holding ability of the suspension system simultaneously. In this paper two traditional FLCs, one for minimizing the sprung mass displacement error and the other for tyre deflection error has been employed. PSO optimized Grey model has been used to design the FLC and the simulation results showed that the proposed controller enhanced both ride comfort and road holding ability simultaneously.

Literature survey reveals that FLC in vehicle suspension offers better performance and it does not guarantee stability. Sliding Mode controller ensures stability by Lyapunov theorem and the major drawback of SMC is the chattering phenomenon which is reduced by combining Fuzzy logic with SMC. The concept of fuzzifying the sliding surface and getting the feedback control gain from an inference rule base is proposed. This paper presents Fuzzy Sliding Mode Controller whose performance is further improved by combining the advantages of grey prediction algorithm discussed in the previous work of the author. Grey Prediction is an effective algorithm in predicting the future output and it requires only four set of data's to predict the future output. The prediction error of grey model is further minimized by optimal choice of weight factor using Particle Swarm Optimization (PSO).

Organization of the paper is as follows. In section 2 the dynamics of the Quarter Car model of Vehicle Suspension System is expounded. Fuzzy Sliding Mode Controller design is detailed in section 3. GFSMC based VSS is proposed in section 4. Simulation Results are presented and discussed in section 5. The final section concludes the paper.

2. QUARTER CAR MODEL

A two Degree of Freedom (DOF), Quarter Car model of VSS, is shown in Fig. 1. Quarter car model is taken up for this study because it is simple but provides accurate information about essential parameters of VSS viz. sprung mass displacement, sprung mass acceleration, suspension deflection and tyre deflection (Rajeswari and Lakshmi, 2010b). Moreover two degree of freedom quarter car model is widely used in automotive industry due to the simplicity and the qualitatively correct information they provide especially for ride and handling studies.

The quarter car model consists of 'sprung mass M_s' which represents the weight of the car body at each wheel. The 'unsprung mass M_u' denotes the equivalent mass due to axle and tyre at any one of the four wheels of the vehicle. The spring and shock absorber of the suspension system, that supports the car body is represented as K_s and B_s respectively. The tyre has been replaced by its equivalent stiffness K_t but tyre damping is neglected. The variables Z_s, Z_u and Z_t represent the vertical displacement from static equilibrium of sprung mass, unsprung mass and the road profile respectively. A controllable actuator F is included between the sprung and the unsprung mass and this actuator is able to both add and dissipate energy from the system.

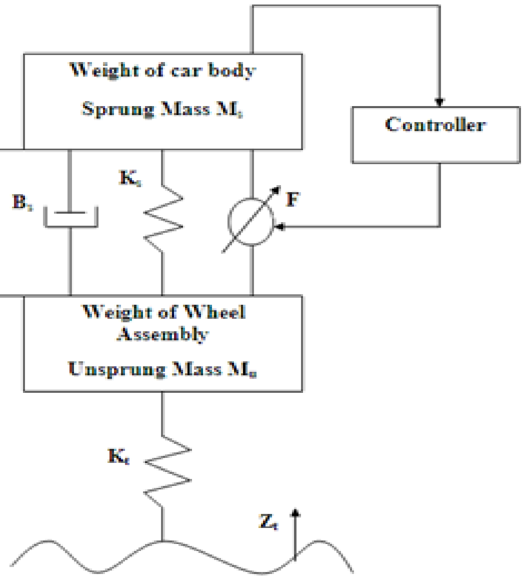

Fig. 1. Two DOF Quarter Car Model.

It is assumed that, the suspension spring stiffness and tyre stiffness are linear in their operating range and the tyre does not leave the ground. Let x(t) be the state vector $x(t)=[x_1\ x_2\ x_3\ x_4]^T$ where, $x_1=Z_S-Z_U$ is the Suspension Deflection, $x_2=\dot{Z}_S$ is the Sprung Mass Velocity, $x_3=Z_U-Z_T$ is the Tyre Deflection, $x_4=Z_U$ is the Unsprung Mass Velocity. The state space representation of the dynamics of Quarter Car model is given as,

$$\dot{X}=Ax(t)+Bu(t)+Gw(t) \tag{1}$$

where, A is the State Matrix, B is the Input Matrix, G is the Disturbance input vector, u(t) is the System Input, x(t) is the State Vector, w(t) is the road disturbance. The state matrix, input vector, disturbance input vector and the state vector are given as,

$$A=\begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_S}{M_S} & -\frac{B_S}{M_S} & 0 & \frac{B_S}{M_S} \\ 0 & 0 & 0 & 1 \\ \frac{K_S}{M_U} & \frac{B_S}{M_U} & -\frac{K_T}{M_U} & -\frac{B_S}{M_U} \end{bmatrix} \qquad B=\begin{bmatrix} 0 \\ \frac{1}{M_S} \\ 0 \\ -\frac{1}{M_U} \end{bmatrix}$$

$$G=\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}; \qquad x(t)=\begin{bmatrix} Z_s-Z_u \\ \dot{Z}s \\ Z_U-Z_T \\ \dot{Z}_U \end{bmatrix} \tag{2}$$

3. SLIDING MODE CONTROLLER

The state space model of VSS discussed in section 2 and (1) shows that the disturbance input is not in phase with the system input, i.e. rank[B]≠rank[B,G], therefore, the system suffers from the mismatched uncertainties. Hence, the controller must be robust enough to overcome the mismatched condition, so that the disturbance would not have significant effect on the performance of the system. Equation (1) can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x,t) \tag{3}$$

where $f(x,t)$ are the uncertainties with mismatched condition. The following assumptions are taken as standard.

Assumption 1 The pair (A, B) is controllable and the input matrix B has full rank.

Assumption 2 There exists a known positive constant η such that $\|f(x,t)\| \leq \eta$, where $\|\bullet\|$ denotes the standard Euclidean norm.

The control law that satisfies the sliding condition is given as

$$u = u_{eq} - k\,sgn(S) \tag{4}$$

where u_{eq} is the equivalent control, ksgn(S) is the discontinuous control, S is the Sliding Surface. The sliding surface for the VSS is given as,

$$S = [C_1\ C_2\ C_3\ C_4] * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{5}$$

and

$$sgn(S) = \begin{cases} -1 & \text{if} \quad S < 0 \\ 0 & \text{if} \quad S = 0 \\ 1 & \text{if} \quad S > 0 \end{cases}$$

3.1 Discontinuous Control Signal

Reaching mode is defined as the movement of state trajectory from an arbitrary point in the state space to the sliding surface and it is achieved by discontinuous control signal. The time varying switching surface, S(x,t) = 0 in the state space is defined as

$$S = C * x(t) \tag{6}$$

where $C=[C_1\ C_2 \ldots, C_{n-1}\ 1]$ is strictly positive real constant and (6) is expanded as

$$S = C_1x_1(t) + C_2x_2(t) + \cdots + C_{n-1}x_{n-1}(t) + x_n(t) = 0 \tag{7}$$

The coefficients C_1, $C_2 \ldots C_{n-1}$ are chosen so that the roots of the system's characteristic equation lie on left half of the complex plane. To ensure stability these coefficients are chosen using Linear Quadratic Regulator technique (Ching-Chang Wong et al., 2001). In discontinuous control law k' is a constant representing the maximum controller output.

3.2 Equivalent Control Signal

Sliding Mode occurs when the trajectory asymptotically tends to the origin of the hyper plane along the sliding surface. Sliding mode is achieved by equivalent control signal and it is derived using Filippov's construction of equivalent dynamics. The dynamics in sliding mode is given as

$$\dot{S} = C * \dot{x} = 0 \tag{8}$$

Substituting (1) in (8) and rearranging,

$$\dot{S} = C * [Ax(t) + Bu(t) + Gw(t)] \tag{9}$$

$$\dot{S} = CAx(t) + CBu_{eq}(t) + CGw(t) = 0 \tag{10}$$

Thus the equivalent control signal is obtained as

$$u_{eq} = (CB)^{-1}[-CAx(t) - CGw(t)] \tag{11}$$

4. GREY FUZZY SLIDING MODE CONTROLLER

4.1 GM(1,1) Model

Grey prediction is used to study uncertainty in the system (Deng, 1989). The general form of a grey model is GM (n,h), where 'n' represents the order of the ordinary differential equation of the Grey model and 'h' represents the input to the Grey model. Prediction time increases exponentially with increasing equation order n and h. Moreover, using large n, h values do not guarantee accuracy. GM(1,1) model is widely employed in various grey systems for prediction application. Accumulated Generating Operation (AGO) and Inverse Accumulated Generating Operation (IAGO) are the two basic tools used in grey modelling.

If {y(0)(k)}, y(0)(k) ≥ 0, k=1,2,…..,N is a time sequence data, then AGO is,

$$y^{(1)}(k) = \sum_{i=0}^{k} y^{(0)}(i) \tag{12}$$

The data sequence y(1)(k) is an accumulation of past and present output information. To derive an approximate growing curve for y(1)(k), four or five sets of data are required to extract the grey model tendency. {y(0)(k)} is not always a positive sequence and so exponential or linear mapping should be employed to change its behavior for obtaining one.

The first-order Ordinary Differential Equation (ODE) used to describe the grey GM(1,1) model is given as ,

$$\frac{dy^{(1)}}{dt} + cy^{(1)} = u \tag{13}$$

where y(1) is the accumulated data computed from (12), c is the developing coefficient and u is the control input. If the sampling interval is one unit, then the differential of the generating sequence y(1) can be described as discrete time sequence,

$$\frac{dy^{(1)}}{dt} = y^{(1)}(k+1) - y^{(1)}(k) = y^{(0)}(k+1) \tag{14}$$

The second term of the first-order grey model can be considered to represent the average of y(1)(k+1) + y(1)(k) . Substituting (14) and replacing the second term by the average value, the first-order ODE given by (13) can be rewritten as

$$y^{(0)}(k+1) = c\left\{-\frac{1}{2}\left[y^{(1)}(k+1) + y^{(1)}k\right]\right\} + u(k+1) \tag{15}$$

The matrix-form corresponding to (15) can be described as

$$Y_N = \emptyset * B \tag{16}$$

where

$$Y_N = \left[y^{(0)}(1)\ y^{(0)}(2)\ \cdots\ y^{(0)}(N)\right] \tag{17}$$

$$B = \begin{bmatrix} -\propto * \left(y^{(1)}(0) + y^{(1)}(1)\right) & 1 \\ -\propto * \left(y^{(1)}(1) + y^{(1)}(2)\right) & 1 \\ \vdots & \vdots \\ -\propto * \left(y^{(1)}(N-1) + y^{(1)}(N)\right. & 1 \end{bmatrix};\ \emptyset = \begin{bmatrix} c \\ u \end{bmatrix} \tag{18}$$

The term α is known as the weight. In traditional GM (1,1) model the value of α in B matrix or data matrix is chosen as 0.5 (Deng, 1989). Least-Square technique is employed to determine the optimal average solution of the parameters c and u. By Least-Square estimation technique, the predicted value of parameter vector $\hat{\phi}$ is given as,

$$\hat{\emptyset} = \begin{bmatrix} c \\ u \end{bmatrix} = (B^T * B)^{-1} B^T Y_N^T \tag{19}$$

The next step output for the accumulated data is the solution of first order ODE represented in (13), is given by the following equation, where $\hat{y}(k+1)$ is the predicted value

$$\hat{y}^{(1)}(k+1) = \left(y^{(0)}(1) - \frac{u}{c}\right) * e^{ck} + \frac{u}{c} \tag{20}$$

The prediction output as the (k+1)th step can be estimated using IAGO

$$\hat{y}^{(0)}(k+1) = \hat{y}^{(1)}(k+1) - \hat{y}^{(1)}(k) \tag{21}$$

4.2 Optimization of Grey Model

The development coefficient c and grey input u in the ODE (13), describing grey model are important parameters in the grey forecast model. Traditional grey model uses least square estimation technique to estimate the coefficients c and u (19). The least square estimation method requires a large amount of data whereas grey model requires only set of 4 or 5 data. Particularly least square method will result in considerable error in estimating the coefficients for data with obvious fluctuations. The prediction error can be minimized by modifying the sampling interval, changing the initial condition and optimizing the value of α (Deng, 1989; Kennedy and Eberhart, 1995).

The initial value $y^{(1)}(0)$ is replaced by accumulated value $y^{(1)}(N)$ and (20) is rewritten as

$$\hat{y}^{(1)}(k+1) = \left(y^{(1)}(n) - \frac{u}{c}\right) * e^{-c(k-n)} + \frac{u}{c} \tag{22}$$

The weight factor is optimized using Particle Swarm Optimization (PSO). Root Mean Square (RMS) value of the prediction error is taken as performance index.

4.3 Particle Swarm Optimization

Craig Reynolds inspired by the social behavior of birds, derived a formula for representation of the flocking behavior of the birds. (Kennedy and Eberhart, 1995) recognized the suitability of this technique for optimization. Particle Swarm Optimization (PSO) is a robust stochastic population based optimization technique. PSO algorithm is becoming popular because it involves fewer parameters to adjust and the ease in implementation. It finds its application in tuning PID controllers, functional optimization, training of artificial neural networks and fuzzy logic control systems.

A number of particles, which constitute a swarm, are initialized in an N-dimensional search space. The particles move in the search space looking for the optimal solution. The position of a particle and the velocity with which the particle moves are determined by its own flying experience and the flying experience of the other particles. Each particle keeps track of the best solution that is achieved by the particle so far and is known as local best position. The best solution achieved by the group is known as global best position. Each particle tries to move towards the optimal solution using the current velocity and the distance between the local best position and the global best position. The new velocity and new position of the particle can be calculated using (Kennedy and Eberhart, 1995)

$$v_i^{k+1} = wv_i^k + c_1 rand_1(pbest_i - x_i) + c_2 rand_2(gbest_i - x_i)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \tag{23}$$

where w is the inertia weight parameter, c_1 is called as the coefficient of the self recognition component, c_2 is called as the coefficient of the social component, $rand_1$ and $rand_2$ are random numbers between 0 and 1, v_i^k and x_i^k are the velocity and position of the particle in k^th^ iteration, v_i^{k+1} and x_i^{k+1} are the velocity and position of the particle in (k+1)^th^ iteration. PSO algorithm has been discussed in the literatures (Rajeswari and Lakshmi, 2010a; Kalaivani and Lakshmi, 2014).

Generating initial populations is the first step of PSO. The population is composed of the particles that are real codes. The fitness function is given by

$$F = \frac{1000}{1+J} \tag{24}$$

where J is the RMS value of the prediction error. After the fitness function has been calculated, the fitness value and the number of the iterations determine the stopping condition. The pbest of each particle and gbest of population (the best movement of all particles) are calculated. Updating the velocity, position, gbest and pbest of particles give a new best position.

4.4 Need for Grey Fuzzy Sliding Mode Controller

Fuzzy controller requires significant effort to find appropriate membership functions and robust fuzzy rules for improving the control performance. Also time delay exists especially when the fuzzy inference system is complex. Consequently, this study introduces the grey predictive theory into the FLC to predict the next output error of the vehicle system and the error change, rather than the current output error of the system and the current error change as the input variables of the FLC. This control strategy is expected to improve the performance of FLC and reduce the difficulty of its implementation. Grey Fuzzy Logic Controller is combined with SMC to develop Grey Fuzzy Sliding Mode Controller (GFSMC).

4.5 GFSMC Based Vehicle Suspension System

The structure of GFSMC based VSS is shown in Fig. 2. The future sprung mass displacement of the active suspension system is predicted using grey prediction algorithm discussed in section 4.1. Based on the predicted value, future error and rate of change of error is calculated and is given as input to the FLC-2. The sliding surface and rate of change of sliding surface are given as inputs to FLC-1. The membership function chosen for this work is triangular and trapezoidal membership function. The normalized domain [-1, 1] is classified into seven fuzzy sets namely, Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZO), Positive Small (PS), Positive Medium (PM), Positive Big (PB). The membership functions of input and output variables of FLC-1 and FLC-2 are shown in Fig. 3.

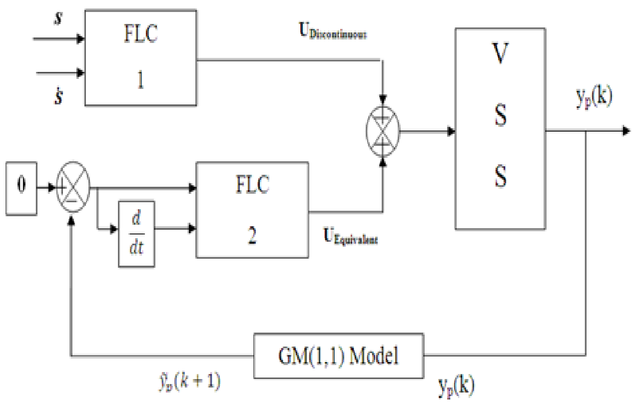

Fig. 2. Structure of Grey Fuzzy Sliding Mode Controller based VSS.

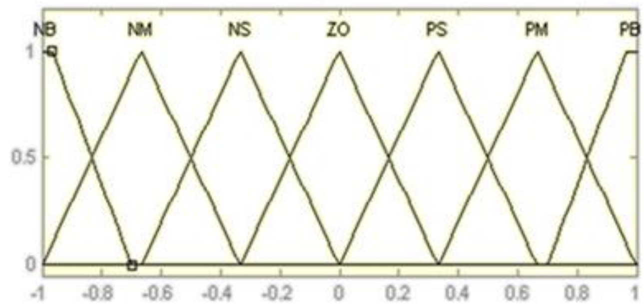

Fig. 3. Membership Function of FLC-1 and FLC-2.

The FLC rule table for FLC-1 is shown in Table. 1. Rule table for FLC 2 is the same as in Table 1 and the input variables are the predicted sprung mass displacement and sprung mass velocity. Mamdani inference system is used for determining the appropriate control signal. Bisector method is used to defuzzify the inferred output. Scaling factors are tuned by trial and error method. The input to FLC-1 and FLC-2 in Fig. 3 is in the normalized domain [-1, 1], hence the output should be de-normalized before giving to the actuator. Thus FLC-1 generates discontinuous control signal and FLC-2 generates equivalent control signal. The output of GFSMC is the actuator force given to the VSS.

Table 1. Rule Table for FLC-1 and FLC-2

Rate of change of sliding surface ($\dot{S}$) / Sprung mass velocity	Sliding surface (S) / sprung displacement						
	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NB	NB	NM	NS	NS	ZO
NM	NB	NB	NM	NM	NS	ZO	PS
NS	NM	NM	NS	NS	ZO	PS	PM
ZO	NM	NM	NS	ZO	PS	PM	PM
PS	NM	NS	ZO	PS	PM	PM	PB
PM	NS	ZO	PS	PM	PM	PM	PB
PB	ZO	PS	PM	PM	PM	PB	PB

5. SIMULATION STUDIES

The parameters of the quarter car model (Rajeswari and Lakshmi, 2010b) are listed below

Sprung mass (MS)	290 kg
Unsprung mass (Mu)	59 kg
Damper coefficient (Bs)	1,000 Ns/m
Suspension stiffness (Ks)	16,812 N/m
Tyre stiffness (Kt)	190,000 N/m

The mathematical model of the quarter car model defined by (1, 2) is simulated, using the parameters listed, for the dual bump road profiles using MATLAB-SIMULINK. Mathematical representation of dual bump input, shown in Fig.4, with amplitude of 10 cm and 5 cm is given by,

$$Z_r(t)=\begin{cases}\frac{a(1-\cos(8\Pi t))}{2} & \text{if } 1.0 \le t \le 1.25 \ \& \ 3.0 \le t \le 3.25 \\ 0 & \text{otherwise}\end{cases} \tag{25}$$

Where a denotes the bump amplitude.

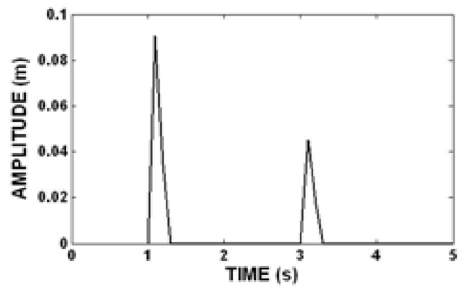

Fig. 4. Dual Bump Input.

5.1 Predicted Output

The prediction error of GM(1,1) model is minimized by changing the initial condition and optimizing the value of weight factor used in the computation of data matrix. The parameters of PSO used for optimization of α are given as no. of swarms 50, bird step 100, w 0.9 and c1, c2 as 2. The convergence plot is shown in Fig. 5.

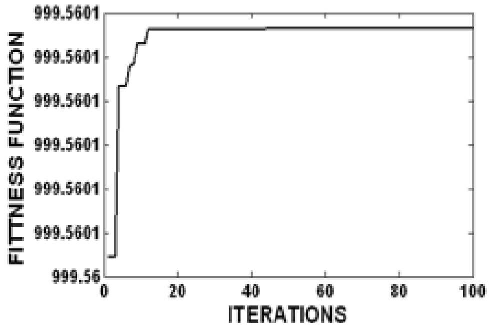

Fig. 5. Convergence plot.

RMS value of prediction error is minimized from $131.1*10^{-5}$ to $13.79*10^{-5}$ m for Sprung Mass Displacement. Fig. 5 shows that the fitness function converges to 999.8621 and the optimal value of α obtained is 0.3656. Fig. 6 shows that the predicted values of PSO-GM(1,1) model is closer to actual value than the prediction output of traditional GM(1,1) model

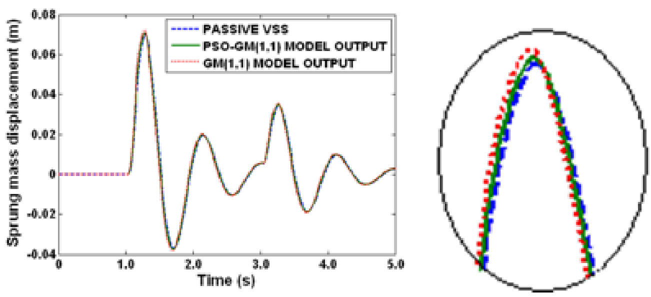

Fig. 6. Grey Predicted Sprung Mass Displacement.

5.2 Suspension Parameters for GFSMC

The C matrix in (14) is determined using LQR technique to ensure stability of the designed controller. The C matrix computed using LQR technique is given as,

$C = [16.441*10^3 \;\; 3.621*10^3 \;\; 28.8412 \;\; 1]$

Simulations are conducted for open loop passive, SMC, FSMC and GFSMC based vehicle suspension system. Fig. 7-10 shows the simulation results of passive, SMC, FSMC and GFSMC vehicle suspension system for dual bump input. Fig.7 shows the sprung mass displacement of GFSMC is significantly reduced by 84% when compared to passive suspension and 32% compared to FSMC. Fig. 8 show that, GFSMC reduces the sprung mass acceleration by 48% when compared to passive suspension system and 15% when compared to FSMC.

Fig. 9 shows suspension deflection of GFSMC is significantly reduced by 30% when compared to passive system and has increased by 7% when compared to FSMC. Also Fig. 10 shows that, GFSMC increases the tyre deflection by 11% and by 6% when compared to passive and FSMC respectively.

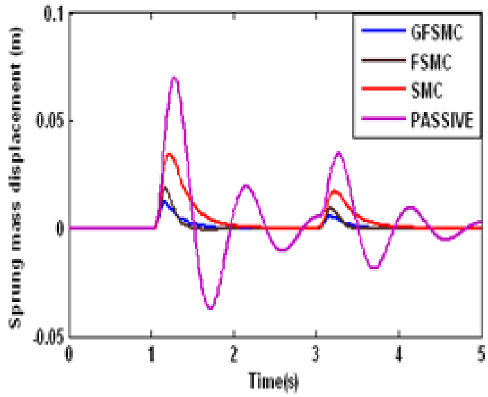

Fig. 7. Sprung Mass Displacement.

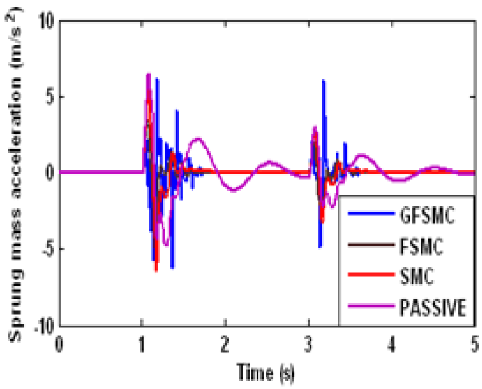

Fig. 8 Sprung Mass Acceleration

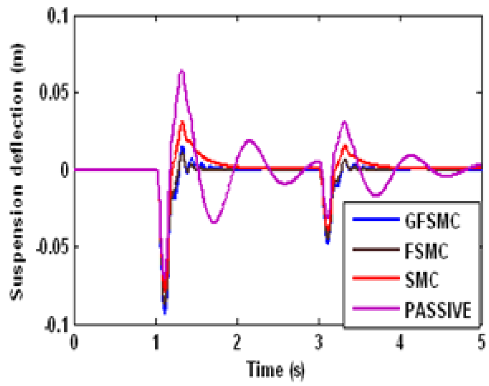

Fig. 9 Suspension deflection

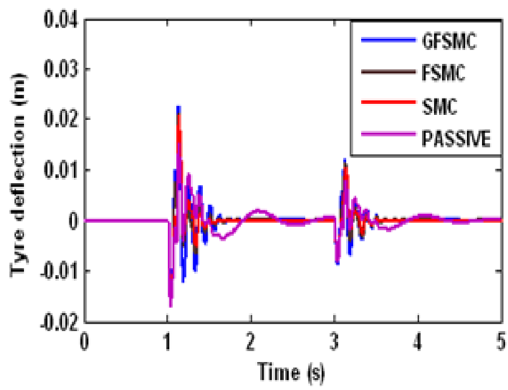

Fig. 10. Tyre Deflection.

Fig. 11 shows the occurrence of chattering phenomena in the control signal of SMC. Fig. 12 shows that chattering effect is eliminated by GFSMC.

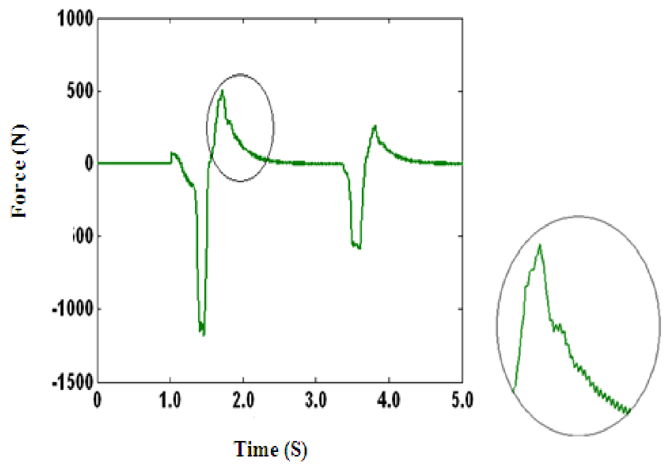

Fig. 11. Control output – SMC.

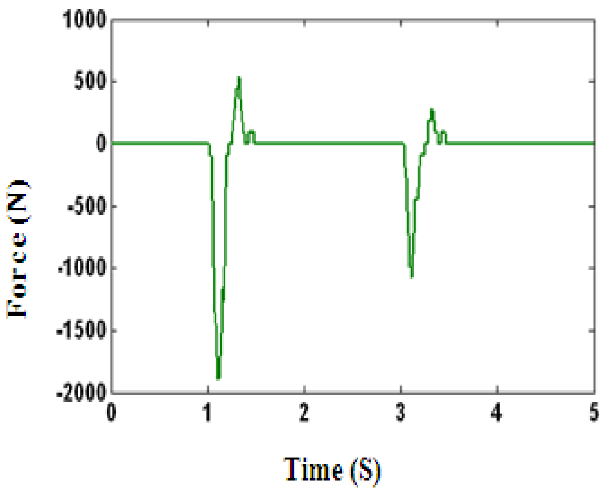

Fig. 12. Control output – GFSMC.

5.3 Robustness Test

Robustness test is performed by considering a different set of values for the simulation and comparing the performance with nominal values or the actual parameters. Under perturbed conditions the sprung mass of the vehicle is assumed to be increased by 30% and the spring constant and damper coefficient are considered to be decreased by 30% from the nominal values. Simulation results under perturbed conditions are discussed in terms of RMS values.

5.4 RMS Values of Suspension Parameters

The RMS values of Sprung Mass Displacement, Sprung acceleration, Suspension Deflection and Tyre Deflection are presented in Table 2. The GFSMC reduces the sprung mass displacement of VSS from 0.01888 meters to 0.002481 meters and sprung mass acceleration from 1.318m^2/s to 0.5733m^2/s. Hence GFSMC offers better ride comfort when compared to open loop passive system, SMC, FSMC.

Table 2. RMS values of Suspension Parameters

Quarter car parameters	Controller	Sprung Mass Displacement $*10^{-3}$(m)	Sprung Mass Acceleration (m^2/s)	Suspension Deflection $*10^{-3}$(m)	Tyre Deflection $*10^{-4}$(m)
Nominal condition	PASSIVE	18.88	1.318	17.59	25.88
	SMC	8.289	0.8633	12.14	24.85
	FSMC	3.317	0.6893	12.35	33.17
	GFSMC	**2.481**	**0.5733**	13.12	31.88
Perturbed condition	PASSIVE	18.06	0.7673	19.9	23.78
	SMC	8.289	0.8633	12.14	24.85
	FSMC	3.17	0.6893	12.35	28.1
	GFSMC	1.843	0.3409	13.52	29.49

5.5 Power Spectral Density

Power Spectral Density (PSD) is the statistical parameter used to describe random signals and it describes the power per unit frequency. Hence the PSD plot shows the distribution of the signal over a range of frequencies. Mathematical description of PSD for a signal X(t) is given as

$$S_x(f) = \int_{-\infty}^{\infty} R_x(t) * e^{-j2\pi f\tau} d\tau \tag{26}$$

where $S_X(f)$ is the PSD of the signal X(t), $R_X(t)$ is the auto correlation function of X(t). Thus PSD is the Fourier transform of the autocorrelation of signal considered. The instantaneous power (the mean or expected value of which is the average power) is then given by

$$P(t) = s(t)^2 \tag{27}$$

for a signal s(t). In the evaluation of vehicle ride quality, the PSD of the sprung mass acceleration as a function of frequency is of prime interest and Fig. 13 shows that the sprung mass acceleration has been brought down within the frequency between 2.8 Hz to 8 Hz by the GFSMC scheme. The ride comfort of the vehicle system is obviously improved in the human sensitive frequency range of 4 to 8Hz. Thus the active suspension with GFSMC scheme could greatly contribute to the improvement of the vehicle ride comfort.

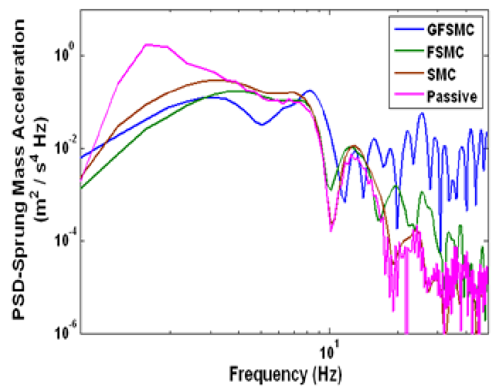

Fig. 13 Power Spectral Density- Sprung mass acceleration

6. CONCLUSION

In this paper GFSMC is designed for Vehicle Suspension System. GFSMC overcomes the drawback of SMC, improves the performance of FSMC by grey prediction algorithm whose weight factor is further tuned by PSO technique. Simulation results demonstrate that GFSMC based active suspension system greatly contributes towards better ride comfort and road handling compared to existing FSMC based suspension system. The proposed controller guarantees the system robustness in presence of uncertainties.

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