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Discrete-time Integral Sliding Mode Control for Large-Scale System with Unmatched Uncertainty

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Abstract: This paper presented a discrete-time integral sliding mode control for a large-scale system with unmatched uncertainty. A new theorem is presented and proved that the controller is able to handle the effect of interconnection for the large-scale systems and unmatched uncertainty, and the system stability is ensured. The controller will ensure the system achieve the quasi-sliding surface and remains on it. The results showed a fast convergence to the desired value and the attenuation of disturbance is achieved.

Keywords: Discrete-time large-scale systems, Robust control, Decentralized system, Variable structure control, Sliding mode control, Nonlinear system control.

1. INTRODUCTION

Large-scale systems usually refer to systems that consists of a large number of state variables, system parametric uncertainties, a complex structure and a strong interaction between subsystems (Siljak, 1978). The development of discrete-time control methodology for large-scale variable structure system such as multi-axis robotic arm or large process control systems is relatively limited as compared to its continuous-time counterpart. Discrete-time controller is important for implementing computerized control technique. (Li et al., 1982) used decentralized control by dynamic programming method to achieve the control of three-reach river pollution problem. This paper addressed the dynamic issue of the interconnections and external disturbance of the systems. (Hou, 2001) has used neural network for dynamic hierarchical optimization of nonlinear discrete-time large-scale system. The challenge for this method is the speed of computation required for systems with fast respond. (Haddad, et al., 2004) developed an analysis framework for discrete-time large-scale dynamical system using vector dissipativity notion. They introduced a generalized definition of dissipativity for large-scale nonlinear discrete-time dynamical systems in terms of a vector inequality involving a vector storage functions and vector supply rates. Subsequently, linear matrix inequality (LMI) technique has been used by (Park and Lee, 2002) to derive a sufficient condition for robust stability in decentralized discrete-time large-scale systems with parametric uncertainty. (Park et al., 2004) applied the dynamic output feedback controller design to a discrete-time large-scale system with delay at subsystem interconnections. Lyapunov method has been combined with LMI technique to develop the dynamic output feedback controller to guarantee the cost stabilization of the systems and achieve asymptotically stable closed-loop system with adequate level of performance. (Ou et.al., 2009) also used LMI method to achieve the stability analysis and H_∞ controller design to achieve disturbance attenuation performance by using Fuzzy Logic approach for the decentralized control of discrete-time large-scale systems.

In the early development stage of discrete-time sliding mode control theory which is also known as variable structure control, the basic conditions for achieving the equivalent of sliding mode as in continuous-time variable strucuture control have been proposed by (Dote and Hoft, 1980; Sarpturk et al., 1987; Milosavljevic, 1985; Furata, 1990). Method for quasi-sliding mode design and the use of reaching law approach to develop the control law for robust control in discrete-time sliding mode method has been proposed by (Gao et al., 1995). Discrete-time integral sliding mode control for sampled data system under state regulation was reported in (Abidi et al., 2007). Subsequently,(Xi and Hesketh, 2010) demonstrated the discrete-time integral sliding mode system to deal with both matched and unmatched uncertainties focused on SISO system.

Discrete-time large-scale systems in variable structure control has been introduced by (Sheta, 1996) with optimum control method. His study focused on the uncertain changes in the interconnection between subsystems and these uncertainties are governed by Markov chain technique. The controller was designed off-line based on a set of expected system failure modes and switched on-line when failure detected. It is relatively fewer literatures that have been focusing in the research of discrete-time sliding mode control for large-scale systems.

In this paper, a new theorem using integral sliding mode control method to control a large-scale discrete-time system with matched and unmatched uncertainties is proposed. Such that the proposed controller renders the large-scale system to be stable and handle the effect of the interconnections with matched and unmatched uncertainties.

This paper is organized into 5 sections, Section 1 as the introduction, followed by the problem statement in Section 2. The controller design and proof of the theorem is given in Section 3. Section 4 presents the simulation results of two examples of large-scale systems under study and the conclusion is given in Section 5.

2. PROBLEM STATEMENTS

This paper consider a discrete-time large-scale system given by,

$$x(k+1) = Ax(k) + Bu(k) + d(k) \tag{1}$$

It is assumed that the system can be decomposed into p subsystems as follows,

$$x_i(k+1) = A_i x_i(k) + B_i\big(u_i(k) + f_{mi}(k)\big) + V_i \sum_{\substack{j=1 \\ j\neq i}}^{p} \emptyset_{ji} z_{ji}(k) + f_{ui}(k) \quad (i = 1,2,\ldots p) \tag{2}$$

where A_i is system parameter, B_i is input parameter, $x_i(k+1) \in R^n$, $u_i(k) \in R^m$, $f_{mi}(k)$ is the matched uncertainty, $f_{ui}(k) = \Delta B_i\big(u_i(k) + f_{mi(k)}\big)$ is the unmatched uncertainty, V_i & $\emptyset_{ji}$ is constant matric with appropriate dimension, $z_{ji}(k) \in R^{q_j}$ is the interconnection between subsystem j *and* i with,

$$z_{ji}(k+1) = A_{zji} z_{ji}(k) + \psi_{ji} y_i(k) \tag{3}$$

$$y_i(k) = c_i x_i(k) \tag{4}$$

$x_i(k) = x_{i0}$, $k = 0$, $\emptyset_{ji}, \psi_{ji}$ *and* A_{zji} are matric with appropriate dimension.

It is assumed that both $f_{mi}(k)$ and $f_{ui}(k)$ are bounded, that is,

$$0 \le f_{mi}(k) \le F_m, \; 0 \le f_{ui}(k) \le F_u$$

and the bounds are known.

Assumption 1,

$G_i B_i$ is invertible (G can be arbitrarily chosen by assuming that the following conditions are met), according to (Xi and Hesketh, 2010),

$$\left|G_i B_i(f_{mi}(k) - \hat{f}_{mi}(k))\right| < N \quad , \; 0 < N < \infty \tag{5}$$

$$\left|G_i(f_{ui}(k) - \hat{f}_{ui}(k))\right| < M \quad , \; 0 < M < \infty \tag{6}$$

$$\left|G_i\left(V_i \sum_{\substack{j=1 \\ j\neq i}}^{p} \emptyset_{ji} z_{ji}(k) - \sum_{\substack{j=1 \\ j\neq i}}^{p} \emptyset_{ji} z_{ji}(k-1)\right)\right| < L, \quad 0 < L < \infty \tag{7}$$

It is assumed that N,M,L is known, and $\hat{f}_{mi}(k)$ and $\hat{f}_{ui}(k)$ are the last value of the disturbance signal.

According to Su et al., 2000), the last value of a disturbance signal can be taken as estimate of its current value if the updated value is not accessible, under the assumption that the disturbance is continuous and smooth:

$$f_{mi}(k) - \hat{f}_{mi}(k) = f_{mi}(k) - f_{mi}(k-1) \tag{8}$$

$$f_{ui}(k) - \hat{f}_{ui}(k) = f_{ui}(k) - f_{ui}(k-1) \tag{9}$$

The objective is then to design a decentralized controller $u_i(k)$ such that the large-scale discrete-time system (2) & (3) can be controlled.

3. INTEGRAL SLIDING MODE CONTROLLER DESIGN

In this paper, a decentralized discrete-time controller $u_i(k)$ based on integral sliding mode control technique is proposed for each subsystem. As in centralized case, the sliding surface is designed for each subsystem followed by the switching controller as presented in the following:

In order to guarantee the existence of sliding mode and reduce chattering effect, the following condition must be satisfied (Sarpturk et al., 1987):

1. $\sigma_i(k)(\sigma_i(k+1) - \sigma_i(k)) \le 0$ (10)
2. $|\sigma_i(k+1)| \le |\sigma_i(k)|$ (11)

3.1 Sliding Surface Design

In this paper, a discrete-time integral sliding surface for each subsystem is designed as:

$$\sigma_i(k) = G_i x_i(k) - G_i x_i(0) + h_i(k) \tag{12}$$

where $h_i(k)$ is iteratively computed as:

$$h_i(k) = h_i(k-1) - (G_i B_i u_{i0}(k-1) + G_i A_i x_i(k-1)) \tag{13}$$

with $\sigma_i(k) \in R^n, h_i(k) \in R^m, h_i(0) = 0, G_i \in R^{1\times n}$.

It is assumed that the control law is given as:

$$u_i(k) = u_{i0}(k) + u_{i1}(k) \tag{14}$$

where the first component, $u_{i0}(k) = -K_i x_i(k)$ is the equivalent control portion after system achieve the quasi-sliding mode. The gain K to be designed later.

By taking $h_i(k+1) = h_i(k) - (G_i B_i u_{i0}(k) + G_i A_i x_i(k))$, the sliding surface dynamics can be obtained from (2), (12) & (13) as follow:

$$\begin{aligned}
\sigma_i(k+1) &= G_i x_i(k+1) - G_i x_i(0) + h_i(k+1) \\
&= G_i(A_i x_i(k) + B_i\big(u_i(k) + f_{mi}(k)\big) + V_i \sum_{\substack{j=1 \\ j\neq i}}^{p} \emptyset_{ji} z_{ji}(k) + f_{ui}(k)) \\
&\quad - G_i x_i(0) + h_i(k) - G_i B_i u_{i0}(k) - G_i A_i x_i(k) \\
&= G_i A_i x_i(k) + G_i B_i\big(u_i(k) + f_{mi}(k)\big) + G_i f_{ui}(k) + G_i V_i \textstyle\sum_{\substack{j=1 \\ j\neq i}}^{p} \emptyset_{ji} z_{ji}(k) - G_i x_i(0) + \\
&\quad h_i(k) - G_i B_i u_0(k) - G_i A_i x_i(k)
\end{aligned} \tag{15}$$

By substituting (14) into (15), the equation of sliding surface becomes:

$$\sigma_i(k+1) = G_i B_i u_{i1}(k) + G_i B_i f_{mi}(k) + G_i f_{ui}(k) + G_i V_i \textstyle\sum_{\substack{j=1 \\ j\neq i}}^{p} \emptyset_{ji} z_{ji}(k) + h_i(k) - G_i x_i(0) \tag{16}$$

From (12) & (16),

$$\sigma_i(k+1)-\sigma_i(k) = G_iB_iu_{i1}(k) + G_iB_if_{mi}(k) + G_if_{ui}(k) + G_iV_i\sum_{\substack{j=1\\ j\neq i}}^{p}\emptyset_{ji}z_{ji}(k) - G_ix_i(k) \tag{17}$$

3.2 Controller Design

The controller is designed according to (14) with two components, $u_i(k) = u_{i0}(k) + u_{i1}(k)$ where the second component is given as,

$$u_{i1}(k) = -(G_iB_i)^{-1}\Big[G_iB_i\hat{f}_{mi}(k) + G_i\hat{f}_{ui}(k) +$$

GiVij=1j≠ipØjizjik−1−Gixik+αikσik+βiksgn(σik) (18)

where,

$$\left.\begin{matrix}\alpha_i(k)=0\\ \beta_i(k)=0\end{matrix}\right\} \textit{when } \sigma_i(k)=0$$

or,

$$\alpha_i(k) = \gamma_i(k)sgn(\sigma_i(k)) + \epsilon_i\ ,$$

with $\gamma_i(k) = \frac{N+M+L}{|\sigma_i(k)|}$ and $0<\epsilon_i<1$

$$\beta_i(k) = \lambda_i|(1-\epsilon_i)\sigma_i(k)|\quad ,0<\lambda_i<1$$

otherwise.

$u_{i1}(k)$ is the reaching mode control component that will ensure the system able to achieve a quasi-sliding mode.

Theorem 1: Subject to Assumption 1 and sliding surface design of (12), the large-scale discrete-time system will achieve quasi-sliding mode and remain in it by having the control input of (14).

Proof: The proof of the theorem is given below:

While $\sigma_i(k)\neq 0$, define a Lyapunov function:

$$J(k) = 0.5\sigma_i^2(k) \tag{19}$$

Ensuring that J(k) is non-increasing is equivalent to ensuring the condition in (10) & (11) (Xi and Hesketh, 2010).

Let,

$$Q_i(k) = G_iB_i(f_{mi}(k)-\hat{f}_{mi}(k)) + G_i(f_{ui}(k)-\hat{f}_{ui}(k)) + G_i(V_i\sum_{\substack{j=1\\ j\neq i}}^{p}\emptyset_{ji}z_{ji}(k) - V_i\sum_{\substack{j=1\\ j\neq i}}^{p}\emptyset_{ji}z_{ji}(k-1)) \tag{20}$$

Substitute (18) into (17) gives,

$$\sigma_i(k+1)-\sigma_i(k) = -\alpha_i(k)\sigma_i(k) - \beta_i(k)sgn(\sigma_i(k)) + G_iB_i(f_{mi}(k)-\hat{f}_{mi}(k) + G_i(f_{ui}(k)-\hat{f}_{ui}(k)) + G_i(V_i\sum_{\substack{j=1\\ j\neq i}}^{p}\emptyset_{ji}z_{ji}(k) - V_i\sum_{\substack{j=1\\ j\neq i}}^{p}\emptyset_{ji}z_{ji}(k-1)) \tag{21}$$

Multiply both sides with $\sigma_i(k)$, gives:

$$\begin{aligned}\sigma_i(k)(\sigma_i(k+1)-\sigma_i(k)) &= -\alpha_i(k)\sigma_i^2(k) - \beta_i(k)\sigma_i(k)sgn(\sigma_i(k)) + Q_i(k)\sigma_i(k)\\ &= \sigma_i(k)(Q_i(k)-\alpha_i(k)\sigma_i(k)) - \beta_i(k)|\sigma_i(k)|\end{aligned} \tag{22}$$

When $\sigma_i(k)<0$, this implies that,

$\sigma_i(k)(\sigma_i(k+1)-\sigma_i(k))<0$, provided $\alpha_i(k)>0$.

Subsequently, following the conditions stated in (5),(6) and (7), it can be shown that:

$$Q_i(k) < \|Q_i(k)\| < M+N+L = \gamma_i(k)|\sigma_i(k)|$$

When $\sigma_i(k)>0$ and since $\gamma_i(k)\sigma_i^2(k) > Q_i(k)\sigma_i(k)$, (22) becomes:

$$\begin{aligned}(\sigma_i(k+1)-\sigma_i(k)) &< (\gamma_i(k)-\alpha_i(k))\sigma_i^2(k) - \beta_i(k)|\sigma_i(k)|\\ &= (\gamma_i(k)sgn\,(\sigma_i(k)) - \alpha_i(k))\sigma_i^2(k) - \beta_i(k)|\sigma_i(k)|\\ &= -\epsilon_i\sigma_i^2(k) - \beta_i(k)|\sigma_i(k)| \ \leq 0\end{aligned}$$

By substituting $Q_i(k)$ as defined in (20) into (21), gives

$$\sigma_i(k+1) = (\sigma_i(k) + Q_i(k) - \alpha_i(k)\sigma_i(k)) - \beta_i(k)sgn(\sigma_i(k))$$

Since it is defined in (18) that, $\gamma_i(k) = \frac{N+M+L}{|\sigma_i(k)|}$ and $\alpha_i(k) = \gamma_i(k)sgn(\sigma_i(k)) + \epsilon_i$, let $\varsigma_i = \gamma_i(k)|\sigma_i(k)| - Q_i(k) = M + N + L - Q_i(k)$, and it is known that $|\sigma_i(k)| = \sigma_i(k)\,sgn\,(\sigma_i(k))$ according to Edwards and Spurgeon (1998). Then substitute $Q_i(k) = \gamma_i(k)|\sigma_i(k)| - \varsigma_i$ and $\epsilon_i = \gamma_i(k)sgn(\sigma_i(k)) - \alpha_i(k)$ into equation below,

$$\begin{aligned}|\sigma_i(k+1)| &= |(\sigma_i(k) + Q_i(k) - \alpha_i(k)\sigma_i(k)) - \beta_i(k)sgn(\sigma_i(k))|\\ &= |(1-\alpha_i(k))\sigma_i(k) + (\gamma_i(k)|\sigma_i(k)| - \varsigma_i) - \beta_i(k)sgn(\sigma_i(k))|\\ &= |(1-\alpha_i(k))\sigma_i(k) + (\gamma_i(k)sgn\,(\sigma_i(k))\sigma_i(k) - \varsigma_i) - \beta_i(k)sgn(\sigma_i(k))|\\ &= |(1-\alpha_i(k) + (\gamma_i(k)sgn\,(\sigma_i(k)))\sigma_i(k) - \varsigma_i - \beta_i(k)sgn(\sigma_i(k))|\\ &= |(1-\epsilon_i)\sigma_i(k) - \varsigma_i - \beta_i(k)sgn(\sigma_i(k))|\end{aligned}$$

Hence,

$$\begin{aligned}|\sigma_i(k+1)| &= |(1-\epsilon_i)\sigma_i(k) - \varsigma_i - \lambda_i|(1-\epsilon_i)\sigma_i(k)|sgn(\sigma_i(k))|\\ &= |(1-\lambda_i)(1-\epsilon_i)\sigma_i(k) - \varsigma_i|\end{aligned}$$

This will ensure that $|\sigma_i(k+1)| \leq |\sigma_i(k)|$ is satisfied when the conditions below are met,

$$\sigma_i(k) < -\left[\frac{\varsigma_i}{\lambda_i+\epsilon_i-\lambda_i\epsilon_i}\right] \quad \text{or,}$$

$$\sigma_i(k) > -\left[\frac{\varsigma_i}{2-(\lambda_i+\epsilon_i-\lambda_i\epsilon_i)}\right]$$

In the case of $\sigma_i(k) < 0$, it should be noted that in order to ensure $\sigma_i(k)\big(\sigma_i(k+1)-\sigma_i(k)\big) < 0$, there is another condition to be met, that is $\alpha_i(k) > 0$. Equation (18) stated that $\alpha_i(k) = \gamma_i(k)sgn\big(\sigma_i(k)\big) + \epsilon_i$, $and\ 0 < \epsilon_i < 1$, therefore, in order to guarantee $\alpha_i(k) > 0$, it is necessary to have,

$$\gamma_i(k) = \frac{M+N+L}{|\sigma_i(k)|} > -1 \tag{23}$$

So (23) imply that when $\sigma_i(k) < 0$, the second condition to ensure the size of $\sigma_i(k)$decreasing is,

$$\sigma_i(k) < -(M+N+L)$$

Since $\varsigma_i = M+N+L - Q_i(k)$ and $Q_i(k) < \|Q_i(k)\| < M+N+L$, this imply that,

$$0 < \varsigma_i < 2(M+N+L)$$

It can be concluded that $\sigma_i(k)$ exhibits a quasi-sliding mode with lower and upper bound ($\overline{U}_l$ and $\overline{U}_u$ respectively) and the band is

$$U_l \le \sigma_i(k) \le U_u$$

$$U_l > \overline{U}_l = min\left(-\frac{2(M+N+L)}{l}, -(M+N+L)\right)$$

$$U_u < \overline{U}_u = \frac{2(M+N+L)}{2-l} \quad with\ l = \lambda_i + \epsilon_i - \lambda_i\epsilon_i$$

While $\sigma_i(k) = 0$,

By substituting $\sigma_i(k) = 0$ into (18),

$$u_{i1}(k) = -(G_iB_i)^{-1}\left[G_iB_i\hat{f}_{mi}(k) + G_i\hat{f}_{ui}(k) + G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) - G_ix_i(k)\right] \tag{24}$$

Since from (12),

$$\sigma_i(k) = G_ix_i(k) - G_ix_i(0) - h_i(k),$$

$$u_{i1}(k) = -\hat{f}_{mi}(k) - (G_iB_i)^{-1}(G_i\hat{f}_{ui}(k) + h_i(k) + G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) - G_ix_i(0)) \tag{25}$$

Substitute (25) into (16) gives,

$$\sigma_i(k+1) = G_iB_i\left(f_{mi}(k) - \hat{f}_{mi}(k)\right) + G_i\left(f_{ui}(k) - \hat{f}_{ui}(k)\right) + G_i(V_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k) - \sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1)) \tag{26}$$

This concludes the proof for Theorem 1.

3.3 Overall System Stability

The overall system stability as the closed-loop performance while travelling along the sliding surface will be discussed in this section, that is when $\sigma_i(k) = 0$. Substituting (14) and (18) into (2), the closed-loop dynamic is derived below:

$$\begin{aligned} x_i(k+1) &= A_ix_i(k) + B_i\big(u_{i0}(k) + u_{i1}(k) + f_{mi}(k)\big) + f_{ui}(k) + V_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k) \\ &= A_ix_i(k) + B_i\left(f_{mi}(k) - \hat{f}_{mi}(k)\right) + B_iu_{i0}(k) + B_i(G_iB_i)^{-1}G_ix_i(0) + (f_{ui}(k) - B_i(G_iB_i)^{-1}G_i\hat{f}_{ui}(k) - B_i(G_iB_i)^{-1}h_i(k) + V_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k) - B_i(G_iB_i)^{-1}G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) \end{aligned} \tag{27}$$

Since $h_i(k) = \sigma_i(k) - G_ix_i(k) + G_ix_i(0)$ and $u_{i0}(k) = -K_ix_i(k)$, (27) becomes,

$$\begin{aligned} x_i(k+1) = {} & (A_i - B_iK_i)x_i(k) + B_i\left(f_{mi}(k) - \hat{f}_{mi}(k)\right) + (f_{ui}(k) - B_i(G_iB_i)^{-1}G_i\hat{f}_{ui}(k) - B_i(G_iB_i)^{-1}(\sigma_i(k) - G_ix_i(k)) \\ & + V_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k) - B_i(G_iB_i)^{-1}G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) \end{aligned} \tag{28}$$

Taking into account (8) and (9) and (26),

$$\sigma_i(k) = G_iB_i\big(f_{mi}(k-1) - f_{mi}(k-2)\big) + G_i\big(f_{ui}(k-1) - f_{ui}(k-2)\big) + G_iV_i\left(\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) - \sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-2)\right) \tag{29}$$

Thus (28) becomes

$$\begin{aligned} x_i(k+1) = {} & (A_i + B_i(G_iB_i)^{-1}G_i - B_iK_i)x_i(k) + B_i[f_{mi}(k) - 2f_{mi}(k-1) + f_{mi}(k-2)] + [f_{ui}(k) - 2B_i(G_iB_i)^{-1}G_if_{ui}(k-1) + B_i(G_iB_i)^{-1}G_if_{ui}(k-2)] \\ & + \left[V_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k) - 2B_i(G_iB_i)^{-1}G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) + B_i(G_iB_i)^{-1}G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-2)\right]h \end{aligned} \tag{30}$$

Owing to the assumption that both $f_{mi}(k)$ and $f_{ui}(k)$ are bounded, it can be assumed that:

$$-W \le B_i[f_{mi}(k) - 2f_{mi}(k-1) + f_{mi}(k-2)] + [f_{ui}(k) - 2B_i(G_iB_i)^{-1}G_if_{ui}(k-1) + B_i(G_iB_i)^{-1}G_if_{ui}(k-2)] + \left[V_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k) - 2B_i(G_iB_i)^{-1}G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-1) + B_i(G_iB_i)^{-1}G_iV_i\sum_{\substack{j=1\\j\ne i}}^{p}\emptyset_{ji}z_{ji}(k-2)\right] = w_i(k) \le W,$$

$$with\; 0 < W < \infty \tag{31}$$

Then the closed-loop system dynamic can be represented by,

$$x_i(k+1) = (A_i + B_i(G_iB_i)^{-1}G_i - B_iK_i)x_i(k) + w_i(k) \tag{32}$$

The gain, K must be selected so that $\bar{\rho}_{max} < 0$ with $\bar{\rho}_{max}$ standing for the largest eigenvalue of $A_i + B_i(G_iB_i)^{-1}G_i - B_iK_i$.

Thus,

$$\begin{aligned} x_i^T(k)[x_i(k+1) - x_i(k)] &= x_i^T(k)(A_i + B_i(G_iB_i)^{-1}G_i - B_iK_i - I)x_i(k) + x_i^T(k)w_i(k) \\ &\le \rho_{max}x_i^T(k)x_i(k) + \|x_i^T(k)w_i(k)\| \\ &\le \rho_{max}\|x_i(k)\|^2 + \|x_i^T(k)W\| \end{aligned} \tag{33}$$

Since it is necessary to have $\rho_{max} < 0$ to ensure the stability of the system, then

$$\rho_{max} < -max\left(\frac{\|x_i^T(k)W\|}{\|x_i(k)\|^2}\right) \tag{34}$$

It can be seen from (33) that when the system is not affected by any uncertainty, that is W=0, a $\rho_{max} < 0$ is sufficient to ensure the system stability. When $W \ne 0$, the larger the uncertainty, W, the more negative ρ_{max} must be to guarantee the stability. This is due to the nature of discrete-time system that $x_i(k)$ will never converge to zero but stays within a band about the origin.

4. EXAMPLES AND SIMULATION RESULTS

4.1 Example 1

In this example, a simulation on balancing double-inverted pendulums connected by a spring as used in (Ou et al., 2009). It is composed of 2 subsystems and the dynamic equation of the subsystems can be written as:

$$x_{i1}(k+1) = x_{i1}(k) + Tx_{i2}(k) \tag{35}$$

$$x_{i2}(k+1) = x_{i2}(k) + T\left[\left(\frac{m_igr}{J_i} - \frac{kr^2}{4J_i}\right)sin(x_{i1}(k)) + \right.$$

vi(k)Ji+ui(k)Ji+j≠i2kr24Jisinxj1k

(36)

$$y_i(k) = x_{i1}(k) \tag{37}$$

where,

$$x_i(k) = \begin{bmatrix} x_{i1}(k) \\ x_{i2}(k) \end{bmatrix} \qquad i = 1,2$$

$x_{i1}(k)$ is the angular displacements of the *i*-th pendulum from the vertical reference. Each pendulum may be positioned by a torque input $u_i(k)$applied by a servomotor at its base. $v_1(k) = 10sin(0.02\pi k)$ and $v_2(k) = 10sin(0.01\pi k)$ are the torque disturbance. It is assumed that $x_{i1}(k)$ and $\dot{x}_{i1}(k)$ (angular position and velocity) is available to the *i*-th controller. The end masses of the pendulums are $m_1 = 2kg\ and\ m_2 = 2.5kg$, the moments of inertia are $J_1 = 0.5kg\ and\ J_2 = 0.625kg$, the constant of the connecting torsional spring is k=100 N/m, the pendulum height is *r*=0.5m, the gravitational acceleration is *g*=9.81m/s^2, the natural length of the spring is *l*=0.5m. The distance between the pendulum hinges is *b*=0.5m, and then the spring is relaxed when the pendulums are all in the upright position. So the origin $x_{11} = x_{12} = x_{21} = x_{22} = 0$ is the equilibrium point of this nonlinear large-scale system. The sampling period is T=0.005s.

The value of $\boldsymbol{\epsilon_i}(k)$ was chosen as $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.1$. The value of $\lambda_i(k)$ was chosen as $\lambda_1 = 0.9$ and $\lambda_2 = 0.99$. In this case, $G_i = B_i^T$. The value of K is obtained as $K_i = place(A_i + B_i(G_iB_i)^{-1}G_i, B_i, [0.4\ 0.7])$, which is $K_1 = [1210\ \ 710]$; $K_2 = [3006\ \ 225]$. The initial contition is $x_1(0) = [0.3\ 0]^T$and $x_2(0) = [0.3\ 0]^T$ for this simulation.

The sliding surfaces, $\sigma_1(k)\ and\ \sigma_2(k)$ are chosen to be of the same parameter. Example below shows the equation of $\sigma_1(k)$ and $\sigma_2(k)$ as implemented in the simulation:

$$\begin{aligned}\sigma_1(k) = [0\quad 0.01]x_1(k) - [0\quad 0.01]x_1(0) + h_1(k-1) \\ - \left([0\quad 0.01]\begin{bmatrix}0\\0.01\end{bmatrix}u_{10}(k-1)\right. \\ \left. + [0\quad 0.01]\begin{bmatrix}1 & 0.005\\0.0356 & 1\end{bmatrix}x_1(k-1)\right)\end{aligned}$$

$$\begin{aligned}\sigma_2(k) = [0\quad 0.01]x_2(k) - [0\quad 0.01]x_2(0) + h_2(k-1) \\ - \left([0\quad 0.01]\begin{bmatrix}0\\0.01\end{bmatrix}u_{20}(k-1)\right. \\ \left. + [0\quad 0.01]\begin{bmatrix}1 & 0.005\\0.0481 & 1\end{bmatrix}x_2(k-1)\right)\end{aligned}$$

where,

$u_{10}(k) = -[1210\ \ 710\ ]x_1(k)$ and $h_1(0) = 0$ and

$u_{20}(k) = -[3006\ \ 225\ ]x_2(k)$ and $h_2(0) = 0$.

$u_{11}(k)$is the control signal for the first inverted pendulum with the present of disturbance and interconnection from the second subsystem.

$$\begin{aligned}u_{11}(k) = -\left([0\quad 0.01]\begin{bmatrix}0\\0.01\end{bmatrix}\right)^{-1}\big[-[0\quad 0.01]x_1(k) + \\ [0\quad 0.01](20sin(0.02\pi k))u_1(k-1) + \\ 0.0625sin(x_{21}(k)) + \alpha_1(k)\sigma_1(k) + \\ \beta_1(k)sgn(\sigma_1(k))\big]\end{aligned}$$

The same conditions apply to the second subsystem with the present of disturbabce and interconnection effect. The control signal, $u_{21}(k)$, is given below:

$$u_{21}(k) = -([0 \quad 0.01]\begin{bmatrix}0\\0.01\end{bmatrix})^{-1}[-[0 \quad 0.01]x_2(k) + [0 \quad 0.01](16sin(0.01\pi k))u_2(k-1) + 0.05sin\big(x_{11}(k)\big) + \alpha_2(k)\sigma_2(k) + \beta_2(k)sgn(\sigma_2(k))\big]$$

The simulation has been done at a period of 2 seconds and the results are shown in figures 1 to 5 below:

It is shown in Figure 1 to Figure 4 that, for both inverted pendulum, the system trajectories under discrete-time integral sliding mode control able to achieve stabilility and reached the desired conditions with attenuated disturbance. The output of subsystems 1 and 2 at steady state is shown in Figure 3 and Figure 4. It can be observed that the magnitude of disturbance has been reduced while the systems are controlled by VSC as compared to systems controlled solely by feedback control. Figure 5 shows the sliding sufaces signal that the quasi-slding mode has been achieved.

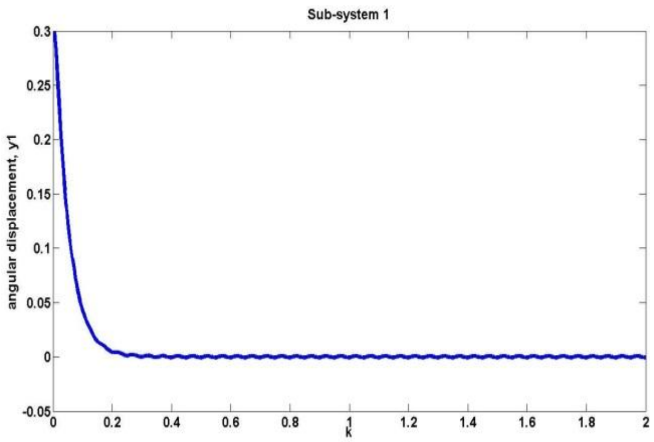

Fig. 1. Angular displacement, $x_{11}(k)$ for first inverted pendulum under discrete-time integral sliding mode control.

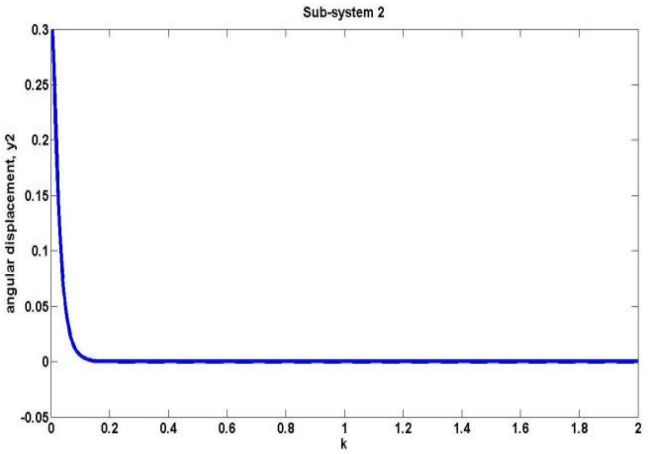

Fig. 2. Angular displacement, $x_{21}(k)$ for second inverted pendulum under discrete-time integral sliding mode control.

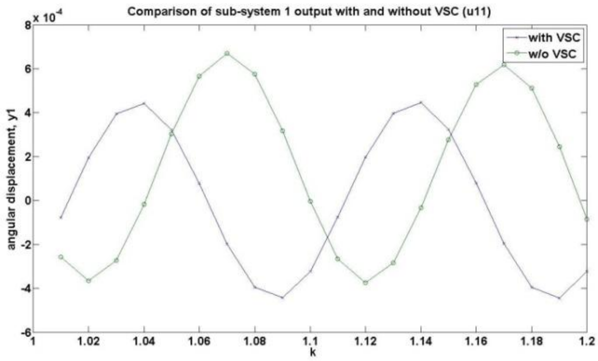

Fig. 3. Comparison of subsystem 1 output with and without discrete-time integral sliding mode controller input (VSC), $u_{11}(k)$.

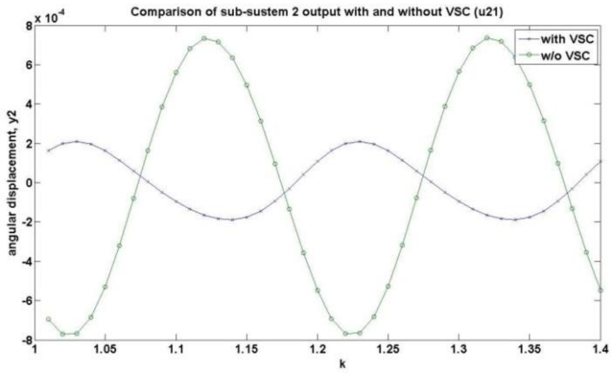

Fig. 4. Comparison of subsystem 2 output with and without discrete-time integral sliding mode controller input (VSC), $u_{21}(k)$.

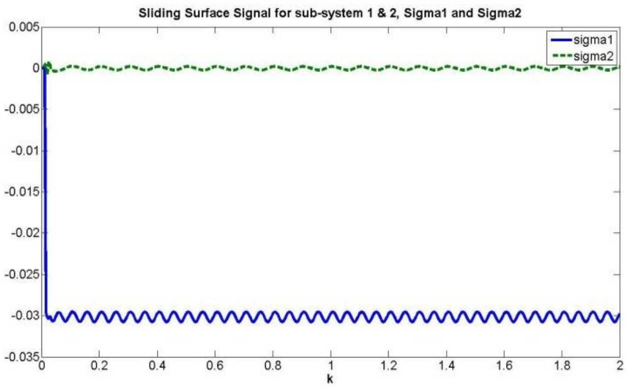

Fig. 5. Sliding surface signal for subsystem 1 and subsystem 2, $\sigma_1(k)$ (sigma1) and $\sigma_2(k)$ (sigma2).

4.2 Example 2

In this example, a simulation of large-scale discrete-time system with three interconnectted subsystems is performed to illustrate the new control strategy. This example taken from Park and Lee (2002) and the dynamic equation of the subsystems can be written as:

$x_1(k+1) =$
$\left(\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} +\right.$
*00.4*cos(*k*)*0.2*sin(*k*)*0x1k+01+00.04*sin(*k*)*u1k+00.10.10+0000.05*cos(*k*)*x2k+0.05000.15+0.01*cos(*k*)*00 0.01sin*(*k*)*x3k* (38)

$$x_2(k+1) = \left(\begin{bmatrix} 0 & 1 \\ 0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}\right) x_2(k) + \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.04\cos(k) \\ 0 \end{bmatrix}\right) u_2(k) + \left(\begin{bmatrix} 0 & 0.2 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0 & 0 \end{bmatrix}\right) x_1(k) + \left(\begin{bmatrix} 0.15 & 0 \\ 0 & 0.1 \end{bmatrix} + \begin{bmatrix} 0.01\cos(k) & 0 \\ 0 & 0.01 sin(k) \end{bmatrix}\right) x_3(k) \quad (39)$$

$$x_3(k+1) = \left(\begin{bmatrix} -0.9 & 0.5 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.1\cos(k) & 0 \\ 0 & 0.2\sin(k) \end{bmatrix}\right) x_3(k) + \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.05\sin(k) \end{bmatrix}\right) u_3(k) + \left(\begin{bmatrix} 0 & 0.05 \\ 0.02 & 0.1 \end{bmatrix} + \begin{bmatrix} 0 & 0.1\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}\right) x_1(k) + \left(\begin{bmatrix} 0 & 0.05 \\ 0 & 0.1 \end{bmatrix} + \begin{bmatrix} 0 & 0.01\cos(k) \\ 0.01\sin(k) & 0 \end{bmatrix}\right) x_2(k) \quad (40)$$

where,

$$x_i(k) = \begin{bmatrix} x_{i1}(k) \\ x_{i2}(k) \end{bmatrix}, u_i(k) = \begin{bmatrix} u_{i1}(k) \\ u_{i2}(k) \end{bmatrix}, \quad i = 1,2,3$$

The initial conditions for this simulation are:

$$x_1(k) = [1 \quad -0.5]^T,\ x_2(k) = [-0.5 \quad -1.5]^T,\ x_3(k) = [-1 \quad 0.5]^T$$

The value of $\boldsymbol{\epsilon_i}(k)$ was chosen as $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, *and* $\epsilon_3 = 0.1$. The value of $\lambda_i(k)$ was chosen as $\lambda_1 = 0.9$, $\lambda_2 = 0.2$, and $\lambda_3 = 0.3$. In this case, $G_1 = [0 \quad 10], G_2 = [10 \quad 1], and\ G_3 = [5 \quad 10]$. The value of K is obtained as
$K_1 = place(A_1 + B_1(G_1B_1)^{-1}G_1, B_1, [0.5\ 0.9]) = [0.45\ 1.6]$,
$K_2 = place(A_2 + B_2(G_2B_2)^{-1}G_2, B_2, [0.05\ 0.1]) = [1.33 - 0.9]$,and
$K_3 = place(A_3 + B_3(G_3B_3)^{-1}G_3, B_3, [0.1\ 0.2]) = [-0.2857\ 1.085]$.

The sliding surfaces, $\sigma_1(k), \sigma_2(k), and\ \sigma_3(k)$ are chosen to be of the same parameter. Example below shows the equation of $\sigma_1(k)$, $\sigma_2(k)$, and $\sigma_3(k)$ as implemented in the simulation:

$$\sigma_1(k) = [0 \quad 10]x_1(k) - [0 \quad 10]x_1(0) + h_1(k-1) - \left([0 \quad 10]\begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{10}(k-1) + [0 \quad 10]\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} x_1(k-1)\right)$$

$$\sigma_2(k) = [10 \quad 1]x_2(k) - [10 \quad 1]x_2(0) + h_2(k-1) - \left([10 \quad 1]\begin{bmatrix} 1 \\ 1 \end{bmatrix} u_{20}(k-1) + [10 \quad 11]\begin{bmatrix} 0 & 1 \\ 0.5 & -0.5 \end{bmatrix} x_2(k-1)\right)$$

$$\sigma_3(k) = [5 \quad 10]x_3(k) - [5 \quad 10]x_3(0) + h_2(k-1) - \left([5 \quad 10]\begin{bmatrix} 1 \\ 1 \end{bmatrix} u_{30}(k-1) + [5 \quad 10]\begin{bmatrix} -0.9 & 0.5 \\ 0 & 1 \end{bmatrix} x_3(k-1)\right)$$

where,

$u_{10}(k) = -[0.45\ \ 1.6\ ]x_1(k)$ and $h_1(0) = 0$,

$u_{20}(k) = -[1.33\ -0.9\ ]x_2(k)$ and $h_2(0) = 0$, and

$u_{30}(k) = -[-0.2857\ \ 1.085\ ]x_3(k)$ and $h_3(0) = 0$.

$u_{11}(k)$is the control signal for the first subsystem with the present of disturbance and interconnection from the second and third subsystems.

$$u_{11}(k) = -\left([0 \quad 10]\begin{bmatrix} 0 \\ 10 \end{bmatrix}\right)^{-1}\Bigg[-[0 \quad 10]x_1(k) + [0 \quad 10]\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0.04\sin(k) \end{bmatrix}\right) u_1(k-1) + [0 \quad 10]\begin{bmatrix} 0 & 0.4\cos(k) \\ 0.2\sin(k) & 0 \end{bmatrix} x_1(k) + [0 \quad 10]\begin{bmatrix} 0 & 0 \\ 0 & 0.05\cos(k) \end{bmatrix} x_2(k) + [0 \quad 10]\begin{bmatrix} 0.01\cos(k) & 0 \\ 0 & 0.01 sin(k) \end{bmatrix} x_3(k) + \alpha_1(k)\sigma_1(k) + \beta_1(k) sgn(\sigma_1(k))\Bigg]$$

The same conditions apply to the second subsystem with the present of disturbance and interconnection effect. The control signal, $u_{21}(k)$and $u_{31}(k)$, are given below:

$$u_{21}(k) = -\left([10 \quad 1]\begin{bmatrix}1\\1\end{bmatrix}\right)^{-1}\Bigg[-[10 \quad 1]x_2(k) + [10 \quad 1]\begin{bmatrix}1\\1\end{bmatrix}\left(\begin{bmatrix}0.04\cos(k)\\0\end{bmatrix}\right)u_2(k-1) + [10 \quad 1]\begin{bmatrix}0 & 0.04\cos(k)\\0.04\sin(k) & 0\end{bmatrix}x_2(k) + [10 \quad 1]\begin{bmatrix}0 & 0.04\cos(k)\\0 & 0\end{bmatrix}x_1(k) + [10 \quad 1]\begin{bmatrix}0.01\cos(k) & 0\\0 & 0.01sin(k)\end{bmatrix}x_3(k) + \alpha_2(k)\sigma_2(k) + \beta_2(k)sgn(\sigma_2(k))\Bigg]$$

$$u_{31}(k) = -\left([5 \quad 10]\begin{bmatrix}1\\1\end{bmatrix}\right)^{-1}\Bigg[-[10 \quad 1]x_3(k) + [5 \quad 10]\begin{bmatrix}1\\1\end{bmatrix}\left(\begin{bmatrix}0\\0.05\sin(k)\end{bmatrix}\right)u_3(k-1) + [5 \quad 10]\begin{bmatrix}0.1\cos(k) & 0\\0 & 0.2\sin(k)\end{bmatrix}x_3(k) + [5 \quad 10]\begin{bmatrix}0 & 0.1\cos(k)\\0.04\sin(k) & 0\end{bmatrix}x_1(k) + [5 \quad 10]\begin{bmatrix}0 & 0.01\cos(k)\\0.01\sin(k) & 0\end{bmatrix}x_2(k) + \alpha_3(k)\sigma_3(k) + \beta_3(k)sgn(\sigma_3(k))\Bigg]$$

The simulation has been done at a period of 50 seconds and the results are shown in figures below:

It is shown in Figure 6 to Figure 8, the system trajectories of all the 3 subsystems of the discrete-time large-scale system under discrete-time integral sliding mode control able to achieve stability and reached the desired conditions with disturbance being rejected. A comparison has been made for the output of the system with only the feedback control input, $u_0(k)$ without the sliding mode controller input, $u_1(k)$. It is clearly shown in the Figure 9 to Figure 11 that the system is unable to be controlled and unstable. Figure 12 has shown that the sliding surface signal for all 3 subsystem achieved quasi-sliding mode with the discrete-time integral sliding mode controller in place.

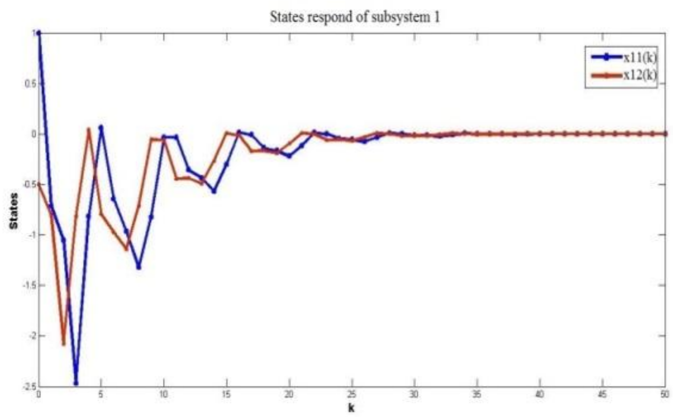

Fig. 6. States respond of subsystem 1, $x_{11}(k)$and $x_{12}(k)$ under discrete-time integral sliding mode control.

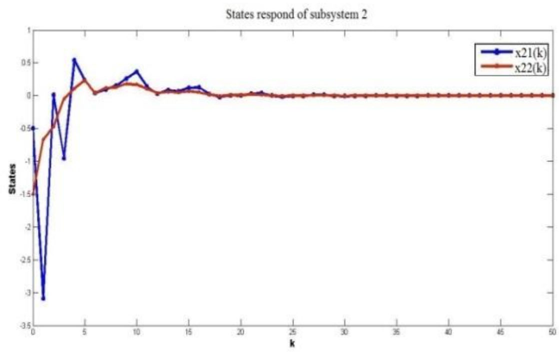

Fig. 7. States respond of subsystem 2, $x_{21}(k)$and $x_{22}(k)$ under discrete-time integral sliding mode control.

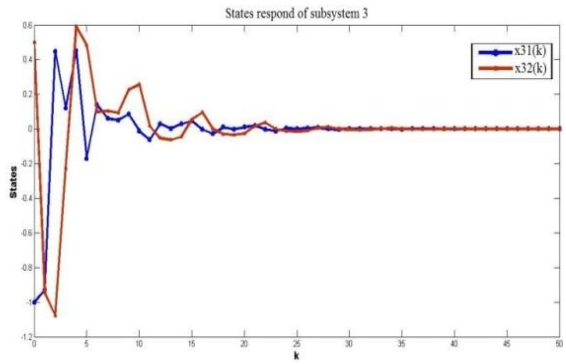

Fig. 8. States respond of subsystem 3, $x_{31}(k)$and $x_{32}(k)$ under discrete-time integral sliding mode control.

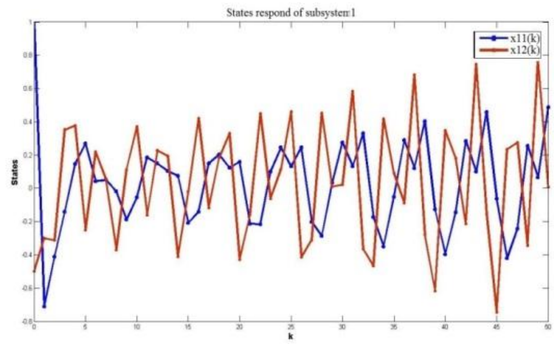

Fig. 9. States respond of subsystem 1, $x_{11}(k)$and $x_{12}(k)$ under feedback control, without discrete-time integral sliding mode control.

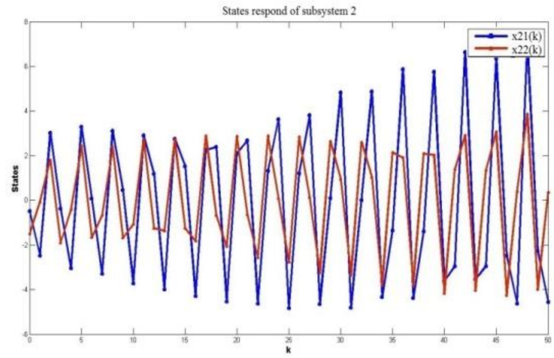

Fig. 10. States respond of subsystem 2, $x_{21}(k)$and $x_{22}(k)$ under feedback control, without discrete-time integral sliding mode control.

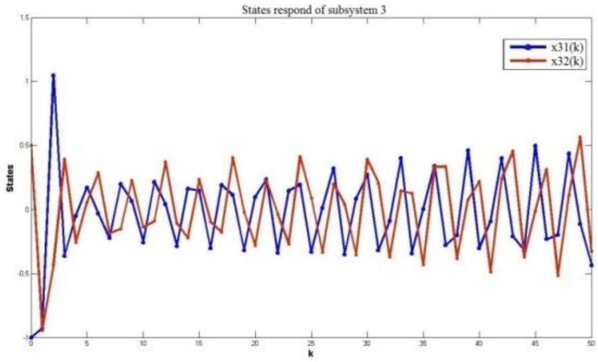

Fig. 11. States respond of subsystem 3, $x_{31}(k)$and $x_{32}(k)$ under feedback control, without discrete-time integral sliding mode control.

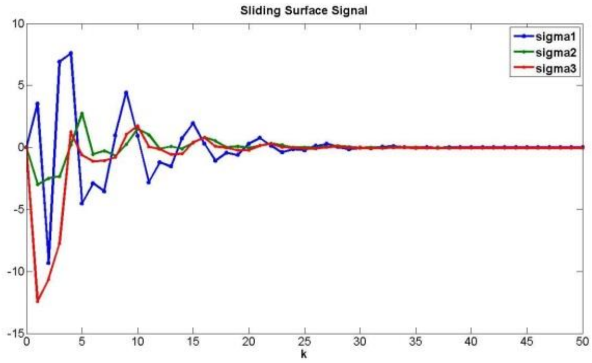

Fig. 12. Sliding surface signal for subsystem ($\sigma_1(k)$, sigma1), subsystem 2 ($\sigma_2(k)$, sigma2), and subsystem 3 ($\sigma_3(k)$, sigma3).

5. CONCLUSIONS

The control of large-scale discrete-time system with matched and unmatched uncertainty using discrete-time integral sliding mode control has been proposed in this paper. A new theorem has been presented and proved that it will ensure the system to achieve the quasi-sliding surface and remains there. The proposed controller showed that the effect of interconnection in large-scale system is being handled well and the system stability is ensured. It is also shown that the effect of matched and unmatched uncertainty in the system also being rejected. Two examples of large-scale systems have been used to evaluate the performance of the controller. It can be seen that the proposed controller is able to control the system to achieve the stability and desired value, and also reduce the effect of disturbance as compared to the system without using the integral sliding mode controller. As the conclusion, it can be concluded that the proposed discrete-time integral sliding mode controller has the advantage in controlling large-scale discrete-time system with matched or unmatched uncertainties and nonlinearities as it is able to handle the effect of interconnection, matched and unmatched uncertainty very well.

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